Theory of Relativity — How to Develop Its Understanding at a Secondary School Level

M. Ryston
Charles University in Prague, Faculty of Mathematics and Physics, Prague, Czech Republic.

Abstract. This article presents first conclusions from the research of available literature concerning both special and general relativity with the intention to use the findings in making a Czech study text that would allow interested upper-secondary school students (students for short) and also graduates a clear basic understanding of relativity. The goal was to identify common themes and crucial topics in textbooks and answer some theoretical questions that arose prior to the research.

Introduction

Although modern relativity is over a century old it is not commonly found in normal secondary school physics education. Special relativity (SR) used to be taught on Czech grammar schools but as it is not included in the Framework Education Programme for Secondary General Education [Balada, 2007] and physics teachers in general deal with having less than ideal number of lessons per week, special relativity tends to be abandoned or, in the best case scenario, moved to special seminars. General relativity (GR) has never been a proper part of secondary school education for obvious reasons, chief among which is its great mathematical difficulty and abstractness. Of course, there have been attempts to explain general relativity to Czech readers (in whom we are most interested). A quick internet search shows a few websites dedicated to popularisation of physics which dedicated some effort even to GR. The main problem is that these treatments are either strictly popular [e.g., Reichl, Všetička, 2006; or Krynický, 2010], ignoring any mathematics or quantitative descriptions, or they introduce complex physical quantities without properly explaining them first [e.g., Aldebaran, 2000], most likely because the presented information is meant for college students with necessary background in physics and mathematics. The situation is similar, yet slightly better, with English websites such as Einstein Online [2015] or Mastin [2009]. Nevertheless, websites remain a secondary source of information as they are less likely to be reviewed. That is why our main focus lies with textbooks so far.

When dealing with physics textbooks, we necessarily need to expand our interest to English literature because of a certain lack of introductory Czech books about general relativity.¹ Even with books written in English, a clear gap can be seen between two groups of sources. One is strictly of a popular nature, avoiding any mathematical or quantitative descriptions, or they introduce complex physical quantities without properly explaining them first [e.g., Aldebaran, 2000], most likely because the presented information is meant for college students with necessary background in physics and mathematics. The situation is similar, yet slightly better, with English websites such as Einstein Online [2015] or Mastin [2009]. Nevertheless, websites remain a secondary source of information as they are less likely to be reviewed. That is why our main focus lies with textbooks so far.

When dealing with physics textbooks, we necessarily need to expand our interest to English literature because of a certain lack of introductory Czech books about general relativity.¹ Even with books written in English, a clear gap can be seen between two groups of sources. One is strictly of a popular nature, avoiding any mathematical treatment and relying heavily on spoken word and analogies accompanied with illustrations. The other group are college level textbooks meant for undergraduate or graduate students. Naturally, these textbooks require a non-trivial knowledge of appropriate mathematics and are therefore not suitable for students’ first encounter with GR.

It is our belief, that this gap can be bridged by a clear and simple, yet at the same time precise approach using appropriate simplified mathematics. What follows are conclusions from the research among selected textbooks that shape the possible approach to the desired exposition of general relativity to (mostly) upper-secondary school students in the form of a study text.

¹ Few exceptions can be mentioned — Kuchař [1968], Dvořák [1984], Ullmann [1986], and Horský et al. [2001]. However, these books are meant for college students used to calculus and basics of special relativity and on their own cannot be recommended to an upper-secondary school student. On the other hand, a not-so-long-ago reedited Czech translation of Einstein’s book [Einstein, 2005] is also available. Although this book is much more accessible because it uses only reasoning and thought experiments, it lacks more concrete mathematical foundation or problem solving. Furthermore, the book mentions so-called classical experiments of GR (red-shift, deflection of light rays, motion of Mercury’s perihelion) and does not include more modern ones like systems of global navigation or other astronomical observations.
In this article, we will discuss various concepts connected with GR such as curvature, spacetime, etc. They will not be properly defined, as certain (not necessarily rigorous) grasp of these concepts is expected from the reader and precise definitions are not necessary for the purposes of this paper.

