Sample Entrance Test

1. Find all real solutions to the quadratic equation $x^2 - 12x + 27 = 0$. Then for each of the following claims mark whether it is correct or not (YES = correct, NO = incorrect).

   (a) All solutions are in the interval $(\pi/2, 10\pi)$.

   (b) All solutions are in the set $(0, 5) \cup (6, 35)$.

   (c) The equation has exactly one solution.

   (d) All solutions are negative.

   (e) The equation has no solution.

2. Let x be a solution of the equation $\log_{10}(\log_{10} x) = 1$. For each of the following claims denote whether it is correct or not.

   (a) $x = 1$,

   (b) $x = 10$,

   (c) $x = 100$,

   (d) $x = 10^{10}$,

   (e) nothing of the above.

3. For each of the following expressions decide whether for every real number x it equals the expression $\sqrt{x^2}$.

   (a) x,

   (b) $|x|$,

   (c) $\sqrt[3]{x^3}$,

   (d) $\max\{x, -x\}$,

   (e) $-x$.

4. Let k be a natural number. A boy said: "I have as many brother as sisters." His sister said: "The number of my brothers is k-times larger than the number of my sisters." How many boys and how many girls are in the family? For each answer mark whether it is correct or not.

   (a) The assignment has a solution for infinitely many values of the parameter k.

   (b) The assignment has a solution for exactly one value of the parameter k.

   (c) The assignment has a solution for at least two values of the parameter k.

   (d) The assignment has a solution for exactly three values of the parameter k.

   (e) There exists a value for the parameter k in the set $\{1, 3, 4\}$ for which the assignment has a solution.

5. Decide what is the number of rectangular parallelepipeds with the following property: the rectangular parallelepiped has three different edge lengths and each edge length is a natural number smaller or equal to 10. The rectangular parallelepipeds that differ by orientation only are counted as equal. For each answer mark whether it is correct or not.

   (a) The number is 120.

(b) The number is 240 or more.

(c) The number is 360.

(d) The number is 720.

(e) Nothing of the above.

6. Four friends arrived one by one to an appointment – Anička, Eliška, Josefína and Barbora. Each of them wore a T-shirt of a favorite color – black, blue, white and green. We know that

   - Eliška came first and she did not wear a white T-shirt.
   - The girl in the blue T-shirt came later than Anička.
   - Bára did not wear a green T-shirt.
   - The girl in the green T-shirt came last.

   Which of the following statements are correct? For each of them mark whether it is correct or not.

   (a) Anička came third.

   (b) Eliška's favorite color is black.

   (c) It is not possible to decide which color is Bara's favorite.

   (d) Bára did not wear a white T-shirt.

   (e) Josefína came the last.

7. Mark all propositions to which the fact that $6 \times 7 = 42$ is a counterexample (YES = a counterexample, NO = not a counterexample).

   (a) The product of two odd numbers is an odd number.

   (b) If the product of two integers a and b is not divisible by 4, then these two numbers are not consecutive (i.e., $|a - b| \neq 1$).

   (c) If the product of two integers a and b is divisible by 4, then these two numbers are not consecutive (i.e., $|a - b| \neq 1$).

   (d) Every even number is a product of two even numbers.

   (e) The product of every two consecutive numbers is divisible by three.

8. The amount of a meal allowance for international business trips (for employees in the Czech republic) is determined as follows. There is a table specifying for each country a standard daily meal allowance. If the employee had a free meal (a breakfast, a lunch or a dinner) the employer can shorten the amount of the meal allowance for the day by up to 25% for each of the specified meals. If the employer does not specify the shortening, the employer is eligible for the full amount of the standard meal allowance. Assume that the employer was on a business trip to Germany and the standard daily meal allowance for Germany is 40 EURO. For each of the following cases decide whether the employee obtained the meal allowance according to the above rule.

   (a) The employer specified the shortening as follows: 25% for each meal. The employee had two breakfasts in the hotel for free. He obtained 60 EURO as the meal allowance.

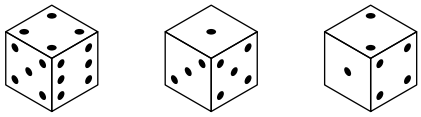
   (b) The employer specified the shortening as follows: 25% for breakfast, 20% for lunch and 30% for dinner. The employee had two breakfasts in the hotel for free and he was invited for a dinner (that he did not pay for). He obtained 48 EURO as the meal allowance.

(c) The employer specified the shortening as follows: 20% for breakfast, 25% for lunch and 25% for dinner. The employee had two breakfasts in the hotel for free and he was invited for a dinner (that he did not pay for). He obtained 80 EURO as the meal allowance.

(d) The employer specified the shortening as follows: 20% for breakfast, 25% for lunch and 25% for dinner. The employee had one breakfast in the hotel for free. He obtained 72 EURO as the meal allowance.

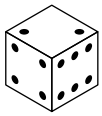
(e) The employer did not specify any shortening. The employee had two breakfasts in the hotel for free. He obtained 80 EURO as the meal allowance.

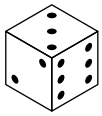
9.

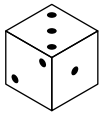


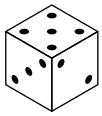
There are three views of a die in the picture. In contrast to the ordinary die, the sum of the pips on the opposite faces of this die need not be seven. Which of the following pictures can be views of the same die? For each of the picture mark whether it can or it cannot be a view of the same die (YES = it can, NO = it cannot).

(a) 

(b) 

(c) 

(d) 

(e) 

10. Let r be a radius of a circle passing through points $(0, 2)$, $(0, -2)$, $(2, 2)$. For each of the following assertions decide whether it is correct or not.

(a) $r = \sqrt{5}$,

(b) $r = 2$,

(c) $r = 3\sqrt{3}$,

(d) $r = 4\sqrt{2}$,

(e) $r = 1$.

Answers

Name:

		Yes	No
1.	(a)	○	○
	(b)	○	○
	(c)	○	○
	(d)	○	○
	(e)	○	○

		Yes	No
2.	(a)	○	○
	(b)	○	○
	(c)	○	○
	(d)	○	○
	(e)	○	○

		Yes	No
3.	(a)	○	○
	(b)	○	○
	(c)	○	○
	(d)	○	○
	(e)	○	○

		Yes	No
4.	(a)	○	○
	(b)	○	○
	(c)	○	○
	(d)	○	○
	(e)	○	○

		Yes	No
5.	(a)	○	○
	(b)	○	○
	(c)	○	○
	(d)	○	○
	(e)	○	○

		Yes	No
6.	(a)	○	○
	(b)	○	○
	(c)	○	○
	(d)	○	○
	(e)	○	○

		Yes	No
7.	(a)	○	○
	(b)	○	○
	(c)	○	○
	(d)	○	○
	(e)	○	○

		Yes	No
8.	(a)	○	○
	(b)	○	○
	(c)	○	○
	(d)	○	○
	(e)	○	○

		Yes	No
9.	(a)	○	○
	(b)	○	○
	(c)	○	○
	(d)	○	○
	(e)	○	○

		Yes	No
10.	(a)	○	○
	(b)	○	○
	(c)	○	○
	(d)	○	○
	(e)	○	○