The Properties of Gamma-ray Images of Supernova Remnants Due to Proton–Proton Interactions

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Abstract. MAGIC and H.E.S.S experiments are the first to produce images of supernova remnants (SNRs) in TeV gamma-rays. The gamma-radiation are produced either by electrons (due to inverse-Compton scatterings) or protons (due to pion decays). We present a method to synthesize gamma-ray images of Sedov SNRs due to hadronic emission. The model is developed in the frame of a classic approach to proton acceleration and hydrodynamics of the shocks in a uniform interstellar medium; it includes energy losses of relativistic protons due to pp interactions. Our calculations show that these losses are important only for large densities of protons as it could be in case of interactions of the supernova shock with molecular cloud. Numerical simulations are used to synthesize radial profiles of hadronic TeV gamma-rays.

Introduction

Cosmic rays (CRs) are widely studied. Supernova remnants (SNRs), the main source of galactic CRs, are excellent objects to study magneto-hydrodynamics of nonrelativistic shocks and acceleration of CRs, namely protons and electrons. These particles radiate from radio to gamma-rays due to different emission processes. Experiments in high-energy astronomy observe all types of these emission.

Most of galactic CRs are believed to be produced by the forward shocks in SNRs. In particular, efficient proton acceleration changes the structure of the shock front and makes plasma more compressible that leads to lower adiabatic index, to increased shock compression factor and to some observed effects: reduced physical separation between the forward shock and the “contact discontinuity” (or reverse shock) [e.g. Warren et al., 2005]; concave shape of the energy spectrum [e.g. Reynolds and Ellison, 1992]; growth of some turbulence modes and to magnetic field amplification in the pre-shock region [e.g. Bell, 2004]; “blinking” X-ray spots originated from such growth of magnetic field [Uchiyama et al., 2007].

Observations are expected to confirm that protons are accelerated in SNRs to very high energy and emit (TeV) gamma-rays. Nevertheless, analysis of the broad-band spectra of SNRs shows that both electrons and protons may be responsible for TeV gamma-rays [e.g. SN 1006: Acero et al., 2010].

The properties of the thickness of the radial profiles of hard X-ray brightness are used to estimate the strength of the post-shock magnetic field [Berezhko et al., 2003]. Radial profiles of the radio brightness may constrain the time evolution of the electron injection efficiency [Petruk et al., 2011a]. In a simple fashion, the radial profiles of hadronic TeV gamma-ray brightness are sensitive to the density of ambient medium. This property is the subject of the present study.

Properties of the nonthermal images of Sedov SNRs due to radiation of accelerated electrons in radio, X-rays and gamma-rays are systematically studied in Reynolds [1998, 2004] and Petruk et al. [2009, 2011b, Papers I and II respectively]. Numerical models for synthesis of maps of adiabatic SNRs in uniform interstellar medium (ISM) and uniform interstellar magnetic field (ISMF) from basic theoretical principles as well as their approximate analytical descriptions are developed in these papers. The main factors determining the azimuthal and radial variation of surface brightness of Sedov SNRs are determined there.

These papers, as the present one, are limited to the test-particle approach because the non-linear theory of diffusive acceleration is not developed for shocks of different obliquity, while the obliquity dependence of various parameters is important for image modelling.

In the present paper, we study properties of the radial profiles of surface brightness in gamma-ray due to proton–proton interactions including of the energy losses of proton.

Model

Our model closely restores that used in Papers I and II. Let us consider an adiabatic SNR in uniform ISM and uniform ISMF. We use quite accurate approximate formula in Lagrange coordinates [Petruk, 2000] for description of hydrodynamics of SNRs in the adiabatic stage of evolution (Sedov solutions [Sedov, 1959]). The magnetic field is described following [Reynolds, 1998]. We do not consider amplification of the ambient field.
At the shock, the spectrum of accelerated protons $N_p$ is taken as

$$N_p(E_p) = K_s E_p^{-s} \exp \left( - \frac{E_p}{E_{p,\text{max}}} \right),$$

where $E_p$ - the energy of proton, $E_{p,\text{max}}$ is the maximum energy of protons, $K_s$, $s$ are constants and we use $s = 2$.

We assume that variation of the maximum energy with obliquity angle (angle between the ambient magnetic field and the shock velocity) is constant and the injection efficiency of protons is isotropic.

