Relaxation of Turbulence Behind Interplanetary Shocks

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Abstract. Relaxation of solar wind turbulence has been a topic of interest for decades. Many phenomenological laws for the energy decay have been derived and subsequently confirmed in MHD simulation studies. However, in-situ observations of decay laws is an inherently difficult task. In this study, we use interplanetary shocks because (a) they enhance the level of turbulent fluctuations and (b) they propagate into a pristine solar wind. Using a few simplifying assumptions, we study the evolution of large-scale energy-containing eddies. We analyze a relaxation of the turbulent energy enhanced by interplanetary shocks and discuss its observed turbulent decay.

Introduction

Although the solar wind is the most studied magnetized plasma, there are still many unresolved issues concerning the expanding solar plasma, mainly its origin and non-adiabatic radial temperature profile [Marsch et al., 1982; Gazis, 1984; Goldstein et al., 1996; Matthaeus et al., 1999b,a]. Moreover, MHD turbulence plays a crucial role in the evolution of solar wind fluctuations [Zank et al., 1996; MacBride et al., 2008]. The first evidence for the turbulent nature of the fluctuations was reported by Coleman [1968]. He analyzed magnetic field and plasma velocity measurements from the Mariner 2 spacecraft and he found that a level of turbulent fluctuations was sufficient to heat the protons at 1 Astronomical Unit (AU). Many other authors attributed the excess of the proton temperature to the transfer of energy from the large scale MHD fluctuations into the ion kinetic scales, $l_i \simeq (\lambda_i, \rho_i)$, where $\lambda_i$ is the inertial length and $\rho_i$ is the proton thermal gyroradius. The first phenomenological description of a turbulent cascade was presented by Iroshnikov [1963] and Kraichnan [1965]. According to their theory of turbulence, the power spectrum density (PSD) of magnetic and velocity fluctuations in the inertial range (where the cascade takes place) has a power law behaviour, $PSD(f) \propto f^n$, with $n \approx -3/2$, $f$ being the spacecraft frame frequency. Goldreich and Sridhar [1995] derived another theory of MHD turbulence, where the PSD of magnetic field fluctuations scales differently in perpendicular, $\alpha_\perp = -5/3$, and parallel, $\alpha_\parallel = -2$, directions relative to a local magnetic field [Horbury et al., 2008]. For more details, see comprehensive reviews on solar wind turbulence by Tu and Marsch [1995] and Bruno and Carbone [2013].

In the theory of hydrodynamic turbulence, there are two important spatial scales that define the range where turbulence operates. First, the outer scale, $L$, denotes the scale on which the energy is injected into the system. Second, the dissipation scale, $l_\eta$, is the scale where the kinetic energy contained within the turbulent eddies transforms into heat. In MHD turbulence, this picture is still valid, but the (weakly) collisionless solar wind indicates that the dissipation mechanism is an unresolved problem up to now. Observations of the Cluster spacecraft show that the PSD of magnetic fluctuations between ion and electron spatial scales is a power law, suggesting another turbulent cascade [Alexandrova et al., 2009; Sahraoui et al., 2009].

It is believed that large-scale ($l \gtrsim L$) magnetic and plasma fluctuations originate in the solar corona and could also be produced by the high-velocity shears between the fast and slow solar wind [Roberts et al., 1992]. A transition between the injection and inertial ranges of turbulence, at frequency $f_L$, is observed as a spectral break in the power spectrum of magnetic field fluctuations [Bruno et al., 2009]. At spacecraft-frame frequencies $f < f_L$, where $f_L$ lies between $10^{-4}$ and $10^{-3}$ Hz at 1 AU, the spectrum is a power law with $\alpha = -1$. The Taylor hypothesis [Taylor, 1938] allows us to determine the spatial scale corresponding to $f_L$ as $l_\alpha = v_\alpha / f_L$, $v_\alpha$ being the bulk solar wind speed.

As mentioned above, the energy contained in large-scale fluctuations is transferred towards smaller scales via a turbulent cascade and this process leads to a decrease of the level of fluctuations in time. Kolmogorov [1941] showed that the decay of the total energy in these fluctuations, $E$, over time, $t$, should have a power-law behavior, $E \propto t^{-n}$, with $n = 10/7$. This phenomenological result is valid for hydrodynamic turbulence, while for MHD turbulence, the decay is still a power law but with $n < 10/7$. Biskamp [2003] summarized the decay laws for various MHD cases of turbulence that differ in the values of magnetic helicity $H_M$ and the energy ratio, $\Gamma = E_K/E_M$ ($E_K$ is the kinetic energy of the turbulent eddies and $E_M$ is the magnetic energy of turbulent fluctuations). The most relevant scenario for the
solar wind is that the magnetic helicity, \( H_M \), is finite but greater than zero and \( 0 < \Gamma < 1 \). The energy decay laws should be \( E \propto t^{-1/2} \), whereas \( E_K \propto t^{-1} \) [Biskamp and Müller, 1999]. Biskamp and Müller [1999] and Biskamp [2003] questioned the validity of a power-law behavior when \( 0 < t \approx 2 \), due to the effect of the initial conditions, and they concluded that it is valid asymptotically for \( t \gtrsim 10 \), where the time \( t \) is in units of the eddy turnover time, defined as \( \tau_{nl} = 1/ku \), \( u \) is the root mean square turbulent amplitude and \( k \) is the wave vector [Matthaeus et al., 2014].

