Selected Techniques for Langmuir Probe Modelling in Low-Temperature Plasmas

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Abstract. The proper understanding of the interaction of plasmas with immersed solids is profitable for Langmuir probe diagnostics. In addition to theoretical and experimental approach, computational simulations can provide more detailed information about the processes that influence the diagnostics. In our contribution, we present some selected techniques used in modelling of the interaction of Langmuir probes with surrounding low-temperature plasmas and we discuss their main features including accuracy, time consumption and hardware requirements. The presented techniques are chosen from three distinct groups: fluid, particle and hybrid models. The comparison of results shows the unsuitability of the basic fluid model for this purpose, whereas the hybrid model appears to be valid and more efficient at certain conditions. Finally, results of simulations are compared with experimental data.

Introduction

Since the plasma-solid interaction is a highly complex system of physical processes, a large number of computational techniques has been developed [Kim et al., 2005]. In this work, fluid, particle and hybrid models are discussed and their capabilities are shown on a simulation of a finite Langmuir probe with a thin-film holder at various pressures in the positive column of a DC glow discharge in argon.

Fluid model

The fluid simulations in low-temperature plasmas are usually based on solving velocity moments of the Boltzmann kinetic equation, especially in drift-diffusion approximation [e.g. Meyyappan et al., 1990]. The set of governing partial differential equations consists at least of transport equations for charged particles, i.e. electrons and ions, and the Poisson’s equation:

\[
\frac{\partial n_e}{\partial t} + \nabla \cdot (\mu_e n_e \nabla \varphi - D_e \nabla n_e) = 0,
\]

\[
\frac{\partial n_i}{\partial t} + \nabla \cdot (-\mu_i n_i \nabla \varphi - D_i \nabla n_i) = 0,
\]

\[
\Delta \varphi = \frac{e}{\varepsilon_0} (n_i - n_e).
\]

Boundary conditions described in [Hrach et al., 2009] were used. The problem was solved using finite element method by COMSOL Multiphysics.

PIC/MCC model

In the Particle-in-Cell / Monte Carlo Collisions model (PIC/MCC) the motion of individual particles determined by electric and magnetic fields is simulated [e.g. Birdsall et al., 1991]. Positions and velocities of particles are updated by solving the Newton equations at every time step. The electric field is calculated at grid points and interpolated to positions of particles. Whereas the electric field is calculated at every time step, the simulation is self-consistent. The magnetic field is neglected in the presented work.

Collisions of charged particles with the neutral background gas are incorporated by Monte Carlo Collisions method. The scattering is determined by cross sections for various collisional
processes, such as elastic collision, excitation and ionization and charge transfer [Bogaerts et al., 1996].

Finally, the interaction of particles with solids, e.g. collection of charge by dielectric surfaces, is simulated at every time step. The whole procedure is repeated, until a desired time or a steady state is reached.

Hybrid model

The hybrid models combine more computational methods to solve the problems with higher efficiency than separate fluid and particle models [Kim et al., 2005]. While there are many hybrid methods, we have selected one of them for the comparison.

The presented hybrid model is based on a combination of a fluid model and a non self-consistent particle model, which are iteratively coupled [Bartoš et al., 2008]. A step of the iterative process starts with a run of the fluid part. The resulting distribution of electric potential is passed to the particle part. The motion of electrons and ions is computed and transport coefficients are updated

\[
\mu_k(\vec{r}) = \frac{q_k}{m_k\nu_k(\vec{r})} \quad \text{and} \quad D_k(\vec{r}) = \frac{k_BT_k(\vec{r})}{m_k\nu_k(\vec{r})},
\]

where \( k \) stands for electrons or ions, \( q \) is the particle charge, \( m \) is the mass, \( k_B \) is the Boltzmann constant, \( T \) is the temperature and \( \nu \) is the mean frequency of elastic collisions. After a few iterations a sufficient steady-state solution is reached.

Comparison of models

The interaction of a finite Langmuir probe was simulated by the presented techniques. The geometry of the problem is shown in Fig. 1. The dimensionality is of the type 2d3v with cylindrical coordinates \( r \) and \( z \). Parameters of the selected models are listed in Table 1.