The surface brightness is calculated integrating emissivities along the line of sight within SNR. Hadronic gamma-rays appear as a consequence of the neutral pion and $\eta$-meson decays produced in inelastic collisions of accelerated protons with thermal protons downstream of the shock; the spatial distribution of the target protons is simply proportional to the plasma density. The hadronic gamma-ray emissivity is calculated as [Kelner et al., 2006]

$$q_\gamma(E_\gamma) = cn_H \int_0^1 \sigma_{pp}(E_\gamma/x)N(E_\gamma/x)F_\gamma(x, E_\gamma/x) \frac{dx}{x},$$

where $x = E_\gamma/E_p$, $E_\gamma$ is energy of gamma-photon and $c$, $n_H$ are speed of light, the proton target density, the cross-section is [Aharonian and Athoyan, 2000]:

$$\sigma_{pp}(E_p) = 28.5 + 1.8 \ln (E_p/1 \text{ GeV}) \ \text{mb},$$

(1 mb = $10^{-27}$ cm$^2$), the function $F_\gamma$ is [Kelner et al., 2006]

$$F_\gamma(x, E_p) = B_\gamma \frac{\ln(x)}{x} \left( \frac{1 - x^{\beta_\gamma}}{1 + k_\gamma x^{\beta_\gamma} (1 - x^{\beta_\gamma})} \right)^4 \times \left[ \frac{1}{\ln(x)} - \frac{4 \beta_\gamma x^{\beta_\gamma}}{1 - x^{\beta_\gamma}} - \frac{4 k_\gamma \beta_\gamma x^{\beta_\gamma} (1 - 2 x^{\beta_\gamma})}{1 + k_\gamma x^{\beta_\gamma} (1 - x^{\beta_\gamma})} \right],$$

where

$$B_\gamma = 1.30 + 0.14 L + 0.011 L^2,$$

$$\beta_\gamma = \frac{1}{1.79 + 0.11 L + 0.008 L^2},$$

$$k_\gamma = \frac{0.801 + 0.049 L + 0.014 L^2}{L},$$

and $L = \ln(E_p/1 \text{ TeV})$.

**Energy losses of protons due to proton–proton interactions**

Modelling the surface brightness distribution of SNRs due to collisions acceleration protons on the shock wave and protons in rest with is important to accurate the energy losses due to pion productions. The losses due to proton collisions are important for higher densities of target protons. It may be shown that the proton collision losses can be described as

$$- \left( \frac{dE_p}{dt} \right)_{pp} = 3\kappa cn_H \sigma_{pp}(E_p)E_p,\text{kin},$$

where the factor 3 accounts for the production of $\pi^0$, $\pi^+$ and $\pi^-$ mesons, respectively and $\sigma_{pp}(E_\pi, E_p)$ the differential cross-section for the interaction of two protons, $3\kappa = 0.51$ if $\kappa = 0.17$ [Aharonian and Athoyan, 2000].

Let us compare the energy losses due to proton–proton interactions with the radiative losses of electrons. The losses of electrons are given by

$$- \left( \frac{dE_e}{dt} \right)_{rad} = \frac{4}{3} \sigma_T c \left( \frac{E_{\text{kin}}}{m_e c^2} \right)^2 \left( \frac{B^2}{8\pi} \right),$$

where $\sigma_T$ is the Thomson cross-section, $m_e$ the mass of electron. The ratio of electron to proton radiative losses is

$$\frac{E_{e,\text{rad}}}{E_{p,\text{rad}}} \simeq \frac{B^2}{m_e c^2} E_{e,\text{TeV}}^{2/3} E_{p,\text{TeV}}^{-1/3},$$

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where we used \( \sigma_{pp} \approx 33 \, mb \), \( B_{\mu G} \) is the magnetic field in \( 10^{-6} \, G \), and \( E_{e,T eV} \) and \( E_{p,T eV} \) are the energy of electrons and protons in \( 10^{12} \, eV \). The losses of electrons end protons are similar when the density of protons is \( 440 \, cm^{-3} \) and the typical galactic magnetic field \( B_{\mu G} = 3 \) the maximum energy of electrons end protons are 30 and 100 TeV respectively. One can see that the losses of protons with energy 1000 TeV are comparable to losses of electrons with energy 30 TeV in magnetic field 3 \( \mu G \), if the number density of target protons is rather high 440 cm\(^{-3}\).

