Modified Method of Asteroid Pairs Convergence

J. Žižka and D. Vokrouhlický
Institute of Astronomy, Charles University, V Holešovičkách 2, 18000 Prague 8, Czech Republic.

Abstract. Analysis of astronomical catalogs has recently revealed existence of a small population of asteroid couples (pairs) which reside on basically the same heliocentric orbit. While a different value of the mean anomaly implies they are two different objects, a common thinking is that they separated very recently from a parent asteroid sharing the same orbit. Indeed, backward integration of the orbits of the asteroids in pairs reveals very close encounters within the past tens to hundreds of kys. In this paper we first improve efficiency of these past encounters detection by computing relative state vectors of asteroid clones of the primary/secondary in the pair with respect to an arbitrary position on orbits of the clones of the secondary/primary in the same pair. In terms of a number of convergent configurations we typically gain several orders of magnitude as compared to the plain comparison of the clone positions and velocities used so far. Next, we apply our new method to the analysis of convergent configurations of asteroid pairs with similar-size components. [Pravec et al., 2010] argue such pairs should not exist, provided their suggested mechanism of rotational fission is correct. We found 7 asteroid pairs which indicate fairly good convergence within the past 500 ky. We propose these bodies should be prime candidates for accurate photometric observations with the goal to determine their absolute magnitudes.

Introduction

Asteroid pairs are couples of asteroids which share basically the same heliocentric orbit. While today they are at different locations of their orbit, the tiny difference in their semimajor axis value implies their mean motion is slightly different and thus the two bodies could approach very closely in the past. Indeed, backward orbital tracking of components in the pairs reveals such very close encounters within the past tens to hundreds of kys for most cases. Examining mutual configuration of the two components at their separation, [Vokrouhlický and Nesvorný, 2008, 2009] proposed several possibilities of the processes that lead to formation of the pairs: (i) catastrophic collision, (ii) rotational fission, and (iii) binary system instability. [Pravec et al., 2010] conducted an observational campaign determining rotation period $P_1$ of the primary component in numerous pairs. They found $P_1$ is correlated with the estimated mass ratio of the two components in the pair and concluded this finding strongly favors the rotational fission hypothesis. One of the implications of the formation model promoted by [Pravec et al., 2010] is that pairs should not have similar-size components. More quantitatively, assuming the same albedo value, difference in the absolute magnitude $H$ values of the components in the pairs should always be larger than one magnitude. Yet, there are several candidate pairs which violate this rule. In this paper we re-examine these particularly interesting cases. First, using the most up-to-date asteroid catalog we anew identify candidates of pairs with closely similar-size components violating the standard model of [Pravec et al., 2010]. Second, we numerically propagate their orbits to the past to confirm their convergence, henceforth strengthening their case as a real pair of asteroids of a common origin. In order to perform this second task as efficiently as possible, we also improve the convergence technique. We finally end-up with a list of seven confirmed asteroid pairs with similar-size components. We propose these asteroids should be carefully observed with the goal to determine their absolute magnitude as accurately as possible.

1[Vokrouhlický and Nesvorný, 2008] proposed the Yarkovsky-O’Keefe-Radzievskii-Paddack (YORP) effect (see, e.g., [Bothe et al., 2006]) is the primary physical mechanism that is capable to efficiently bring small asteroids to the rotational fission limit.
Selection of candidate pairs

Discovery of an asteroid pair proceeds in two steps: (i) first, its preliminary identification is based on proximity of the two orbits in the five dimensional space of orbital elements, and (ii) second, its confirmation is based on detailed orbital tracking of the two asteroid orbits backward in time. Note that (ii) is needed to justify a candidate couple to be a real pair, because random fluctuations in the asteroid distribution in the orbital space may also result in two very similar orbits. In this section we only briefly comment on the step (i), while a novel technique for (ii) is discussed in the next Section.

In order to quantitatively define proximity of two orbits in the space of orbital elements [Vokrouhlický and Nesvorný, 2008] introduced a metric $D_M$
\[
\left( \frac{D_M}{na} \right)^2 = \kappa_a \left( \frac{\delta a}{a} \right)^2 + k_e (\delta e)^2 + k_i (\delta \sin i)^2 + k_\Omega (\delta \Omega)^2 + k_\varpi (\delta \varpi)^2 ,
\]
where $(\delta a, \delta e, \delta \sin i, \delta \varpi, \delta \Omega)$ is the difference vector of Keplerian orbital elements (semimajor axis $a$, eccentricity $e$, inclination $i$, longitude of node $\Omega$ and pericenter $\varpi$), $n$ is the mean motion and $\kappa$ are numerical coefficients. Our choice of the first three elements, $k_a = 5/4$, $k_e = k_i = 2$ is based on the classical work of [Zappalà et al., 1990]. Following [Vokrouhlický and Nesvorný, 2008], we choose $k_\Omega = k_\varpi = 10^{-4}$ for the relative weight factors of the secular angles in the metric $D_M$. However, unlike [Vokrouhlický and Nesvorný, 2008], we use mean orbital elements rather than osculating elements of a given epoch. This is based on discussion of [Rožek et al., 2011], who found that short-period perturbations in the osculating elements produce large oscillations in $D_M$ and thus could obscure selection of the real pairs. The choice of mean orbital elements provides a more stable set of orbital parameters and it can serve to detect asteroid pairs with age up to My.

