Particle Diffusion in Tokamak Edge Layer

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Abstract. In our paper we study character of the motion of charged particles both ions and electrons in tokamak’s edge vacuum magnetic field with resonant magnetic perturbations. We are interested in this area because of very interesting and important phenomena of edge localized modes (ELMs) mitigation by resonant magnetic perturbations (RMP) method. For integration of particle’s trajectories we use drift equation of motions. From our simulations follows that for particular strength of magnetic field perturbation there exists difference in character of the motion of light ions and electrons respectively. While ions are confined to magnetic surfaces, electrons move away in radial direction. We later discuss what follows from that observation: creation of radial electric field, poloidal rotation of plasma edge. At last we present our futures plans.

Introduction

Plasma pulse in the H-Mode, which is supposed to be ITER’s baseline scenario, is a mode in which at the plasma edge there are internal transport barriers caused by sheared E cross B flows [Terry, 2000]. As a result of this improved confinement on the edge there is an enhanced pedestal height and a few centimetres wide area of a steep plasma pressure gradient called pedestal. The pedestal height is important for fusion efficiency. The higher the pedestal height, the higher the efficiency, so the highest possible pedestal height is desired.

The obstacle to reach the highest possible achievable pedestal height is the fact, that a steep pressure gradient and/or current enables coupled pressure gradient driven ballooning modes and coupled current density driven peeling modes to grow (see pressure gradient-current stability diagram at Fig. 1). These modes with very high grow rate develop to abrupt density and energy changes, called Edge Localized Modes (ELMs). Based on the experimental observations a heuristic categorization of ELMs was created. We therefore distinguish ELM type I, II, III. [Wesson, 2004]. Type I ELMs are giant and present the largest threat to a divertor, type III ELMs are much smaller with higher frequency. Type II ELMs are intermediate category between them [Wesson, p. 409]. A frequency of ELM is 10–150 Hz and typical duration is 10–250 µs [Becoulet, 2010].

During an ELM’s event up to 20% of pedestal energy gets lost [Becoulet, EPS 2010]. It represents up to 20MJ in ITER size facility. This obstacle can restrain a long lasting operation of a future fusion power facility. At the divertor surfaces this effect must be reduced below the technologically feasible, maximum perpendicular heat flux for actively cooled structures, typically 10 MW m⁻², normal to the surface in a steady state or up to 20 MW m⁻² during transient events [Loarte, 2007, p.207].

Currently there are tested several methods for ELMs mitigation: vertical plasma kicks, pellets injection, Resonant Magnetic Perturbations (RMP). These techniques were developed recently and are experimentally tested on JET and other tokamaks, but their theoretical understanding is still not fully developed.

As to RMP method, which is subject-matter of this article, there are mixed results and therefore further investigation is needed. This method was successfully tried for the first time on DIII-D tokamak [Evans, 2008, 2005, etc]. JET experiments show another picture of RMP
influence. The ELMs are not suppressed but some changes are observed. ELM frequency is increased and energy of particular burst is lowered. [Liang, 2007]. Contrary to these observations on the MAST spherical tokamak experiments, no suppression was observed [Nardon, 2009].

As follows from experiments and recent non linear MHD simulation the particle transport in perturbed magnetic field is bigger than the heat transport. The density perturbation predominantly follows the ballooning mode convection cells leading to density filaments. The temperature perturbation, due to the large parallel conduction, follows the magnetic field perturbation [Huysmans, 2009].

Current extrapolations of ELM size in ITER size tokamak are based on inter-machine extrapolation, but experiments as well as advanced numerical simulations show that that this simple extrapolation give not correct estimates [Snyder, 2009]. Understanding and controlling ELMs is currently a major objective of fusion research.

In our work we study influence of Resonant Magnetic Perturbations on motion of electrons and light ions and we discuss results of our simulation.

Drift Equation of Motion

We study the influence of stochastic magnetic field on particle’s motion by a test particle approach. Collisions of particles are not taken into account. We use vacuum approach even though the actual magnetic field plasma depends at least on plasma resistivity and plasma rotation. It is computationally inefficient to integrate the full motion of charged particle motion in (electro-)magnetic field, i.e. to include small cyclotron motion (Larmor radius for 10 keV electron is typically $10^{-2}$ mm, for proton typically $10^{-1} - 10^{0}$ mm (at $B \approx 5T$)). Since the size of magnetic structures is $\approx 10$ mm it is safe to use drift approximation given by following equations [Cary, 2009]:

$$\dot{u} = \frac{e}{m} \frac{\mathbf{B}^* \cdot \mathbf{E}^*}{B^*_{||}}$$  (1)

$$\dot{\mathbf{X}} = u \frac{\mathbf{B}^*}{B^*_{||}} + \frac{\mathbf{E}^* \times \mathbf{b}}{B^*_{||}}$$  (2)

where $u$ is velocity parallel to magnetic field line and $\mathbf{X}$ is the position of a guiding centre. $\mathbf{E}^*$ and $\mathbf{B}^*$ are defined as follows

$$\mathbf{E}^* = -\nabla \Phi^* - \frac{1}{c} \frac{\partial \mathbf{A}^*}{\partial t}$$  (3)

$$\mathbf{B}^* = \nabla \times \mathbf{A}^*$$  (4)
Potentials $\Phi^*$ and $\vec{A}^*$ are given by
\begin{align}
e e\Phi^* &= e\Phi + \mu B - \frac{m}{2} v_E^2 \\
\vec{A}^* &= \vec{A} + \frac{mc}{e} u \vec{b}
\end{align}

For the time independent magnetic field and zero electric field this equations can be simplified as follows
\begin{align}
e e\Phi_0^* &= \mu B \\
\vec{A}_0^* &= \vec{A} + \frac{mc}{e} u \vec{b}
\end{align}
and
\begin{align}E_0^* &= \mu B \\
\vec{B}_0^* &= \vec{\nabla} \times (\vec{A} + \frac{mc}{e} u \vec{b}) = \vec{B} + \frac{mc}{e} u \vec{\nabla} \times \vec{b}
\end{align}

There are two integrals of motion: the total particle’s energy and particle’s magnetic moment $\mu$. This 3D space and 1D velocity equations of motion are integrated by adaptive Runge-Kutta method 853 in our numerical code [Press et al., 2007].

