The First Czech Textbooks of Mathematical Analysis

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Abstract. This article focuses on textbooks of mathematical analysis written in the Czech language in the second half of the 19th century and in the early 20th century. It describes the appropriate books written by Václav Šimerka (1819–1887), František Josef Studnička (1836–1903), Eduard Weyr (1852–1903) and Josef Úlehla (1852–1933). These books are compared and put into the historical context.

Introduction

The emergence of Czech textbooks of mathematical analysis represents an interesting chapter in the history of teaching mathematics in our country. The need for them arose in the 1860s, when our secondary schools and universities began to teach in the Czech language.[1]

Textbooks of mathematical analysis were mostly written for university students. The following text describes publications by Václav Šimerka (1819–1887), František Josef Studnička (1836–1903), Eduard Weyr (1852–1903) and Josef Úlehla (1852–1933).

Přídavek k algebře pro vyšší gymnasia by Václav Šimerka

The first Czech textbook of mathematical analysis was published by the grammar school teacher, philosopher and priest Václav Šimerka[2] under title *Přídavek k algebře pro vyšší gymnasia* (*Supplement to Algebra for Higher Grammar Schools*) [*Šimerka*, 1864].[3] This book was also the only one in this article designated for grammar school students. Nevertheless, it was not generally used at schools, because the calculus has not yet taught at the grammar schools.

Přídavek has 56 pages and it is divided into 6 chapters.[4] The first four of them are devoted to the differential calculus of real functions of one variable, applications of the differential calculus in geometry and Taylors series. The fifth chapter describes the integral calculus of real functions and methods of calculation of indefinite and definite integrals. The last one shows the use of definite integrals.

Textbooks of mathematical analysis by František Josef Studnička

Professor František Josef Studnička[5] was an important Czech mathematician and teacher at the Czech Polytechnic in Prague (from 1864 to 1871) and at the Czech University in Prague (from 1871 to 1903). We will describe his six textbooks, which cover the whole mathematical analysis that was taught at that time.

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[1] In 1848 the Czech and German languages were equalized thanks to the National Revival. At high schools it started to teach in Czech (against the current German) in 1860s, at the Czech Polytechnic in Prague in 1864 and at Czech University in Prague in 1871.

[2] More about the work and life of Václav Šimerka see, for example, in Fiala J.: *Síla přesvědčení Václava Šimerky* (*Strength of Conviction of Václav Šimerka*). In Pátý L. (ed.): *Jubilejní almanach jednoty čs. matematiků a fyziků 1862–1987* (*Jubilee Almanac Union of Czechoslovak Mathematicians and physicists*). Jednota československých matematiků a fyziků, Prague, 1987, pp. 97–106.

[3] V. Šimerka wrote this textbook as the addition for his *Algebra čili počtářství obecné pro vyšší gymnasia* (*Algebra or General Calculating for Higher Grammar Schools*).

[4] *Differencialy daných úkonů* (*Differentials of Operations*). *Proměňování úkonů v řady* (*Transformation of Operations into Series*). *Úkony trigonometrické* (*Trigonometrically Operations*). *Taylorova poučka a její následky* (*Taylors Theorem and its Corollaries*). *Základy počtu integrálného* (*Foundations of Integral Calculus*) and *Upotřebení počtu nekonečného v geometrii* (*Use of Infinitesimal Calculus in Geometry*).

[5] More about the work and life of František Josef Studnička see, for example, in Němcová M.: *František Josef Studnička 1836–1903*. Edition *Dějiny matematiky*, vol. 10, Prometheus, Prague, 1998.

Vyšší mathematika v úlohách

Vyšší mathematika v úlohách (*Higher Mathematics in Exercises*) [*Studnička*, 1866] was written for polytechnic students and it was a collection of mathematical problems. Altogether, there are 1 051 exercises (without solutions). The book has 48 pages and it is divided into three parts.[6] The first one covers the problems concerning differential calculus and its applications, the second part contains exercises on integration and on applications of the integral calculus. The last part is devoted to problems from the field of calculus of variations. Eight years later, the second edition [*Studnička*, 1874] of the book was published.

Základové vyšší mathematiky

Základové vyšší mathematiky (*Foundations of Higher Mathematics*) [*Studnička*, 1867–1871] is a three-part textbook of mathematical analysis. F. J. Studnička prepared it originally with professor Gustav Skřivan (1831–1866),[7] but their cooperation was stopped by Skřivan's early death. F. J. Studnička first published the third part, which was already finished, then the first one and finally the second part.

