Simulations of Runaway Electrons

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Abstract. In this paper we discuss current knowledge of phenomena of runaway electrons in tokamaks. We summarize experimental facts and investigate theoretical understanding of runaway electrons. We also present result of our recent work concerning influence of JET’s Error Filed Correction Coils on runaway electrons dynamics. At the end of our paper we describe our plan to study runaway electrons dynamic in time and space varying magnetic field during plasma disruption as obtained from global MHD code JOREK.

1. Introduction—Runaway Electrons

It is known from experience that relativistic runaway electrons (REs) can be produced during plasma disruptions as well as during an operation at low plasma densities or sawtooth instabilities when a significant parallel electric field is induced \[\text{e.g. Knoepfel, 1979}\]. Their energy is 10–50 MeV in the present large tokamaks and is supposed to be up to 500 MeV in ITER in the worst case scenario \[\text{Jaspers, 1996}\]. Conversion rate of \[\text{IRE/IPLASMA}\] is usually 50–70 %, under some particular conditions up to 80 % \[\text{Martin-Solis, 2006}\]. There is a large uncertainty about their spatial and energy distribution which lead also to an uncertainty in predicting of heat loads on the first wall of tokamak. In unfavourable cases they can lead to melting and ablation of plasma facing components (PFC) of the first wall or divertor. Estimates predict that heat load can reach locally \[\approx 50 \text{ MJ/m}^2/0.3 \text{ s}\] what is much more than technically sustainable level during transient events (e.g. ELMs) \[\approx 8 \text{ MW/m}^2\]. It is therefore necessary to develop an effective mitigating mechanism for their suppression before ITER starts to work. Otherwise it is very probably that REs will substantially decrease lifetime of ITER plasma facing components and frequent, timely and costly repairs of PFC would be needed.

Our paper is organized as follows: in chapter two brief descriptions of possible theoretical approaches are described, chapter three summarizes a more detailed description of physical processes involved in REs physics and contains quite a long list of references to relevant papers. Chapter four briefly mention possible mitigation methods for REs generation. In chapter five we present out recent results and in chapter six there are some future plans outlined.

2. Runaway Electrons Dynamics Description

2.1 Kinetic description

It is generally known that kinetic description is the most accurate statistical description of many particles system. In strongly magnetized plasma even the knowledge of a reduced distribution function \[f(r,z,\varphi,p_\parallel,p_\perp,t)\] (since Larmor radius of electron is small) would allow us to determine all practically needed quantities and information, e.g. current density, energy distribution, heat load and heat load patterns. Of course, we must further couple (drift-, gyro-) kinetic equation to space and time dependent Maxwell equations which should give us reliable physical results. This makes the problem non-linear and considering that we need to solve time evolution in whole tokamak vessel and that our exact knowledge of thermal plasma is limited, we find out that full problem is non-tractable unless we use some degree of approximation.

2.2 Reduced models—Bounce averaged kinetic equation

To overcome difficulties connected with the solution of kinetic equation, some authors used simplified models \[\text{Eriksson, 2003, Harvey, 2000}\]. Dimensional reduction to one spatial coordinate, two coordinates in velocity space and time averaging of kinetic equation over a bounce period greatly simplifies the problem. Schematic form of a bounce averaged relativistic Fokker–Planck equation for electron distribution function \[f(r,p_\parallel,p_\perp,t)\] may be written as

\[
\frac{\partial f}{\partial t} = \left\langle L_E(f) \right\rangle + \left\langle C(f) \right\rangle + \left\langle L_{\text{synch}}(f) \right\rangle + \left\langle L_{sl}(f) \right\rangle + \left\langle S \right\rangle + \left\langle I_B \right\rangle ,
\]

(1)
where \( \langle \cdots \rangle \) means bounce averaging operator and terms on the right hand side of (1) represent terms causing acceleration by electric field, collisions, synchrotron radiation, radial diffusion caused by magnetic field, sources of REs and loss of electrons from bulk thermal plasma respectively. Solution of the REs physics by this approach represent today's most advanced and powerful tool of investigation of REs. In chapter 3 we will give some terms explicitly.

2.3 Other descriptions

Many other studies are based on further simplified kinetic equations and geometries used and give insight on particular aspect of REs physics, especially as concerns birth rate of REs. Test particle approach in velocity space is used in non-relativistic case [e.g. Fuchs, 1986] and relativistic case [e.g. Martin-Solis, 1998]. More detailed discussion and references will be given in the following chapter.

3. Physical Processes Involved in Runaway Electrons Physics

3.1 Generation mechanism

There are few different mechanisms leading to creation of REs.