We used metrics (1) to search for asteroid pairs in the most recent catalog of mean orbital elements of asteroids as of April 2011. Given the motivation outlined in Section 1, our pool of candidates had to satisfy: (i) $D_M < 10$ m/s (which sets the quantitative threshold of orbital proximity of our pairs; see [Rožek et al., 2011]), and (ii) difference $\Delta H$ in absolute magnitudes of the secondary $H_2$ and primary $H_1$ components in the pair be $\Delta H < 1$ magnitude. We obtained about 15 candidate cases, out of which only some passed a more severe criterion of true orbital convergence discussed in the next Section. Basic parameters of those which successfully passed both criteria are given in Table 1. The absolute magnitudes $H_1$ and $H_2$ were taken from the AstDyS\(^2\) catalog. These values may have an uncertainty up to $\pm 0.5$ magnitude (e.g., [Galád, 2010]).

<table>
<thead>
<tr>
<th>Asteroid pair</th>
<th>$H_1$ [mag]</th>
<th>$H_2$ [mag]</th>
<th>$\Delta H$ [mag]</th>
<th>$D_M$ [m/s]</th>
<th>$D_P$ [m/s]</th>
<th>$T_{conv}$ [ky]</th>
</tr>
</thead>
<tbody>
<tr>
<td>180906</td>
<td>217266</td>
<td>17.4</td>
<td>17.4</td>
<td>0.0</td>
<td>0.21</td>
<td>0.30</td>
</tr>
<tr>
<td>195479</td>
<td>2008 WK70</td>
<td>16.3</td>
<td>17.1</td>
<td>0.8</td>
<td>0.23</td>
<td>0.00</td>
</tr>
<tr>
<td>165389</td>
<td>2001 VN61</td>
<td>16.3</td>
<td>16.8</td>
<td>0.5</td>
<td>0.48</td>
<td>0.09</td>
</tr>
<tr>
<td>60677</td>
<td>142131</td>
<td>15.7</td>
<td>16.0</td>
<td>0.3</td>
<td>0.66</td>
<td>0.09</td>
</tr>
<tr>
<td>2005 OS5</td>
<td>268305</td>
<td>16.9</td>
<td>16.7</td>
<td>0.1</td>
<td>0.90</td>
<td>0.54</td>
</tr>
<tr>
<td>10484</td>
<td>44645</td>
<td>13.7</td>
<td>14.6</td>
<td>0.9</td>
<td>2.51</td>
<td>0.28</td>
</tr>
<tr>
<td>80218</td>
<td>213471</td>
<td>16.5</td>
<td>16.6</td>
<td>0.1</td>
<td>4.14</td>
<td>1.41</td>
</tr>
</tbody>
</table>

Backward numerical integrations and convergence conditions

In order to verify that a selected pair of asteroids is real, as opposed to a random fluke in the background population, we perform backward numerical integration of their orbit seeking conditions of their very close approach in Cartesian space. Details of this technique have been given in [Vokrouhlický and Nesvorný, 2008] or Supplementary information of [Pravec et al., 2010]. In what follows we thus only

\(^2\)http://hamilton.dm.unipi.it/astdys2
briefly recall basic steps and dwell on one improvement of identification of the convergent configurations developed in this paper.

