The Golden Ratio
Determined Using
a Ruler and Compass
The Golden Ratio
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1 Introduction and Definitions

1.1 What This Book Is About
This book focuses on how to construct the golden ratio

$$\phi = \frac{\sqrt{5} + 1}{2}$$

using a ruler (straightedge) and a compass. This book will not discuss the history or occurrences of the golden ratio in nature, arts, architecture, etc.; numerous other books are devoted to such topics.

1.2 Why We Are Writing This Book
The golden ratio has been known for more than two thousand years, yet new ways to construct and describe the ratio are still discovered (e.g., [7]). Therefore, we have decided to determine the smallest number of steps required to construct the golden ratio using different types of tools and to compile a complete list of these short constructions.
The construction of the golden ratio, the number of steps, the shortest construction and the tools utilized will be described later; in this section, note the following notation:

"Golden section" refers to either the number $\phi$ or the division of a given segment into two parts with lengths in this ratio.
In this book, we use the name golden ratio to indicate the ratio $\phi$.

1.3 Terms
First, let us define the terms we will use:

Step is a construction (drawing) of a single line (straight line or circle) and its corresponding intersection points (required, but not always all) with already existing lines.
The definition for one step allows us to consider the length of the construction.

Types of lines are the following:

$p(A, B)$ a straight line passing through the given points;
$k(A, B)$ a circle with a given center and passing through the given point; and
$k(A, |BC|)$ a circle with a given center and a radius that is a distance between two points.

Types of steps corresponds to drawing the line of the given type (therefore, three total) and we will denote them identically to the type of the line, e.g., step of the type $p(A, B)$.

Construction process is a sequence of steps.
Construction is a set of lines and points that is a result of the construction process.

Length of the construction is the number of lines that equals the number of steps in every construction process of this construction.

Different processes Two processes are different if they are not permutations of each other, varying in more than the order of steps.

The identical construction (set of straight lines and circles) can sometimes be created by different processes by varying e.g., the position the pair of points forming the radius of the circle was selected from. Because we are describing the list of constructions, not the list of processes, from these processes we describe always one.

Similar constructions are constructions sets of lines (not points or names of lines) of which can be mapped onto each other by translation, scaling, rotation and/or reflection.

Distance contained in the construction is the distance between two points in the construction.

Occurrence of the golden ratio in the construction is the couple of distances contained in the construction, ratio of which (distances) equals to $\phi$.

Construction of the golden ratio is the construction containing the occurrence of the golden ratio.

This book contains a list of all of the shortest constructions of the golden ratio; these constructions are divided into classes to organize similar constructions. For every construction, we show one process of construction.

1.4 Types of Constructions

We previously mentioned construction by ruler and compass and we defined three types of steps: one step draws a straight line and two steps draw a circle.

Euclid in [1] uses a compass that collapses when lifted from the page. This compass we consider a collapsible compass, and a collapsible compass cannot be used for the steps $k(A, |BC|)$.

Mascheroni ([2]) and Mohr ([3]) stated that every ruler and compass construction can be constructed (without straight lines) by a compass only. These so-called Mohr-Mascheroni constructions do not use the steps of the type $p(A,B)$.

Other authors use other tools such as toothpicks or rusty compasses ([9], [10]). We distinguish four types of constructions, and we indicate them using the following two- or three-letter abbreviations:

RC ruler and compass constructions, can exploit all types of steps

RCC ruler and collapsible compass constructions, can use steps of type $p(A,B)$ and $k(A,B)$, do not use steps of type $k(A,|BC|)$
1.5 The Shortest Constructions

MM Mohr-Mascheroni constructions use only a compass, can use steps of type $k(A, B)$ and $k(A, |AB|)$, do not use steps of type $p(A, B)$

CCO collapsible compass only constructions, can use only steps of type $k(A, B)$

1.5 The Shortest Constructions

The goal of this book is to determine the minimal length required to construct the golden ratio for all four types of constructions. Additionally, this book presents the complete list of minimal length constructions of the golden ratio for each type.

In a construction, the golden ratio can be found at multiple different places. We will present only different constructions, and for every construction, we will show all possibilities how to find the golden ratio.

1.6 Naming the Constructions

To identify individual constructions, we label the constructions with a prefix indicating the type of construction (RC, RCC, MM or CCO; see above) and a serial number.

1.7 Content of This Book

After this introduction, a list of all shortest constructions will follow. The list is divided into four parts for the four types of construction. Following the list, the ideas of the proofs of the constructions are provided.

1.8 Number of the Shortest Constructions

For individual types of constructions, the following shortest constructions are found:

RC minimal length: 4 steps
- number of golden ratio constructions for the minimal length: 4
- number of processes of constructions for the minimal length: 8
- number of classes for similar constructions: 1

RCC minimal length: 5 steps
- number of golden ratio constructions for the minimal length: 457
- number of processes of constructions for the minimal length: 2266
- number of classes for similar constructions: 69

MM minimal length: 5 steps
- number of golden ratio constructions for the minimal length: 368
- number of processes of constructions for the minimal length: 988
- number of classes for similar constructions: 80

CCO minimal length: 5 steps
- number of golden ratio constructions for the minimal length: 114
- number of processes of constructions for the minimal length: 1048
- number of classes for similar constructions: 21
1.9 Relations between the Lists of Constructions of the Individual Types

Individual lists of the shortest constructions, except for constructions of the kind RC, that are one step shorter, are not mutually disjoint.

Every construction that can be drawn by a collapsible compass also belongs to constructions that can be created using a collapsible compass and a ruler (that will not be used).

The list of the shortest constructions of CCO is therefore (a proper) subset of the list of the shortest constructions of RC.

Similarly, every construction of CCO is also a construction of MM because constructions of both types do not use a ruler and can be constructed by a collapsible compass or an ordinary compass.

The list of the shortest constructions of CCO is therefore (also a proper) subset of the list of the shortest constructions of MM. Additionally, the list of the shortest constructions of CCO is the intersection of the lists of the shortest constructions of RC and MM.

Thus, in the list of constructions of RCC, only constructions that are not found in the constructions of CCO will be shown. Additionally, we do not show constructions of CCO in the list of the shortest constructions of MM. The numbering of constructions is not affected, and only some constructions from the lists for RCC and MM are omitted and are mentioned later with the corresponding construction in the list for CCO.

2 All of the Shortest Constructions

2.1 Description of a Construction

The description of every construction has identical format. For every construction, we present the following:

• name
• drawing in which the given point $B$ is to the right of the given point $A$
• the one process of construction creating this construction
• list of distances belonging to any of couples forming the golden ratio in this construction
• list of couples of distances (occurrences of the golden ratio) in this construction.

2.2 Description of a Process of Construction

The process of construction is described as a list of steps using lines and intersections. We use the following notation:

• $k_1 = k(A, B)$ circle with the center $A$ passing through the point $B$;
• $k_2 = k(A, |BC|)$ circle with the center $A$ and the radius equal to the distance of the points $B$ and $C$;
2.3 Overview of All Classes of Similarity

\[ p_3 = p(A, B) \] straight line passing through points \( A \) and \( B \); and

\[ E \in k_4 \cap p_5 \] point of intersection of two lines.

Individual lines in the description of the process of construction are marked \( p_i \) (straight lines) or \( k_i \) (circles), where index \( i \) is the number of the line in the construction. The points are indicated by the capitalized letters of the alphabet in ascending order.

2.3 Overview of All Classes of Similarity

All constructions form classes of mutually similar constructions. In total, 715 different constructions are noted; however, only 129 similar classes are described. To easily find any specific construction, on the following page we present a table of representative images for all classes.

In these drawings, we do not indicate names of points or lines because of the readability. Additionally, the similarity of constructions concerns lines, not naming.

The 129 images must be small to be displayed on one page. Nevertheless, electronic versions of this book allow zooming and magnifying. However, to search the specific construction, visualizing the small pictures on one page before the list of bigger images on multiple pages allows for easier access.

Drawings have no numbers of classes or sections, but individual pictures have links embedded. Clicking on the picture transfers the reader to the appropriate section.
2.4 Overview of Classes of Similarity — Images
2.5 RC

Construction processes presented in this section use the strongest tools of all four types: ruler (straight line passing through the given points) and a compass that can measure the distance between two arbitrary points and use this distance as a radius (a circle with a given center and radius). Accordingly, constructions from this part are the shortest; to construct the golden ratio, they require just four steps.

Only four shortest constructions of the golden ratio are available (resulting from eight different processes). These constructions differ only by the roles of points $A$ and $B$ and by selecting one of the two points as the intersection of the circle and the straight line for the center of the large circle. Therefore, all constructions of this type are similar.

The constructions are named from RC1 to RC4; construction RC1 is identical with the construction described by K. Hofstetter [7].
2.6 Class of Similar Constructions No. 1

Contained Constructions
RC1, RC2, RC3, RC4
2.6.1 Construction RC1

**Construction Process**

- **A, B given initial points**
- 1. \( p_1 = p(A, B) \)
- 2. \( k_2 = k(A, |AB|) \)
  - \( C \in p_1 \cap k_2 \)
- 3. \( k_3 = k(A, |BC|) \)
  - \( D \in p_1 \cap k_3 \)
- 4. \( k_4 = k(D, |BC|) \)
  - \( E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4 \)

**Distances**

- \( d_0 = |EH| = |FG| \) (red) \( \ldots 2 \)
- \( d_1 = |CG| = |CH| \) (blue) \( \ldots 2 \)
- \( d_2 = |EG| = |FH| \) (green) \( \ldots 2 \)

**Occurrences of the Golden Ratio**

- \( \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \) (8 occurrences)
2.6.2 Construction RC2

**Construction Process**

A, B given initial points

1. \( p_1 = p(A, B) \)
2. \( k_2 = k(A, |AB|) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(A, |BC|) \)
   \( D \in p_1 \cap k_3 \)
4. \( k_4 = k(D, |BC|) \)
   \( E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4 \)

**Distances**

\[ d_0 = |EH| = |FG| \text{ (red)} \ldots 2 \]
\[ d_1 = |BG| = |BH| \text{ (blue)} \ldots 2 \]
\[ d_2 = |EG| = |FH| \text{ (green)} \ldots 2 \]

**Occurrences of the Golden Ratio**

\[ \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)} \]
2.6 Class of Similar Constructions No. 1

2.6.3 Construction RC3

Construction Process

A, B given initial points
1. \( p_1 = p(A,B) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(B, |AC|) \)
   \( D \in p_1 \cap k_3 \)
4. \( k_4 = k(D, |AC|) \)
   \( E \in k_2 \cap k_4 , F \in k_2 \cap k_4 , G \in k_3 \cap k_4 , H \in k_3 \cap k_4 \)

Distances

\( d_0 = |EH| = |FG| \) (red) \( \ldots 2 \)
\( d_1 = |AG| = |AH| \) (blue) \( \ldots 2 \)
\( d_2 = |EG| = |FH| \) (green) \( \ldots 2 \)

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \) (8 occurrences)
2.6.4 Construction RC4

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(B, |AC|) \)
   \( D \in p_1 \cap k_3 \)
4. \( k_4 = k(D, |AC|) \)
   \( E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4 \)

Distances

\[ d_0 = |EH| = |FG| \text{ (red)} \ldots 2 \]
\[ d_1 = |CG| = |CH| \text{ (blue)} \ldots 2 \]
\[ d_2 = |EG| = |FH| \text{ (green)} \ldots 2 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)} \]
2.7 RCC

The construction processes presented in this section use more limited tools than processes in the previous section: a ruler (straight line passing through the given points) and a collapsible compass that cannot draw a circle with an arbitrary measured diameter and only allows drawing circles with a given center passing by another given point.

A compass drawing only circles $k(A,B)$ cannot construct more ratios than a compass creating circles with an arbitrary measured diameter $k(A, |BC|)$. Fewer are constructed; therefore, the construction of the golden ratio by these tools requires at least 5 steps.

In total, 457 shortest constructions can be constructed by 2266 different processes forming 69 classes of mutually similar constructions.

The constructions are labeled RCC1 to RCC457.

From this list (the initial numbering is retained), we omit 114 constructions that do not require a ruler. These constructions are mentioned in the list of the constructions of CCO.

The presented constructions are numbered by the similarity of their process and divided into classes by similarity of the construction. Similarity between two constructions indicates that the straight lines and circles of the one construction can be mapped onto those of other by translation, scaling, rotation and reflection.

Construction RCC27 is identical to the construction described in [4] by K. Hofstetter; construction RCC52 to the construction of the same author described in [6].

Remarks

From the constructions RC3 and RCC35, in which are constructed (apart from the names of lines and points) ratios of equally placed couples of points, we display the limited possibilities of the constructions using collapsible compasses.

In RC3, we construct point $D$ together with the large circle $k_3 = k(B, |AC|)$ concentric with circle $k_2$ because we can measure and transfer the distance $AC$.

In RCC35, however, we have to first construct the point $D$ using the circle (here $k_3 = k(A,B)$), and thereafter construct the large circles (the circle concentric with the circle $k_2$ passing through this point $D$ ($k_5 = k(B,D)$) and the circle with the center in the point $D$ ($k_4 = k(D,B)$).

(The order of constructing circles $k_3$ and $k_4$ in RC3 and circles $k_4$ and $k_5$ in RCC35 does not matter; the order can be reversed).

