Current Carriers and Electric Field in Tilted Current Sheets

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Abstract. Based on statistics of 23 tilted current sheets observed in the Earth magnetotail by Cluster, we investigate current carriers and electrostatic field, which appears in current sheet due to electron-ion decoupling. We have found that in tilted current sheets current flows near the neutral plane almost along the field lines. Moreover, in tilted current sheets the largest part of the current density is provided by electrons. We have found that electric field normal to tilted current sheets ($\approx E_y$ GSM) can be separated into dawn-dusk electric field related to the earthward (tailward) plasma convection and Hall electrostatic field. Electrostatic potentials have parabolic profile across current sheet (U-shape profile) with potential drops between current sheet’s boundary and neutral plane in the range from several hundreds eV to several keV. We have developed a theoretical model, which describes electrostatic potential profiles across current sheet.

Introduction

The natural state of the Earth magnetotail current sheet (CS) is characterized by the dominance of the dawn-dusk component of the current density. This current results in the magnetic field reversal, i.e. the magnetic field has opposite directions in the southern and northern lobes [Ness, 1965]. However there are regular observations of rather strange CSs with the dominant north-south component (or $Z$ component) of the current density [Sergeev et al., 2004; Runov et al., 2006; Petrukovich et al., 2008] hereinafter we use GSM coordinate system.

From the theoretical point of view, there always exists the normal to the CS electrostatic field component [Zelenyi et al., 2004; Yoon and Lui, 2004], which appears due to electron-ion decoupling. However, since Cluster measures the electric field only in the spin plane $XY$ one cannot investigate this electric field in the usually observed non-tilted CSs with the dominant $Y$ component of the current density (when CS’s normal vector points along the $Z$ axis). On contrary, this field can be observed reliably in tilted CSs, since the normal vector is along the $Y$ axis.

Cluster’s four-point measurements make it possible to probe the local structure of the CS. In this paper we concentrate on tilted CSs. An assumed geometry of the tilted CS is shown in Fig. 1. The right panel includes projections of two magnetic field lines on the $YZ$ plane (lines 1 and 2). Magnetic field lines are supposed to lie on almost vertical planes $Y =$ const. However, contrary to an ordinary situation the neutral plane is tilted, i.e. the field lines lying in neighboring planes $Y =$ const, are shifted relative to each other in the $Z$ direction [Petrukovich et al., 2006; Rong et al., 2010]. This effect is illustrated in the bottom panel ($XZ$ plane), where the same two magnetic field lines 1 and 2 are shown. Fig. 1 shows that the normal vector to the tilted CS is almost along the $Y$ direction.

Data and methods

Our investigation is based on 23 crossings of the CS by Cluster spacecraft in 2001, 2002 and 2004 years. Following the general selection criteria [Runov et al., 2006] we have picked out events, where magnetic field varies similarly at four spacecraft. We have used the following data from Cluster Active Archive (http://caia.estec.esa.int/caia/): FGM magnetic field [Balogh et al., 2001], CIS/CODIF proton moments [Reme et al., 2001], PEACE electron moments [Owen et al., 2001], EFW electric field [Gustafsson et al., 2001].

The unified treatment of the CS structure can be done in the local coordinate system (see Fig. 1). We use the four-point magnetic field measurements to calculate the current density via curlometer technique $\vec{j}_{curl} = (e/4\pi) \nabla \times \vec{B}$ [Dunlop et al., 2002]. Then following the method by Russell et al.,[1983], Runov et al., [2006], we determine the local coordinate system ($\vec{l}, \vec{m}, \vec{n}$) for each selected crossing: $\vec{l}$ is directed along the maximum variance eigenvector (from MVA (maximum variance analysis) applied to the magnetic field in the barycenter $\vec{B}(bc)$) that is predominantly along the $X$ axis for all CSs from our dataset; $\vec{m}$ is determined by using current density $\vec{j}_{curl}$ averaged over the center of current sheet, where $|B_l| < 5$ nT, i.e. $\vec{m} = \left[ \vec{l} \times \vec{j}_{curl} \times \vec{l} \right] / |\vec{j}_{curl}|$ and $\vec{n} = \left[ \vec{l} \times \vec{m} \right]$. In the local coordinate system $\vec{B} = (B_l, B_m, B_n)$ and $\vec{j} = (j_l, j_m, j_n)$. All selected CSs have proven to be approximately one-dimensional structures, i.e. $|j_{l,n}/j_m| \ll 1$ and $\nabla_n B_{n,m} \approx 0$. We conclude therefore that in CSs selected, the gradient $\nabla_n B_l$ along the normal vector is supported by $j_m$. Alternatively the normal vector $\vec{n}$,
can be determined by the timing technique [Runov et al., 2006]. In selected CSs the both normals are close to each other: \((\vec{n}, \vec{n}_e) > 0.95\). The timing technique allows to determine the flapping velocity of CSs. We use flapping velocity \(V_n\) averaged over the central region of the CS \(|B_t| < 5 \text{ nT}\) to obtain CS spatial scale.