(a) The first mechanism is gradual process of acceleration of electrons from the tail of distribution function. When an energy gain of electron on its mean free path caused by the electric field is larger than the loss suffered by collisions electron runs away, because collisional frequency starts to fall down with velocity. For determination of critical velocity, at which accelerating and drag force are in equilibrium, we look for steady state solution of kinetic equation when accelerating force is equal to collisional drag force:

\[
- \frac{eE}{m_e} \frac{\partial f}{\partial v} = C(f) ,
\]

where \( f \) is electron distribution function, \( m_e \) is mass of electron, \( e \) is electron charge, \( E \) is applied electric field and \( v \) is velocity of electron.

For more detailed information see basic text books [e.g. Wesson, 2004] or original paper dealing with non-relativistic case [Dreicer, 1960]. If relativistic effects are taken into account, one finds that the friction force on a fast electron does not fall all the way to zero at high energy, but remains finite at the speed of light. Consequently, no runaway generation can occur unless the electric field exceeds the critical field

\[
E_c = \frac{4 \pi e^3 \ln \Lambda}{m_e c^2} ,
\]

where \( c \) is the speed of light in vacuum and \( \ln \Lambda \) is Coulomb logarithm.

Birth rate of REs is expressed by the following equation [e.g. Feher, 2011]:

\[
\left( \frac{dn_r}{dt} \right) \approx \frac{n_e}{\tau} \left( \frac{m_e c^2}{2 T_e} \right)^{3/2} \left( \frac{E_D}{E} \right)^{3(\ln Z_{eff})/16} e^{-\frac{E_D}{4E} \sqrt{\frac{(\ln Z_{eff})/16}{E}}} ,
\]

where \( n_e \) is electron density, \( T_e \) temperature, \( Z_{eff} \) effective ion charge, \( E_D = m_e c^3 / (e \tau T_e) \) is the Dreicer field and \( \tau = 4 \pi e^2 m_e c^3 / (n_e e^4 \ln \Lambda) \) is the relativistic electron collision time.

(b) Second mechanism deal with the case when there is already present a non-negligible amount of REs population. In that case there is non-zero probability that REs will encounter very close collision with thermal plasma electron, during which the thermal electron obtain energy high enough to pass the critical level given by (2). Because of nature of this mechanism (\( dn_r/dt \sim n_r \)), leading to exponential growth of REs density, it is also called avalanche mechanism. This mechanism plays a key role in tokamaks where plasma current is bigger than \( \approx 1 \text{ MA} \). For the birth rate Rosenbluth and Putvinski derived from relativistic Fokker–Planck kinetic equation the following approximate/fitted analytical formula [Rosenbluth, 1997]:

\[
\left( \frac{dn_r}{dt} \right) \approx \frac{(E - 1)n_r}{\tau \ln \Lambda} \sqrt{\frac{\pi \varphi}{3(Z_{eff} + 5)}} \left( 1 - \frac{1}{E} + \frac{4\pi(Z_{eff} + 1)^2}{3\varphi(Z_{eff} + 5)(E^2 + 4/\varphi^2 - 1)} \right)^{-1/2}
\]

where \( \varphi = \left( 1 + 1.46e^{1/2} + 1.72e \right)^{-1} \) and \( e = r / R \) denotes the inverse aspect ratio.
(c) Currently there appeared also another possibly important source of REs, so called hot-tail process. For details see [Feher, 2011].

(d) In ITER, tritium decay and Compton scattering of γ-rays emitted by the activated wall are also possible primary RE sources.

3.2 Acceleration by electric field

The role of an accelerating electric field (in case we already know this quantity) is straightforward:

\[
\langle L_E(f) \rangle = -\frac{eE}{m_ec} \frac{\partial f}{\partial p_{II}}.
\]  

Toroidal electric field

Main contribution to the toroidal electric field \(E_{II}\) following from Maxwell equations is given by:

\[
E \approx -\frac{L}{2\pi R} \frac{dI}{dt},
\]  

where \(L\) is plasma inductance, \(R\) is major radius, \(I\) plasma current and \(t\) time.

At least one remark is needed at this place. Assumption that toroidal electric field is constant across poloidal cross section and is equal to that measured by external voltage loop is misguiding and led to false determination when first seed fraction of REs population is firstly created during disruption [James, 2010]. Therefore it is necessary to take this fact of inhomogeneity of electric field into account in future works aiming to correctly predict a real experimental REs current.

3.3 Loss mechanisms

We can recognize a few distinct phenomena leading to the loss of REs energy, subsequently leading to a decay of RE current and final recombination with positive ions.

Synchrotron radiation

Loss of kinetic energy as a consequence of accelerated motion of charged particle in electro-magnetic field is described by well-known Abraham-Lorentz equation:

\[
\langle \dot{p} \rangle = \left\langle \frac{\gamma E}{m_c^2 \dot{p}} \right\rangle = -\frac{e^2 \gamma^5}{6\pi\varepsilon_0 m_c^3} \frac{\langle |\dot{v}|^2 \rangle}{p},
\]  

where \(\gamma\) is usual relativistic factor, \(p\) is momentum of particle, \(\varepsilon_0\) is the vacuum permittivity.