Therefore, RCC35 requires one more line than RC3.
2.8 Constructions Omitted Due to Identity

RCC266 ( MM2 CCO1), RCC267 ( MM1 CCO2), RCC268 ( MM4 CCO3),
RCC269 ( MM3 CCO4), RCC270 ( MM12 CCO5), RCC271 ( MM11 CCO6),
RCC272 ( MM10 CCO7), RCC273 ( MM9 CCO8), RCC274 ( MM23 CCO9),
RCC275 ( MM24 CCO10), RCC276 ( MM5 CCO11), RCC277 ( MM6 CCO12),
RCC278 ( MM19 CCO13), RCC279 ( MM20 CCO14), RCC280 ( MM21 CCO15),
RCC281 ( MM22 CCO16), RCC282 ( MM14 CCO17), RCC283 ( MM13 CCO18),
RCC284 ( MM15 CCO19), RCC285 ( MM16 CCO20), RCC286 ( MM17 CCO21),
RCC287 ( MM18 CCO22), RCC288 ( MM7 CCO23), RCC289 ( MM8 CCO24),
RCC314 ( MM53 CCO25), RCC315 ( MM52 CCO26), RCC316 ( MM51 CCO27),
RCC317 ( MM50 CCO28), RCC318 ( MM64 CCO29), RCC319 ( MM65 CCO30),
RCC320 ( MM46 CCO31), RCC321 ( MM47 CCO32), RCC322 ( MM40 CCO33),
RCC323 ( MM61 CCO34), RCC324 ( MM62 CCO35), RCC325 ( MM63 CCO36),
RCC326 ( MM55 CCO37), RCC327 ( MM54 CCO38), RCC328 ( MM56 CCO39),
RCC329 ( MM57 CCO40), RCC330 ( MM58 CCO41), RCC331 ( MM59 CCO42),
RCC332 ( MM48 CCO43), RCC333 ( MM49 CCO44), RCC348 ( MM263 CCO45),
RCC349 ( MM258 CCO46), RCC351 ( MM325 CCO47), RCC353 ( MM339 CCO48),
RCC354 ( MM335 CCO49), RCC356 ( MM347 CCO50), RCC357 ( MM344 CCO51),
RCC358 ( MM325 CCO52), RCC360 ( MM353 CCO53), RCC362 ( MM326 CCO54),
RCC364 ( MM348 CCO55), RCC365 ( MM343 CCO56), RCC367 ( MM340 CCO57),
RCC368 ( MM336 CCO58), RCC369 ( MM328 CCO59), RCC371 ( MM354 CCO60),
RCC372 ( MM260 CCO61), RCC375 ( MM264 CCO62), RCC376 ( MM259 CCO63),
RCC379 ( MM267 CCO64), RCC381 ( MM277 CCO65), RCC383 ( MM293 CCO66),
RCC384 ( MM289 CCO67), RCC386 ( MM301 CCO68), RCC387 ( MM298 CCO69),
RCC389 ( MM307 CCO70), RCC390 ( MM279 CCO71), RCC392 ( MM280 CCO72),
RCC393 ( MM302 CCO73), RCC395 ( MM297 CCO74), RCC397 ( MM294 CCO75),
RCC398 ( MM290 CCO76), RCC400 ( MM308 CCO77), RCC401 ( MM282 CCO78),
RCC402 ( MM268 CCO79), RCC403 ( MM152 CCO80), RCC404 ( MM147 CCO81),
RCC406 ( MM212 CCO82), RCC408 ( MM228 CCO83), RCC409 ( MM224 CCO84),
RCC411 ( MM236 CCO85), RCC412 ( MM233 CCO86), RCC413 ( MM214 CCO87),
RCC415 ( MM242 CCO88), RCC417 ( MM215 CCO89), RCC419 ( MM237 CCO90),
RCC420 ( MM232 CCO91), RCC422 ( MM229 CCO92), RCC423 ( MM225 CCO93),
RCC424 ( MM217 CCO94), RCC426 ( MM243 CCO95), RCC427 ( MM149 CCO96),
RCC430 ( MM153 CCO97), RCC431 ( MM148 CCO98), RCC434 ( MM156 CCO99),
RCC436 ( MM166 CCO100), RCC438 ( MM182 CCO101), RCC439 ( MM178 CCO102),
RCC441 ( MM190 CCO103), RCC442 ( MM187 CCO104), RCC444 ( MM196 CCO105),
RCC445 ( MM168 CCO106), RCC447 ( MM169 CCO107), RCC449 ( MM191 CCO108),
RCC450 ( MM186 CCO109), RCC452 ( MM183 CCO110), RCC453 ( MM179 CCO111),
RCC455 ( MM197 CCO112), RCC456 ( MM171 CCO113), RCC457 ( MM157 CCO114),

Omitted 114 constructions.
Contains 343 constructions.
2.9 Class of Similar Constructions No. 2

Contained Constructions
RCC1, RCC35, RCC66, RCC127, RCC159, RCC190
2.9 Class of Similar Constructions No. 2

2.9.1 Construction RCC1

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(A, B) \)
   \( D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3 \)
4. \( k_4 = k(C, A) \)
   \( G \in k_3 \cap k_4, H \in k_3 \cap k_4 \)
5. \( k_5 = k(A, C) \)
   \( I \in k_4 \cap k_5, J \in k_4 \cap k_5 \)

Distances

\[
\begin{align*}
&d_0 = |GJ| = |HI| \quad \text{(red)} \quad \ldots \quad 2 \\
&d_1 = |BI| = |BJ| = |CE| = |CF| = |DE| = |DF| = |EF| \quad \text{(blue)} \quad \ldots \quad 7 \\
&d_2 = |GI| = |HJ| \quad \text{(green)} \quad \ldots \quad 2
\end{align*}
\]

Occurrences of the Golden Ratio

\[
\phi = \frac{d_0}{d_1} = \frac{d_2}{d_4} \quad (28 \text{ occurrences})
\]
2.9 Class of Similar Constructions No. 2

2.9.2 Construction RCC35

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(A, B) \)
   \( D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3 \)
4. \( k_4 = k(D, B) \)
   \( G \in k_2 \cap k_4, H \in k_2 \cap k_4 \)
5. \( k_5 = k(B, D) \)
   \( I \in k_4 \cap k_5, J \in k_4 \cap k_5 \)

Distances

\[ d_0 = |GJ| = |HI| \text{ (red)} \ldots 2 \]
\[ d_1 = |AI| = |AJ| = |CE| = |CF| = |DE| = |DF| = |EF| \text{ (blue)} \ldots 7 \]
\[ d_2 = |GI| = |HJ| \text{ (green)} \ldots 2 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (28 occurrences)} \]
2.9.3 Construction RCC66

Construction Process

A, B given initial points
1. $p_1 = p(A, B)$
2. $k_2 = k(B, A)$
   \[ C \in p_1 \cap k_2 \]
3. $k_3 = k(C, A)$
4. $k_4 = k(A, C)$
   \[ D \in k_3 \cap k_4, \ E \in k_3 \cap k_4 \]
5. $k_5 = k(C, B)$
   \[ F \in p_1 \cap k_5, \ G \in k_2 \cap k_5, \ H \in k_2 \cap k_5, \ I \in k_4 \cap k_5, \ J \in k_4 \cap k_5 \]

Distances

\[
\begin{align*}
\text{d}_0 &= |DJ| = |EI| \quad \text{(red)} \ldots 2 \\
\text{d}_1 &= |AG| = |AH| = |BD| = |BE| = |FG| = |FH| = |GH| \quad \text{(blue)} \ldots 7 \\
\text{d}_2 &= |DI| = |EJ| \quad \text{(green)} \ldots 2
\end{align*}
\]

Occurrences of the Golden Ratio

\[
\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \quad \text{(28 occurrences)}
\]
2.9.4 Construction RCC127

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(C, B) \)
   \( D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3 \)
4. \( k_4 = k(D, B) \)
   \( G \in k_2 \cap k_4, H \in k_2 \cap k_4 \)
5. \( k_5 = k(B, D) \)
   \( I \in k_4 \cap k_5, J \in k_4 \cap k_5 \)

Distances

\[ d_0 = |GJ| = |HI| \quad \text{(red) \ldots 2} \]
\[ d_1 = |AE| = |AF| = |CI| = |CJ| = |DE| = |DF| = |EF| \quad \text{(blue) \ldots 7} \]
\[ d_2 = |GI| = |HJ| \quad \text{(green) \ldots 2} \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \quad \text{(28 occurrences)} \]
2.9 Class of Similar Constructions No. 2

2.9.5 Construction RCC159

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(A, B) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(C, A) \)
   \( D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3 \)
4. \( k_4 = k(C, B) \)
5. \( k_5 = k(B, C) \)
   \( G \in k_3 \cap k_5, H \in k_3 \cap k_5, I \in k_4 \cap k_5, J \in k_4 \cap k_5 \)

Distances

\( d_0 = |GJ| = |HI| \) (red) \( \cdots 2 \)
\( d_1 = |AI| = |AJ| = |BE| = |BF| = |DE| = |DF| = |EF| \) (blue) \( \cdots 7 \)
\( d_2 = |GI| = |HJ| \) (green) \( \cdots 2 \)

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \) (28 occurrences)
2.9.6 Construction RCC190

Construction Process

1. \( p_1 = p(A, B) \)
2. \( k_2 = k(A, B) \)
   
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(C, A) \)
   
   \( D \in p_1 \cap k_3 , \ E \in k_2 \cap k_3 , \ F \in k_2 \cap k_3 \)
4. \( k_4 = k(D, A) \)
   
   \( G \in k_2 \cap k_4 , \ H \in k_2 \cap k_4 \)
5. \( k_5 = k(A, D) \)
   
   \( I \in k_4 \cap k_5 , \ J \in k_4 \cap k_5 \)

Distances

\[
\begin{align*}
   d_0 &= |GJ| = |HI| \text{ (red)} \ldots 2 \\
   d_1 &= |BE| = |BF| = |CI| = |CJ| = |DE| = |DF| = |EF| \text{ (blue)} \ldots 7 \\
   d_2 &= |GI| = |HJ| \text{ (green)} \ldots 2
\end{align*}
\]

Occurrences of the Golden Ratio

\[
\phi = \frac{d_0}{d_1} = \frac{d_4}{d_2} \text{ (28 occurrences)}
\]
2.10 Class of Similar Constructions No. 3

Contained Constructions
RCC2, RCC32, RCC33, RCC34, RCC87, RCC156, RCC165, RCC219
2.10 Class of Similar Constructions No. 3

2.10.1 Construction RCC2

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(A, B) \)
   \( D \in p_1 \cap k_3 \)
4. \( k_4 = k(C, A) \)
   \( E \in k_3 \cap k_4, \quad F \in k_3 \cap k_4 \)
5. \( k_5 = k(D, A) \)
   \( G \in k_3 \cap k_5, \quad H \in k_3 \cap k_5 \)

Distances

\( d_0 = |EH| = |FG| \) (red) \ldots 2
\( d_1 = |BE| = |BF| \) (blue) \ldots 2
\( d_2 = |EG| = |FH| \) (green) \ldots 2

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} = \frac{d_2}{d_1} \] (8 occurrences)
2.10.2 Construction RCC32

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(A, B) \)
   \( D \in p_1 \cap k_3 \)
4. \( k_4 = k(C, B) \)
   \( E \in k_2 \cap k_4 \), \( F \in k_2 \cap k_4 \)
5. \( k_5 = k(D, B) \)
   \( G \in k_2 \cap k_5 \), \( H \in k_2 \cap k_5 \)

Distances

\[ d_0 = |EH| = |FG| \text{ (red)} \ldots 2 \]
\[ d_1 = |AG| = |AH| \text{ (blue)} \ldots 2 \]
\[ d_2 = |EG| = |FH| \text{ (green)} \ldots 2 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} = \frac{d_2}{d_2} \text{ (8 occurrences)} \]
2.10.3 Construction RCC33

Construction Process

\( A, B \) given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(A, B) \)
   \( D \in k_2 \cap k_3, E \in k_2 \cap k_3 \)
4. \( k_4 = k(C, B) \)
   \( F \in p_1 \cap k_4 \)
5. \( k_5 = k(F, B) \)
   \( G \in k_2 \cap k_5, H \in k_2 \cap k_5 \)

Distances

\[ d_0 = |DH| = |EG| \quad \text{(red)} \quad \ldots 2 \]
\[ d_1 = |CG| = |CH| \quad \text{(blue)} \quad \ldots 2 \]
\[ d_2 = |DG| = |EH| \quad \text{(green)} \quad \ldots 2 \]

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \quad \text{(8 occurrences)} \)
2.10.4 Construction RCC34

Construction Process

1. $p_1 = p(A, B)$
2. $k_2 = k(B, A)$
3. $k_3 = k(A, B)$
   
   $C \in p_1 \cap k_3$, $D \in k_2 \cap k_3$, $E \in k_2 \cap k_3$
4. $k_4 = k(C, A)$
   