Since the best time resolution of the electron measurements is delivered by C2 and the most accurate proton distribution function is delivered by C1 and C4, we use proton bulk velocity \(\vec{V}_p\) from C4, electron density \(n_e\) and bulk velocity \(\vec{V}_e\) from C2. We obtain the current densities \(j_{mp}\) carried by protons and electrons as \(en_e(\vec{V}_p\vec{m})\) and \(−en_e(\vec{V}_e\vec{m})\). We use proton and electron current densities averaged over the central region of CSs \(|B_t| < 5 \text{ nT}\).

The thickness of CS \(L\) (Fig. 1) is determined by the variation of the magnetic field \(B_t\) during the crossing and current density determined by curlometer technique \(j_{curl} = \vec{j}_{curl}\vec{m}\). The variation of magnetic field during the crossing is characterized by the maximum value of \(|B_t|\) during the crossing, which we denote \(B_{0t}\). The separation between Cluster spacecraft in years 2001, 2002 and 2004 were 2000 km, 3500 km and 1000 km correspondingly.

In our dataset CS thicknesses are comparable with the separations between spacecraft during the corresponding year (there are no CSs much thinner than separation between spacecraft). Therefore curlometer technique give the reliable estimate of the current density. We note that \(L\) determines the variation of the magnetic field along the normal vector \(\vec{n}\), while the parameter \(L_z = L/\sin \beta\) (Fig. 1) determines the variation of the magnetic field along the field lines. The lobe value of magnetic field \(B_{ext}\) can be determined from the vertical pressure balance \(B_{ext}^2 = 8\pi(n_eT_p + n_eT_e) + B_m^2 + B_e^2\), where \(T_p, T_e\) are proton and electron temperatures in CS’s neutral plane. We use values of the electron perpendicular and parallel temperatures to calculate the components of the electron pressure tensor \(p_{\perp e}\) and \(p_{\| e}\). We follow the approach proposed by Zelenyi et al., [2010] and Artemyev et al., [2011] and approximate \(p_{\perp e}\) as the function of \(B_t\), namely \(p_{\perp e} = p_{\perp,0}(1 - \alpha_\perp B_t^2/B_{ext}^2)\), where \(p_{\perp,0}, \alpha_\perp\) are determined by the least squares method.

We use electric field data from the four Cluster spacecraft to determine the normal electric field \(E_n^{(\alpha)}\), \((\alpha = 1, 2, 3, 4\text{-spacecraft number})\) and calculate the normal electric field in the spacecraft barycenter as \(E_{n}^{(bc)} = \frac{1}{4} \sum_{\alpha=1}^{4} E_n^{(\alpha)}\). Due to strong inclination of the CSs one can use only measurements of \(E_y\), i.e. \(E_n \approx n_y E_y\).

Fig. 2a shows the typical profile of the electric field \(E_n\) normal to the CS together with magnetic field profile \(B_t\) (measurements of C4 in CS observed on 20 October 2001 during 9:55–10:02 UT are shown here). One can note that \(E_n\) does not equal to zero in the neutral plane, around 9:58 UT. We define the value of the normal component of the electric field in the neutral plane \(E_{0n}^{(\alpha)} = E_{n}^{(\alpha)}|_{B_t^{(\alpha)}=0}\) Then we calculate the profile of the electric potential across a CS as:

\[
\phi^{(\alpha)}(r_n) = \int_0^{r_n} (E_{n}^{(\alpha)} - E_{0n}^{(\alpha)}) dr_n = -\int_{t_0}^{t_n} (E_{n}^{(\alpha)} - E_{0n}^{(\alpha)}) V_n dt,
\]
where \( r_n = V_n(t - t_0) \) is the distance from the neutral plane across CS and \( t_0 \) is the moment of \( B_t \approx 0 \) crossing. Fig. 2b shows the potential profile \( \phi^{(4)}(t) \) obtained for the electric field from Fig. 2a. It has a characteristic U-shape profile, a parabolic profile as a function of \( B_t \), with a minimum in the neutral plane. In Fig. 2b we have introduced the potential drops \( \phi_{0+}^{(1)} \) and \( \phi_{0-}^{(1)} \) between the CS’s boundaries and the neutral plane. In the present work we use potential \( \phi^{(1)}(t) \) derived from spacecraft C1.

In the end of this Section we list the criteria of CSs’ selection. We have selected crossings, which satisfy the following requirements: (1) CS is strongly tilted, i.e. \( n_y > 0.85 \); (2) CS can be assumed one-dimensional; (3) CS is observed during quiet periods with small proton bulk velocity in the \( X \) direction \( |V_{px}| < 150 \) km/s in 21 of 23 CSs and \( |V_{px}| < 250 \) km/s in other two CSs; (4) parabolic U-shape profile of electrostatic field across CS is observed.

Electric field profiles and particle currents

Using the electric field measurements from C1 and C4 we have found the electric field in the neutral plane \( E_{0y}^{(1)} \) and \( E_{0y}^{(4)} \) (see Fig. 2a). Although sometimes these constants differ from each other, the statistical distributions of \( E_{0y}^{(1)} \approx n_y^{-1} E_{0y}^{(1)} \) and \( E_{0y}^{(4)} \approx n_y^{-1} E_{0y}^{(4)} \) quantitatively coincide (see Fig. 3a,b). As one can see in the majority of cases this electric field is positive with a mean value \( \sim 0.25 \) mV/m. This electric field provides the earthward (tailward) convection in the magnetotail [Angelopoulos et al., 1993]. The conclusion is supported by a substantial correlation (for C1 measurements correlation coefficient is 0.81) between the proton earthward velocity \( V_{px} \) and the electric drift velocity \(-eE_{0y}^{(1)} / B_t \) (see Fig. 3c).

Then we subtract \( E_{0y}^{(1)}(\alpha) \) from the measured electric field \( E_{0y}^{(1)}(\alpha) \). We assume that the resulting difference may be due to an ion-electron decoupling in the CS. The potential drops between the neutral sheet and CS’s boundaries vary from several hundred eV to several keV.

Fig. 4a shows that the tilted CSs are electron-driven, i.e. \( j_{me} > j_{mp} \). Proton contribution to the total current density is usually small. Moreover it is negative in more than a half of the CSs (Fig. 4a). Triangular symbols show CSs, where we observe strong deviation of \( j_{me} + j_{mp} \) from \( j_{curl} \) (\( j_{me} + j_{mp} < 0.3j_{curl} \)). We suppose that such strong deviation of the particle current from the curlometer current is due to errors in the electron bulk velocity \( V_e \). In these CSs electron current density can be estimated as \( j_{me} = j_{curl} - j_{mp} \). We have found that tilted CSs are characterized by rather large north-south component \( B_m \approx B_z \) of the magnetic field (with an average value \( |B_m| = 8 \) nT) and a smaller component \( B_n \approx B_y \) (\( |B_n| = 2 \) nT). Therefore near the neutral plane, while \( B_t \) is smaller than \( B_m \), field aligned electron-driven current dominates.

Model of electrostatic field

In this section we develop an analytical model to explain the observed electric field. Since the electrons are magnetized we describe them in the MHD approximation with an anisotropic pressure tensor. The CS is assumed to be locally one-dimensional and all quantities depend only on the normal coordinate \( r_n \). The force balance has
Figure 3. Distributions of electric field $E_{y0}^{(a)}$ (electric field in the neutral plane $E_0^{(a)}$ converted back from $(\vec{l}, \vec{m}, \vec{n})$ to GSM) determined from measurements on C1(a) and C4(b) and the correlation between observed proton convection velocity $V_{px}$ and drift velocity due to electric field $E_{y0}^{(1)}$ (c). Above the dashed line $V_{px}$ is larger than $-cE_{y0}^{(1)}/B_z$, while below the dashed line $V_{px}$ is smaller than $-cE_{y0}^{(1)}/B_z$.

