# Entrance examination, MFF UK in Prague

## Study programme for Bachelor of Computer Science

## 2016, version A

For each of the following ten problems five possible answers a, b, c, d, e are offered. For each problem your task is to mark for each answer whether it is true or false, or whether the statement holds or does not hold. The duration of the test is **75 minutes**.

**Awarding points.** 10 points are assigned to each problem. You gain 10 points for a given problem if and only if you mark each of the five offered answers correctly.[1] For a given problem, if one of the answers is marked incorrectly you will be awarded 0 points, even if some of the other answers for the problem have been marked correctly. For each problem where no answer is marked incorrectly you obtain 2 points for each correctly marked answer. If you mark all five answers correctly, you gain the maximum score of 10 points.

**How to mark answers and how to make a correction.** The answer you choose should be marked by filling the corresponding circle. If you have already marked an answer and wish to make a correction, you can cancel your choice by making a large cross over the filled circle, and then correct it by filling the other circle. It is not possible to choose an answer again where the circle has been already crossed out. Answers marked in any other way will be regarded as non-marked. Notice in the following example that the answers in the last two columns are the same, as they differ only by the corrections made to them.

**Example.** As an example we show the scoring for four markings for the problem "The sum of $1 + 1$ is ":

| | | Answers | | Answers | | Answers | | Answers | |
|---|---|---|---|---|---|---|---|---|---|
| | | Yes | No | Yes | No | Yes | No | Yes | No |
| (a) | 2 | ● | ○ | ● | ○ | ○ | ○ | ○ | ○ |
| (b) | 3 | ○ | ● | ○ | ○ | ○ | ● | ~~●~~ | ● |
| (c) | Less than 12 | ● | ○ | ● | ○ | ○ | ○ | ~~●~~ | ○ |
| (d) | Positive | ● | ○ | ○ | ○ | ● | ○ | ● | ~~●~~ |
| (e) | 1 | ○ | ● | ● | ○ | ○ | ● | ~~●~~ | ● |
| Points: | | **10** | | **0** | | **6** | | **6** | |

[1]A correctly marked answer is one where the right answer is Yes and you only mark Yes or the right answer is No and you only mark No. An incorrect answer is one where the right answer is Yes and you only mark No or the right answer is No and you only mark Yes. All other possibilities are regarded as being unanswered.

For each of the following problems decide which assertions hold and which do not (Yes = Holds, No = Does not hold). Notation for intervals: $(a, b) = \{x \in \mathbb{R} : a < x < b\}$ and $[a, b] = \{x \in \mathbb{R} : a \leq x \leq b\}$.

**1.** Consider the function $f(x) = e^{\sin x}$. Decide which of the following statements about the function $f$ are valid:

(a) $f$ is even.
(b) $f$ is odd.
(c) $f$ is periodic.
(d) $f$ is increasing.
(e) $f$ is injective.

**2.** Decide which of the following statements about the number

$$x = \frac{2(\sqrt{2} + \sqrt{6})}{3\sqrt{2 + \sqrt{3}}}$$

are valid:

(a) $x$ is greater than 1.
(b) $x$ is rational.
(c) $x = \frac{4}{3}$
(d) $x = \frac{2\sqrt{2}}{3}$
(e) $x = \frac{2\sqrt{3}}{3}$

**3.** Alice, Beatrix, Clara, Diana, Eve, Fred, George, Harry, and Igor are queuing up for lunch. Let $b$ denote the number of ways they can stand in line so that all of the girls are in front of all of the boys. Further, let $a$ be the number of ways in which the girls are ahead of all the boys and, in addition, Alice is first.

Decide which of the following statements are valid:

(a) $b > 4a$
(b) $a$ is the square of an integer.
(c) $\sqrt{a} > 25$
(d) $\sqrt{a} > 35$
(e) $b > 3000$

**4.** The first six terms of a geometric series $(a_n)_{n=1}^{\infty}$ satisfy the following two conditions:

$$a_1 - a_2 + a_3 = 9$$
$$a_4 - a_5 + a_6 = 72$$

Determine which of the following statements are correct:

(a) The sum $a_1 + a_2 + a_3 + a_4 + a_5 + a_6$ is greater than 180.
(b) $a_6 > 100$
(c) $a_5 = 48$
(d) $a_2 = 2$
(e) $a_1 + a_6 < 100$

**5.** Let $M$ be the set of all real solutions to the equation

$$\ln(\operatorname{tg} x) = \ln(\operatorname{cotg} x)$$

(here $\ln$ denotes the natural logarithm, $\operatorname{tg}$ the tangent and $\operatorname{cotg}$ the cotangent). Decide which of the following claims about the set $M$ are valid:

(a) If $x \in M$ then $-x \in M$.
(b) If $x \in M$ then $(x + \pi/2) \in M$.
(c) If $x \in M$ then $(x + \pi) \in M$.
(d) If $x \in M$ then $3x \in M$.
(e) If $x \in M$ then $5x \in M$.