Research questions

During the research, we particularly looked for the answers to following questions:

1. How much (if any) of SR is necessary before introducing GR?
2. What are the core ideas of GR?
3. What mathematics is really needed for these ideas?
4. What prior (classical) knowledge should the reader have before starting with relativity?

Let us now go through each question at a time.

SR before GR

If we look at GR textbooks, it is quite common among the authors to include a brief review of SR at the beginning of the book. That is understandable because GR historically and conceptually arises from SR, so starting with SR seems traditional. However, is it really necessary?

The short answer to this question based on our research is: Yes, it is. The only mention of GR before SR found comes from Rindler [1994]. Rindler’s ideas are put forward using variation principle, a tool of mathematical physics that is not available to students and explaining it just for the benefit of the exposition of GR before SR doesn’t seem practical.

There is a very good reason for SR to be included before dealing with GR. As it was said, GR uses many concepts used in SR and arose as a natural extension of the previous theory. Therefore, if we want to take the potential reader through a discovering process of forming the theory along with how and why it was formed, it is essential to start with SR. The more important question then is how much of SR is necessary before GR? In our research, we focus on the latter, so providing and answer to this question is key to accomplish the required clarity and brevity of proposed study materials. To do so, we need to identify the core concepts of GR, which brings us to the next question.

Core concept(s) of GR

The reader will probably agree that an introductory text about a subject cannot be an 800-page treatise that would discourage even those bravest and most curious among students. To be as concise as possible, we must therefore identify the most important pieces of GR that we want to put forward. This will also help us to pick the essential parts of SR that need to be included without reserving too much space and time for it. As was stated previously, our main focus is GR, mostly because much less material on it is available to eager students and graduates.

Hartle [2006] formulated three main ideas in GR: Gravity is curvature of spacetime,2 mass—energy is the source of spacetime curvature and free mass moves on straight paths in curved spacetime. We will focus on the first statement which is the most essential part. Let us inspect it more closely.

Gravity is the phenomenon that we want to illustrate, so we are left with two concepts, curvature and spacetime. SR does not deal with curvature. Although the geometry of the Minkowski spacetime differs from classical view of space and time, it is still flat, i.e., it has zero curvature. However, what SR already deals with is the concept of spacetime. Therefore we can form the hypothesis that the crucial role of SR is to develop students’ understanding of spacetime and make them used to it. This is usually done by showing that time and distance are not invariant and different observers can measure these quantities differently. Further details go outside the intended range of this paper. Suffice it to say, using SR to introduce the concept of spacetime is often done with already existing techniques and there are a number of modern books (both popular and more rigorous) focusing on SR [e.g., Lieber, 2008; Mermin, 2009; Steane, 2011; etc.].

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We are now left with curvature. Its treatment can be divided into two obvious parts, physical and mathematical. The latter is a subject of the next research question. The former is very important as it is crucial to convince the student (reader) of the validity and certain physical necessity of our claims. Although we would like to guide the students on a discovery-based journey, we are to some extent still presenters of physical ideas and it is our task to convince our students that presented ideas are at least plausible, if not already true. We will leave this topic open for now for two reasons. Firstly, the research itself is not finished yet, and although general conclusions (with which this paper is concerned) are not likely to change, the exact nature of the GR exposition probably will. It is therefore pointless to talk about it now. Secondly, it is a large topic that needs to be addressed in detail and doing so would take us too far from the subject of this paper.

**Essential mathematics for core GR ideas**

What mathematics do we expect from the student? Certainly we can expect secondary school mathematics such as goniometric functions, rearranging mathematical expressions, etc. It is problematic to use basic calculus (such as derivatives and easy integration), because not every secondary school includes these topics, they are sometimes left for a mathematics seminar or left out completely. Moreover, it would be impractical to limit the group of potential students to third or fourth year students, because basic calculus is often discussed in later years of upper-secondary school.

Of course, the concept of derivatives or rather infinitesimal calculations is important in differential geometry, which is the main mathematical field for describing curvature, but in the spirit of focusing only on the most needed concepts of mathematics we will shift our attention to a more physics related concept and try to use it for our purposes without necessarily going through calculus.