An optimal way of investigation of the decay laws in the solar wind is to use a straight-line configuration of three spacecraft, so that the radial expansion of the solar wind and the decay of turbulent fluctuations can be studied directly [Bruno et al., 2009]. Another possibility is to analyze data coming from a spacecraft moving radially to (or from) the Sun, which observes the same plasma stream at different heliocentric distances [Bavassano et al., 1982]. Finally, a novel approach is to use interplanetary (IP) shocks because they are naturally occurring in the solar wind with a cadence of about one per day [Webb and Howard, 2012].

IP shocks have been intensively studied for decades and many aspects of shock physics are understood [see Balogh and Treumann, 2013, for a comprehensive review], nevertheless, many open questions remain. We know that an IP shock dissipates the bulk kinetic energy straight into heat in a thin region around the shock ramp (a few hundred kms wide). Moreover, after the IP shock passage (in the downstream region), the power within the turbulent fluctuations is enhanced approximately tenfold relative to its upstream values [Luttrell et al., 1984; Pitña et al., 2016].

In this preliminary study, we investigate the decay of the turbulent energy behind IP shocks. We perform a statistical analysis of a small set of IP shocks (13) and we present the implications of our study for the MHD decay laws. We discuss a relaxation of turbulence of both magnetic and kinetic energy constituents behind IP shocks.

Data used

We used the data from the BMSW instrument on board Spektr-R. A description of the BMSW instrument has been reported by Šafráňková et al. [2013] and Zastenker et al. [2013], thus only a brief reminder is given here. The principle of measurements lies in the use of six Faraday Cups (FCs) in a special configuration. The instrument operates in two modes, adaptive and sweep modes. The former mode used in this paper allows us to determine the plasma moments with a high cadence. This is achieved by acquiring three points of the distribution function\(^2\). One FC exhibits a control grid without an applied voltage and on control grids of next two FCs, a voltage is applied so that their currents are fractions (0.3 and 0.7) of the first FC. The time resolution of the density, thermal and solar wind speeds is 31 ns. For a comparison, we used the data from the Solar Wind Experiment (SWE) instrument on board Wind [Ogilvie et al., 1995] for the density and solar wind velocity estimates. However, Wind rotates every 3 seconds, therefore the measured cadence is much lower than that of BMSW, namely 92 s.

Finally, we used the data from the Magnetic Field Investigation (MFI) Wind instrument [Lepping et al., 1995] with the resolution of \( \approx 0.1 \) s.

In our statistics, the number of cases is limited mainly by the BMSW data coverage. We analyzed the BMSW data between September 9, 2011 and June 23, 2015, finding only 13 shocks with a sufficiently high-time resolution and with an adequately long time of downstream measurements. For these IP shocks, we analyzed the kinetic energy from BMSW and Wind and the magnetic energy from Wind.

Data analysis

First, we present a method of an analysis of downstream IP shock fluctuations. The instruments (BMSW,SWE) measure the density, \( N \), and the bulk and thermal solar wind speeds, \( v_{sw} \) and \( v_{th} \), respectively, and the measurements of the magnetic field, \( \vec{B} \), are from MFI.

A unit of time that is relevant for the MHD decay analysis is the eddy turnover time, \( \tau_{nl} \) [Zhou et al., 2004]. The time measured at the spacecraft, \( t_{sp} \), is transformed into the time in units of \( \tau_{nl} \), \( t_{nl} \), according to

\[
t_{nl} = t_{sp} \cdot K_{sh}/\tau_{nl}.
\]

where \( K_{sh} \) is the multiplying factor accounting for the fact that the IP shock propagates into the unshocked region slower than the upstream speed of the solar wind. Assuming that (a) the IP shock

\(^1\)But strongly dependent on the solar cycle.