   $F \in p_1 \cap k_4$
5. $k_5 = k(F, A)$
   
   $G \in k_3 \cap k_5$, $H \in k_3 \cap k_5$

Distances

$d_0 = |DH| = |EG|$ (red) \ldots 2
$d_1 = |CG| = |CH|$ (blue) \ldots 2
$d_2 = |DG| = |EH|$ (green) \ldots 2

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2}$ (8 occurrences)
2.10.5 Construction RCC87

Construction Process

A, B given initial points
1. $p_1 = p(A, B)$
2. $k_2 = k(B, A)$
   $C \in p_1 \cap k_2$
3. $k_3 = k(A, C)$
4. $k_4 = k(C, B)$
   $D \in p_1 \cap k_4$, $E \in k_3 \cap k_4$, $F \in k_3 \cap k_4$
5. $k_5 = k(D, C)$
   $G \in k_4 \cap k_5$, $H \in k_4 \cap k_5$

Distances

- $d_0 = |EH| = |FG|$ (red) \dots 2
- $d_1 = |BE| = |BF|$ (blue) \dots 2
- $d_2 = |EG| = |FH|$ (green) \dots 2

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_1} = \frac{d_4}{d_2}$ (8 occurrences)
2.10.6 Construction RCC156

Construction Process

1. $p_1 = p(A,B)$
2. $k_2 = k(B,A)$
   $C \in p_1 \cap k_2$
3. $k_3 = k(C,B)$
   $D \in p_1 \cap k_3$, $E \in k_2 \cap k_3$, $F \in k_2 \cap k_3$
4. $k_4 = k(D,C)$
   $G \in p_1 \cap k_4$
5. $k_5 = k(G,C)$
   $H \in k_3 \cap k_5$, $I \in k_3 \cap k_5$

Distances

$d_0 = |EI| = |FH|$ (red) \ldots 2
$d_1 = |DH| = |DI|$ (blue) \ldots 2
$d_2 = |EH| = |FI|$ (green) \ldots 2

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2}$ (8 occurrences)
2.10.7 Construction RCC165

Construction Process

1. \( p_1 = p(A, B) \)
2. \( k_2 = k(A, B) \)
3. \( k_3 = k(C, A) \)
4. \( k_4 = k(B, C) \)
5. \( k_5 = k(D, C) \)

Distances

\[ d_0 = |EH| = |FG| \text{ (red)} \ldots 2 \]
\[ d_1 = |AE| = |AF| \text{ (blue)} \ldots 2 \]
\[ d_2 = |EG| = |FH| \text{ (green)} \ldots 2 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} = \frac{d_2}{d_5} \text{ (8 occurrences)} \]
2.10.8 Construction RCC219

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(A, B) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(C, A) \)
   \( D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3 \)
4. \( k_4 = k(D, C) \)
   \( G \in p_1 \cap k_4 \)
5. \( k_5 = k(G, C) \)
   \( H \in k_3 \cap k_5, I \in k_3 \cap k_5 \)

Distances

\( d_0 = |EI| = |FH| \) (red) \ldots 2
\( d_1 = |DH| = |DI| \) (blue) \ldots 2
\( d_2 = |EH| = |FI| \) (green) \ldots 2

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \) (8 occurrences)
2.11 Class of Similar Constructions No. 4

Contained Constructions
RCC3, RCC5, RCC37, RCC39, RCC89, RCC91, RCC128, RCC130, RCC167, RCC169, RCC191, RCC193
2.11 Class of Similar Constructions No. 4

2.11.1 Construction RCC3

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
\( C \in p_1 \cap k_2 \)
3. \( k_3 = k(A, B) \)
\( D \in p_1 \cap k_3 \), \( E \in k_2 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(C, A) \)
\( G \in k_3 \cap k_4 \), \( H \in k_3 \cap k_4 \)
5. \( p_5 = p(A, E) \)
\( I \in p_5 \cap k_3 \), \( J \in p_5 \cap k_4 \)

Distances

\( d_0 = |HJ| \) (red) \ldots 1
\( d_1 = |GI| \) (blue) \ldots 1
\( d_2 = |BI| = |BJ| = |CE| = |CF| = |DE| = |DF| = |EF| \) (green) \ldots 7
\( d_3 = |BG| = |BH| \) (cyan) \ldots 2
\( d_4 = |GJ| \) (magenta) \ldots 1
\( d_5 = |HI| \) (yellow) \ldots 1

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \) (18 occurrences)
2.11 Class of Similar Constructions No. 4

2.11.2 Construction RCC5

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(A, B) \)
   \( D \in p_1 \cap k_3 \) , \( E \in k_2 \cap k_3 \) , \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(C, A) \)
   \( G \in k_3 \cap k_4 \) , \( H \in k_3 \cap k_4 \)
5. \( p_5 = p(A, F) \)
   \( I \in p_5 \cap k_3 \) , \( J \in p_5 \cap k_4 \)

Distances

\[ d_0 = |GJ| \text{ (red)} \ldots 1 \]
\[ d_1 = |HI| \text{ (blue)} \ldots 1 \]
\[ d_2 = |BI| = |BJ| = |CE| = |CF| = |DE| = |DF| = |EF| \text{ (green)} \ldots 7 \]
\[ d_3 = |BG| = |BH| \text{ (cyan)} \ldots 2 \]
\[ d_4 = |HJ| \text{ (magenta)} \ldots 1 \]
\[ d_5 = |GI| \text{ (yellow)} \ldots 1 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_4}{d_4} = \frac{d_3}{d_4} = \frac{d_5}{d_5} \text{ (18 occurrences)} \]
2.11.3 Construction RCC37

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(A, B) \)
   \( D \in p_1 \cap k_3 , E \in k_2 \cap k_3 , F \in k_2 \cap k_3 \)
4. \( k_4 = k(D, B) \)
   \( G \in k_2 \cap k_4 , H \in k_2 \cap k_4 \)
5. \( p_5 = p(B, E) \)
   \( I \in p_5 \cap k_2 , J \in p_5 \cap k_4 \)

Distances

\[
\begin{align*}
  d_0 &= |HJ| \text{ (red)} \ldots 1 \\
  d_1 &= |GI| \text{ (blue)} \ldots 1 \\
  d_2 &= |AJ| = |AJ| = |CE| = |CF| = |DE| = |DF| = |EF| \text{ (green)} \ldots 7 \\
  d_3 &= |AG| = |AH| \text{ (cyan)} \ldots 2 \\
  d_4 &= |GJ| \text{ (magenta)} \ldots 1 \\
  d_5 &= |HI| \text{ (yellow)} \ldots 1
\end{align*}
\]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_4}{d_3} = \frac{d_3}{d_4} = \frac{d_5}{d_4} \] (18 occurrences)
2.11 Class of Similar Constructions No. 4

2.11.4 Construction RCC39

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(A, B) \)
   \( D \in p_1 \cap k_3 , E \in k_2 \cap k_3 , F \in k_2 \cap k_3 \)
4. \( k_4 = k(D, B) \)
   \( G \in k_2 \cap k_4 , H \in k_2 \cap k_4 \)
5. \( p_5 = p(B, F) \)
   \( I \in p_5 \cap k_2 , J \in p_5 \cap k_4 \)

Distances

\[ d_0 = |GJ| \text{ (red)} \ldots 1 \]
\[ d_1 = |HI| \text{ (blue)} \ldots 1 \]
\[ d_2 = |AI| = |AJ| = |CE| = |CF| = |DE| = |DF| = |EF| \text{ (green)} \ldots 7 \]
\[ d_3 = |AG| = |AH| \text{ (cyan)} \ldots 2 \]
\[ d_4 = |HJ| \text{ (magenta)} \ldots 1 \]
\[ d_5 = |GI| \text{ (yellow)} \ldots 1 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (18 occurrences)} \]
2.11.5 Construction RCC89

Construction Process

\( A, B \) given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
    \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(A, C) \)
4. \( k_4 = k(C, B) \)
    \( D \in p_1 \cap k_4 , E \in k_2 \cap k_4 , F \in k_2 \cap k_4 , G \in k_3 \cap k_4 , H \in k_3 \cap k_4 \)
5. \( p_5 = p(C, E) \)
    \( I \in p_5 \cap k_3 , J \in p_5 \cap k_4 \)

Distances

\( d_0 = |HI| \) (red) \ldots 1
\( d_1 = |GJ| \) (blue) \ldots 1
\( d_2 = |AE| = |AF| = |BI| = |BJ| = |DE| = |DF| = |EF| \) (green) \ldots 7
\( d_3 = |BG| = |BH| \) (cyan) \ldots 2
\( d_4 = |GI| \) (magenta) \ldots 1
\( d_5 = |HI| \) (yellow) \ldots 1

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_2} = \frac{d_4}{d_5} = \frac{d_5}{d_5} = \frac{d_5}{d_5} \) (18 occurrences)
2.11.6 Construction RCC91

Construction Process

\(A, B\) given initial points
1. \(p_1 = p(A, B)\)
2. \(k_2 = k(B, A)\)
   \(C \in p_1 \cap k_2\)
3. \(k_3 = k(A, C)\)
4. \(k_4 = k(C, B)\)
   \(D \in p_1 \cap k_4, E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4\)
5. \(p_5 = p(C, F)\)
   \(I \in p_5 \cap k_3, J \in p_5 \cap k_4\)

Distances

\(d_0 = |GI| \) (red) \ldots 1
\(d_1 = |HJ| \) (blue) \ldots 1
\(d_2 = |AE| = |AF| = |BI| = |BJ| = |DF| = |EF| \) (green) \ldots 7
\(d_3 = |BG| = |BH| \) (cyan) \ldots 2
\(d_4 = |HI| \) (magenta) \ldots 1
\(d_5 = |GJ| \) (yellow) \ldots 1

Occurrences of the Golden Ratio

\(\phi = \frac{d_0}{d_2} = \frac{d_2}{d_4} = \frac{d_4}{d_5} = \frac{d_5}{d_5} \) (18 occurrences)
2.11.7 Construction RCC128

Construction Process

1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(C, B) \)
   \( D \in p_1 \cap k_3 , E \in k_2 \cap k_3 , F \in k_2 \cap k_3 \)
4. \( k_4 = k(D, B) \)
   \( G \in k_2 \cap k_4 , H \in k_2 \cap k_4 \)
5. \( p_5 = p(B, E) \)
   \( I \in p_5 \cap k_2 , J \in p_5 \cap k_4 \)

Distances

\[
\begin{align*}
  d_0 &= |HJ| \quad \text{(red)} \ldots 1 \\
  d_1 &= |GI| \quad \text{(blue)} \ldots 1 \\
  d_2 &= |AE| = |AF| = |CI| = |DE| = |DF| = |EF| \quad \text{(green)} \ldots 7 \\
  d_3 &= |CG| = |CH| \quad \text{(cyan)} \ldots 2 \\
  d_4 &= |GJ| \quad \text{(magenta)} \ldots 1 \\
  d_5 &= |HI| \quad \text{(yellow)} \ldots 1 
\end{align*}
\]

Occurrences of the Golden Ratio

\[
\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \quad \text{(18 occurrences)}
\]
2.11.8 Construction RCC130

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(C, B) \)
   \( D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3 \)
4. \( k_4 = k(D, B) \)
   \( G \in k_2 \cap k_4, H \in k_2 \cap k_4 \)
5. \( p_5 = p(B, F) \)
   \( I \in p_5 \cap k_2, J \in p_5 \cap k_4 \)

Distances
\[ d_0 = |GJ| \quad (\text{red}) \quad \ldots 1 \]
\[ d_1 = |HI| \quad (\text{blue}) \quad \ldots 1 \]
\[ d_2 = |AE| = |AF| = |CI| = |CJ| = |DE| = |DF| = |EF| \quad (\text{green}) \quad \ldots 7 \]
\[ d_3 = |CG| = |CH| \quad (\text{cyan}) \quad \ldots 2 \]
\[ d_4 = |HJ| \quad (\text{magenta}) \quad \ldots 1 \]
\[ d_5 = |GI| \quad (\text{yellow}) \quad \ldots 1 \]

Occurrences of the Golden Ratio
\[ \phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \quad (18 \text{ occurrences}) \]
2.11.9 Construction RCC167

Construction Process

A, B given initial points
1. $p_1 = p(A, B)$
2. $k_2 = k(A, B)$
   $C \in p_1 \cap k_2$
3. $k_3 = k(C, A)$
   $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
4. $k_4 = k(B, C)$
   $G \in k_3 \cap k_4, H \in k_3 \cap k_4$
5. $p_5 = p(C, E)$
   $I \in p_5 \cap k_3, J \in p_5 \cap k_4$

Distances

\[ d_0 = |H J| \quad (\text{red}) \ldots 1 \]
\[ d_1 = |G I| \quad (\text{blue}) \ldots 1 \]
\[ d_2 = |A I| = |A J| = |B E| = |B F| = |D E| = |D F| = |E F| \quad (\text{green}) \ldots 7 \]
\[ d_3 = |A G| = |A H| \quad (\text{cyan}) \ldots 2 \]
\[ d_4 = |G J| \quad (\text{magenta}) \ldots 1 \]
\[ d_5 = |H I| \quad (\text{yellow}) \ldots 1 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_4}{d_3} = \frac{d_5}{d_4} = \frac{d_2}{d_5} \quad (18 \text{ occurrences}) \]
2.11 Class of Similar Constructions No. 4

