V zimním semestru 2018/2019 pobývá na naší sekci na pozici hostujícího profesora významný odborník v harmonické analýze

Prof. Dr. Hans Georg Feichtinger

(University of Vienna, Austria)

V rámci svého pobytu u nás povede dva kurzy, určené po řadě studentům Bc studia a Mgr/PhD studia.

Neváhejte, přečtěte si obsahy přednášek a získejte své kredity u jedné ze špiček evropské matematiky.

Přednáška č. 1

From Linear Algebra to Fourier Analysis

NMAG359

- **Místo a čas:** **úterý 12:20, K10A**
  (s možným přesunutím do sem. míst. MÚUK)
- **Přednášející:** **Prof. Dr. Hans Georg Feichtinger**
  (přednáška bude anglicky)
- **Vhodné pro studenty Bc. studia**

Abstrakt:

2.1 Course 1: From Linear Algebra to Fourier Analysis

This course will be the more elementary one, requiring from the participating students only a general background in linear algebra, and knowledge of the Riemann integral, ideally with some interest in numerical computations (if not, the examples which will be presented can be viewed just as illustrations of concrete linear algebra applications, providing concrete numerical results, with some discussion of numerical efficiency). In this sense the course should be accessible to students after the first year of studies, ideally in the third year of studies and further on, but even advanced (say PhD) students will benefit from the un-orthodox view on linear algebra concepts, as realized using the programming language MATLAB, which is used widely by applied mathematicians, engineers or physicists. Previous knowledge of MATLAB will not be required, although certainly helpful. Also, following the course without the desire to learn or use MATLAB is also possible.

The goal of the course will be to introduce all the key concepts of time frequency analysis, in particular of what is called Gabor Analysis (a kind of discretization of the time-frequency plane, also called phase-space). The starting point will be linear algebra and polynomials as an important special case.

The participants will gain access to the NuHAG based MATLAB toolbox and also learn about the key features of the so-called LTFAT toolbox (Large Time-Frequency Toolbox) hosted by ARI (Acoustic Research Institute, Vienna).

Question of time-frequency analysis over finite Abelian groups are related to problems in digital signal processing. For the one-dimensional we are concerned with audio signals, for the two-dimensional case it is about digital image analysis. The course will lead up to ideas about time-variant (resp. space-variant) filtering or the principles of mobile communication, viewed from a linear algebra point-of-view.

The course should also be *accessible to computer science and physics or engineering students who want to learn about (modern) Fourier analysis.*

Přednáška č. 2

A „mild“ Theory of Distributions with Applications

Koná se v rámci „Výběrové přednášky Matematické modelování 1“ NMMO498

- **Místo a čas:** středa 9:00, seminární místnost KMA
- **Přednášející:** Prof. Dr. Hans Georg Feichtinger (přednáška bude anglicky)

- **Vhodné spíše pro studenty Mgr nebo PhD studia**

Abstrakt je na následující straně

2.2 Course 2: A "mild" Theory of Distributions with Applications

In contrast to the usual approach to *generalized functions*, which start from test functions of infinitely differentiable functions, forming a topological vector spaces by using countable families of (complicated) seminorms time-frequency (TF) analysis allows to introduce a relatively simple Banach space of test functions, the Segal algebra $\left(S_0(\mathbb{R}^d), \|\cdot\|_{S_0}\right)$ (also named Feichtinger's algebra in the literature) which shares many properties known for the Schwartz space of rapidely descreasing functions $\boldsymbol{\mathcal{S}}(\mathbb{R}^d)$, whose dual is known as Laurent Schwartz' space $\boldsymbol{\mathcal{S}}'(\mathbb{R}^d)$ of tempered distributions. One has among others the following crucial properties

1. $\mathcal{F}(S_0(\mathbb{R}^d)) = S_0(\mathbb{R}^d)$ (Fourier Invariance);
2. $(S_0'(\mathbb{R}^d) * S_0(\mathbb{R}^d)) \cdot S_0(\mathbb{R}^d) \subset S_0(\mathbb{R}^d)$ (Regularization);
3. The so-called *kernel theorem*.For every bounded linear operator from $S_0(\mathbb{R}^d)$ to $S_0'(\mathbb{R}^d)$ there exists a uniquely determined distribution $\sigma \in S_0'(\mathbb{R}^{2d})$ such that
$$\sigma(f \otimes g) = Tf(g), \quad \forall f, g \in S_0(\mathbb{R}^d).$$

The fact that one has $S_0(\mathbb{R}^d) \subset \boldsymbol{L}^p(\mathbb{R}^d) \subset S_0'(\mathbb{R}^d)$ for any $p \in [1, \infty]$ helps to derive various statements concerning these (more traditional) spaces.

The course will start from the concept of *translation invariant linear systems*, modelled as bounded linear operators on the Banach space $\left(C_0(\mathbb{R}^d), \|\cdot\|_\infty\right)$ of continuous functions on $\mathbb{R}^d$, vanishing at infinity. Each such system turns out the be a "moving average" or equivalently a *convolution operator* by some linear functon, called bounded measure $\mu \in M_b(\mathbb{R}^d) := \left(C_0'(\mathbb{R}^d), \|\cdot\|_{C_0'}\right)$. From this identification one can start to define the convolution of bounded measures (e.g. probability measures, related to random variables) and even come up with a description of the Fourier Stieltjes transform (up to the convolution theorem) without making use of Lebesgue integration theory.

On this basis the short-time Fourier transform (STFT) can be introduced which is then used in order to properly define the Banach space $\left(S_0(\mathbb{R}^d), \|\cdot\|_{S_0}\right)$ and its dual space (via integrability resp. boundedness of their STFTs (with Gaussian window functions).

The course will then introduce the so-called Banach Gelfand Triple $(S_0, \boldsymbol{L}^2, S_0')(\mathbb{R}^d)$, which will be explained in analogy with the triple of number systems, namely $\mathbb{Q} \subset \mathbb{R} \subset \mathbb{C}$ (rational, real and complex numbers). It will then be shown how many important concepts (in particular the (generalized) Fourier transform, the kernel theorem, the spreading representation or the Kohn-Nirenberg symbol of a pseudo-differential operator) can be explained resp. understood using these not yet so familiar function spaces.

The pre-requests for participants will be the knowledge of the Riemann integral (no Lebesgue integral needed) and the fundamental concepts of Functional Analysis (Banach spaces, bounded linear functionals and operators, dual spaces, Hilbert spaces, but all these concepts will can be learned in parallel with the course using written material), hence the course should be accessible to mathematics students from the master level onwards, but also the mathematically minded graduate students or PostDocs from the Applied Sciences.