\(^2\)Assuming the Maxwellian proton particle distribution function.
propagates into a stationary medium and (b) the IP shock speed along its normal is a constant, then 
\[ \Delta \theta_{sh} = \theta_{sh}/(\theta_{d} - \theta_{sh}), \]
where \( \theta_{sh} \) is the shock speed and \( \theta_{d} \) is the speed of the solar wind downstream of the shock in the spacecraft frame of reference\(^3\). The eddy turnover time is computed as
\[
\tau_{nl} = \frac{\theta_{d,0} \Delta t}{2\pi \sqrt{\sum_{i=0}^{2} \text{var}_{\Delta t}(V_i)}},
\]
where \( \Delta t \) is the temporal scale for estimations of the moments of any physical quantity of interest, \( \theta_{d,0} \) is the immediate downstream solar wind speed, \( V_i \), \( i=0,1,2 \) are GSE components of the velocity measured by the spacecraft, and \( \text{var}_{\Delta t} \) denotes the variance of the designed quantity over the time scale \( \Delta t \).

In order to estimate the energy contained within turbulent fluctuations, we assume that the velocity and magnetic fields could be expressed as \( \vec{V} = \vec{v}_0 + \vec{v}_1 \) and \( \vec{B} = \vec{b}_0 + \vec{b}_1 \), where \( \vec{v}_0 \) is the average solar wind speed, \( \vec{v}_1 \) the velocity field of turbulent fluctuations, \( \vec{b}_0 \) the ambient magnetic field, and \( \vec{b}_1 \) the fluctuating magnetic field, respectively. Then, we express the kinetic, \( E_K \), and magnetic, \( E_M \), energies as
\[
E_K = \frac{1}{2} \int_V \rho v_1^2 \, dV \quad \text{and} \quad E_M = \frac{1}{2} \int_V \mu_0 b_1^2 \, dV,
\]
and we compute them from the measurements as
\[
E_K = \frac{1}{2} \rho_{\Delta t} \sqrt{\sum_{i=0}^{2} \text{var}_{\Delta t}(V_i)} \quad \text{and} \quad E_M = \frac{1}{2\mu_0} \sum_{i=0}^{2} \text{var}_{\Delta t}(B_i),
\]
where \( \rho_{\Delta t} \) is the average density, \( B_i \) is defined as \( V_i \), and \( \mu_0 \) is the vacuum permeability. All variances and the mean values were computed on a time scale of \( \Delta t = 17 \) minutes.

**Case study of an IP shock**

As an example of an IP shock analysis, we present the IP shock that was detected by BMSW on June 22, 2015 at 0536 UT (0505 UT at Wind). Its basic parameters are: the magnetosonic Mach number, \( M_{\text{ms}} = 2.6 \), the angle between the shock normal and upstream magnetic field, \( \theta_{Bn} = 78^\circ \), the shock speed in the spacecraft frame, \( \theta_{sh} = 483 \) km·s\(^{-1} \), the ratio of the downstream to upstream density, \( \rho_d/\rho_u = 1.8 \), the difference between downstream and upstream solar wind speeds, \( \theta_{d} - \theta_{u} = 60 \) km·s\(^{-1} \) and the ratio between downstream and upstream magnitudes of the magnetic field, \( B_d/B_u = 1.7 \) (propagated IMF from Wind).

For this particular IP shock, we have 12 hours of BMSW and WIND data measurements downstream of the shock, after it, a second IP shock was detected\(^4\). We analyze the data immediately upstream (\( \approx 1 \) hour) and 12 hours downstream and calculate the kinetic and magnetic energies using eq. (4). The time shift between two consecutive evaluations is 1 min. We rescale the spacecraft time \( t_{sp} \) by eq. (1) with \( K_{sh} = 5.1 \) and \( \tau_{nl} = 1.7 \) hours. Figure 1 shows the evolution of the kinetic and magnetic energies as a function of \( t_{nl} \). The black and green lines showing the kinetic energies from BMSW and SWE, respectively, resemble themselves rather closely. The blue line (magnetic energy computed from Wind) is above the green and black lines indicating that the energy contained in magnetic fluctuations is higher than that in the velocity fluctuations, i.e., \( \Gamma < 1 \).

In Figure 1, despite large fluctuations, a decreasing trend with time is quite visible in all three quantities. For this reason, we performed a statistical study; although a number of appropriate IP shocks in the analysis is relatively small.

**Superposed epoch analysis**

As already mentioned, in our limited dataset, most of the shocks have short intervals of downstream data (1–2 hours in the spacecraft time). On the other hand, a number of eddy turnover times could be as high as 10 even for 1 hour, so it could be still enough time for observations of the energy decay.

An important point about the validity of a power law as a good model for ‘young’ turbulence was already commented. The turbulent energy in the upstream solar wind is decaying by some rate, an IP

\(^3\)For the sake of simplicity, we assumed that the shock normal is parallel to upstream and downstream solar wind speeds.

