Trapping Charged Microparticles in the Linear Quadrupole Trap

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Abstract. Investigating charging processes of a single grain under controlled conditions in laboratory is an unique way to understanding the behavior of dust grains in complex plasma both in space and in the laboratory or in technological applications. Trapping a single grain can be realized by an electrodynamic trap. An electrodynamic trap can be utilized for both holding a single grain and continuously measuring its charge-to-mass ratio. We have proposed the novel design of the quadrupole trap having some noticeable advantages. [Beránek et al., 2007] In this paper, we present the results of the experimental verification of the functionality of this trap and the results of our measurement showing the possibility of a precise measurement of the charge-to-mass ratio of the grain in the trap.

Introduction

Dust grains can be found in various plasma environments. They can reach relatively high charge-to-mass ratio by various charging processes. Therefore, their motion will be significantly affected by the electromagnetic forces and knowing their charge is essential for understanding their motion [Horányi, 1996; Smirnov et al., 2007]. The grains can transport atoms and molecules of surrounding gas, alter the distribution function of various particles in the plasma by catching and further releasing them in different state. The dust surface can serve as a template for various multibody chemical reaction in the space [Beranek et al., 2010; Williams et al., 2002].

The charging and discharging processes are principally the common ones: collection of electrons and ions from surrounding plasma, secondary electron emission, photoemission, field emission, etc. But the small size of the grain and high curvature of the surface make it hard to extrapolate the results from bulk samples. Some of the principal differences is, for example, the total yield of secondary electron emission. It can reach very high values for hot electrons due to increased yield of scattered primaries and the sufficiently small grain can reach extremely high positive surface potential entirely by bombardment by hot electrons [Richterová et al., 2010]. Conversely, due to the high curvature of the surface, charged grains can easily reach high enough surface field and discharge itself by field electron emission or other similar processes.

To understand the nature of the charging processes, it is handy to be able to investigate the single known grain under controlled conditions in laboratory. This can be achieved in an experimental setup involving electrodynamic trap. Using such trap, we can hold the single grain in free space in vacuum, the charging conditions can be set by electron and/or ion gun or UV lamp and we can measure a momentary charge-to-mass ratio of the grain.

It is necessary to damp or stabilize the oscillations of a grain in trap, especially when rapid charging or discharging is occurring. The interaction with the residual gas in the trap damps oscillations when the pressure is high enough. But the gas can affect particle beams and the investigated processes therefore we prefer an active damping based on the optical detection of position and motion of a grain. The charge-to-mass ratio of a grain can be computed utilizing either a measured response of a grain to an applied auxiliary electric field or an eigenfrequency of a motion in a trap. The second technique offer significantly better precision especially for highly charged grains. The present experimental set-up taking use of both active damping and eigenfrequency detection is described in our previous papers Čermak [1994]; Pavlu et al. [2008].
It is necessary to keep the amplitude of oscillations of a grain high enough for optical detection of momentary position and speed. Therefore the trap should form a known parabolic effective potential [Gerlich, 1992] around its center. A cylindrically symmetrical trap with the electrodes of hyperbolic shape forms almost perfect quadrupole field, but the trap surrounds an investigated grain almost completely and it is not easy to reach a grain with particle beams and get rid of a background electrons scattered on the electrodes. The study of photoemission is virtually impossible in such trap, because scattered light will generate background photoelectrons from the electrodes and the current of these electrons will exceed the effect of photoemission from the grain.

It is possible to modify a trap described above and reduce the surface of electrodes. This design have been used for trapping grains ([for example Schlemmer et al., 2001]), but the deviation of an electric field from the quadrupole one is high and oscillations have to be damped to get harmonic motion.