The first part, entitled *O počtu differenciálním* (*On Differential Calculus*), was published in 1868. It is designated (as well as the remaining parts) for polytechnic students. The textbook has 240 pages and it is divided into four parts: *Úvod* (*Introduction*) and *Kniha I.–III.* (*Book I–III*).[8] It contains the theory of differential calculus, rules of calculating with derivatives, applications in the graphing of functions and in the representation of a function as an infinite series. The second edition [*Studnička*, 1878] of this volume of the textbook was published 10 years later.

The second part, entitled *O počtu integrálním* (*On Integral Calculus*) was issued in 1871. It has 216 pages and it is structured into three chapters: *Úvod* (*Introduction*), *Kniha I.–II.* (*Book I–II*).[9] It deals with the theory of integral calculus and it describes the relation between derivative and integral, the integration by parts, substitution methods of integration and geometrical applications of integral calculus.

In 1867, the third part called *O integrování rovnic differenciálních a počtu variačním* (*On integration of differential equations and on calculus of variations*) was printed. It has 296 pages and it is divided into five parts *Kniha I.–IV.* (*Book I–IV*) and *Dodatek* (*Supplement*). The book contains the exposition of first order differential equations, second order linear differential equations, *n*-order homogeneous linear differential equations and partial differential equations. In *Dodatek* provides the basics of calculus of variations, its applications and history. It was later divided into two parts and published again as separate brochures [*Studnička*, 1871 and 1872].[10]

Všeobecné tvarosloví algebraické

F. J. Studnička wrote also two textbooks of mathematical analysis for university students. The first one has the whole title *Všeobecné tvarosloví algebraické čili nauka o konečných i nekonečných součtech čili řadách, součinech a podílech čili řetězcích* (*General Algebraic Accidence or Theory of Finite and Infinite Sums, Series, Products and Quotients or Chains*) [*Studnička*, 1880] and it has

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[6] *Počet differenciální a jeho upotřebení* (*Differential Calculus and its Applications*), *Počet integrální a jeho upotřebení* (*Integral Calculus and its Applications*) and *Dodatek* (*Supplement*).

[7] More about the work and life of Gustav Skřivan see, for example, in Slavík A.: *Životní příběh prof. Gustava Skřivana* (*Life Story of Gustav Skřivan*). In Bečvář J., Bečvářová M. (ed.): *32. mezinárodní konference Historie matematiky* (*The 32nd International Conference on the History of Mathematics*). Matfyzpress, Prague, 2011, pp. 245–254.

[8] *O differencování a differenciálních poměrech vůbec* (*On Differentiate and Differential Quotients*). *O upotřebení počtu differenciálního při řešení úloh vyšší algebry* (*On Applications of Differential Calculus in Solving Problems of Higher Algebra*) and *O upotřebení počtu differenciálního při řešení úloh vyšší geometrie* (*On Applications of Differential Calculus in Solving Problems of Higher Geometry*).

[9] *O integrování výrazů differenciálních* (*On Integration of Differential expressions*) and *O upotřebení počtu integrálního k řešení úloh vyšší geometrie* (*On Applications of Integral Calculus in Solving Problems of Higher Geometry*).

[10] *Přednáška o původu a rozvoji počtu variačního* (*Lecture on Origin and Development of Calculus of Variations*) and *O počtu variačním* (*On Calculus of Variations*).

239 pages. This book is divided into two parts,[11] it is devoted to finite arithmetic, geometric and harmonic sequences, binomial and polynomial theorem, finite and infinite continued fractions, convergent, divergent and oscillating infinite series. *Všeobecné tvarosloví algebraické* is the first Czech textbook that comprehensively described the theory of series.

Výklady o funkcích monoperiodických

The second university textbook is from 1892, it has 179 pages and it is called *Výklady o funkcích monoperiodických neboli o nižších funkcích transcendentních* (*Expositions on Monoperiodic Functions or Lower Transcendent Functions*) [*Studnička*, 1892] and it has 179 pages. The text is structured into parts *Úvod* (*Introduction*)*, Oddělení I.–V.* (*Section I–V*)[12] and *Dodatek* (*Supplement*). It contains the theory of limits of functions and methods of calculating the limits. Then it focuses on the theory of trigonometric, hyperbolic, and inverse trigonometric functions of a real variable and expression of functions using series. The last part deals with monoperiodic functions of a complex variable and it describes the basics of complex analysis, which was not yet exposed in the Czech literature.

F. J. Studnička's work was inspired by European textbooks of mathematical analysis. Author studied from publications by Oscar Schlömilch, Joseph Bertrand, Leonhard Euler, Augustin Louis Cauchy, Moritz Stern or Jakub Filip Kulik.