2.11.10 Construction RCC169

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(A, B) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(C, A) \)
   \( D \in p_1 \cap k_3, \ E \in k_2 \cap k_3, \ F \in k_2 \cap k_3 \)
4. \( k_4 = k(B, C) \)
   \( G \in k_3 \cap k_4, \ H \in k_3 \cap k_4 \)
5. \( p_5 = p(C, F) \)
   \( I \in p_5 \cap k_3, \ J \in p_5 \cap k_4 \)

Distances

\[ d_0 = |GJ| \] (red) . . . 1
\[ d_1 = |HI| \] (blue) . . . 1
\[ d_2 = |AJ| = |AE| = |BE| = |BF| = |DE| = |DF| = |EF| \] (green) . . . 7
\[ d_3 = |AG| = |AH| \] (cyan) . . . 2
\[ d_4 = |HJ| \] (magenta) . . . 1
\[ d_5 = |GJ| \] (yellow) . . . 1

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \] (18 occurrences)
2.11.11 Construction RCC191

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(A, B) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(C, A) \)
   \( D \in p_1 \cap k_3 , E \in k_2 \cap k_3 , F \in k_2 \cap k_3 \)
4. \( k_4 = k(D, A) \)
   \( G \in k_2 \cap k_4 , H \in k_2 \cap k_4 \)
5. \( p_5 = p(A, E) \)
   \( I \in p_5 \cap k_2 , J \in p_5 \cap k_4 \)

Distances

\[ d_0 = |HJ| \ (\text{red}) \ldots 1 \]
\[ d_1 = |GI| \ (\text{blue}) \ldots 1 \]
\[ d_2 = |BE| = |BF| = |CI| = |CJ| = |DE| = |DF| = |EF| \ (\text{green}) \ldots 7 \]
\[ d_3 = |CG| = |CH| \ (\text{cyan}) \ldots 2 \]
\[ d_4 = |GJ| \ (\text{magenta}) \ldots 1 \]
\[ d_5 = |HI| \ (\text{yellow}) \ldots 1 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \ (18 \ \text{occurrences}) \]
2.11.12 Construction RCC193

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(A, B) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(C, A) \)
   \( D \in p_1 \cap k_3 , E \in k_2 \cap k_3 , F \in k_2 \cap k_3 \)
4. \( k_4 = k(D, A) \)
   \( G \in k_2 \cap k_4 , H \in k_2 \cap k_4 \)
5. \( p_5 = p(A, F) \)
   \( I \in p_5 \cap k_2 , J \in p_5 \cap k_4 \)

Distances

\( d_0 = |GJ| \) (red) \( \ldots 1 \)
\( d_1 = |HI| \) (blue) \( \ldots 1 \)
\( d_2 = |BE| = |BF| = |CI| = |CJ| = |DE| = |DF| = |EF| \) (green) \( \ldots 7 \)
\( d_3 = |CG| = |CH| \) (cyan) \( \ldots 2 \)
\( d_4 = |HJ| \) (magenta) \( \ldots 1 \)
\( d_5 = |GI| \) (yellow) \( \ldots 1 \)

Occurrences of the Golden Ratio

\( \phi = \frac{d_4}{d_2} = \frac{d_5}{d_3} = \frac{d_4}{d_4} = \frac{d_5}{d_5} \) (18 occurrences)
2.12 Class of Similar Constructions No. 5

Contained Constructions
RCC4, RCC6, RCC36, RCC38, RCC88, RCC90, RCC129, RCC131, RCC166, RCC168, RCC192, RCC194, RCC252, RCC253, RCC264, RCC265, RCC300, RCC301, RCC312, RCC313
2.12 Class of Similar Constructions No. 5

2.12.1 Construction RCC4

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(A, B) \)
   \( D \in p_1 \cap k_3 \), \( E \in k_2 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(C, A) \)
   \( G \in k_3 \cap k_4 \), \( H \in k_3 \cap k_4 \)
5. \( k_5 = k(E, A) \)
   \( I \in k_2 \cap k_5 \), \( J \in k_3 \cap k_5 \), \( K \in k_4 \cap k_5 \)

Distances

\( \phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} = \frac{d_4}{d_6} \) (22 occurrences)
2.12.2 Construction RCC6

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(A, B) \)
   \( D \in p_1 \cap k_3 , E \in k_2 \cap k_3 , F \in k_2 \cap k_3 \)
4. \( k_4 = k(C, A) \)
   \( G \in k_3 \cap k_4 , H \in k_3 \cap k_4 \)
5. \( k_5 = k(F, A) \)
   \( I \in k_2 \cap k_5 , J \in k_3 \cap k_5 , K \in k_4 \cap k_5 \)

Distances

\[ d_0 = |GK| \text{ (red)} \ldots 1 \]
\[ d_1 = |GJ| \text{ (blue)} \ldots 1 \]
\[ d_2 = |AI| = |BJ| = |BK| = |CE| = |CF| = |DE| = |DF| = |EF| = |JK| \text{ (green)} \ldots 9 \]
\[ d_3 = |BG| = |BH| \text{ (cyan)} \ldots 2 \]
\[ d_4 = |HK| \text{ (magenta)} \ldots 1 \]
\[ d_5 = |HJ| \text{ (yellow)} \ldots 1 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_4}{d_5} = \frac{d_5}{d_2} \text{ (22 occurrences)} \]
2.12.3 Construction RCC36

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(A, B) \)
   \( D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3 \)
4. \( k_4 = k(D, B) \)
   \( G \in k_2 \cap k_4, H \in k_2 \cap k_4 \)
5. \( k_5 = k(E, A) \)
   \( I \in k_2 \cap k_5, J \in k_3 \cap k_5, K \in k_4 \cap k_5 \)

Distances

\[
\begin{align*}
\phi &= \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \quad (\text{22 occurrences})
\end{align*}
\]
2.12.4 Construction RCC38

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(A, B) \)
   \( D \in p_1 \cap k_3 \), \( E \in k_2 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(D, B) \)
   \( G \in k_2 \cap k_4 \), \( H \in k_2 \cap k_4 \)
5. \( k_5 = k(F, A) \)
   \( I \in k_2 \cap k_5 \), \( J \in k_3 \cap k_5 \), \( K \in k_4 \cap k_5 \)

Distances

\[ d_0 = |GK| \text{ (red)} \ldots 1 \]
\[ d_1 = |GI| \text{ (blue)} \ldots 1 \]
\[ d_2 = |AI| = |AK| = |BJ| = |CE| = |CF| = |DE| = |DF| = |EF| = |IK| \text{ (green)} \ldots 9 \]
\[ d_3 = |AG| = |AH| \text{ (cyan)} \ldots 2 \]
\[ d_4 = |HK| \text{ (magenta)} \ldots 1 \]
\[ d_5 = |HI| \text{ (yellow)} \ldots 1 \]

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \) (22 occurrences)
### 2.12.5 Construction RCC88

#### Construction Process

1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
3. \( C \in p_1 \cap k_2 \)
4. \( k_4 = k(C, B) \)
5. \( k_5 = k(E, B) \)

Distances

- \( d_0 = |HJ| \) (red) \ldots 1
- \( d_1 = |HK| \) (blue) \ldots 1
- \( d_2 = |AE| = |AF| = |BJ| = |BK| = |CI| = |DE| = |DF| = |EF| = |JK| \) (green) \ldots 9
- \( d_3 = |BG| = |BH| \) (cyan) \ldots 2
- \( d_4 = |GJ| \) (magenta) \ldots 1
- \( d_5 = |GK| \) (yellow) \ldots 1

#### Occurrences of the Golden Ratio

\[
\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \quad (22 \text{ occurrences})
\]
2.12.6 Construction RCC90

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \) 
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(A, C) \)
4. \( k_4 = k(C, B) \)
   \( D \in p_1 \cap k_4 , E \in k_2 \cap k_4 , F \in k_2 \cap k_4 , G \in k_3 \cap k_4 , H \in k_3 \cap k_4 \)
5. \( k_5 = k(F, B) \)
   \( I \in k_2 \cap k_5 , J \in k_3 \cap k_5 , K \in k_4 \cap k_5 \)

Distances

\[
\begin{align*}
d_0 &= |GJ| \text{ (red)} \ldots 1 \\
d_1 &= |GK| \text{ (blue)} \ldots 1 \\
d_2 &= |AE| = |AF| = |BJ| = |BK| = |CI| = |DE| = |DF| = |EF| = |JK| \text{ (green)} \ldots 9 \\
d_3 &= |BG| = |BH| \text{ (cyan)} \ldots 2 \\
d_4 &= |HJ| \text{ (magenta)} \ldots 1 \\
d_5 &= |HK| \text{ (yellow)} \ldots 1 \\
\end{align*}
\]

Occurrences of the Golden Ratio

\[
\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} = \frac{d_4}{d_5} \text{ (22 occurrences)}
\]
2.12.7 Construction RCC129

Construction Process

A, B given initial points

1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(C, B) \)
   \( D \in p_1 \cap k_3 \, , \, E \in k_2 \cap k_3 \, , \, F \in k_2 \cap k_3 \)
4. \( k_4 = k(D, B) \)
   \( G \in k_2 \cap k_4 \, , \, H \in k_2 \cap k_4 \)
5. \( k_5 = k(E, B) \)
   \( I \in k_2 \cap k_5 \, , \, J \in k_3 \cap k_5 \, , \, K \in k_4 \cap k_5 \)

Distances

\[ d_0 = |HK| \text{ (red)} \ldots 1 \]
\[ d_1 = |HI| \text{ (blue)} \ldots 1 \]
\[ d_2 = |AE| = |AF| = |BJ| = |CI| = |CK| = |DE| = |DF| = |EF| = |IK| \text{ (green)} \ldots 9 \]
\[ d_3 = |CG| = |CH| \text{ (cyan)} \ldots 2 \]
\[ d_4 = |GK| \text{ (magenta)} \ldots 1 \]
\[ d_5 = |GI| \text{ (yellow)} \ldots 1 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (22 occurrences)} \]
2.12.8 Construction RCC131

Construction Process

A, B given initial points
1. \( p_1 = p(A,B) \)
2. \( k_2 = k(B,A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(C,B) \)
   \( D \in p_1 \cap k_3 \), \( E \in k_2 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(D,B) \)
   \( G \in k_2 \cap k_4 \), \( H \in k_2 \cap k_4 \)
5. \( k_5 = k(F,B) \)
   \( I \in k_2 \cap k_5 \), \( J \in k_3 \cap k_5 \), \( K \in k_4 \cap k_5 \)

Distances

\( d_0 = |GK| \) (red) \( \ldots 1 \)
\( d_1 = |GI| \) (blue) \( \ldots 1 \)
\( d_2 = |AE| = |AF| = |BJ| = |CT| = |CK| = |DE| = |DF| = |EF| = |IK| \) (green) \( \ldots 9 \)
\( d_3 = |CG| = |CH| \) (cyan) \( \ldots 2 \)
\( d_4 = |HK| \) (magenta) \( \ldots 1 \)
\( d_5 = |HI| \) (yellow) \( \ldots 1 \)

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \) (22 occurrences)
2.12.9 Construction RCC166

Construction Process

- Given initial points $A, B$
- $p_1 = p(A, B)$
- $k_2 = k(A, B)$
  - $C \in p_1 \cap k_2$
- $k_3 = k(C, A)$
  - $D \in p_1 \cap k_3 \ , \ E \in k_2 \cap k_3 \ , \ F \in k_2 \cap k_3$
- $k_4 = k(B, C)$
  - $G \in k_3 \cap k_4 \ , \ H \in k_3 \cap k_4$
- $k_5 = k(E, A)$
  - $I \in k_2 \cap k_5 \ , \ J \in k_3 \cap k_5 \ , \ K \in k_4 \cap k_5$

Distances

- $d_0 = |HK|$ (red) \ldots 1
- $d_1 = |HJ|$ (blue) \ldots 1
- $d_2 = |AJ| = |AK| = |BE| = |BF| = |CI| = |DE| = |DF| = |EF| = |JK|$ (green) \ldots 9
- $d_3 = |AG| = |AH|$ (cyan) \ldots 2
- $d_4 = |GK|$ (magenta) \ldots 1
- $d_5 = |GJ|$ (yellow) \ldots 1

Occurrences of the Golden Ratio

- $\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5}$ (22 occurrences)
2.12.10 Construction RCC168

Construction Process

1. \( p_1 = p(A, B) \)
2. \( k_2 = k(A, B) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(C, A) \)
   \( D \in p_1 \cap k_3 \), \( E \in k_2 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(B, C) \)
   \( G \in k_3 \cap k_4 \), \( H \in k_3 \cap k_4 \)
5. \( k_5 = k(F, A) \)
   \( I \in k_2 \cap k_5 \), \( J \in k_3 \cap k_5 \), \( K \in k_4 \cap k_5 \)

Distances

\[
\begin{align*}
d_0 &= |GK| \text{(red)} \ldots 1 \\
d_1 &= |GJ| \text{(blue)} \ldots 1 \\
d_2 &= |AJ| = |AK| = |BE| = |BF| = |CI| = |DE| = |DF| = |EF| = |JK| \\
& \text{(green)} \ldots 9 \\
d_3 &= |AG| = |AH| \text{(cyan)} \ldots 2 \\
d_4 &= |HK| \text{(magenta)} \ldots 1 \\
d_5 &= |HJ| \text{(yellow)} \ldots 1
\end{align*}
\]