Another approach is a well-known linear trap routinely used as a filter of mass spectrometers. But a charged grain can move freely along the axis of trap. To capture it some modifications are necessary. The obvious possibility is to add a DC field blocking the movement along the axis. The DC voltage is often added to main electrodes which are divided into two [Raizen et al., 1992] or three [Drewsenn et al., 2000] segments. The disadvantage is a distorted field and anharmonicity of higher amplitude oscillations, because radial component of the DC field which distorts the radial quadrupole field can not be avoided.

Probably the most open design is a planar trap [Pearson et al., 2006], but the field is far from ideal one and it makes oscillations of a grain anharmonic.

Therefore, we have designed a novel type of electrodynamic trap based on the linear rod quadrupole. This trap allows three dimensional confinement of a grain without affecting oscillations in radial direction. The proposal has been published together with theoretical derivation of parameters of the trap in Beránek et al. [2007].

Design of the electrodynamic trap

The trap is formed from four cylindrical shaped electrodes, each of them split in the half into two isolated cylinders on the common axis. The proposed geometry is much more open then the trap with hyperbolic electrodes, still allowing the trapped particle to oscillate in the almost purely quadratic effective potential even in relatively big distance from the center of the trap. The quadratic potential is the prerequisite for the harmonic oscillations of the trapped grain and the harmonic oscillations are necessary for exact measurement of the charge-to-mass ratio.

Moreover, it is possible to form either a trap or a particle guide without losing the confinement to the principal axis of the trap only by changing the voltage on the electrodes. Therefore there is a possibility to build an experiment allowing to guide preselected grain to the trap, perform a certain measurement, and move the grain onward then.

The dimensions of the trap, we performed the tests with, are following: the radius of each of four rods, $R$, is 6 mm, the rods are positioned around an inner empty cylinder of radius $r_0 = 7.5$ mm. The total length of the trap is 90 mm and the gaps dividing the rods are 0.5 mm wide. See Fig. 1.

Electric field inside the trap

The high voltage supplying the trap features the central symmetry (the voltage on the electrodes and the potential inside the trap does not change if we invert the coordinates). The lowest multipole term that such field can form, is the quadrupole term. The total field $\varphi$ inside the trap is a result of superposition of three partial fields referred here as $\varphi_x$, $\varphi_y$ and $\varphi_z$. The field $\varphi_z$ is generated by alternating voltage of amplitude $V_z$ applied to the rods according to the Fig. 2. This field does not change the sign if we invert direction of the $z$ axis but change the sign
Figure 1. The geometry and dimensions of the trap. The origin of coordinates is situated in the center of the trap.

Figure 2. Voltage on the electrodes forming field $\varphi_z$, $\varphi_y$ and $\varphi_x$ respectively.

If the direction of any of remaining two axis is inverted. Therefore only possible quadrupole term is proportional to $xy$. Introducing the geometrical factor $\lambda_z$ and neglecting higher multipole terms, we can write the equation:

$$\varphi_z = \lambda_z V_z \cdot \frac{x y}{r_0^2} \cdot \cos \omega t ,$$

where $r_0$ is the radius of the inner void cylinder and $\omega$ the angular frequency of the applied voltage.

In the analogical way, we take advantage of the symmetry of the other components of the field (see Fig. 2). The geometrical factor for both of them is the same and we define $\lambda_{xy} = \lambda_z = \lambda_y$. The both fields are formed by the voltage of amplitude $V_{xy}$. The respective equations are:

$$\varphi_x = \lambda_{xy} V_{xy} \cdot \frac{y z}{r_0^2} \cdot \cos \omega t ,$$

$$\varphi_y = \lambda_{xy} V_{xy} \cdot \frac{x z}{r_0^2} \cdot \cos \omega t .$$

In Beránek et al. [2007], we have shown that the superposition of such three fields forms an effective potential with a minimum in the center of the trap. Because of the numerical errors in the previous paper we will briefly derive the expression for such field and the eigenvectors of the oscillations here again.