Figure 4. (a) Electron current density $j_{me}$ versus proton current density $j_{mp}$. Triangle symbols represent CSs, where $j_{mp} + j_{me} < 0.3 j_{curl}$. Above the dashed line $j_{me} > j_{mp}$, while $j_{me} < j_{mp}$ below it. (b) Theoretical estimates of potential drop $\phi_{th}$ versus the mean potential drop $(\phi_{0+}^{(1)} + \phi_{0-}^{(1)})/2$ determined from measurements of C1. Above the dashed line $\phi_{th}$ is larger than the experimental potential drop, while below it $\phi_{th}$ is smaller than the experimental potential drop.

\begin{align*}
0 &= -en_e\vec{E} - \nabla \tilde{p}_e + \frac{1}{c^2} \vec{j}_e \times \vec{B}, \\
\text{where } \tilde{p}_e &= p_{\perp e} \hat{1} + (p_{|| e} - p_{\perp e}) \hat{b} b \text{ is the electron pressure tensor. Taking into account the electron earthward convection and omitting } E_0^{(a)} \text{ in (1) we can rewrite force balance (1) in the form:} \\
en_e \nabla \phi &= \nabla p_{\perp e} + \nabla || (p_{|| e} - p_{\perp e}) + \frac{p_{|| e} - p_{\perp e}}{R_c^2} \tilde{R}_e - \frac{p_{|| e} - p_{\perp e}}{B} \nabla \parallel B + \frac{1}{c} j_{me} B l \hat{n} - \frac{1}{c} j_{me} B n \hat{l},
\end{align*}

where $B = |\vec{B}|$ and $\tilde{R}_e$ is the curvature radius of the magnetic field lines. In the following we make several simplifications. We note that:

\begin{align*}
|\nabla p_{\perp e}| &\sim p_{\perp e} / L, \\
|\nabla || (p_{|| e} - p_{\perp e})| &\sim (p_{|| e} - p_{\perp e}) \sin \beta / L_z, \\
(p_{|| e} - p_{\perp e}) R_c^{-2} |\tilde{R}_e| &\sim (p_{|| e} - p_{\perp e}) \sin \beta / R_e, \\
(p_{|| e} - p_{\perp e}) |\nabla \parallel \ln B| &\sim (p_{|| e} - p_{\perp e}) \sin \beta / L_z,
\end{align*}
while \( R_c \approx L_c B_m / B_0 \approx LB_m / (B_0 \sin \beta) \). The smallness of projections on \( \vec{n} \) of the second, third and fourth terms in equation (2) in comparison with the first one is characterized by parameters \((p_l / p_{\perp e}) - 1) \sin^2 \beta \) and \((p_{\parallel e} / p_{\perp e} - 1) B_0 / B_m \sin^2 \beta \). The observed values of these ratios are rather small due to a large CSs’ inclination and a relatively small anisotropy \( p_{\parallel e} / p_{\perp e} \sim 1.15 \). Taking into account these approximations, after the projection of (2) onto the normal \( \vec{n} \) it can be written as

\[
e n_e \frac{\partial \phi}{\partial n} = \frac{\partial p_{\perp e}}{\partial r_n} + \frac{1}{e} j_{me} B_l \tag{3}
\]

The proton contribution into the total current is characterized by the parameter \( \chi = j_{mp} / j_{me} \). In CSs with \( j_{mp} + j_{me} \) strongly deviating from \( j_{curt} \) the parameter \( \chi \) can be estimated as \( \chi = j_{mp} / (j_{curt} - j_{mp}) \). Then using approximation for pressure \( p_{\perp e} \) we can rewrite (3) in the form:

\[
e \frac{\partial \phi}{\partial r_n} = -\alpha_{\perp} \frac{p_{\perp 0}}{n_e} \frac{\partial}{\partial r_n} B_0^2 + \frac{1}{1 + \chi} \frac{1}{n_e} \frac{\partial}{\partial r_n} B_0^2 = 1 \frac{B_0^2}{B_{ext}^2} \tag{4}
\]

