Descriptive Geometry Exercises of Advanced Level

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Abstract. The aim of this article is to present some interesting descriptive geometry exercises which were commonly taught at secondary schools\(^1\) and used during the graduation examinations in the 2\(^{nd}\) half of the 19\(^{th}\) century and at the beginning of the 20\(^{th}\) century. However, the same exercises may seem very difficult for present time students. We will also have a look at the secondary school system and graduation exams in connection with descriptive geometry in the mentioned period.

Introduction

Descriptive geometry appeared as a subject in the timetable at secondary schools in 1850s at realschules\(^2\), before that, geometry was taught only as a part of drawing and mathematics. In the 19\(^{th}\) century descriptive geometry was taught only at realschules, later also at real gymnasiums\(^3\).

The first realschule in the Habsburg Monarchy was established in 1770 in Wien (Real-Handlung-Akademie). At the end of the 18\(^{th}\) and at the beginning of the 19\(^{th}\) century other realschules were established but they served as commercial schools or preparatory courses for studies at polytechnic schools. Until 1849 realschules were not conceptualized as secondary schools, in Bohemia there was only one type of secondary schools (where graduation enabled university studies) – gymnasia. Nevertheless, in 1849 the Exner–Bonitz reform was carried, a part of which was the Entwurf der Organization der Gymnasien und Realschulen in Oesterreich [Outline of the organization of gymnasiums and realschules in Austria]. In accordance with this Outline realschules were changed and they started to exist as a kind of secondary schools.\(^4\) The first realschule of this new type was established in 1849 in Prague (the First Czech Realschule in Prague in Nové Město).\(^5\)

Teaching of descriptive geometry at realschules

In 1849 realschules were instituted as six-year secondary schools. The number of classes was increased to seven in 1869 and in the same year the graduation exams were introduced at some of these schools (and also at real gymnasiums). Descriptive geometry was taught three lessons per week from the fifth to the seventh class (in the 20\(^{th}\) century three lessons per week in the fifth and sixth class and only two lessons per week in the last class but later it was also taught in the forth class). The teachers of descriptive geometry were mostly graduates from Prague (or Wien) Polytechnic.

The curriculum for the realschules was very often changed (oftener than for the gymnasiums) but it had always had a large extent, there were taught parts which are not taught today at universities, yet alone at secondary schools.

\(^1\)In this article, the term secondary schools is used to refer to schools for students between the ages from approximately 11 to 18 years.

\(^2\)For this type of school the English term does not exist, therefore here is used the term realschule (realschules – pl.) from German die Realschule (in Czech – ‘reálnka’). More see [Chmelíková, 2009].

\(^3\)Real gymnasia is a special kind of secondary school that connects realschule and gymnasium. The first real gymnasium was established in 1862 in Tábor.

\(^4\)More about history and development of realschules see [Kádner, 1929, 1931] and [Šafránek, 1898, 1918].

\(^5\)More about the First Czech Realschule in Prague in Nové Město see [Vávra, 1902].
Graduation examinations

The first graduation examinations at realschules were in 1869, from 1972 these exams were set by law (before that graduation exams could only be taken at gymnasiums). At realschules descriptive geometry was one of the obligatory subjects. Exams in it were written, they lasted five hours. Students solved three or four exercises (generally three, but in some cases one of them had two different parts – (a) and (b)), which were verbally assigned. The main topics, which were often repeated, were different constructions in the orthogonal (Monge’s) and the central projection. Later, in the 20th century, there were rarely some exercises on the oblique projection and the axonometry too, because these parts started to be taught.

Examples of descriptive geometry exercises

The examples, which are given below, have been choosen from the textbook Základy deskriptivní geometrie [Elements of Descriptive Geometry] [Pithardt, Seifert, 1923, 1925] by Josef Pithardt and Ladislav Seifert. This textbook was published with little changes in several editions for realschules and real gymnasiums. We can find similar exercises in earlier textbooks on descriptive geometry but in this one there is also the oblique projection. We will present three typical exercises which were often used in graduation exams and from the present point of view we can consider them as difficult exercises. These are:

- the lighting of polyhedrons or quadric solids of revolution (or other quadric solids),
- the intersection of solids,
- the lighting of groups of solids.