***Počet differenciální* by Eduard Weyr**

At the turn of the 19th and 20th centuries *Jednota českých mathematiků* (*Union of Czech Mathematicians*) asked Eduard Weyr,[13] professor of Czech university in Prague and the Prague Czech Polytechnic for writing new textbooks of mathematical analysis. He started to prepare the books *Počet differenciální* (*Differential Calculus*) and *Počet integrální* (*Integral Calculus*), but the second one was never finished.

Počet differenciální [*Weyr*, 1902] contains 416 pages and it is divided into 12 chapters.[14] It deals with rational, real and complex numbers, limits of sequences, theory of elementary functions, differential calculus of real functions of one and more variables, expression of functions using series, first order differential equations, use of differential calculus (limits, l'Hospital theorem, graphing of functions etc.) and elementary functions of complex variable.

Ed. Weyr came from books by Angelo Genocchi, Rudolf Lipschitz, Francisco Teixeira, Joseph Serret or also F. J. Studnička, but some parts he just copied and unfortunately with some mistakes. Approximately one half year after the publication *Počet differenciální* Jan Vilém Pexider (1874–1914)[15]

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[11] *O konečných tvarech algebraických* (*On Finite Algebraic Expressions*) and *O nekonečných tvarech algebraických* (*On Infinite Algebraic Expressions*) .

[12] *O původu a významu exponenciální funkce* (*On Origin and Meaning of Exponential Function*). *O funkcích hyperbolických* (*On Hyperbolic Functions*). *O logarithmech* (*On Logarithms*). *O funkcích kyklických neb goniometrických* (*On Cyclic or Goniometric Functions*) and *O funkcích kyklometrických* (*On Cyclometric Functions*).

[13] More about the work and life of Eduard Weyr see, for example, in Bečvář J. et al: *Eduard Weyr 1852–1903*. Edition *Dějiny matematiky*, vol. 2, Prometheus, Prague, 1995.

[14] *Čísla racionálná, irracionálná, komplexní* (*Rational, Irrational and Complex Numbers*). *Limity a meze* (*Limits and Bounds*). *Nekonečné řady a součiny* (*Infinite Series and Products*). *O funkcích, hlavně jedné proměnné, a zvlášť o elementárných funkcích* (*On Functions, Especially of One Variable and Separately on Elementary Functions*). *Derivace a differenciály funkcí jedné proměnné* (*Derivatives and Differentials of Function of One Variable*). *Rozvoj funkce jedné proměnné v řadu mocninnou* (*Representation of Function of One Variable by the Power Series*). *Funkce více proměnných a funkce implicitní* (*Function of Several Variables and Implicit Functions*). *Vznik rovnic differenciálných* (*Genesis of Differential Equations*). *Neurčité tvary* (*Indeterminate Expressions*). *Zavádění nových proměnných* (*Defining New Variables*). *Homogenní funkce* (*Homogeneous Function*). *Funkcionálné determinanty* (*Functional Determinants*). *Maxima a minima* (*Maxima and Minima*). *Theorie čar v rovině* (*Theory of Lines in the Plane*). *Theorie křivosti čar v prostoru* (*Theory of Curvature of Lines in the Space*). *Theorie křivosti ploch* (*Theory of Curvature of Surfaces*) and *Elementárné funkce komplexní proměnné* (*Elementary Functions of Complex Variable*).

[15] More about work and life of Jan Vilém Pexider see, for example, in Bečvář J. (ed.): *Jan Vilém Pexider 1874–1914*. Edition *Dějiny matematiky*, vol. 5, Prometheus, Prague, 1997. About his dispute with Ed. Weyr see in Bečvář J. et al.: *Eduard Weyr 1852–1903*. Edition *Dějiny matematiky*, vol. 2, Prometheus, Prague, 1995, pp. 143–162.

sharply criticized it. It led to a great dispute, which was known also outside the mathematical community.

***Počet infinitesimální* by Josef Úlehla**

A unique calculus textbook, entitled *Počet infinitesimální* (*Infinitesimal Calculus*) [*Úlehla*, 1906], published secondary school teacher Josef Úlehla[16] in 1906. This publication was not an official school textbook. Úlehla wrote it on his own initiative and designed it for autodidacts.

The book has 130 pages and it is divided into two main parts. The first one, *Počet diferenciální* (*Differential Calculus*), is devoted to differential calculus of real functions of one variable, Maclaurin, Newton and Taylors series and applications in geometry. The second volume *Počet integrální* (*Integral* Calculus), describes the methods of integration and especially the use of definite integral: determining the area under a graph, length of a curve, surface and volume of a solid of revolution. The supplement of the book contains a brief note about differential equations, algebraic solution of cubic equations and the list of analytical expressions of curves.