We define the effective potential, i.e. the potential of the virtual field affecting the motion of a charged grain inside the alternating electric field of high enough frequency, according to Gerlich [1992]:

$$u_{\text{eff}}(r) = \frac{Q^2}{4m \omega^2} \cdot |E_0(r)|^2 ,$$

where $Q$ and $m$ are the charge and mass of the trapped grain respectively. The amplitude of the electric field inside the trap is $E_0(r)$, and the amplitude of the electric potential is $\varphi_0(r)$.

The free oscillations of the grain in the trap can be described by eigenvectors and eigenvalues of the operator $(-\nabla u_{\text{eff}})$ (the operator of the effective force acting on the trapped grain):

$$(-\nabla u_{\text{eff}}) \begin{pmatrix} x \\ y \\ z \end{pmatrix} = -\frac{Q^2}{4m \omega^2} \cdot |E_0(r)|^2 = -\frac{Q^2}{4m \omega^2} \cdot |\nabla \varphi_0(r)|^2 =$$

$$= -\frac{Q^2}{4m \omega^2 r_0} \cdot \nabla |(\lambda_{xy} V_{xy} z + \lambda_z V_z y, \lambda_{xy} V_{xy} z + \lambda_z V_z x, \lambda_{xy} V_{xy} (x + y))|^2 .$$
We define the ratio of components of electric field:

\[ C = \frac{\lambda_{xy} V_{xy}}{\lambda_z V_z} \]  

and we apply this relation to the previous expression:

\[
\left(-\nabla u_{\text{eff}}\right) \begin{pmatrix} x \\ y \\ z \end{pmatrix} = -\frac{Q^2 \lambda_z^2 V_z^2}{4m\omega^2 r_0^2} \cdot \nabla |(Cz + y, Cz + x, C(x + y))|^2 = \\
-\frac{Q^2 \lambda_z^2 V_z^2}{2m\omega^2 r_0^2} \cdot \begin{pmatrix} C^2 + 1 & C^2 & C \\ C^2 & C^2 + 1 & C \\ C & C & 2C^2 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix}. 
\]  

We can rotate the coordinate system in such way that the matrix representation of the operator will be diagonal. We get a new coordinate system \((x', y', z')\) first by rotating by \(\pi/4\) around the axis \(z\) and then by rotating by \(\theta\) around axis \(x'\). The exact value of \(\theta\) depends on \(C\). The transformation matrix from the coordinates in system \((x, y, z)\) to the coordinates in system \((x', y', z')\) is:

\[
\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta \\ 0 & \sin \theta & \cos \theta \end{pmatrix} \cdot \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix},
\]  

where

\[ \cos \theta = \sqrt{\frac{1}{2} - \frac{1}{2\sqrt{8C^2 + 1}}}. \]  

The operator \((-\nabla u_{\text{eff}})\) is purely diagonal in the new coordinate system and its eigenvectors and thus the principal directions of oscillations are the base vectors of the new coordinate system:

\[
\left(-\nabla u_{\text{eff}}\right) = -\frac{Q^2 \lambda_z^2 V_z^2}{4m\omega^2 r_0^2} \cdot \begin{pmatrix} 2 & 0 & 0 \\ 0 & 4C^2 + 1 - \sqrt{8C^2 + 1} & 0 \\ 0 & 0 & 4C^2 + 1 + \sqrt{8C^2 + 1} \end{pmatrix}. 
\]  

The eigenvalues, \(\alpha_i\), of this operator determine the frequency of oscillations of the grain along the respective axis (secular frequency), \(\Omega_i\):

\[
\Omega_i = \sqrt{-\frac{\alpha_i}{m}} = \frac{Q}{m} \cdot \frac{\lambda_z V_z}{2\omega r_0^2} \cdot \begin{cases} \sqrt{2} & \text{for } i = 1, \\ \sqrt{4C^2 + 1 - \sqrt{8C^2 + 1}} & \text{for } i = 2, \\ \sqrt{4C^2 + 1 + \sqrt{8C^2 + 1}} & \text{for } i = 3. \end{cases}
\]
The geometry factor \( \lambda_z \) is easy to compute because it is a geometry factor of an ordinal quadrupole consisting of four cylindrical rods. We have diverted our design a bit from the optimal ratio of rod diameter and rod spacing [Douglas et al., 1999] to get a more open trap. We have shown in the previous paper that such deviation should not significantly affect the characteristics of the trap. The value of the geometrical factor of our trap setup is

\[
\lambda_z = 0.99. 
\]