Occurrences of the Golden Ratio

\[
\phi = \frac{d_2}{d_4} = \frac{d_1}{d_3} = \frac{d_3}{d_5} = \frac{d_5}{d_2} \text{ (22 occurrences)}
\]
2.12.11 Construction RCC192

Construction Process

$A, B$ given initial points

1. $p_1 = p(A, B)$
2. $k_2 = k(A, B)$
   $C \in p_1 \cap k_2$
3. $k_3 = k(C, A)$
   $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
4. $k_4 = k(D, A)$
   $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
5. $k_5 = k(E, A)$
   $I \in k_2 \cap k_5, J \in k_3 \cap k_5, K \in k_4 \cap k_5$

Distances

$d_0 = |HK|$ (red) \ldots 1
$d_1 = |HI|$ (blue) \ldots 1
$d_2 = |AJ| = |BE| = |BF| = |CI| = |CK| = |DE| = |DF| = |EF| = |IK|$ (green) \ldots 9
$d_3 = |CG| = |CH|$ (cyan) \ldots 2
$d_4 = |GK|$ (magenta) \ldots 1
$d_5 = |GI|$ (yellow) \ldots 1

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} = \frac{d_4}{d_5}$ (22 occurrences)
2.12 Class of Similar Constructions No. 5

2.12.12 Construction RCC194

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(A, B) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(C, A) \)
   \( D \in p_1 \cap k_3 , E \in k_2 \cap k_3 , F \in k_2 \cap k_3 \)
4. \( k_4 = k(D, A) \)
   \( G \in k_2 \cap k_4 , H \in k_2 \cap k_4 \)
5. \( k_5 = k(F, A) \)
   \( I \in k_2 \cap k_5 , J \in k_3 \cap k_5 , K \in k_4 \cap k_5 \)

Distances

- \( d_0 = |GK| \) (red) \ldots 1
- \( d_1 = |GI| \) (blue) \ldots 1
- \( d_2 = |AJ| = |BE| = |BF| = |CI| = |CK| = |DE| = |DF| = |EF| = |IK| \) (green) \ldots 9
- \( d_3 = |CG| = |CH| \) (cyan) \ldots 2
- \( d_4 = |HK| \) (magenta) \ldots 1
- \( d_5 = |HI| \) (yellow) \ldots 1

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_4}{d_3} = \frac{d_2}{d_4} = \frac{d_4}{d_5} \] (22 occurrences)
2.12.13 Construction RCC252

Construction Process

\( A, B \) given initial points
1. \( k_1 = k(B,A) \)
2. \( k_2 = k(A,B) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( p_3 = p(A,C) \)
   \( E \in p_3 \cap k_2 \)
4. \( k_4 = k(C,A) \)
   \( F \in k_1 \cap k_4 \), \( G \in k_2 \cap k_4 \), \( H \in p_3 \cap k_4 \)
5. \( k_5 = k(E,C) \)
   \( I \in k_1 \cap k_5 \), \( J \in k_4 \cap k_5 \), \( K \in k_4 \cap k_5 \)

Distances

\( d_0 = |IJ| \) (red) \ldots 1
\( d_1 = |FJ| \) (blue) \ldots 1
\( d_2 = |AF| = |AI| = |BE| = |BG| = |BH| = |CD| = |EG| = |FI| = |GH| \) (green) \ldots 9
\( d_3 = |AJ| = |AK| \) (cyan) \ldots 2
\( d_4 = |IK| \) (magenta) \ldots 1
\( d_5 = |FK| \) (yellow) \ldots 1

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_2} = \frac{d_3}{d_4} = \frac{d_4}{d_5} = \frac{d_5}{d_0} \) (22 occurrences)
2.12.14 Construction RCC253

Construction Process

A, B given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2, \ D \in k_1 \cap k_2 \)
3. \( p_3 = p(A, C) \)
   \( E \in p_3 \cap k_2 \)
4. \( k_4 = k(C, A) \)
   \( F \in k_1 \cap k_4, \ G \in k_2 \cap k_4, \ H \in p_3 \cap k_4 \)
5. \( k_5 = k(H, A) \)
   \( I \in k_1 \cap k_5, \ J \in k_2 \cap k_5, \ K \in k_2 \cap k_5 \)

Distances

\( d_0 = |IJ| \) (red) ... 1
\( d_1 = |DJ| \) (blue) ... 1
\( d_2 = |AF| = |BE| = |BG| = |BH| = |CD| = |CI| = |DI| = |EG| = |GH| \) (green) ... 9
\( d_3 = |CJ| = |CK| \) (cyan) ... 2
\( d_4 = |IK| \) (magenta) ... 1
\( d_5 = |DK| \) (yellow) ... 1

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \) (22 occurrences)
2.12.15 Construction RCC264

Construction Process

A, B given initial points
1. $k_1 = k(B, A)$
2. $k_2 = k(A, B)$
   $C \in k_1 \cap k_2$, $D \in k_1 \cap k_2$
3. $k_3 = k(C, A)$
   $E \in k_1 \cap k_3$, $F \in k_2 \cap k_3$
4. $p_4 = p(B, C)$
   $G \in p_4 \cap k_1$, $H \in p_4 \cap k_3$
5. $k_5 = k(G, C)$
   $I \in k_2 \cap k_5$, $J \in k_3 \cap k_5$, $K \in k_3 \cap k_5$

Distances

$d_0 = |IK|$ (red) \ldots 1
$d_1 = |FK|$ (blue) \ldots 1
$d_2 = |AE| = |AG| = |AH| = |BF| = |BI| = |CD| = |EG| = |EH| = |FI|$
(green) \ldots 9
$d_3 = |BJ| = |BK|$ (cyan) \ldots 2
$d_4 = |IJ|$ (magenta) \ldots 1
$d_5 = |FJ|$ (yellow) \ldots 1

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_4}{d_5}$ (22 occurrences)
2.12.16 Construction RCC265

Construction Process

\[ A, B \text{ given initial points} \]
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2, \ D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, A) \)
   \( E \in k_1 \cap k_3, \ F \in k_2 \cap k_3 \)
4. \( p_4 = p(B, C) \)
   \( G \in p_4 \cap k_1, \ H \in p_4 \cap k_3 \)
5. \( k_5 = k(H, B) \)
   \( I \in k_1 \cap k_5, \ J \in k_1 \cap k_5, \ K \in k_2 \cap k_5 \)

Distances

\[ d_0 = |JK| \text{ (red)} \ldots 1 \]
\[ d_1 = |DJ| \text{ (blue)} \ldots 1 \]
\[ d_2 = |AE| = |AG| = |AH| = |BF| = |CD| = |CK| = |DK| = |EG| = |EH| \text{ (green)} \ldots 9 \]
\[ d_3 = |CI| = |CJ| \text{ (cyan)} \ldots 2 \]
\[ d_4 = |IK| \text{ (magenta)} \ldots 1 \]
\[ d_5 = |DI| \text{ (yellow)} \ldots 1 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_4}{d_5} \text{ (22 occurrences)} \]
2.12.17 Construction RCC300

Construction Process

- A, B given initial points
- $k_1 = k(B, A)$
- $k_2 = k(A, B)$
- $C \in k_1 \cap k_2$, $D \in k_1 \cap k_2$
- $p_3 = p(A, D)$
- $E \in p_3 \cap k_2$
- $k_4 = k(D, A)$
- $F \in k_1 \cap k_4$, $G \in k_2 \cap k_4$, $H \in p_3 \cap k_4$
- $k_5 = k(E, D)$
- $I \in k_1 \cap k_5$, $J \in k_4 \cap k_5$, $K \in k_4 \cap k_5$

Distances

- $d_0 = |IJ|$ (red) \ldots 1
- $d_1 = |FJ|$ (blue) \ldots 1
- $d_2 = |AF| = |AI| = |BE| = |BG| = |BH| = |CD| = |EG| = |FI| = |GH|$ (green) \ldots 9
- $d_3 = |AJ| = |AK|$ (cyan) \ldots 2
- $d_4 = |IK|$ (magenta) \ldots 1
- $d_5 = |FK|$ (yellow) \ldots 1

Occurrences of the Golden Ratio

- $\phi = \frac{d_0}{d_2} = \frac{d_4}{d_3} = \frac{d_4}{d_2} = \frac{d_5}{d_2}$ (22 occurrences)
2.12 Class of Similar Constructions No. 5

2.12.18 Construction RCC301

Construction Process

A, B given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2 , \ D \in k_1 \cap k_2 \)
3. \( p_3 = p(A, D) \)
   \( E \in p_3 \cap k_2 \)
4. \( k_4 = k(D, A) \)
   \( F \in k_1 \cap k_4 , \ G \in k_2 \cap k_4 , \ H \in p_3 \cap k_4 \)
5. \( k_5 = k(H, A) \)
   \( I \in k_1 \cap k_5 , \ J \in k_2 \cap k_5 , \ K \in k_2 \cap k_5 \)

Distances

\[ d_0 = |IJ| \text{ (red)} \ldots 1 \]
\[ d_1 = |CJ| \text{ (blue)} \ldots 1 \]
\[ d_2 = |AF| = |BE| = |BG| = |BH| = |CD| = |CI| = |DI| = |EG| = |GH| \text{ (green)} \ldots 9 \]
\[ d_3 = |DJ| = |DK| \text{ (cyan)} \ldots 2 \]
\[ d_4 = |IK| \text{ (magenta)} \ldots 1 \]
\[ d_5 = |CK| \text{ (yellow)} \ldots 1 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_4}{d_2} = \frac{d_4}{d_3} = \frac{d_2}{d_4} = \frac{d_4}{d_5} \text{ (22 occurrences)} \]
2.12.19 Construction RCC312

Construction Process

A, B given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, A) \)
   \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( p_4 = p(B, D) \)
   \( G \in p_4 \cap k_1 \), \( H \in p_4 \cap k_3 \)
5. \( k_5 = k(G, D) \)
   \( I \in k_2 \cap k_5 \), \( J \in k_3 \cap k_5 \), \( K \in k_3 \cap k_5 \)

Distances

\[
\begin{align*}
d_0 &= |IK| \text{ (red)} \ldots 1 \\
d_1 &= |FK| \text{ (blue)} \ldots 1 \\
d_2 &= |AE| = |AG| = |AH| = |BF| = |BI| = |CD| = |EG| = |EH| = |FI| \\
     &\text{ (green)} \ldots 9 \\
d_3 &= |BJ| = |BK| \text{ (cyan)} \ldots 2 \\
d_4 &= |IJ| \text{ (magenta)} \ldots 1 \\
d_5 &= |FJ| \text{ (yellow)} \ldots 1
\end{align*}
\]

Occurrences of the Golden Ratio

\[
\phi = \frac{d_3}{d_2} = \frac{d_4}{d_3} = \frac{d_5}{d_4} = \frac{d_3}{d_4} \text{ (22 occurrences)}
\]
2.12 Class of Similar Constructions No. 5

2.12.20 Construction RCC313

Construction Process

A, B given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, A) \)
   \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( p_4 = p(B, D) \)
   \( G \in p_4 \cap k_1 \), \( H \in p_4 \cap k_3 \)
5. \( k_5 = k(H, B) \)
   \( I \in k_1 \cap k_5 \), \( J \in k_1 \cap k_5 \), \( K \in k_2 \cap k_5 \)

Distances

\( d_0 = |JK| \) (red) \ldots 1
\( d_1 = |CJ| \) (blue) \ldots 1
\( d_2 = |AE| = |AG| = |AH| = |BF| = |CD| = |CK| = |DK| = |EG| = |EH| \) (green) \ldots 9
\( d_3 = |DJ| = |DJ| \) (cyan) \ldots 2
\( d_4 = |IK| \) (magenta) \ldots 1
\( d_5 = |CI| \) (yellow) \ldots 1

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \) (22 occurrences)
2.13 Class of Similar Constructions No. 6

Contained Constructions

RCC7, RCC16, RCC40, RCC49, RCC93, RCC102, RCC132, RCC141, RCC171, RCC180, RCC195, RCC204
2.13 Class of Similar Constructions No. 6

2.13.1 Construction RCC7

Construction Process

A, B given initial points
1. \( p_1 = p(A,B) \)
2. \( k_2 = k(B,A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(A,B) \)
   \( D \in p_1 \cap k_3, \ E \in k_2 \cap k_3, \ F \in k_2 \cap k_3 \)
4. \( k_4 = k(C,A) \)
   \( G \in k_3 \cap k_4, \ H \in k_3 \cap k_4 \)
5. \( p_5 = p(A,G) \)
   \( I \in p_5 \cap k_2, \ J \in p_5 \cap k_3 \)

Distances

\[ d_0 = |EJ| \text{ (red)} \ldots 1 \]
\[ d_1 = |BG| = |BH| = |DI| = |DJ| \text{ (blue)} \ldots 4 \]
\[ d_2 = |FJ| \text{ (green)} \ldots 1 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)} \]
2.13.2 Construction RCC16