**Downstream evolution of the proton distribution**

Let the energy of proton at the time \( t_i \), when it leave the region of acceleration, is \( E_{pi} \). Then it is smaller at the present time \( t \),

\[
E_p = E_{pi} \epsilon_{ad}(\bar{a})^\mu(\bar{a}) \epsilon_{pp}(E_p, \bar{a}),
\]

because the terms responsible for the adiabatic \( \epsilon_{ad} \) and collisional \( \epsilon_{pp} \), losses are equal to or smaller than unity; \( \bar{a} = a/R \), \( a \) the Lagrangian coordinate, \( R \) the radius of SNR,

\[
\epsilon_{ad}(\bar{a}) = \bar{n}(\bar{a})^{1/3},
\]

where \( \bar{n} = n/n_s \), index “s” denotes the value immediately post-shock,

\[
\epsilon_{pp}(E_p, \bar{a}) = (E_p/1 GeV)^{1-\mu(\bar{a})} I(\bar{a}),
\]

\( \mu(\bar{a}) \) and \( I(\bar{a}) \) are dimensionless self-similar functions

\[
\mu(\bar{a}) = \exp \left[ \zeta \int_\bar{a} x^{3/2} p \left( \frac{\bar{a}}{x} \right) dx \right],
\]

\[
I(\bar{a}) = \exp \left[ \zeta \int_\bar{a} x^{3/2} q \left( \frac{\bar{a}}{x} \right) \mu \left( \frac{\bar{a}}{x} \right) dx \right],
\]

\[
p = 1.8\bar{n}, \quad q = -\bar{n}(28.5 + 1.8 \ln \bar{n}^{1/3}),
\]

where \( \zeta = 5tc_1/2 = 1.21 \cdot 10^{-6}t_3n_{Hs}, c_1 = 3 \times 10^{-27} \kappa C n_{Hs}, t_3 = t/1000 \) yrs. It is clear from here that \( \epsilon_{pp} \) is effective only when the density of target protons is large, at least \( n_{Hs} \approx 10^6 \) cm\(^{-3}\).

The energy spectrum of protons downstream of the shock evolves self-similarly

\[
N_p(E_p, \bar{a}, t) = K(\bar{a}, t) E_p^{-p} \mu(\bar{a}) \epsilon_{pp}(E_p, \bar{a})^{s-1}
\]

\[
\times \exp \left[ -\left( \frac{E_p \bar{a}^{3q/2}}{E_{p,max} \epsilon_{ad}(\bar{a})^\mu(\bar{a}) \epsilon_{pp}(E_p, \bar{a})} \right)^{\alpha} \right].
\]

with \( K(\bar{a}, t) = K_s K(\bar{a}), K(\bar{a}) = \bar{a}^{3q/2} \bar{n}(\bar{a})^{1+\mu(\bar{a})/(s-1)/3}. \) The downstream distribution of relativistic protons are modified by the adiabatic expansion of SNR. Losses due to inelastic collisions affects the distribution only when \( \zeta \) is not small, i.e. when \( n_{Hs} \) is large.

**Results and conclusion**

Analysis of the formula (8) shows that the thickness of radial profiles of the TeV gamma-ray surface brightness distributions are only function of density of target protons. If density of target protons is small then we can neglect the energy losses due to proton–proton interactions. However, when the density of target protons is more than \( 400 \) cm\(^{-3}\) one can not neglect these losses. However,they reveal themselves in the radial profiles for even higher densities.

In the fig.1, we show the influence of target protons density on the thickness of radial profiles of surface brightness. If the density increases the energy losses of protons increase as well and the thickness of the radial profile decreases. Since the downstream density is proportional to the pre-shock density, thus may be used as a probe of the ambient medium density.
Figure 1. Radial profiles of the gamma-ray surface brightness due to hadronic emission for different densities of the target protons: $1 \text{ cm}^{-3}$ (line 1), $10^4 \text{ cm}^{-3}$ (line 2), $10^5 \text{ cm}^{-3}$ (line 3), $10^6 \text{ cm}^{-3}$ (line 4). $E_{\text{max}} = 1000 \text{ TeV}$, energy of gamma-rays is $1 \text{ TeV}$. $S_{\text{max}}$ is the peak value of $S_{\text{pp}}(\rho)$.

References


