On Transfer of Mass and Angular Momentum from Accretion Disk onto Black Hole

J. Hamerský
Charles University, Faculty of Mathematics and Physics, Prague, Czech Republic.

V. Karas
Astronomical Institute of Academy of Sciences, Prague, Czech Republic.

Abstract. We study the properties of accretion tori orbiting Kerr black hole. Our approach to this problem comes from the solving of general relativistic magnetohydrodynamic equations, which follow from conservation of the energy-momentum tensor, the particle number and from Maxwell’s equations. We solve these equations by numerical methods. We are interested in tori with constant density of angular momentum and those with radially increasing angular momentum density. We study accretion rates in these tori when the mass of black hole is increased suddenly and so the equilibrium in the torus is perturbed. We also study the influence of the presence of toroidal magnetic field on accretion rates.

Introduction

It is commonly thought that many astrophysical systems contain relativistic plasma with a dynamically significant magnetic field. Examples include accreting black holes in black hole binaries, galactic nuclei, gamma-ray bursts, the cores of massive stars undergoing core collapse, isolated neutron stars, and neutron stars in binary systems. The kinematical gas analysis with HST (The Hubble Space Telescope) makes a convincing argument that the masses of central black holes in galaxies ranges from $10^5 M_\odot$ to more than $10^9 M_\odot$ [Kormendy and Richstone, 1995]. Astrophysical black holes do not support their own magnetic field, however they are embedded in cosmic magnetic field of external origin, with which they interact [Begelman et al., 1984]. So we expect that the interaction of magnetized plasma with the gravitational fields of black holes permeate the Universe and the astrophysical consequences should be significant. From the observations of SgrA* the magnetic field strength can reach the value around 40G [Eckart et al., 2008].

Fluid mechanics is well described by hydrodynamics but including magnetic fields into the flow of fluids, gases and plasmas creates forces unknown to hydrodynamics. Thus, we have much more complicated problem which requires a new subject, magnetohydrodynamics (MHD). Even more complicated situation occurs when we combine both magnetic and gravitational interactions with plasma flows. The subject which studies such problems is called GRMHD (General Relativistic Magnetohydrodynamics) [Punsly, 2008].

It is not surprising that there is an interest in numerical methods for solving the equations of GRMHD. For our simulations we use 2D version of program HARM developed by [Gammie et al., 2003] that solves hyperbolic partial differential equations in conservative form. HARM has been configured to solve the relativistic magnetohydrodynamic equations of motion on a stationary black hole spacetime in Kerr-Schild coordinates to evolve an accretion torus model.

We apply the code to study axially symmetric fluid tori of Abramowicz et al. We first implement the appropriate boundary conditions and reproduce the non-magnetized stationary solutions, including the critical configurations with the relativistic cusp. Then we go over to study the case of toroidal magnetic field permeating the stationary torus.
Fluid tori orbiting Kerr black hole

The magnetized ideal fluid can be described by energy-momentum tensor

\[
T_{\mu\nu} = (w + b^2)u^\mu u^\nu + \left(p + \frac{1}{2}b^2\right)g_{\mu\nu} - b^\mu b^\nu,
\]

where \(w\) denotes the specific enthalpy \((w = \rho_0 + p + u)\), \(\rho_0\) is the rest mass density, \(p\) is the pressure, \(u\) is the density of internal energy and \(b^\mu\) is the projection of magnetic field vector.

From the energy-momentum tensor conservation \(T_{\mu\nu}^{\;\;;\nu} = 0\) for the case of purely axially rotating fluid follow [Abramowicz, 1977]

\[
\ln |u_t| - \ln |u_{t_0}| + \int_0^P \frac{dp}{w} - \int_0^l \frac{\Omega dl}{1 - \Omega l} + \int_{\tilde{p}_m}^{\tilde{p}} \frac{d\tilde{p}_m}{\tilde{w}} = 0,
\]

where \(u_t\) denotes covariant time component of four-velocity, \(u_{t_0}\) corresponds to the inner edge of the torus, \(\Omega = \frac{u_\phi}{u_t}\) is angular velocity, \(l = -\frac{u_\phi}{u_t}\) is angular momentum density, \(\tilde{w} \equiv \mathcal{L}w\), \(\tilde{p}_m \equiv \mathcal{L}p_m\), \(\mathcal{L}(r, \theta) \equiv g_{t\phi}^2 - g_{tt}g_{\phi\phi}\) and \(p_m = \frac{b^2}{2}\) is the magnetic pressure. It can be shown that for components of \(b^\mu\) following expressions are valid [Komissarov, 2006]:

\[
b^\phi = \pm \sqrt{\frac{2p_m}{A}}
\]

and

\[
b^t = lb^\phi,
\]

where \(A \equiv g_{\phi\phi} + 2lg_{t\phi} + l^2 g_{tt}\).