The main tool for quantitative description of curved spaces, the metric, is essential if we want to do some basic calculations to illustrate the effects of gravity more concretely. Therefore a thorough and clear introduction of intrinsic geometry is necessary. Metric coefficients then come up in the spacetime interval to which reader should be used from the treatment of SR. Of course, we cannot immediately jump to the curvature of spacetime or even space, it is necessary to start with curvature of surface, which can be much easily comprehended, being imbedded in a tree-dimensional Cartesian space. It is natural to discuss the geometry of a sphere to introduce basic ideas of curvature [e.g., Lieber, 2008; Cheng, 2010] because sphere is arguably the most comprehensible curved surface for people. By the means of a sphere we can introduce metric coefficients which can be then generalized to higher dimensions simply by raising the number of dimensions. This is useful because we can avoid trying to imagine what a curved three-dimensional space looks like and instead we can rely on the solid foundation of mathematical expressions.

Now let’s go back to basic calculus. It is clear that using it would significantly widen the range of possible calculations and physical problems we can do; however, it would also limit possible audience. The most logical approach seems to be a compromise. It is necessary to present the students with understandable exposition of GR using curvature without basic calculus, but at the same time additional material involving simple calculus calculations should be provided for those versed in doing derivatives and easy integration. The decision should be up to the students themselves, whether they want to follow an easier and more qualitative line of thought or delve into more precise calculations involving interesting problems.

Finally, we need to address one more point. Introducing curvature, which is a product of differential geometry, without using proper calculus tools seems problematic. And indeed, if we want to avoid using differentials etc., we have to spend some time discussing that we need to deal with curvature on a very small scale, small enough so we can make some simplifications. With enough

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3 By curved we mean that it has non-zero Gaussian curvature. For example a cylindrical surface is curved in Euclidean space (we talk about extrinsic curvature) but has zero Gaussian curvature (related to intrinsic curvature). Informally said, a cylindrical surface can be unfolded into a Euclidean plane whereas a spherical surface cannot. This important distinction between intrinsic and extrinsic curvature will also be part of the intended study text.
effort this can be done and we can spare the student some difficult mathematical tools for the sake of clarity. At the same time, however, we can refer more knowledgeable students to the proper mathematical tools. Again, we would be leaving the choice to the student, which is essential for us.

Necessary prior knowledge

Many of the researched textbooks [e.g., Geroch, 1980; Wolfson, 2003; Hartle, 2003; Cheng, 2010] include a classical beginning before they start with relativity (be it SR, GR or both). We can try to identify the reasons why. Relativity brings many unusual and counterintuitive results. To accept them, we need to rely heavily on our most basic principles in physics like measurement of time and distance, observation, experimentation, etc. The problem is that many of these ideas are not stressed enough during school education or perhaps there are some misconceptions about them. Because authors cannot be sure what the students have been introduced to and what not, they tend to lay down the physical foundation right at the start. This also helps to focus the student on the specific topics of the exposition.

Another reason for such classical introductions is, that, with introducing relativity, we often tend to “tear down the old ideas” and replace them with new ones. So, for the purposes of a well-structured explanation, it is useful to remind the student of these “old ideas.”

What is often included in these classical parts of course varies from one author to the next. As the most important we could choose said measurement of time and distance, frames of reference, synchronization of clocks or relativity of velocity. Another important aspect to start with are coordinate transformations that have a lot to do with relativity (and indeed are very often used in SR) and are seldom dealt with during normal physics education. We consider all these above-mentioned concepts to be necessary prior knowledge for a student and it is practical to include them in our treatment of GR.

Maths first vs. physics first

Let us mention one more idea that is important for the introduction to relativity. It was put forward by Hartle [2006]. Hartle describes two different approaches to teaching physics. We will only briefly comment on this, for more details see the referred paper.