**6.** For a real number $a$ let $M_a$ denote the set of all real solutions to the equation

$$ax^3 - a^2|x| = 0\,.$$

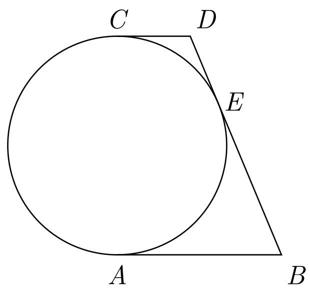
Decide which of the following statements are valid:

(a) There is a real number $a$ for which $|M_a| = 1$.
(b) There is a real number $a$ for which $|M_a| = 2$.
(c) There is a real number $a$ for which $|M_a| = 3$.
(d) There is a real number $a$ for which $|M_a| > 3$.
(e) For every real number $b$ there is a real number $a$ such that $M_a \cap (-\infty, b]$ has exactly one element.

**7.** The parabola P1 is given by the equation $y = 2x^2 - 5x + 2$ and the parabola P2 by the equation $y = 7x^2 + 5x + 7$. Decide which of the following are valid:

(a) The parabolas P1 and P2 intersect in two points.
(b) The parabolas P1 and P2 have at least one point in common.
(c) The parabola P1 intersects the $x$-axis.
(d) The parabola P2 intersects the $x$-axis.
(e) The vertex of the parabola P1 has a positive $x$-coordinate.

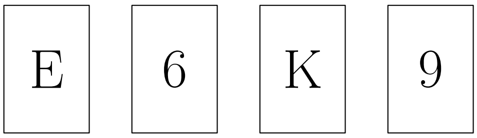
**8.** Two parallel tangents $AB$ and $CD$ to a circle are given, as shown in the figure ($A$ and $C$ are the points of tangency). The length of $AB$ is 9; the length of $CD$ is 4. The line segment $BD$ touches the circle at the point $E$.




Decide which of the following statements are valid:

(a) The radius of the circle is greater than 6.
(b) The radius of the circle cannot be determined from the given data.
(c) The length of $BD$ is 13.
(d) The area of the quadrilateral $ABCD$ is 39.
(e) The triangle $CDE$ is isosceles.

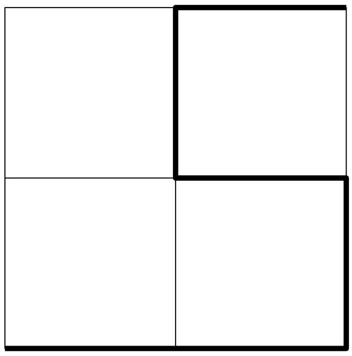
**9.** In front of us are four cards on a table, as shown in the figure. We know that each card has a letter on one side and an integer on the other side.




The reverse side of each of the cards on the table cannot be seen. Paul claims that the cards obey the following rule: “If a vowel is on one side of the card then an even number is on the other side.” Which cards do we need to turn over in order to verify the validity of Paul’s claim?

(a) We must turn all four cards over.
(b) We must turn over any two cards.
(c) We must turn over the first two cards.
(d) We must turn over the two cards on which we see a letter.
(e) We must turn over the first card and the last card.

**10.** We wish to count paths in a $2 \times 2$ lattice that go from the bottom left corner to the top right corner. Paths must run along the edges of the small squares and cannot go through any point more than once. One such path of length $6$ is depicted in the diagram.



Let $C_k$ denote the number of such paths of length $k$.
Decide which of the following statements are valid:

(a) $C_4 > 5$
(b) $C_5 > 4$
(c) $C_6 > 4$
(d) $C_8 < 4$
(e) $C_8 > 4$

## Solutions

**1.** Correct answers: c.

**2.** By simplifying $x^2$ we easily find that $x = 4/3$.
Correct answers: a, b, c.

**3.** Easy combinatorics gives us $b = 5! \cdot 4! = 120 \cdot 24$ and $a = 4!^2 = 24^2$.
Correct answers: a, b.

**4.** The sequence starts with 3, 6, 12, 24, 48, 96.
Correct answers: a, c, e.

**5.** The solution consists of all numbers $\frac{\pi}{4} + 2k\pi$ for an integer $k$.
Correct answers: c, e.

**6.** It is easy to solve the equation, if we solve three cases separately: $a = 0$, $a > 0$, $a < 0$.
Correct answers: b, d, e.

**7.** By solving several quadratic equations we find that there is a unique common point of the parabolas and it has coordinates $[-1, 9]$. Further, the parabola $P_1$ intersects the $x$-axis in points $x = (5 \pm 9)/4$; $P_2$ does not intersect the $x$-axis.
Correct answers: b, c, e.

**8.** By adding the foot of the perpendicular from $D$ to $AB$ and using the Pythagoras law, we easily find that the radius of the circle is $6$.
Correct answers: c, e.

**9.** We must check card with the vowel E (to see if the other side has indeed an even number on it) and the one with 9 (to verify there is not a vowel on the other side – then the rule would be invalid as well).
Correct answers: e.

**10.** $C_4 = \binom{4}{2} = 6$ (from four steps we must pick which two are up and which two right), $C_5 = 0$ (the path cannot have odd length), finally easy case-study gives $C_6 = 4$ and $C_8 = 2$.
Correct answers: a, d.