Axi-symmetric accretion of magnetized tori

For constructing the torus we need to solve (2). In this way we obtain a stationary accretion torus. The only integral in equation (2) which can be evaluated analytically is \(\int_0^P \frac{dp}{w}\). For a case of constant angular momentum density inside the torus the equation (2) can be solved analytically if we impose following relation:

\[
\int_0^P \frac{dp}{w} = \text{const} \cdot \int_0^{\tilde{p}_m} \frac{d\tilde{p}_m}{\tilde{w}},
\]

If we assume angular momentum not to be constant and relation (4) to be fulfilled we finally get solution of (2) (for equation of state \(p = \kappa \cdot \rho^\gamma\)) in this form

\[
p = \left(\left(\frac{\ln(u_{t_{in}})}{\ln(u_t)}\right) \cdot e^{\int_0^l \frac{dl}{u_t}} \frac{\text{const}}{\text{const} + \text{const}} - 1\right)^4 \cdot 25^4 \cdot 10^{-8} \cdot \kappa^{-3},
\]

where we assumed \(\gamma = \frac{4}{3}\). In our case we focused on tori with a relativistic cusp (which is an analogy to Roche lobe). It means that the angular momentum on the inner edge is equal to Keplerian angular momentum. The profile of such torus is shown in Figure 1.

We assume that the initial stationary state is perturbed by increasing the black hole mass and so the accretion occurs. Since we know that the tori with radially increasing angular momentum density are more stable against these sorts of perturbation [Abramowicz et al., 1998], we restrict ourselves to study influence of magnetic field on accretion rate. The mass of black hole was increased by two percent. After some time \(\delta t\) mass \(\delta M\) and angular momentum \(\delta L\) fall through a horizon. We update parameters of black hole: \(M \rightarrow M + \delta M\), \(L \rightarrow L + \delta L\).

For a torus with radially increasing angular momentum \((l \sim r)\) in Kerr spacetime with \(a = 0.1\), the dependance of mass of torus on time for different values of ratio between thermodynamical
and magnetic pressure $\beta$ ($\beta = \frac{P_T}{P_{Mag}}$) is shown in Figure 2. From the graph we see, that the accreted mass is significantly different only for $\beta = 3$. For $\beta = 25, 80, 190$ values of accreted mass are more similar. From the graph one can also see that the accretion is not monotonic but there are changing phases of bigger accretion with phases where the mass of torus remains almost constant. The same situation is also for cases with $a = 0.3$ and $a = 0.5$. We also did a simulation for different angular momentum dependance on radius for comparison. On the left picture in Figure 3 there is shown accretion process for $l \sim \sqrt{r}$. One can see that these tori are not as stable as those with $l \sim r$. We also compared the case with constant background in time and the case when we update metric coefficients according to values of mass and angular momentum which fell down the horizon. This comparison is shown on the right image in Figure 3. It is done for non-magnetized case, $a = 0.5, l \sim r$.

Discussion

We have presented our latest results of studying accretion properties of magnetized tori with radially increasing angular momentum density. We have done our simulations for the value
Figure 3. On the left image there is dependance of mass of torus on time for the case when $l \sim \sqrt{r}$. On the right image there is comparison of the case with constant metric (blue line) with the case when the metric is updated (red line).

Figure 4. Accretion of mass for $\beta = 25$, $a = 0.1$. 
of black hole spin $a = 0.1$, $a = 0.3$ and $a = 0.5$. At first we assumed constant Kerr background and then we did the same simulations when the metric was updated. It means that the metric coefficients of Kerr spacetime were changing according to value of mass and angular momentum which was accreted (which fell down across the horizon). We found that the accretion is not monotonic and that there are changing phases of bigger accretion with phases where the mass of torus remains almost constant. This behaviour is typical for all cases which we assumed (different values of spin of black hole $a$ and values of $\beta$). Since the time step of updating of metric coefficients is much more shorter than the typical time of this “stair” profile it is obvious that it can not be source of this behaviour.

**Conclusion**

We confirmed that the tori with radially increasing angular momentum density are more stable against perturbations compared to those tori with constant angular momentum density. From our simulations it is obvious that the influence of magnetic field on accretion rate is significant if the magnetic pressure is comparable with thermodynamical pressure. It was also shown that the bigger magnetic pressure is (bigger $\beta$) the bigger value of mass is accreted. However the differences are really small for $\beta = 25, 80, 190$. We also compared the case when the spacetime background is constant with the case when the metric coefficients are updated in each time step according to values of mass and angular momentum which fell down through the horizon. In this place we have to emphasize that initially we suppose the torus to be “test” torus which does not influence the metric but when the mass is accreted it influences the metric. Since the mass of torus is approximately 6 percent of black hole mass, there are not big differences between these two cases. But one can see that in the case when the metric is updated the bigger value of mass is accreted. The difference can be seen better at the latter phases of accretion which does not surprise us because the effect of the change of metric coefficients is bigger.

**Acknowledgments.** I thank the GAUK 139810 for financial support.

**References**