Finite accuracy of astrometric observations implies the current orbits of asteroids in pairs, as given in catalogs, have some degree of uncertainty. This is typically expressed with a covariance matrix $\Sigma$ of the orbital element $E$ solution, determining probability distribution $p(E) \sim \exp\left[-\frac{1}{2}\Delta E \cdot \Sigma \cdot \Delta E\right]$ with $\Delta E = E - E^*$ and $E^*$ the best-fit orbital values. Any solution with high-enough $p(E)$ value are statistically equivalent and may represent true orbit of the body. With this perspective, we need to consider multiple realizations of the past orbital histories of a given asteroid, all starting from an initial data region with high $p(E)$ (typically a six dimensional ellipsoid). While these variants of the orbital evolution – that we call “geometrical clones” – are usually very close each other at current epoch, they rapidly diverge in the past. [Vokrouhlický and Nesvorný, 2008] also recognized, that the standard dynamical model which uses only planetary gravitational perturbations for the orbital evolution is not accurate enough for the purpose of convergence study of asteroids in pairs. Namely, the role of thermal accelerations, known as the Yarkovsky effect (e.g., [Bottke et al., 2006]), is important. This is because the Yarkovsky effect may secularly change the value of semimajor axis over the timescale of the pair age by a value larger than the initial uncertainty of this element for the two orbits. Since the strength and sign of the Yarkovsky effect depends on apriori unknown parameters, such as surface thermal conductivity or rotation pole position, we must consider all possible values of the Yarkovsky effect. In practice, we assign to each geometrical clone a spectrum of expected Yarkovsky-effect values (modeled as an along-track acceleration producing semimajor axis change with a rate in an estimated interval of values $(\langle da/dt\rangle_{\text{min}}, \langle da/dt\rangle_{\text{max}})$; e.g., [Vokrouhlický, 1999]). We speak about the “Yarkovsky clones” to denote these variants of geometrical clones with different values of the Yarkovsky effect. In all examples given below we used 30 geometric clones and 40 Yarkovsky clones for each asteroid, thus altogether 1200 different variants of the possible past orbital evolutionary history for each body. Our integrations were performed using SWIFT software package [Levison and Duncan, 1994], where we included the effects of thermal accelerations. We used a fixed timestep of 5 d and performed integrations to 500 ky in the past.

A standard procedure for identification of convergent configurations of the numerically propagated clones of the two asteroids in the pair is as follows. At every timestep we compare heliocentric state vectors of all clones of the primary with all clones of the secondary component in the pair. In particular, let $(r_1, v_1)$ and $(r_2, v_2)$ are those state vectors, we compute (i) Cartesian distance $D_{\text{rel}} = |r_1 - r_2|$, and (ii) relative velocity $V_{\text{rel}} = |v_1 - v_2|$. We consider the configuration convergent, when both $D_{\text{rel}}$ and $V_{\text{rel}}$ are smaller than some quantitative threshold. In the case of distance, we require $D_{\text{rel}} \leq R_{\text{Hill}}$, where

$$R_{\text{Hill}} = r_1 \left(\frac{M_{\text{bin}}}{M_{\text{Sun}}}\right)^{1/3}$$

$M_{\text{bin}}$ is the total estimated mass of the asteroids in the pair, $M_{\text{Sun}}$ is the solar mass and $r_1$ is the heliocentric distance of the primary component in the pair. Recall that $R_{\text{Hill}}$ is a measure of distance between the two asteroids when their mutual gravitational interaction becomes more important than the gravitational attraction by the Sun. In the case of velocity, we require $V_{\text{rel}} \leq V_{\text{esc}}$, where $V_{\text{esc}}$ is the estimated escape velocity from a spherical body of mass $M_{\text{bin}}$. For reference, we note that $R_{\text{Hill}}$ is typically of the order of several hundreds to thousands of km, while $V_{\text{esc}}$ is typically few m/s.

An improved method for clone convergence

A major drawback of the convergence criterion outlined above is that the uncertainty ellipsoids occupied by the geometrical and Yarkovsky clones rapidly expand as the time increases to the past. So while their sampling is satisfactory at the current epoch, they become quickly under-sampled by the clones some tens to hundreds of thousands years ago. To solve the problem, we would need to use orders of magnitude more clones, which is not possible due to the CPU limitations. In this paper, we propose a compromise solution. Put in simple words, our strategy is as follows. When the clones of primary and secondary are close enough, we evaluate distance of a first chosen clone to the orbit of the second clone and vice-versa (see Fig. 1). This approach assumes the closest point on the orbit to a given clone of the first asteroid would have been occupied with a hypothetical clone of the second asteroid, were we able to consider a huge number of them. In the same time, this approach is computationally more efficient than determination of the orbital MOID (The Minimum Orbit Intersection Distance) for the two clone orbits (e.g., [Gronchi, 2000]).

Put in more quantitative terms we proceed as follows. At a given timestep of our numerical propagation of clones for the primary and secondary components in the pair we compute their mutual distances (as in the classical approach outlined above). When the distance $D_{\text{rel}}$ of two particular clones is less than a specified threshold $D_{\text{thresh}}$, $\approx 0.003$ AU in our case, we switch to a mode which computes distance of
a clone to the orbit of its counterpart clone. For instance, let \((r_1, v_1)\) be the heliocentric state vector of the first clone (primary, say) and \((r_2, v_2)\) be the heliocentric state vector of the second clone (secondary, say), we