Model of Tokamak Magnetic Field with Resonant Magnetic Perturbation

Further we use the following model of circular large aspect ratio tokamak magnetic field. An amplitude of toroidal magnetic field at the magnetic axis $B_0$ is taken to be 5.0 T and its dependence on radius satisfies radial dependence $\propto 1/R$, which follows from Ampere law, where $R$ is distance from tokamak’s axis. The model magnetic field contains poloidal sheared magnetic filed. Q-profile is taken quadratic $q(r) = q_0 + (q_a - q_0) r^2 / a^2$ with boundary values $q_0 = 1.0$ and $q_a = 3.5$, where $a$ is a minor radius. Major radius $R_0 = 5.0$ m, minor radius $a = 0.5$ m.

In the time independent case and no electric field, we do not need to know vector potential, but instead we can use poloidal magnetic flux function $\Psi_0$ (defined for example in [Wesson, 2004, p.108]).
\begin{align}B_r &= -\frac{1}{R} \frac{\partial \Psi_0}{\partial z} \\
B_z &= \frac{1}{R} \frac{\partial \Psi_0}{\partial R}
\end{align}

The particular form of function $\Psi_0$ and its derivation used to get the desired q-profile is rather lengthy and tedious operation, so I do not describe it here expressly.

Model example of resonant magnetic fluctuation is created by “Fourier-like” method by adding small functions in the form as follows
\begin{equation}\Psi_1 = \sum B_{1i} \cos (n_i \varphi + m_i \theta) \end{equation}
where $\varphi, \theta$ is toroidal, poloidal angle respectively. Numerical value of a ratiois chosen close to value 3, so that resonant surfaces (and if Chirikov criterion is grater than 1 then also ergodic layer) are created at the plasma edge. Ratio $B_1 / B_0 = 5 \cdot 10^{-5}$ is taken to be the same order as value present in DIII-D experiment. On the figure 2 there is shown Poincaré section of magnetic field lines (red dots). We can see how magnetic surfaces are destroyed around radius, where q has value $\approx 3$. Other two colours show two electrons with the same energy, but different initial conditions. We can see how substantially the character of the motion in the presence of ergodic layer is changed.
Figure 2. The Poincaré plot of magnetic field lines shows structural changes in magnetic field caused by the added perturbation. We have used the following coefficients of resonant perturbation \((m, n) = (10, 3), (9, 3), (8, 3)\).

Simulation Results

On the following chart (Fig. 3) we show how the average distance \(< r >\) from magnetic axis depends on time, when we set up initial positions of particles at the inner side of ergodic layer. We take an average from 104 particles for ions and 103 particles for electrons, with initial positions taken in space rectangle at \(r = (3.90–3.93 \text{ m}), z = (-0.01 \text{ m}, 0.01 \text{ m})\), toroidal angle \(\varphi = 0\) and with randomly selected direction of velocity. Energy of particles is equal to 1 keV and the rate of parallel to perpendicular kinetic energy equal to or bigger than 0.8. The last condition ensures, that we consider only passing particles, instead of trapped particles [Wesson, 2004, p.129].

We can see that light ions (\(H^+, D^+\)) remain quit firmly confined about their initial magnetic flux surface, but totally different picture is valid for electrons. They travel during their fast toroidal motion through magnetic surfaces outward and back, similarly as magnetic field lines, what is consistent with the intuitive theory that light particles move along field lines. We do not stop or take away electrons, when they reach some maximal radius as would be more realistic in case of divertor configuration.

Discussion

The qualitatively different behaviour of electrons and light ions as \(H^+\) or \(D^+\) mentioned above, should lead (if we assume that there is no other effect than this free particle motion on vacuum magnetic background) to a deficiency of electrons in the inner edge of resonant layer, what simultaneously means the creation of a radial electric field. This conclusion is supported by experimental data, where even reversion of radial electric field from negative to positive
values is observed by application of sufficient current in resonant magnetic coils [Unterberg, 2007]. The corresponding E cross B drift in poloidal plane should cause poloidal rotation of the plasma edge which adds up to the spontaneous plasma rotation or by NBI heating induced rotation. The consequences of this changed/enhanced rotation are impossible to deal within our current test particle schema, as well as to calculate how large E cross B flow this mechanism can evoke. It is clear that it is impossible to deal with in our model affects of collision, turbulence, response of electro-magnetic field etc.

Conclusion

We have created a numerical model and shown on it that by the appropriate choice of perturbation coefficient we can get a very different behaviour of motion of ions and electrons respectively, which should lead to the creation of a radial electric field and enhancement of poloidal rotation of the plasma edge. However, unfortunately the quantitative estimate of the intensity of radial electric field is unclear and will need further investigation, which might be unreachable in this kind of test particle/vacuum field simulation. Nevertheless, despite of our model's limited predictive power of such a complicated phenomenon as ELMs and ELM mitigation are, this test particle simulation provided to us a solid background and improved our technical knowledge of how to deal with more advanced and more complicated methods. Currently we consider two options for further continuation of our work. The first one is a development of our own parallel PIC 3D model of plasma edge layer and the second one is to initiate cooperation with already existing MHD or gyrokinetic models of some foreign plasma modellers group.
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