J. Úlehla did not specify the references. In the introduction he just mentioned the above-described textbooks with the remark that Šimerka's publication was too short and Studnička's and Weyr's books were too difficult for beginners. In 1944, Úlehla's textbook was published again under the title *Vyšší matematika bez učitele* (*Higher Mathematics without Teacher*) [*Úlehla*, 1944].

Comparison

The content and form of mentioned textbooks was influenced mainly by those for whom they were intended. The books also flash back their author and the used literature. In the following, let us compare the texts and discuss their historical importance. For each publication, we will also focus on definition of the derivative, one of the main concept in mathematical analysis.

Šimerka's *Přídavek k algebře pro vyšší gymnasia* [*Šimerka*, 1864] documented the efforts to introduce teaching of calculus into secondary schools. The book was written very freely, it was not structured into definitions, theorems and proofs. The main term in concerning derivative was differential:

Nesmírně čili nekonečně malá část, o niž spojitou proměnnou veličinu (x, y, z, atd.) růsti necháváme, jmenuje se differencial (lišné, rozčinek) veličiny této, a znamená písmenou δ před veličinu onu postavenou (δx, δy, δz, atd.). [*Šimerka*, 1864, p. 1]

(*Extremely or infinitely small part by which we let grow continuous variable (x, y, z etc.) is called differential of this variable and means the letter δ before that variable (δx, δy, δz, etc.)*.)

Šimerka did not use the idea of a limit. He described the derivative as a quotient of differentials:

$$\frac{dy}{dx}, \frac{d^2y}{dx^2}, \frac{d^3y}{dx^3}, \ldots, \frac{d^ry}{dx^r}$$
$$f^1, \quad f^2, \quad f^3, \ldots, \quad f^r$$

[*Šimerka*, 1864, p. 7]

F. J. Studnička explained the derivative in the first part of his *Základové vyšší mathematiky* [*Studnička*, 1878]. He used the term limit, but he did not give its exact definition using ε and δ arithmetic. He defined the derivative by the limit supposing that $\lim \Delta x = 0$:

$$f'(x) = \lim \frac{f(x + \triangle x) - f(x)}{\triangle x}$$

[*Studnička*, 1878, p. 12]

Základové vyšší mathematiky was the first Czech university textbook of mathematical analysis and it had played an important role in the formation of the Czech mathematical community in the last quarter of the 19th century.

[16] More about work and life of Josef Úlehla see, for example, in Vízek L.: *Josef Úlehla and His Calculus Textbook*. In J. Šafránková and J. Pavlů (eds.): *WDS'11 Proceedings of Contributed Papers: Part I – Mathematics and Computer Sciences*. Matfyzpress, Prague, 2011, pp. 91–94.

Ed. Weyr described the derivative in his *Počet differenciální* [*Weyr*, 1902] similarly as F. J. Studnička:

Klademe-li přírůst proměnné $\Delta x = h$, *jest přírůst funkce*
(*If we put the increment of variable* $\Delta x = h$, *the increment of the function is*)

$$\triangle f(x) = f(x+h) - f(x)$$

a
(*and*)

$$f'(x) = \lim \frac{\triangle f(x)}{\triangle x} = \lim \frac{f(x+h) - f(x)}{h}$$

pro lim $h = 0$ *derivace funkce f.* [*Weyr*, 1902, p. 131]
(*for lim* $h = 0$ *the derivative of the function f.*)

Ed. Weyr based the explanation of mathematical topics mostly in definitions, theorems and proofs. The historical importance of his work lies in the precision and exactness of the interpretation.

Počet infinitesimální [*Úlehla*, 1906] by J. Úlehla is a simple contraption for "doing" the calculus where is explained how to calculate without any theory. The derivative is described in a similar way like in the book by V. Šimerka:

$$f'(x) = \frac{dy}{dx}$$

[*Úlehla*, 1906, p. 42]

Úlehla's textbook was good and useful for self-taught persons. This is confirmed by the second edition from 1944 [*Úlehla*, 1944] published in the time of the Second World War, when all Czech universities were closed and books for autodidacts were especially necessary.

Conclusion

The present-day reader of the above-described textbooks will probably demur to the inaccuracy in definitions of basic terms or to the lack of the contemporary structure of explanation of mathematics. But when we evaluate these publications, we should have in mind the historical context.

We can get a lot of inspiration from these old books. There are many interesting examples and a lot of applications of calculus, which are important and which are sometimes not comprehended in contemporary textbooks.

References

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