- first determine orbital elements of the second clone and from them we compute the orbit-attached orthonormal basis \((e_p, e_q, e_n)\) such that \(e_p\) is directed to the osculating pericenter, \(e_n\) is directed along the osculating angular momentum vector and \(e_q = e_n \times e_p\);
- we determine the transformation matrix \(T\) from the heliocentric coordinate system used in our orbital propagator to the \((e_p, e_q, e_n)\) frame, and transform the state vector of the first clone using \(r_1 \rightarrow R_1 = T \cdot r_1\) and \(v_1 \rightarrow V_1 = T \cdot v_1\);
- the elliptical orbit of the second clone obviously reads \(R_2(E) = (X_2(E), Y_2(E), 0)^T\) in the \((e_p, e_q, e_n)\) frame, such that \(X_2(E) = a_2 (\cos E + e_2)\) and \(Y_2(E) = a_2 \sqrt{1 - e_2^2} \sin E\), where \(a_2\) and \(e_2\) are semimajor axis and eccentricity, and \(E \in (0, 2\pi)\) is the eccentric anomaly;
- we seek the point \(R_2(E^*)\) which minimizes \(D^2 = (R_1 - R_2) \cdot (R_1 - R_2)\) by iteratively solving \(E^*\) from \((R_1 = (X_1, Y_1, Z_1)^T)\)

\[
Y_1 \sqrt{1 - e_2^2} \cos E^* + a_2 e_2^2 \sin E^* \cos E^* - (X_1 + a_2 e_2) \sin E^* = 0
\]

(2)

(obviously, we make sure to determine global minimum of the distance function \(D(E)\));

- the relative velocity of the first clone to the closest point on the orbit of the second clone is then given by \(V_* = V(E^*) = |V_1 - V(E^*)|\) with

\[
V(E^*) = \sqrt{\frac{GM}{a_2}} \begin{pmatrix}
- \sin E^* \\
\sqrt{1 - e_2^2} \cos E^* \\
1 - e_2 \cos E^*
\end{pmatrix}.
\]

(3)

We repeat the procedure twice, each time choosing one of the two clones as a reference point and the latter represented by its orbit. The resulting minimum distance and velocity values \((D_*, V_*)\) are used in the statistical considerations instead of the plain values \((D_{rel}, V_{rel})\). While improving previous results by detecting more encounter configurations, our new method is obviously only approximate. Most importantly, it is not optimized to analyze clones convergence for pairs older than couple of hundreds of kys. This is because, when clones for either primary or secondary components spread over the whole heliocentric orbit, our criterion of determining \((D_*, V_*)\) only when \(D_{rel} \leq D_{thresh}\) is too restrictive. We plan to improve this issue in the forthcoming work.

Figure 1. Geometrical insight into the new convergence method. a) The ellipses (i) and (j) are the osculating trajectories of the i-th clone of the primary and j-th clone of the secondary components in the pair. Points P and Q indicate their position at a given time as provided by direct numerical integration. Taking the clone Q as a reference, we seek point M on the osculating orbit of the clone P which has the minimum distance to Q. b) Q transformed into the reference frame \((e_p, e_q, e_n)\) and clone P osculating orbit.
Figure 2. Results of convergence efforts for clones in the candidate pairs: 180906-217266 (top and left), 165389-2001 VN61 (top and right), 60677-142131 (bottom and left) and 80218-213471 (bottom and right). The abscissa is time to the past in ky. Upper panel shows distribution of number of convergent solutions binned in 1.5 ky to 10 ky intervals of time, henceforth providing statistical information about the age of the pair (the distribution has been normalized to unity at the most occupied bin). The bottom panel shows the same information but in cumulative form. The second and the third panels show ($D_*, V_*$) values for converging clones; the black line is an average computed over a 10 y running window.

Asteroid pairs with $\Delta H < 1$: a brief analysis

We applied the above mentioned method to backward tracking of components in the selected candidates of similar-size asteroid pairs ($\Delta H < 1$ mag). Out of these, only 7 revealed solid convergence of clones and age less than 500 ky. Figures 2 and 3 show fundamental properties of these successful solutions. The first-shown pair, 180906-217266, is somewhat exceptional because of its young and well-defined age. The only caveat is that these bodies are rather small, we estimate their size to $\sim 1.5$ km only, such that their observation will require a middle-class telescope (1.5 m mirror size and more).
Conclusions

We found 7 asteroid pairs which indicate a solid convergence within the past 500 ky and which have formally similar-size components (as derived from their absolute magnitude values). This is apparently in contradiction with the currently standard model of their separation as individual bodies (cf. [Pravec et al., 2010]). There are two possibilities of solution: (i) either the absolute magnitudes of the two components in these pairs were not determined accurately enough, or (ii) the formation scenario of the pairs needs modifications. Obviously the second possibility is more interesting, but we need first to rule out the first possibility. For that reason we propose the pairs identified in this paper need to be carefully observed with the goal to determine their absolute magnitude values with an uncertainty of \( \pm 0.05 \) mag.

References