We neglect the variation of the plasma density and parameter \( \chi \) across the central region of the sheet \( \nabla_n n_e \approx 0, \nabla n \chi \approx 0 \). We take into account an approximate vertical pressure balance \( B_{ext}^2 \approx 8 \pi n_e (T_p + T_{\perp e}) \) and definition \( p_{\perp e} = n_e T_{\perp e} \), to rewrite relation (4) as:

\[
e \frac{\phi}{T_p} = \lambda \frac{B_0^2}{B_{ext}^2}, \tag{5}
\]

where

\[
\lambda = \frac{1}{1 + \chi} \left( 1 + \frac{T_{\perp e}}{T_p} \right) - \alpha_{\perp} \frac{T_{\perp e}}{T_p} \tag{6}
\]

Relation (5) shows that the electrostatic potential is changing as \( B_0^2 \) across CS, which is in agreement with the observed parabolic profile of the electric potentials (Fig. 2b). As one can see in equation (3), the electrostatic field \( E_n = -\partial \phi / \partial r_n \) is formed due to electron pressure gradient \( \partial p_{\perp e} / \partial r_n \) and \( \vec{j} \times \vec{B} \) force. The competition of these two terms is presented in equation (6) for \( \lambda \), where the first term on the right hand side corresponds to \( \vec{j} \times \vec{B} \) force and the second one corresponds to the pressure gradient. Firstly we note that in our dataset \( \alpha_{\perp} \) (we recall that \( p_{\perp} = p_{\perp 0}(1 - \alpha_{\perp} B_0^2 / B_{ext}^2) \)) is basically positive (in 22 of 23 CSs), equals on average to \( \sim 1 \) (with standard deviation \( \sim 0.8 \)) and electron temperature is always less than proton one. If substantial part of the current is carried by protons, then \( \chi = j_{mp} / j_{me} \gg 1 \) and hence \( \lambda < 0 \). On the other hand, for electron dominated CSs \( (\chi < 1) \) or for CSs with \( \chi \sim 1 \) the first term in the right hand side of (6) is larger than the second one resulting in \( \lambda > 0 \). One can therefore conclude that in proton-driven CSs the domination of the electron pressure gradient leads to electric potentials with a maximum in the neutral plane, while in electron dominated ones (and CSs with comparable contributions from electrons and protons) electric potential with a minimum in the neutral plane appears due to Lorenz force. One can say therefore that the observed U-shape electric potentials appear due to the Hall effect.

It is possible to compare mean experimental potential drops \( \frac{e \phi_{th}}{T_p} = \lambda T_p (B_0 / B_{ext})^2 \), Fig. 4b shows the experimental potential drops versus corresponding theoretical values. A regression analysis \((\phi_{th}^{(a)} + \phi_{th}^{(b)}) / 2 = a \phi_{th} + b, \) where \( a \) and \( b \) are fit parameters) shows that for potentials observed on C1 (\( \alpha = 1, \) Fig. 4b) correlation coefficient is 0.83, while the slope \( a \) equals to 0.56.

**Conclusions**

In the present work we have investigated 23 tilted CSs observed in the Earth magnetotail. The CS’s inclination allows us to investigate electrostatic effects in these CSs, since Cluster can measure \( E_y \) component of the electric field on reliable basis. Our conclusions are summarized below.

(a) Electrons carry the largest part of current in tilted CSs, while the proton contribution is small and often negative.

(b) Near the neutral plane current flows almost parallel to the magnetic field. Electron current cannot be therefore explained by a drift motion.

(c) We observe parabolic U-shape electrostatic potentials across CS. We have developed a theoretical model, which is quantitatively consistent with the Cluster observations.
Acknowledgments. Authors would like to acknowledge Cluster Active Archive and Cluster instrument teams, in particular FGM, CIS, PEACE, EFW for data processing. The work was supported by the RF Presidential Program for the State Support of Leading Scientific Schools (project NSh-3200.2010.2.), by the Russian Foundation for Basic Research (projects 10-05-91001, 11-02-01166).

References


Gustafsson G., André M., Carozzi T. et. al., First results of electric field and density observations by Cluster EFW based on initial months of operation, Ann. Geophys., 19, 1219–1240, 2001.