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(A, B) \)
   \( D \in p_1 \cap k_3 , E \in k_2 \cap k_3 , F \in k_2 \cap k_3 \)
4. \( k_4 = k(C, A) \)
   \( G \in k_3 \cap k_4 , H \in k_3 \cap k_4 \)
5. \( p_5 = p(A, H) \)
   \( I \in p_5 \cap k_2 , J \in p_5 \cap k_3 \)

Distances

\( d_0 = |FJ| \) (red) \ldots 1
\( d_1 = |BG| = |BH| = |DI| = |DJ| \) (blue) \ldots 4
\( d_2 = |EJ| \) (green) \ldots 1

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} = \frac{d_2}{d_4} \) (8 occurrences)
2.13.3 Construction RCC40

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(A, B) \)
   \( D \in p_1 \cap k_3, \ E \in k_2 \cap k_3, \ F \in k_2 \cap k_3 \)
4. \( k_4 = k(D, B) \)
   \( G \in k_2 \cap k_4, \ H \in k_2 \cap k_4 \)
5. \( p_5 = p(B, G) \)
   \( I \in p_5 \cap k_2, \ J \in p_5 \cap k_3 \)

Distances

\( d_0 = |EI| \) (red) \( \ldots 1 \)
\( d_1 = |AG| = |AH| = |CI| = |CJ| \) (blue) \( \ldots 4 \)
\( d_2 = |FI| \) (green) \( \ldots 1 \)

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} = \frac{d_2}{d_2} \) (8 occurrences)
2.13.4 Construction RCC49

Construction Process

$A$, $B$ given initial points
1. $p_1 = p(A, B)$
2. $k_2 = k(B, A)$
   $C \in p_1 \cap k_2$
3. $k_3 = k(A, B)$
   $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
4. $k_4 = k(D, B)$
   $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
5. $p_5 = p(B, H)$
   $I \in p_5 \cap k_2, J \in p_5 \cap k_3$

Distances

$d_0 = |FI|$ (red) ... 1
$d_1 = |AG| = |AH| = |CI| = |CJ|$ (blue) ... 4
$d_2 = |EI|$ (green) ... 1

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_1} = \frac{d_2}{d_2}$ (8 occurrences)
2.13.5 Construction RCC93

Construction Process

\(A, B\) given initial points

1. \(p_1 = p(A, B)\)
2. \(k_2 = k(B, A)\)
   \(C \in p_1 \cap k_2\)
3. \(k_3 = k(A, C)\)
4. \(k_4 = k(C, B)\)
   \(D \in p_1 \cap k_4, E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4\)
5. \(p_5 = p(C, G)\)
   \(I \in p_5 \cap k_2, J \in p_5 \cap k_4\)

Distances

\[d_0 = |EJ| \text{ (red)} \ldots 1\]
\[d_1 = |BG| = |BH| = |DI| = |DJ| \text{ (blue)} \ldots 4\]
\[d_2 = |FJ| \text{ (green)} \ldots 1\]

Occurrences of the Golden Ratio

\[\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)}\]
2.13.6 Construction RCC102

Construction Process

A, B given initial points

1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(A, C) \)
4. \( k_4 = k(C, B) \)
   \( D \in p_1 \cap k_4 , E \in k_2 \cap k_4 , F \in k_2 \cap k_4 , G \in k_3 \cap k_4 , H \in k_3 \cap k_4 \)
5. \( p_5 = p(C, H) \)
   \( I \in p_5 \cap k_2 , J \in p_5 \cap k_4 \)

Distances

\( d_0 = |FJ| \) (red) ... 1
\( d_1 = |BG| = |BH| = |DI| = |DJ| \) (blue) ... 4
\( d_2 = |EJ| \) (green) ... 1

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \) (8 occurrences)
2.13.7 Construction RCC132

Construction Process

1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \) 
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(C, B) \)
   \( D \in p_1 \cap k_3 , E \in k_2 \cap k_3 , F \in k_2 \cap k_3 \)
4. \( k_4 = k(D, B) \)
   \( G \in k_2 \cap k_4 , H \in k_2 \cap k_4 \)
5. \( p_5 = p(B, G) \)
   \( I \in p_5 \cap k_2 , J \in p_5 \cap k_3 \)

Distances

\[ d_0 = |EI| \text{ (red) } \cdots 1 \]
\[ d_1 = |AI| = |AJ| = |CG| = |CH| \text{ (blue) } \cdots 4 \]
\[ d_2 = |FI| \text{ (green) } \cdots 1 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)} \]
2.13.8 Construction RCC141

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(C, B) \)
   \( D \in p_1 \cap k_3 \), \( E \in k_2 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(D, B) \)
   \( G \in k_2 \cap k_4 \), \( H \in k_2 \cap k_4 \)
5. \( p_5 = p(B, H) \)
   \( I \in p_5 \cap k_2 \), \( J \in p_5 \cap k_3 \)

Distances

\[
\begin{align*}
  d_0 & = |FI| \quad \text{(red)} \ldots 1 \\
  d_1 & = |AI| = |AJ| = |CG| = |CH| \quad \text{(blue)} \ldots 4 \\
  d_2 & = |EI| \quad \text{(green)} \ldots 1 
\end{align*}
\]

Occurrences of the Golden Ratio

\[
\phi = \frac{d_0}{d_1} = \frac{d_2}{d_2} \quad 8 \text{ occurrences}
\]
2.13.9 Construction RCC171

Construction Process

A, B given initial points
1. $p_1 = p(A, B)$
2. $k_2 = k(A, B)$
   $C \in p_1 \cap k_2$
3. $k_3 = k(C, A)$
   $D \in p_1 \cap k_3$, $E \in k_2 \cap k_3$, $F \in k_2 \cap k_3$
4. $k_4 = k(B, C)$
   $G \in k_3 \cap k_4$, $H \in k_3 \cap k_4$
5. $p_5 = p(C, G)$
   $I \in p_5 \cap k_2$, $J \in p_5 \cap k_3$

Distances

$d_0 = |EJ|$ (red) ... 1
$d_1 = |AG| = |AH| = |DI| = |DJ|$ (blue) ... 4
$d_2 = |FJ|$ (green) ... 1

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_1} = \frac{d_2}{d_1}$ (8 occurrences)
2.13.10 Construction RCC180

Construction Process

A, B given initial points
1. \( p_1 = p(A,B) \)
2. \( k_2 = k(A,B) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(C,A) \)
   \( D \in p_1 \cap k_3 , E \in k_2 \cap k_3 , F \in k_2 \cap k_3 \)
4. \( k_4 = k(B,C) \)
   \( G \in k_3 \cap k_4 , H \in k_3 \cap k_4 \)
5. \( p_5 = p(C,H) \)
   \( I \in p_5 \cap k_2 , J \in p_5 \cap k_3 \)

Distances

\( d_0 = |FJ| \) (red) \( \ldots 1 \)
\( d_1 = |AG| = |AH| = |DI| = |DJ| \) (blue) \( \ldots 4 \)
\( d_2 = |EJ| \) (green) \( \ldots 1 \)

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \) (8 occurrences)
2.13.11 Construction RCC195

Construction Process

\( A, B \) given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(A, B) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(C, A) \)
   \( D \in p_1 \cap k_3 , E \in k_2 \cap k_3 , F \in k_2 \cap k_3 \)
4. \( k_4 = k(D, A) \)
   \( G \in k_2 \cap k_4 , H \in k_2 \cap k_4 \)
5. \( p_5 = p(A, G) \)
   \( I \in p_5 \cap k_2 , J \in p_5 \cap k_3 \)

Distances

\[ d_0 = |EI| \quad \text{(red)} \ldots 1 \]
\[ d_1 = |BI| = |BJ| = |CG| = |CH| \quad \text{(blue)} \ldots 4 \]
\[ d_2 = |FI| \quad \text{(green)} \ldots 1 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \quad \text{(8 occurrences)} \]
2.13.12 Construction RCC204

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(A, B) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(C, A) \)
   \( D \in p_1 \cap k_3 , E \in k_2 \cap k_3 , F \in k_2 \cap k_3 \)
4. \( k_4 = k(D, A) \)
   \( G \in k_2 \cap k_4 , H \in k_2 \cap k_4 \)
5. \( p_5 = p(A, H) \)
   \( I \in p_5 \cap k_2 , J \in p_5 \cap k_3 \)

Distances

\[ d_0 = |FI| \text{ (red)} \ldots 1 \]
\[ d_1 = |BI| = |BJ| = |CG| = |CH| \text{ (blue)} \ldots 4 \]
\[ d_2 = |EI| \text{ (green)} \ldots 1 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} = \frac{d_2}{d_2} \text{ (8 occurrences)} \]
2.14 Class of Similar Constructions No. 7

Contained Constructions

RCC8, RCC17, RCC41, RCC50, RCC94, RCC103, RCC133, RCC142, RCC172, RCC181, RCC196, RCC205
2.14 Class of Similar Constructions No. 7

2.14.1 Construction RCC8

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(A, B) \)
   \( D \in p_1 \cap k_3 , \ E \in k_2 \cap k_3 , \ F \in k_2 \cap k_3 \)
4. \( k_4 = k(C, A) \)
   \( G \in k_3 \cap k_4 , \ H \in k_3 \cap k_4 \)
5. \( k_5 = k(G, A) \)
   \( I \in p_1 \cap k_5 , \ J \in k_2 \cap k_5 , \ K \in k_3 \cap k_5 , \ L \in k_3 \cap k_5 \)

Distances

\[ d_0 = |CK| \text{ (red)} \] ... 1
\[ d_1 = |DL| = |FJ| = |FK| = |JK| \text{ (blue)} \] ... 4
\[ d_2 = |CE| = |CF| = |DE| = |DF| = |EF| = |KL| \text{ (green)} \] ... 6
\[ d_3 = |IK| \text{ (cyan)} \] ... 1
\[ d_4 = |BG| = |BH| = |EJ| = |EK| = |FL| = |IJ| \text{ (magenta)} \] ... 6
\[ d_5 = |CL| \text{ (yellow)} \] ... 1
\[ d_6 = |EI| = |FI| \text{ (grass)} \] ... 2
\[ d_7 = |DK| = |EJ| = |EL| = |JL| \text{ (sea)} \] ... 4
\[ d_8 = |IL| \text{ (darkorange)} \] ... 1

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_1}{d_4} = \frac{d_2}{d_5} = \frac{d_3}{d_6} = \frac{d_4}{d_8} = \frac{d_6}{d_8} \] (64 occurrences)
2.14.2 Construction RCC17

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(A, B) \)
   \( D \in p_1 \cap k_3 , E \in k_2 \cap k_3 , F \in k_2 \cap k_3 \)
4. \( k_4 = k(C, A) \)
   \( G \in k_3 \cap k_4 , H \in k_3 \cap k_4 \)
5. \( k_5 = k(H, A) \)
   \( I \in p_1 \cap k_5 , J \in k_2 \cap k_5 , K \in k_3 \cap k_5 , L \in k_3 \cap k_5 \)

Distances

\[ d_0 = |CK| \text{ (red)} \ldots 1 \]
\[ d_1 = |DL| = |EJ| = |EK| = |JK| \text{ (blue)} \ldots 4 \]
\[ d_2 = |CE| = |CF| = |DE| = |DF| = |EF| = |KL| \text{ (green)} \ldots 6 \]
\[ d_3 = |JK| \text{ (cyan)} \ldots 1 \]
\[ d_4 = |BG| = |BH| = |CJ| = |EL| = |FK| = |IJ| \text{ (magenta)} \ldots 6 \]
\[ d_5 = |CL| \text{ (yellow)} \ldots 1 \]
\[ d_6 = |EI| = |FI| \text{ (grass)} \ldots 2 \]
\[ d_7 = |DK| = |FJ| = |FL| = |JL| \text{ (sea)} \ldots 4 \]
\[ d_8 = |IL| \text{ (darkorange)} \ldots 1 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_1}{d_4} = \frac{d_2}{d_5} = \frac{d_3}{d_7} = \frac{d_4}{d_8} = \frac{d_6}{d_8} \text{ (64 occurrences)} \]
2.14.3 Construction RCC41

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
3. \( k_3 = k(A, B) \)
   \( C \in p_1 \cap k_2 \)
4. \( k_4 = k(D, B) \)
   \( D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3 \)
5. \( k_5 = k(G, B) \)
   \( I \in p_1 \cap k_5, J \in k_2 \cap k_5, K \in k_2 \cap k_5, L \in k_3 \cap k_5 \)

Distances

\[
\begin{align*}
  d_0 &= |DK| \text{ (red)} \ldots 1 \\
  d_1 &= |CJ| = |FK| = |FL| = |KL| \text{ (blue)} \ldots 4 \\
  d_2 &= |CE| = |CF| = |DE| = |DF| = |EF| = |JK| \text{ (green)} \ldots 6 \\
  d_3 &= |IK| \text{ (cyan)} \ldots 1 \\
  d_4 &= |AG| = |AH| = |DL| = |EK| = |FJ| = |IL| \text{ (magenta)} \ldots 6 \\
  d_5 &= |DJ| \text{ (yellow)} \ldots 1 \\
  d_6 &= |EI| = |FI| \text{ (grass)} \ldots 2 \\
  d_7 &= |CK| = |CE| = |EL| = |JL| \text{ (sea)} \ldots 4 \\
  d_8 &= |IJ| \text{ (darkorange)} \ldots 1 \\
\end{align*}
\]