The lighting of polyhedrons or quadric solids of revolution or other quadric solids

The lighting of polyhedrons is usually constructed using light rays through vertices of the polyhedron and their intersections with planes of projections. These exercises do not require a profound reflection but their solving is often lengthy and demands a high level of accuracy. Moreover, the lighting of quadric solids (of revolution) demands the knowledge of other rules of projection of these solids and their shadows.

In Figure 1, there is the lighting of a road–fence in the orthogonal projection, in Figure 2, there is the same problem but it is constructed in the oblique projection. These exercises are explained in the first volume (this textbook has four volumes in total) of the Pithard’s and Seifert’s textbook on pages 85–87 [Pithardt, Seifert, 1923].

The Intersection of solids

If you find the intersection of solids, the solving of the problem depends on the kind of these solids. The easiest case is the intersection of polyhedrons. In this case you have to find

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6 The others were the language of tuition (it was usually Czech or German), French, mathematics, homeland study and physics; some of them were tested orally only. More about graduation examinations at secondary schools see [Morkes, 2003].

7 Gaspard Monge, Comte de Péluse, was born on 10th May 1746 in Beaumée. He became professor of mathematics at Mézières in 1768 and professor of physics in 1771 at the same place. Later he was made professor at the École Polytechnique in Paris where he gave lectures on descriptive geometry. These lectures were published in 1800 in the form of a textbook entitled Géométrie descriptive. Because of this textbook Monge is considered to be the founder of descriptive geometry. He died on 28th July 1818 in Paris.

8 The first Czech textbook on descriptive geometry was Zobrazující měřictví [Descriptive Geometry] [Ryšavý, 1862–3] by Dominik Ryšavý. The second was Deskriptivní geometrie pro vysší školy reálné [Descriptive Geometry for Higher Realschules] [Jarolímek, 1875–7] (which was also translated into Bulgarian and it had five editions in Czech) and the last textbook on descriptive geometry written in the 19th century was Deskriptivní geometrie pro vysší třídy škol středních [Descriptive Geometry for Higher Classes of Secondary Schools] [Šanda, 1877] by František Šanda. More about these descriptive geometry textbooks see [Chmelíková, 2009].
the meeting points of the edges of the first solid with the faces of the next solid and conversely. The exercise with two quadric solids (of revolution) is less easy. In this case we usually find the meeting points of two concurrent lines which lie on these solids (the first line on the first solid, the second line on the second solid) but only in case the solids are cylinders or cones (of revolution), or if you like they are not solids but surfaces created with movement of a line. In case the solids are not created by a moving (rotating) line, we have to use some auxiliary planes, circles or similar.

In Figure 3, there is the intersection of two oblique cylindrical surfaces. This exercise is published in the third volume of Pitard’s and Seifert’s textbook on page 60. In the textbook there are other similar exercises, mainly on the intersections of cylinders and cones of revolution.
The lighting of groups of solids

In exercises on the lighting of groups of solids there are combined both of the above-described problems – the lighting of solids and the intersection of solids. Except we find shadows on planes of projection, we have to find the way one of the solids shadows on the next solid. In case the solids are not polyhedrons, it can be a very difficult exercise.

In Figure 4, there is an exercise on the lighting of a cylinder of revolution (with very short high) and a cone of revolution. Both of them shadow on the planes of projection, moreover, the cylinder shadows on the cone. Therefore we have to find the intersection of the light cylindrical surface (defined by the cylinder of revolution and by the direction of light rays) with the cone of revolution. We find this exercise in Pithard’s and Seifert’s textbook on pages 63–65.

Conclusion

When we confront the above exercises with the concrete exercises assigned during the graduation exams, we can see that almost every year at every realschules students had to solve at least one of the above-mentioned types of exercises. For example, in the assignments for the graduation examinations at the First Czech Realschule in Prague in Nové Město in the period 1871–1900, there were used six exercises on the lighting of polyhedrons, more than twenty exercises on the lighting of quadric solids (of revolution) and thirteen exercises on the intersection of quadric solids (of revolution) or on the lighting of a group of solids. Moreover, many of these exercises were demanded to construct in the central projection.

As we can see, one hundred years ago learning of descriptive geometry at realschules was not easy for students. At the present time, not only secondary school students would probably have difficulty solving graduation exams exercises but also some graduates from technical universities.
Figure 4. The lighting of a cylinder of revolution and a cone of revolution.

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