Decomposing of REs motion into fast gyro-motion and slower guiding center motion around the torus we can derive appropriate expression for radiation losses for electrons in toroidal geometry [Anderson, 2001]. From relation (8) we can obtain also a limitation on maximal momentum which can RE gain, so called “synchrotron limit” [Anderson, 2001]:

\[
p_{II} < \left( \frac{R}{\rho_0} \right)^{1/2} \left( \frac{E - 1}{\sigma} \right)^{1/4},
\]  

where \(\rho_0 = m_c/eB, \sigma = \tau / \tau_c\) and \(\tau_c = 6\pi\varepsilon_0 (m_c)^3 / e^4B^2\) is the radiation time scale.

Collisions

Basically, REs with their high velocity \(\approx c\) are collisionless. Their mean free path is strongly increasing function of velocity [e.g. Wesson, 2004]. Generally one of the most exact forms of collisional operator (next to the Fokker-Planck collisional operator) represents Landau collisional operator. Relativistic expression was derived by Klimontovich [1967] and further reduced for the case \(m_e/m_i \ll kT/m_e c^2\), i.e. for the case of REs by Connor and Hastie [Connor, 1975]. Collisional operator is in this case represented by two parts giving collisional drag and pitch angle (\(\xi = p_{II} / p\)) scattering:

\[
\langle C(f) \rangle = \frac{1}{\tau} \left[ \frac{1}{p^2} \frac{\partial}{\partial \xi} \left( 1 + p^2 \right) f + \frac{1}{2} \frac{Z_{eff}}{p^3} \sqrt{1 + \frac{p^2}{\xi}} \frac{\partial}{\partial \xi} \left( 1 + \xi^2 \right) \frac{\partial f}{\partial \xi} \right],
\]  

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where $\tau = 4\pi e^2 m_e^2 c^3 / n_e e^4 \ln \Lambda$ is the collision time for relativistic electrons. Pitch angle scattering process is important in dynamics of REs because it increases the electron radiation losses. For the importance of this process see references [Jayakumar et al., 1993, Rosenbluth et al., 1997].

Effect of close collisions is described by the leading order term in the quantum-mechanical relativistic electron–electron (Moller) scattering formula [Rosenbluth, 1997 and references therein]:

$$S = \frac{n_e}{4\pi \tau \ln \Lambda} \delta(\xi - \xi_2) \frac{1}{p^2} \frac{\partial}{\partial p} \left( \frac{1}{1 - \sqrt{1 + p^2}} \right), \quad (11)$$

where $\xi_2 = p / \sqrt{1 + p^2 + 1}$ and $n_e$ is the density of runaway electrons.

### Radial diffusion of runaways caused by magnetic fluctuations

All aforementioned processes happen in a velocity space. In order to take into account space development of distribution function we have to deal with space inhomogeneities of electromagnetic field. In reference [Hauf, 2009] there is justified that the solely use of magnetic perturbations for REs transport is satisfactory even if it is well known that in the plasma there are present also fluctuations of electric field.

Knowledge of detailed structure of magnetic field in tokamak plasma (deviations from equilibrium value) is experimentally hardly obtainable quantity. Currently the most advanced models of REs [Eriksson, 2003] therefore use Rochester–Rosenbluth estimate for flux surface averaged radial shift of REs in the presence of magnetic fluctuations [Rochester, 1978]:

$$L_{av.}(f) = \frac{1}{r} \frac{\partial}{\partial r} \left( r D_{RR} \frac{\partial n_e}{\partial r} \right), \quad (12)$$

where diffusion coefficient is equal to $D_{RR} = \frac{\pi q R v (\delta B / B)^2}{v}$, $q$ is the safety factor, $v$ is the parallel velocity, $R$ is the major radius and $\delta B / B$ is the normalized magnetic perturbation amplitude. Typical values for $\delta B / B$ are $10^{-4}$–$10^{-3}$. There are probably another effects which could lead to another radial diffusion but they are not usually taken into account in present models, e.g. influence of magnetic ripple [Martin-Solis, 2000], resonances with magnetic modes (islands), an outward drift of the electron orbit when its energy increases (as consequence of (approximate) generalized toroidal momentum conservation) and others.