To compare the capabilities of the models, various criteria and characteristics can be used. In Fig. 2A, a dependency of “sheath thickness”, characterised by a decrease of electric field in the cross-section plane defined in Fig. 1, on the pressure is shown. The error of sheath thickness estimation is better than 5%. Since the PIC/MCC models are generally used as a benchmark, a relative error of sheath thickness estimation with respect to results of the PIC/MCC model is shown in Fig. 2B. It can be seen that the error of the hybrid model depends strongly on the ratio of the electron mean free path \( \lambda_e \) and the probe radius \( r_p \). The presented hybrid model is therefore limited by the condition \( \lambda_e < l \) and \( \lambda_i < l \), where \( l \) is the smallest measure of the problem, which is implied by the drift-diffusion approximation. Results of the fluid model differ...
Figure 2. A: pressure dependence of sheath thickness computed by the selected models, B: relative error of sheath thickness estimation as a function of $\lambda_e/r_p$ ratio. The error of the hybrid model increases significantly, when the electron mean free path is smaller than the probe radius, which is the smallest characteristic dimension of the geometry.

Table 1. Main parameters of studied models.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>General parameters</td>
<td></td>
</tr>
<tr>
<td>Number density of charged particles</td>
<td>$n_0 = 1 \times 10^{15}, \sqrt{p/p_0}$ m$^{-3}$, where $p_0 = 133$ Pa</td>
</tr>
<tr>
<td>Electron temperature</td>
<td>$T_e = 23,200$ K</td>
</tr>
<tr>
<td>Ion temperature</td>
<td>$T_i = 300$ K</td>
</tr>
<tr>
<td>Probe bias</td>
<td>$U_p = 10$ V</td>
</tr>
<tr>
<td>Probe radius</td>
<td>$r_p = 0.1$ mm</td>
</tr>
<tr>
<td>Probe length</td>
<td>$l_p = 5.0$ mm</td>
</tr>
<tr>
<td>Dimensions of working area</td>
<td>$L_z = 20.0$ mm</td>
</tr>
<tr>
<td></td>
<td>$L_r = 10.0$ mm</td>
</tr>
<tr>
<td>Electron mean free path</td>
<td>$\lambda_e = 1.18 \times 10^{-4} \cdot p_0/p$ m</td>
</tr>
<tr>
<td>Ion mean free path</td>
<td>$\lambda_i = 2.82 \times 10^{-5} \cdot p_0/p$ m</td>
</tr>
<tr>
<td>Fluid model Ion mobility</td>
<td>$\mu_i = 0.33$ m$^2$ s$^{-1}$ V$^{-1}$</td>
</tr>
<tr>
<td>Ion diffusion coefficient</td>
<td>$D_i = 0.012$ m$^2$ s$^{-1}$</td>
</tr>
<tr>
<td>Electron mobility</td>
<td>$\mu_e = 110$ m$^2$ s$^{-1}$ V$^{-1}$</td>
</tr>
<tr>
<td>Electron diffusion coefficient</td>
<td>$D_e = 300$ m$^2$ s$^{-1}$</td>
</tr>
<tr>
<td>Particle model</td>
<td>Mesh</td>
</tr>
<tr>
<td></td>
<td>$400 \times 200$ cells</td>
</tr>
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</table>
The efficiency of the hybrid model allowed us to study the Langmuir probe characteristics. Results of simulations and experiments were published in [Hrubý et al., 2010]. In Fig. 3B, the electron current as a function of the probe bias is shown. The hybrid model of a finite probe is in good correspondence with the experimental data, while the approximation of an infinite probe seems to be insufficient for both the model and the Druyvesteyn method [Hippler et al., 2000] used in an inverse manner.

**Conclusion**

Three distinct computational techniques were described and their results were compared. In comparison with the PIC/MCC model, the accuracy of the hybrid model is sufficient, only if conditions for the drift-diffusion approximation are fulfilled, however, the hybrid model is at least one order of magnitude faster than the PIC/MCC model. The results of the fluid model were significantly less accurate. The hybrid model was successfully applied to study of the Langmuir probe characteristics.

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**References**