Occurrences of the Golden Ratio

\[
\phi = \frac{d_0}{d_2} = \frac{d_4}{d_6} = \frac{d_2}{d_4} = \frac{d_4}{d_6} = \frac{d_6}{d_8} = \frac{d_8}{d_0} \text{ (64 occurrences)}
\]
2.14.4 Construction RCC50

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(A, B) \)
   \( D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3 \)
4. \( k_4 = k(D, B) \)
   \( G \in k_2 \cap k_4, H \in k_2 \cap k_4 \)
5. \( k_5 = k(H, B) \)
   \( I \in p_1 \cap k_5, J \in k_2 \cap k_5, K \in k_2 \cap k_5, L \in k_3 \cap k_5 \)

Distances

\[ d_0 = |DK| \text{ (red)} \ldots 1 \]
\[ d_1 = |CJ| = |EK| = |EL| = |KL| \text{ (blue)} \ldots 4 \]
\[ d_2 = |CE| = |CF| = |DE| = |DF| = |EF| = |JK| \text{ (green)} \ldots 6 \]
\[ d_3 = |IK| \text{ (cyan)} \ldots 1 \]
\[ d_4 = |AG| = |AH| = |DL| = |EJ| = |FK| = |IL| \text{ (magenta)} \ldots 6 \]
\[ d_5 = |DJ| \text{ (yellow)} \ldots 1 \]
\[ d_6 = |EI| = |FI| \text{ (grass)} \ldots 2 \]
\[ d_7 = |CK| = |FJ| = |FL| = |JL| \text{ (sea)} \ldots 4 \]
\[ d_8 = |IJ| \text{ (darkorange)} \ldots 1 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} = \frac{d_4}{d_6} = \frac{d_5}{d_8} \text{ (64 occurrences)} \]
2.14.5 Construction RCC94

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(A, C) \)
4. \( k_4 = k(C, B) \)
   \( D \in p_1 \cap k_4 \), \( E \in k_2 \cap k_4 \), \( F \in k_2 \cap k_4 \), \( G \in k_3 \cap k_4 \), \( H \in k_3 \cap k_4 \)
5. \( k_5 = k(G, C) \)
   \( I \in p_1 \cap k_5 \), \( J \in k_2 \cap k_5 \), \( K \in k_4 \cap k_5 \), \( L \in k_4 \cap k_5 \)

Distances

\( d_0 = |AL| \) (red) \... 1
\( d_1 = |DK| = |FJ| = |FL| = |JL| \) (blue) \... 4
\( d_2 = |AE| = |AF| = |DE| = |DF| = |EF| = |KL| \) (green) \... 6
\( d_3 = |IL| \) (cyan) \... 1
\( d_4 = |AJ| = |BG| = |BH| = |EL| = |FK| = |IJ| \) (magenta) \... 6
\( d_5 = |AK| \) (yellow) \... 1
\( d_6 = |EI| = |FI| \) (grass) \... 2
\( d_7 = |DL| = |EJ| = |EK| = |JK| \) (sea) \... 4
\( d_8 = |IK| \) (darkorange) \... 1

Occurrences of the Golden Ratio

\( \phi = \frac{d_6}{d_2} = \frac{d_1}{d_4} = \frac{d_4}{d_2} = \frac{d_4}{d_6} = \frac{d_1}{d_6} \) (64 occurrences)
2.14.6 Construction RCC103

Construction Process

1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
3. \( C \in p_1 \cap k_2 \)
4. \( k_3 = k(A, C) \)
5. \( k_4 = k(C, B) \)
6. \( D \in p_1 \cap k_4 , E \in k_2 \cap k_4 , F \in k_2 \cap k_4 , G \in k_3 \cap k_4 , H \in k_3 \cap k_4 \)
7. \( I \in p_1 \cap k_5 , J \in k_2 \cap k_5 , K \in k_4 \cap k_5 , L \in k_4 \cap k_5 \)

Distances

\begin{align*}
  d_0 &= |AL| \ (red) \ ... \ 1 \\
  d_1 &= |DK| = |EJ| = |EL| = |JL| \ (blue) \ ... \ 4 \\
  d_2 &= |AE| = |AF| = |DE| = |DF| = |EF| = |KL| \ (green) \ ... \ 6 \\
  d_3 &= |IL| \ (cyan) \ ... \ 1 \\
  d_4 &= |AJ| = |BG| = |BH| = |EK| = |FL| = |IJ| \ (magenta) \ ... \ 6 \\
  d_5 &= |AK| \ (yellow) \ ... \ 1 \\
  d_6 &= |EI| = |FI| \ (grass) \ ... \ 2 \\
  d_7 &= |DL| = |FJ| = |FK| = |JK| \ (sea) \ ... \ 4 \\
  d_8 &= |IK| \ (darkorange) \ ... \ 1 \\
\end{align*}

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_2} = \frac{d_4}{d_4} = \frac{d_5}{d_5} = \frac{d_6}{d_6} = \frac{d_8}{d_8} \ (64 \ occurrences) \)
2.14.7 Construction RCC133

Construction Process

1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
3. \( C \in p_1 \cap k_2 \)
4. \( k_3 = k(C, B) \)
5. \( D \in p_1 \cap k_3 \), \( E \in k_2 \cap k_3 \), \( F \in k_2 \cap k_3 \)
6. \( k_4 = k(D, B) \)
7. \( G \in k_2 \cap k_4 \), \( H \in k_2 \cap k_4 \)
8. \( I \in p_1 \cap k_5 \), \( J \in k_2 \cap k_5 \), \( K \in k_2 \cap k_5 \), \( L \in k_3 \cap k_5 \)

Distances

\[
\begin{align*}
d_0 &= |DJ| \text{ (red)} \ldots 1 \\
d_1 &= |AK| = |FJ| = |FL| = |JL| \text{ (blue)} \ldots 4 \\
d_2 &= |AE| = |AF| = |DE| = |DF| = |EF| = |JK| \text{ (green)} \ldots 6 \\
d_3 &= |IJ| \text{ (cyan)} \ldots 1 \\
d_4 &= |CG| = |CH| = |DL| = |EL| = |FK| = |IL| \text{ (magenta)} \ldots 6 \\
d_5 &= |DK| \text{ (yellow)} \ldots 1 \\
d_6 &= |EI| = |FI| \text{ (grass)} \ldots 2 \\
d_7 &= |AJ| = |EK| = |EL| = |KL| \text{ (sea)} \ldots 4 \\
d_8 &= |JK| \text{ (darkorange)} \ldots 1
\end{align*}
\]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_4}{d_6} = \frac{d_7}{d_5} = \frac{d_8}{d_6} = \frac{d_4}{d_6} = \frac{d_8}{d_6} \text{ (64 occurrences)} \]

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2.14.8 Construction RCC142

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(C, B) \)
   \( D \in p_1 \cap k_3 \), \( E \in k_2 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(D, B) \)
   \( G \in k_2 \cap k_4 \), \( H \in k_2 \cap k_4 \)
5. \( k_5 = k(H, B) \)
   \( I \in p_1 \cap k_5 \), \( J \in k_2 \cap k_5 \), \( K \in k_2 \cap k_5 \), \( L \in k_3 \cap k_5 \)

Distances

- \( d_0 = |DJ| \) (red) . . . 1
- \( d_1 = |AK| = |EJ| = |EL| = |JL| \) (blue) . . . 4
- \( d_2 = |AE| = |AF| = |DE| = |DF| = |EF| = |JK| \) (green) . . . 6
- \( d_3 = |J| \) (cyan) . . . 1
- \( d_4 = |CG| = |CH| = |DL| = |EK| = |FJ| = |IL| \) (magenta) . . . 6
- \( d_5 = |DK| \) (yellow) . . . 1
- \( d_6 = |EI| = |FI| \) (grass) . . . 2
- \( d_7 = |AJ| = |FK| = |FL| = |KL| \) (sea) . . . 4
- \( d_8 = |IK| \) (darkorange) . . . 1

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_1}{d_4} = \frac{d_2}{d_5} = \frac{d_3}{d_6} = \frac{d_4}{d_7} = \frac{d_5}{d_8} \] (64 occurrences)
2.14.9 Construction RCC172

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(A, B) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(C, A) \)
   \( D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3 \)
4. \( k_4 = k(B, C) \)
   \( G \in k_3 \cap k_4, H \in k_3 \cap k_4 \)
5. \( k_5 = k(G, C) \)
   \( I \in p_1 \cap k_5, J \in k_2 \cap k_5, K \in k_3 \cap k_5, L \in k_3 \cap k_5 \)

Distances

\[
\begin{align*}
  d_0 &= |BK| \quad \text{(red)} \quad \ldots 1 \\
  d_1 &= |DL| = |FJ| = |FK| = |JK| \quad \text{(blue)} \quad \ldots 4 \\
  d_2 &= |BE| = |BF| = |DE| = |DF| = |EF| = |KL| \quad \text{(green)} \quad \ldots 6 \\
  d_3 &= |IK| \quad \text{(cyan)} \quad \ldots 1 \\
  d_4 &= |AG| = |AH| = |BJ| = |EK| = |FL| = |IJ| \quad \text{(magenta)} \quad \ldots 6 \\
  d_5 &= |BL| \quad \text{(yellow)} \quad \ldots 1 \\
  d_6 &= |EI| = |FI| \quad \text{(grass)} \quad \ldots 2 \\
  d_7 &= |DK| = |EJ| = |EL| = |JL| \quad \text{(sea)} \quad \ldots 4 \\
  d_8 &= |IL| \quad \text{(darkorange)} \quad \ldots 1
\end{align*}
\]

Occurrences of the Golden Ratio

\[
\phi = \frac{d_6}{d_2} = \frac{d_4}{d_2} = \frac{d_2}{d_2} = \frac{d_4}{d_4} = \frac{d_6}{d_6} \quad (64 \text{ occurrences})
\]
2.14.10 Construction RCC181

Construction Process

A, B given initial points
1. $p_1 = p(A,B)$
2. $k_2 = k(A,B)$
3. $k_3 = k(C,A)$
   - $D \in p_1 \cap k_3$ , $E \in k_2 \cap k_3$ , $F \in k_2 \cap k_3$
4. $k_4 = k(B,C)$
   - $G \in k_3 \cap k_4$ , $H \in k_3 \cap k_4$
5. $k_5 = k(H,C)$
   - $I \in p_1 \cap k_5$ , $J \in k_2 \cap k_5$ , $K \in k_3 \cap k_5$ , $L \in k_3 \cap k_5$

Distances

\[ d_0 = |BK| \text{ (red)} \ldots 1 \]
\[ d_1 = |DL| = |EJ| = |EK| = |JK| \text{ (blue)} \ldots 4 \]
\[ d_2 = |BE| = |BF| = |DE| = |DF| = |EF| = |KL| \text{ (green)} \ldots 6 \]
\[ d_3 = |IK| \text{ (cyan)} \ldots 1 \]
\[ d_4 = |AG| = |AH| = |BJ| = |EL| = |FK| = |IJ| \text{ (magenta)} \ldots 6 \]
\[ d_5 = |BL| \text{ (yellow)} \ldots 1 \]
\[ d_6 = |EI| = |FI| \text{ (grass)} \ldots 2 \]
\[ d_7 = |DK| = |FJ| = |FL| = |JL| \text{ (sea)} \ldots 4 \]
\[ d_8 = |IL| \text{ (darkorange)} \ldots 1 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_6}{d_4} = \frac{d_4}{d_2} = \frac{d_2}{d_4} = \frac{d_6}{d_4} = \frac{d_4}{d_2} = \frac{d_6}{d_4} (64 \text{ occurrences}) \]
2.14 Class of Similar Constructions No. 7

2.14.11 Construction RCC196

Construction Process

\[ A, B \text{ given initial points} \]
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(A, B) \)
   \[ C \in p_1 \cap k_2 \]
3. \( k_3 = k(C, A) \)
   \[ D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3 \]
4. \( k_4 = k(D, A) \)
   \[ G \in k_2 \cap k_4, H \in k_2 \cap k_4 \]
5. \( k_5 = k(G, A) \)
   \[ I \in p_1 \cap k_5, J \in k_2 \cap k_5, K \in k_2 \cap k_5, L \in k_3 \cap k_5 \]

Distances

\[ d_0 = |DK| \quad (\text{red}) \ldots 1 \]
\[ d_1 = |BJ| = |FK| = |FL| = |KL| \quad (\text{blue}) \ldots 4 \]
\[ d_2 = |BE| = |BF| = |DE| = |DF| = |EF| = |JK| \quad (\text{green}) \ldots 6 \]
\[ d_3 = |IK| \quad (\text{cyan}) \ldots 1 \]
\[ d_4 = |CG| = |CH| = |DL| = |EK| = |FJ| = |IL| \quad (\text{magenta}) \ldots 6 \]
\[ d_5 = |DJ| \quad (\text{yellow}) \ldots 1 \]
\[ d_6 = |EI| = |FI| \quad (\text{grass}) \ldots 2 \]
\[ d_7 = |BK| = |EJ| = |EL| = |JL| \quad (\text{sea}) \ldots 4 \]
\[ d_8 = |JJ| \quad (\text{darkorange}) \ldots 1 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_1}{d_4} = \frac{d_2}{d_6} = \frac{d_4}{d_7} = \frac{d_6}{d_8} \quad (64 \text{ occurrences}) \]
2.14 Class of Similar Constructions No. 7