### Remark on magnetic field and disruptions

At the end of this subchapter about REs loss mechanisms we put a necessary note for the time development of magnetic field during disruptions. In first predisruptive phase (1–100 ms) plasma becomes unstable and we can usually see on diagnostics (e.g. magnetic) some precursors of a disruption. In a second quite rapid phase ($\approx 1$ ms) called “thermal quench” substantial part of thermal and magnetic energy of plasma is released. This phase is characteristic by abrupt decrease of temperature, very short period of increase of plasma current (as consequence of plasma profile redistribution) and probably destruction of nested topology of magnetic surfaces, where estimate (12) is only very rough and needs to be improved in future works aiming at a good level of reliability. Last stage called “current quench” is characterized by decrease of total plasma current on much longer time scale than thermal quench lasts typically 5–50 ms. Current quench or a period closely before is time when the first seed fraction of REs appears in tokamak [Wesson, 2004; Reux, 2010].

### 4. Runaway Electrons Mitigation Methods

In view of the apparent risk of large energy deposition at a localized area of the first wall in reactor tokamaks, it will be important to develop mitigation procedures and scenarios that avoid REs production. Mitigation methods are based on two different principles. Firstly they are based on enhanced deconfinement due to destruction of nested (closed) magnetic surfaces and secondly on enhanced collisional drag due to strong and very fast increase of plasma density or impurities (usually some noble gases; increase of density is 10–100 fold compared to the former plasma density) [IPB, 1999, PIPB, 2007]. Method using Resonant Magnetic Perturbations [Finken, 2007; Yoshino, 2000] belongs to first group, methods of massive gas injections [Hender, 2007] or (killer) pellets injection usually led to combination of both principles.

### 5. JET's Error Filed Correction Coils

In this chapter we present description and results of our recent studies. We have investigated dynamics, mainly radial motion of REs out of the center region of plasma, under (a) influence of JET’s Error Field
Correction Coils (EFCCs) (see Fig. 1) and (b) influence of toroidal magnetic ripple caused by finite number of toroidal coils. Our final goal was determination whether EFCCs are capable to mitigate REs creation, as explained in chapter 4, or not. We simulated REs with three different initial energies 5, 10, and 20 MeV for \(2 \cdot 10^5\) toroidal turns. Initial position of REs were placed in outboard midplane in a range from the magnetic axis up to 0.8 m outwards from the axis, i.e. up to the vicinity of LCFS as can be seen in Fig. 3 (left). We considered neither accelerating electric field nor any loss mechanism. We just followed one RE by means of drift relativistic Hamiltonian. We used simple vacuum approach, i.e. no plasma response was taken into account. In case a) we also had two different modes of EFCCs in configuration generating different toroidal mode numbers \(n=1\) and \(n=2\) of magnetic perturbations and different coil current in a range 20.8 kAt – 90.6 kAt. In both case we examined two equilibriums with quite different q profiles obtained by EFIT code. One was corresponding to a peaked current profile, another one to a hollow profile (see Fig. 2).

Our code produced large series of Poincare sections, which were later visually analyzed. Two examples of Poincare sections are depicted in Fig. 3. Usually EFCCs perturbations did not even destroyed regular KAM surfaces, sometimes there were formed smaller islands at experimentally accessible range 20.8 kAt – 41.6 kAt. Only for the speculative current 90.6 kAt (obtainable only after substantial hardware upgrade) there appeared a chaotic region at the edge of the plasma, which unfortunately does not have direct influence on REs created mainly at the core plasma region. Overall we made conclusion that neither EFCCs at realistic coil currents nor the toroidal ripple can cause, under assumptions of our simplified model, strong radial motion of REs out of plasma center, which would be desired for REs mitigation. Our more detailed results can be seen in [Cahyna, 2010].

6. Dynamics of Runaway Electrons in Time and Space Varying Magnetic Field

Motivation for our future work follows from the fact that there is currently no precise way how to determine dynamics of REs motion in a rapidly changing magnetic field during disruptions. This lack of knowledge restricts in general an ability to predict correctly REs currents in real experiments where avalanche mechanism plays a very important role. We therefore plan to use an output of global non-linear resistive MHD code JOREK [Huysmans, 2007] and make a similar analysis as we did it for the time independent magnetic field taken from EFIT (see chapter 4) and determine how different disruption magnetic fields (normal disruption, massive gas injection, resonant magnetic perturbations) influence the radial take away of REs from the plasma core.

Figure 1. Schematic visualization of the shape of Error Field Correction Coils installed in tokamak JET.

Figure 2. “q-profiles” of two different equilibriums. Blues line corresponding to a peaked current profile, red line to a hollow current profile.
7. Summary and Conclusion

In this paper we have summarized some experimental as well as theoretical facts about REs. In chapters 2–4 we have given basic (not at all comprehensive) overview of REs physics. In chapter two we have discussed possible theoretical concepts for description of REs physics, in chapter three processes of REs generation, acceleration and energy loss mechanisms are described and in chapter four possible mitigation methods are mentioned. In chapter five we have presented our recent work of REs simulation in the presence of EFCCs and the toroidal ripple of JET tokamak. In the last chapter we have outlined our future plans concerning the post-processing of outputs of global non-linear MHD code JOREK.

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