The other geometrical factor, \( \lambda_{xy} \), is not so easy to compute, because it belongs to the component of the field generated by the voltage between the two parts of a single rod and solving such field is a three-dimensional problem. Additionally, this field contains higher multipole terms of relatively high amplitude, therefore the oscillations along the axis \( y' \) and \( z' \) will show much higher anharmonicity than the oscillation along the axis \( x' \).

We intend to use the trap for measuring the charge-to-mass ratio of microparticles by means of measuring the secular frequency. Therefore, we will damp the oscillations along the axis \( y' \) and \( z' \) and we will measure the \( \Omega_1 \). Due to this fact, we do not need to know the precise value of \( \lambda_{xy} \).

**Measurement**

**Experiment set-up**

All electrodes of the trap were powered by a synchronous signal generated by a single tunable sine generator. The output of the generator was amplified by several factors to form the appropriate signal for each rod. Finally, these signals were amplified 500\( \times \) by eight independent linear high voltage amplifiers. We were able to freely change the ratio \( V_{xy}/V_z \) during the measurement. An arbitrary low voltage signal could be added to each high voltage output. This feature was used to compensate for the gravitation force and to control the amplitude of the oscillations of the grain in the trap.

In this first experiment, we were limited to frequencies lower than 1 kHz and voltages up to approximately 1 kV. This was the limit of the available amplifiers and the isolation strength between the electrodes of trap. We plan to make some improvements and increase these values up to 100 kHz and 5 kV\text{peak} with a new amplifier designed especially for this trap.

The trap was mounted inside a vacuum chamber pumped down to high vacuum (10\(^{-5}\) Pa). We were able to charge and discharge a grain using the electron gun.

Spherical \( \text{SiO}_2 \) grains, 3-8 \( \mu \text{m} \) in diameter, were dropped into the powered trap from a dispenser and charged during the fall by the electron beam until one of them had been trapped. The charge-to-mass ratio of the trapped grain was in order of tens mC/kg. The oscillations of the trapped grain was damped by buffer gas (helium) and then the chamber had been pumped down. The motion of the grain was detected and controlled by an optical detection system and low voltage auxiliary electric signals were applied to the electrodes of the trap. The electronic system for motion detection and damping was the one normally used in our present dust charging experiment (for more details, see Cermak [1994]; Pavlu et al. [2008]).

**Results**

The experiment confirmed that the motion of a grain in the trap is almost perfectly adiabatic. The amplitude of the oscillations of the grain did not significantly increase or decrease after several minutes with electrical damping switched off.

We measured all three secular frequencies of a single grain at various settings of the trap supply (supply voltage and frequency). We plotted the ratio of these frequencies (see Fig. 4) and fitted with the theoretical curve (11). The result of this fit allows us to estimate the geometrical coefficient \( \lambda_{xy} \):

\[
C = 0.92 \cdot V_{xy}/V_z \quad \Rightarrow \quad \lambda_{xy} = 0.91. 
\]
We observed the motion of the grain by a video camera oriented roughly in the direction of the axis $x$ (diverted 17° toward the axis $z$). The trap supply was set in such way that $C = 0.914$ and the three principal modes of oscillations observed by the camera are shown in Fig. 5 and the expected direction is drawn according to the theory presented above.

We have shown in our previous paper that because of the symmetry of the trap the oscillations along the direction $x'$ are driven by the “ordinary” field of the linear quadrupole, $\varphi_z$, and the frequency should not be affected by the remaining components of the electric field.