As the name suggests, maths first approach includes building up the necessary mathematical apparatus beforehand then illustrating general physics equations (such as field equations), which we then solve for some particular interesting examples and compare our findings with experiment. This approach can be often seen in college-level textbooks about GR, which aim to be rigorous and general. A nice example of this given by Hartle is electromagnetism, where by this approach we would first deal with vector and differential calculus, then postulate Maxwell’s field equations and finally solve them for some interesting cases, deriving for example the electric field of a charged point particle. This is of course not how things are done in introductory college courses or at secondary schools.

Instead, it is customary to postulate simple solutions without any rigorous mathematical derivation (such as simple electric and magnetic fields) which are used to illustrate various physical properties or solve problems and can be compared with experiments. Only later can our findings be used to arrive at a more general theoretical level, often represented by a set of general equations describing given theory. This is what Hartle calls physics first.

The advantages and disadvantages of both approaches are easy to see. Physics first is useful for introductory courses, first glances at given physics. It omits higher mathematics that is not necessary at the time and focuses on key physical properties using special cases and limited amount of mathematics. Maths first is more abstract, it immediately deals with more difficult and more powerful mathematical tools, which can sometimes cloud the rest of the exposition for an inexperienced student. That’s why maths first is more suitable for advanced treatments of a given subject.

If we were to apply physics first approach to GR, we could start with postulating the simplest curved spacetime, i.e., the space time of a non-rotating spherical source of gravitation, a so-called Schwarzschild’s solution (of Einstein’s equations). The only “new” mathematics we need for this are
the mentioned metric coefficients in the spacetime metric. We don’t need higher mathematics such as tensor calculus, which is often introduced in college-level textbooks and is considered the difficult part. With this simple, yet astronomically relevant case we can already start illustrating some interesting phenomena of relativistic gravity, even more so if we want to include basic calculus (see above). We therefore don’t necessarily need to derive equations of motion, which are tensor in nature.

A very good implementation of this approach is used by Taylor and Wheeler [2000]. The book starts by introducing Schwarzschild’s solution and the rest are various treatments and probes into the properties of this type of spacetime, which is, in our opinion, more beneficial for a first-time reader than deriving equations of motion or general field equations.

Conclusion

We have given a concise review of the conclusion drawn from the research in literature regarding a possible approach to introducing relativity (with emphasis to general relativity) to upper-secondary school students and graduates. Although the research is not yet complete at this time, further study will most likely only add more details and will not change these underlying ideas. Focusing on the most important idea of general relativity, that gravitation is curvature of spacetime, we can identify the least amount of SR that needs to be introduced before starting with GR. That is enough to properly illustrate what spacetime is and introduce spacetime interval. That is not to say that explaining basic ideas of GR is simple. A thorough treatment of the subject requires combining several both physical and mathematical “components” such as local flatness, principle of equivalence, the difference between intrinsic and extrinsic curvature and so on.

As the key mathematical component of GR we identified the concept of curvature, for which we need to introduce metric as a way to describe intrinsic geometry of spacetime.

Even before diving into relativity itself, it is necessary to review some basic physical concepts that are crucial in our further discussion (such as measurement of time and space, frames of reference, coordinate transformations, etc.). This is mainly to make up for the possible lack in students’ prior education which we cannot control.

Finally, it is a common concept in other parts of physics (secondary school physics particularly) to start with simple physics applications and leave out the more difficult parts that require complex mathematical tools. This issue is crucial in GR where the mathematics is very abstract and can prove challenging even to the most enthusiastic students. In that regard, leaving out equations of motion or Einstein’s equations from an introductory text (or at least leaving them for much later) seems like a suitable approach. Instead, we should focus on the simplest of curved spacetimes, the Schwarzschild’s solution, itself before going to more challenging general topics.

In the future, we intend to make a website containing a thorough treatment of the exposition of relativity from classical through special to general with the emphasis on general relativity, divided into several layers of difficulty and focused on flexibility towards readers (i.e., readers should be able to adjust the difficulty of the text by the means of additional information such as calculations or more advanced topics). The main advantage of a website presentation is not only its availability but also the possibility of using animations, applets and videos to accompany the text. The absence of such moving aids is arguably the biggest disadvantage of ordinary textbooks.

References