\(^4\)It is detected by BMSW on July 22, 2015 at 1828 UT.
shock enhances turbulence and the energy within the fluctuations increases. The question is what is the proper interpretation of the ‘time’ in a power law model \( t^{-n} \) immediately downstream of the shock? There are at least two possibilities: (a) the IP shock ‘resets’ the time and we start from \( t_{nl} \approx 0 \); and (b) there is one master power law \( E = E_0 t_{nl}^{-n} \) and we jump into different ‘times’ given by a downstream energy level. We argue that the later is closer to the truth because downstream turbulence seems to be well developed.

We perform a superposed epoch analysis using the method described in the previous section for each IP shock. Because the downstream energy levels of different shocks vary more than a decade, we normalize the energy profiles by the energy immediately downstream of the shock, \( E_0 \). Another reason for such normalization is that the expected decreasing trend should be more clear. In Figure 2, the trends of these normalized energy profiles are reported for each IP shock. The kinetic energies estimated at both spacecraft (panels b and c in Figure 2) have a slight tendency to decrease in time confirming the trends observed in the case study. The decrease of the magnetic energy (panel 2a) is less evident, but notable. On the last panel (d) of Figure 2, we show the Alfvén ratio, \( r_A \), defined as the ratio between the kinetic and magnetic energies. The profiles are normalized by the first values immediately downstream of the shocks.

**Discussion and Summary**

Let us discuss assumptions and approximations made through an estimation of kinetic and magnetic energies. The most important assumption is a stationarity of solar wind conditions for the whole downstream periods. We do not know a priori whether the observed decrease of a level of the turbulent energy is caused by its actual decrease or by a lower energy level of the yet unshocked plasma. However, it is reasonable to assume that for an interval spanning only a few hours, a probability of encountering discontinuities that significantly change the energy levels is relatively low.

According to eq. (1), a rough estimate of the downstream time interval affected by the shock can be made. The mean value of \( K_{sh} \) is about 6 and the standard travel time for the solar wind plasma to reach the Earth is around 3–4 days. From this, it follows that one should not consider the measurements older that \( \approx 12 \) hours after the passage of an IP shock because this solar wind was not affected by the shock. On the other hand, a longer data interval gives a higher chance to observe the decay of the turbulent energy. Therefore, we made a compromise and analyzed usually 2–3 hour-long time intervals and Figures 1 and 2 confirm a validity of such approach.

The large fluctuations in time profiles of the kinetic and magnetic energies are caused by the crossings of various magnetic and plasma discontinuities. The energy levels prior to and after them could be the same but a jump of the magnetic or velocity fields artificially increases the level (eq. 4). Nevertheless, an overall decreasing trend is visible in Figure 2, although the observed decay of the turbulent energy is not statistically significant due to a small number of samples.

Finally, we mention the role of the inverse turbulent cascade that can take place in the solar wind. The inverse cascade means that if we inject a physical quantity of interest, \( Q(k) \), \( k \) being the wave
number, into the system at wave number $k_0$, it will diffuse in $k$-space towards lower $k$. In 2D and 3D MHD turbulence, the only ideal invariant that shows an inverse cascade is the magnetic city, $H_M$, the total energy and cross-helicity cascade directly [Biskamp, 2003]. In situ observations show that the magnetic helicity in the solar wind is not a zero [Matthaeus et al., 1982]. This implies that the inverse cascade plays some role in the turbulent evolution of the solar wind and its role in the energy decay should be examined. Magnetic and kinetic energies cascade to lower scales (higher $k$) and are eventually dissipated, however, the magnetic helicity cascades to higher scales (lower $k$). On the other hand, an IP shock “injects” the energy in a wide range of scales (from kinetic up to inertial) [Pitnia et al., 2016]. It is not clear, what is the impact of the energy injected by an IP shock on the inverse cascade. One could argue that the “original” injection scale and the inverse cascade are unaffected by the IP shock passage and the only difference is that the turbulent energy at the lower $k$-end of the inertial range is enhanced. Another possibility is that the injected energy cascades in both direct and inverse directions. In any case, we can use the results of Christensson et al. [2001]. They performed a 3D MHD simulation with a focus on the inverse cascade. They found that a freely decaying MHD turbulence with a finite magnetic helicity shows both direct and inverse cascades of the magnetic energy. They also found that the asymptotic decay rates of magnetic and kinetic energies are close to $t^{-0.7}$ and $t^{-1.1}$, respectively. This is consistent with the panels (a)–(c) in Figure 2. However, normalized $r_A$ should increase slightly in time but this is not observed in the panel (d) in Figure 2.

As a conclusion, we analyzed a relaxation of the turbulent energy behind IP shocks and found that the turbulent energy decay can be observed. The slight decrease in the kinetic energy suggests that the magnetic energy decreases slower in time than the kinetic energy, in agreement with Biskamp and Müller [1999]. It seems that an enhanced superposed epoch analysis could allow us to estimate the energy decay from the observed data with a reasonable significance. In a following study, we plan to increase the number of analyzed IP shocks.

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References