By measuring the angular frequency $\Omega_1$, we can compute the charge-to-mass ratio of the grain:

$$\frac{Q}{m} = \frac{\sqrt{2}\omega_0^2}{\lambda_z} \cdot \frac{\Omega_1\omega}{V_z}.$$  

The equation (14) is based on the approximation of the effective potential which is not exact. There is a necessary assumption that the supply voltage frequency is “high enough” compared to the motion of the grain. Therefore, the error decreases with decreasing ratio of the secular frequency to the supply voltage frequency.

The exact solution of the motion in an alternating quadrupole field employs the Mathieu’s differential equation. Solving this equation we can obtain an additional factor correcting the result (14). This term is

$$\frac{1}{\sqrt{1 + (k \cdot \frac{\Omega_1\omega}{\omega})^2}}; \quad k = 1.8,$$  

Figure 4. Ratio of measured frequencies of three modes of oscillations at various settings of the trap. The curves are computed according to the theory shown above, coefficient $\lambda_{xy}$ is determined by least squares fit.

Figure 5. The principal modes of oscillations inside the trap observed by the video camera. The ratio of voltages was such that $C = 0.914$. 

We have shown in our previous paper that because of the symmetry of the trap the oscillations along the direction $x'$ are driven by the “ordinary” field of the linear quadrupole, $\varphi_z$, and the frequency should not be affected by the remaining components of the electric field.
where $\Omega_1$ is the angular frequency of the oscillations of the grain and $\omega$ is the angular frequency of the supply of the trap.

We measured the secular frequency of the same grain (constant charge-to-mass ratio) under various values of the frequency of supply voltage (therefore varying the $\omega/\Omega_1$ ratio). Fig. 6 shows the relative change of the computed charge-to-mass ratio of the grain computed without and with the correction.

We measured the change in the secular frequency (and therefore the computed charge-to-mass ratio of the grain) while varying the voltage $V_{xy}$. We have mentioned before that, according to the theory, there should be no dependence at all as long as the field inside the trap is symmetrical. The real trap has some asymmetry because of its design (the gap in the middle of each rod) and because of manufacturing imperfections. The actual measured data shows a linear dependence of the relative error of the ratio $V_{xy}/V_z$ (see Fig. 7). We are not able to measure the secular frequency when the voltage $V_{xy}$ is too low because the particle would not be safely trapped. Instead of this we extrapolated the measured linear dependence.

Conclusion

We approved experimentally that our proposed design of a new electrodynamic trap is functional and that it is possible to catch and hold a charged grain inside such trap and measure its charge-to-mass ratio. The motion of the grain in the trap is almost adiabatic and it is possible to control and damp oscillations of the grain by small auxiliary voltages applied on the rods of the trap. There is no need for buffer gas to keep the oscillations damped.

The reproducibility of the measured charge-to-mass ratio is good enough for the planned dust charging experiment. With some corrections, it should be possible to get the results reproducible with a relative error in the order of $10^{-3}$ over various operational settings of the trap.

We compute the charge-to-mass ratio according to the equation (14). The ratio is directly proportional to the reciprocal value of the supply voltage thus the relative error of the voltage will contribute to the total error. We suppose, that the errors of the amplitude of voltage will be the major source of the errors. Either due to the long term drifts and due to the inconstant amplification factor over the range of frequencies. In our present experimental set-up the long
term stability is approximately $10^{-2}$. Therefore, the nouveau design of the trap should not decrease an accuracy of an experiment.

The trap is functional over a wide range of values of the $V_{xy}/V_z$ ratio. It is even possible to turn down the “axial” field completely letting the grain to leave the trap in a controlled way along the axis. This feature could be utilized in a wide range of experiments where it is desirable to make another measurement on a particular grain before or after it is investigated in the quadrupole trap.

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References


BERÁNEK ET AL.: TRAPPING MICROPARTICLES IN THE LINEAR QUADRUPOLE TRAP