2.14.12 Construction RCC205

Construction Process

A, B given initial points

1. \( p_1 = p(A, B) \)
2. \( k_2 = k(A, B) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(C, A) \)
   \( D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3 \)
4. \( k_4 = k(D, A) \)
   \( G \in k_2 \cap k_4, H \in k_2 \cap k_4 \)
5. \( k_5 = k(H, A) \)
   \( I \in p_1 \cap k_5, J \in k_2 \cap k_5, K \in k_2 \cap k_5, L \in k_3 \cap k_5 \)

Distances

\( d_0 = |DK| \) (red) \ldots 1
\( d_1 = |BJ| = |EK| = |EL| = |KL| \) (blue) \ldots 4
\( d_2 = |BE| = |BF| = |DE| = |DF| = |EF| = |JK| \) (green) \ldots 6
\( d_3 = |IK| \) (cyan) \ldots 1
\( d_4 = |CG| = |CH| = |DL| = |EJ| = |FK| = |IL| \) (magenta) \ldots 6
\( d_5 = |DJ| \) (yellow) \ldots 1
\( d_6 = |EI| = |FI| \) (grass) \ldots 2
\( d_7 = |BK| = |FJ| = |FL| = |JL| \) (sea) \ldots 4
\( d_8 = |JJ| \) (darkorange) \ldots 1

Occurrences of the Golden Ratio

\( \phi = \frac{d_4}{d_2} = \frac{d_4}{d_2} = \frac{d_4}{d_2} = \frac{d_4}{d_2} = \frac{d_4}{d_2} \) (64 occurrences)
2.15 Class of Similar Constructions No. 8

Contained Constructions
RCC9, RCC18, RCC42, RCC51, RCC92, RCC101, RCC134, RCC143, RCC170, RCC179, RCC197, RCC206
2.15.1 Construction RCC9

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(A, B) \)
   \( D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3 \)
4. \( k_4 = k(C, A) \)
   \( G \in k_3 \cap k_4, H \in k_3 \cap k_4 \)
5. \( k_5 = k(G, C) \)
   \( I \in k_2 \cap k_5, J \in k_3 \cap k_5, K \in k_4 \cap k_5, L \in k_4 \cap k_5 \)

Distances

- \( d_0 = |IL| \) (red) \ldots 1
- \( d_1 = |AL| \) (blue) \ldots 1
- \( d_2 = |EJ| \) (green) \ldots 1
- \( d_3 = |CE| = |CF| = |DE| = |DF| = |EF| \) (cyan) \ldots 5
- \( d_4 = |AF| = |BG| = |BH| = |DJ| \) (magenta) \ldots 4
- \( d_5 = |AK| \) (yellow) \ldots 1
- \( d_6 = |FJ| \) (grass) \ldots 1
- \( d_7 = |IK| \) (sea) \ldots 1

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} = \frac{d_4}{d_6} = \frac{d_6}{d_7} \] (20 occurrences)
2.15.2 Construction RCC18

Construction Process

A, B given initial points
1. $p_1 = p(A, B)$
2. $k_2 = k(B, A)$
   $C \in p_1 \cap k_2$
3. $k_3 = k(A, B)$
   $D \in p_1 \cap k_3$, $E \in k_2 \cap k_3$, $F \in k_2 \cap k_3$
4. $k_4 = k(C, A)$
   $G \in k_3 \cap k_4$, $H \in k_3 \cap k_4$
5. $k_5 = k(H, C)$
   $I \in k_2 \cap k_5$, $J \in k_3 \cap k_5$, $K \in k_4 \cap k_5$, $L \in k_4 \cap k_5$

Distances

\[ d_0 = |IL| \text{ (red)} \quad 1 \]
\[ d_1 = |AL| \text{ (blue)} \quad 1 \]
\[ d_2 = |FJ| \text{ (green)} \quad 1 \]
\[ d_3 = |CE| = |CF| = |DE| = |DF| = |EF| \text{ (cyan)} \quad 5 \]
\[ d_4 = |AF| = |BG| = |BH| = |DJ| \text{ (magenta)} \quad 4 \]
\[ d_5 = |AK| \text{ (yellow)} \quad 1 \]
\[ d_6 = |EJ| \text{ (grass)} \quad 1 \]
\[ d_7 = |IK| \text{ (sea)} \quad 1 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_1}{d_4} = \frac{d_2}{d_3} = \frac{d_4}{d_5} = \frac{d_6}{d_7} \quad (20 \text{ occurrences}) \]
2.15.3 Construction RCC42

Construction Process

A, B given initial points
1. $p_1 = p(A, B)$
2. $k_2 = k(B, A)$
   $C \in p_1 \cap k_2$
3. $k_3 = k(A, B)$
   $D \in p_1 \cap k_3$, $E \in k_2 \cap k_3$, $F \in k_2 \cap k_3$
4. $k_4 = k(D, B)$
   $G \in k_2 \cap k_4$, $H \in k_2 \cap k_4$
5. $k_5 = k(G, D)$
   $I \in k_2 \cap k_5$, $J \in k_3 \cap k_5$, $K \in k_4 \cap k_5$, $L \in k_4 \cap k_5$

Distances

\[ d_0 = |JK| \text{ (red) } \ldots 1 \]
\[ d_1 = |BK| \text{ (blue) } \ldots 1 \]
\[ d_2 = |EI| \text{ (green) } \ldots 1 \]
\[ d_3 = |CE| = |CF| = |DE| = |DF| = |EF| \text{ (cyan) } \ldots 5 \]
\[ d_4 = |AG| = |AH| = |BJ| = |CI| \text{ (magenta) } \ldots 4 \]
\[ d_5 = |BL| \text{ (yellow) } \ldots 1 \]
\[ d_6 = |FI| \text{ (grass) } \ldots 1 \]
\[ d_7 = |JL| \text{ (sea) } \ldots 1 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} = \frac{d_4}{d_6} = \frac{d_5}{d_7} \text{ (20 occurrences) } \]
2.15.4 Construction RCC51

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(A, B) \)
   \( D \in p_1 \cap k_3 , \ E \in k_2 \cap k_3 , \ F \in k_2 \cap k_3 \)
4. \( k_4 = k(D, B) \)
   \( G \in k_2 \cap k_4 , \ H \in k_2 \cap k_4 \)
5. \( k_5 = k(H, D) \)
   \( I \in k_2 \cap k_5 , \ J \in k_3 \cap k_5 , \ K \in k_4 \cap k_5 , \ L \in k_4 \cap k_5 \)

Distances

\[
\begin{align*}
  d_0 &= |JK| \text{ (red)} \ldots 1 \\
  d_1 &= |BK| \text{ (blue)} \ldots 1 \\
  d_2 &= |FI| \text{ (green)} \ldots 1 \\
  d_3 &= |CE| = |CF| = |DE| = |DF| = |EF| \text{ (cyan)} \ldots 5 \\
  d_4 &= |AG| = |AH| = |BJ| = |CI| \text{ (magenta)} \ldots 4 \\
  d_5 &= |BL| \text{ (yellow)} \ldots 1 \\
  d_6 &= |EI| \text{ (grass)} \ldots 1 \\
  d_7 &= |JL| \text{ (sea)} \ldots 1 
\end{align*}
\]

Occurrences of the Golden Ratio

\[
\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} = \frac{d_4}{d_6} = \frac{d_5}{d_7} \quad (20 \text{ occurrences})
\]
2.15.5 Construction RCC92

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(A, C) \)
4. \( k_4 = k(C, B) \)
   \( D \in p_1 \cap k_4, E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4 \)
5. \( k_5 = k(G, A) \)
   \( I \in k_2 \cap k_5, J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5 \)

Distances

\[
\begin{align*}
&d_0 = |IJ| \quad \text{(red)} \quad \ldots 1 \\
&d_1 = |CJ| \quad \text{(blue)} \quad \ldots 1 \\
&d_2 = |EL| \quad \text{(green)} \quad \ldots 1 \\
&d_3 = |AE| = |AF| = |DE| = |DF| = |EF| \quad \text{(cyan)} \quad \ldots 5 \\
&d_4 = |BG| = |BH| = |CI| = |DL| \quad \text{(magenta)} \quad \ldots 4 \\
&d_5 = |CK| \quad \text{(yellow)} \quad \ldots 1 \\
&d_6 = |FL| \quad \text{(grass)} \quad \ldots 1 \\
&d_7 = |IK| \quad \text{(sea)} \quad \ldots 1
\end{align*}
\]

Occurrences of the Golden Ratio

\[
\phi = \frac{d_0}{d_2} = \frac{d_3}{d_4} = \frac{d_5}{d_6} = \frac{d_6}{d_7} = \frac{d_7}{d_8} \quad \text{(20 occurrences)}
\]
2.15.6 Construction RCC101

**Construction Process**

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(A, C) \)
4. \( k_4 = k(C, B) \)
   \( D \in p_1 \cap k_4 , E \in k_2 \cap k_4 , F \in k_2 \cap k_4 , G \in k_3 \cap k_4 , H \in k_3 \cap k_4 \)
5. \( k_5 = k(H, A) \)
   \( I \in k_2 \cap k_5 , J \in k_3 \cap k_5 , K \in k_3 \cap k_5 , L \in k_4 \cap k_5 \)

**Distances**

\[
\begin{align*}
  d_0 &= |IJ| \text{ (red)} \ldots 1 \\
  d_1 &= |CJ| \text{ (blue)} \ldots 1 \\
  d_2 &= |FL| \text{ (green)} \ldots 1 \\
  d_3 &= |AE| = |AF| = |DE| = |DF| = |EF| \text{ (cyan)} \ldots 5 \\
  d_4 &= |BG| = |BH| = |CI| = |DL| \text{ (magenta)} \ldots 4 \\
  d_5 &= |CK| \text{ (yellow)} \ldots 1 \\
  d_6 &= |EL| \text{ (grass)} \ldots 1 \\
  d_7 &= |IK| \text{ (sea)} \ldots 1
\end{align*}
\]

**Occurrences of the Golden Ratio**

\[
\phi = \frac{d_0}{d_2} = \frac{d_2}{d_4} = \frac{d_4}{d_6} = \frac{d_4}{d_6} = \frac{d_5}{d_7} = \frac{d_6}{d_8} \text{ (20 occurrences)}
\]
2.15.7 Construction RCC134

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(C, B) \)
   \( D \in p_1 \cap k_3 , E \in k_2 \cap k_3 , F \in k_2 \cap k_3 \)
4. \( k_4 = k(D, B) \)
   \( G \in k_2 \cap k_4 , H \in k_2 \cap k_4 \)
5. \( k_5 = k(G, D) \)
   \( I \in k_2 \cap k_5 , J \in k_3 \cap k_5 , K \in k_4 \cap k_5 , L \in k_4 \cap k_5 \)

Distances

\[
\begin{align*}
    d_0 &= |JL| \quad \text{(red)} \ldots 1 \\
    d_1 &= |BL| \quad \text{(blue)} \ldots 1 \\
    d_2 &= |EI| \quad \text{(green)} \ldots 1 \\
    d_3 &= |AE| = |AF| = |DF| = |EF| \quad \text{(cyan)} \ldots 5 \\
    d_4 &= |AJ| = |BJ| = |CG| = |CH| \quad \text{(magenta)} \ldots 4 \\
    d_5 &= |BK| \quad \text{(yellow)} \ldots 1 \\
    d_6 &= |FI| \quad \text{(grass)} \ldots 1 \\
    d_7 &= |JK| \quad \text{(sea)} \ldots 1
\end{align*}
\]

Occurrences of the Golden Ratio

\[
\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} = \frac{d_4}{d_6} = \frac{d_5}{d_7} \quad (20 \text{ occurrences})
\]
2.15.8 Construction RCC143

Construction Process

\(A, B\) given initial points

1. \(p_1 = p(A, B)\)
2. \(k_2 = k(B, A)\)
   \(C \in p_1 \cap k_2\)
3. \(k_3 = k(C, B)\)
   \(D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3\)
4. \(k_4 = k(D, B)\)
   \(G \in k_2 \cap k_4, H \in k_2 \cap k_4\)
5. \(k_5 = k(H, D)\)
   \(I \in k_2 \cap k_5, J \in k_3 \cap k_5, K \in k_4 \cap k_5, L \in k_4 \cap k_5\)

Distances

\(d_0 = |JL| (\text{red}) \ldots 1\)
\(d_1 = |BL| (\text{blue}) \ldots 1\)
\(d_2 = |FI| (\text{green}) \ldots 1\)
\(d_3 = |AE| = |AF| = |DE| = |DF| = |EF| (\text{cyan}) \ldots 5\)
\(d_4 = |AI| = |BJ| = |CG| = |CH| (\text{magenta}) \ldots 4\)
\(d_5 = |BK| (\text{yellow}) \ldots 1\)
\(d_6 = |EI| (\text{grass}) \ldots 1\)
\(d_7 = |JK| (\text{sea}) \ldots 1\)

Occurrences of the Golden Ratio

\(\phi = \frac{d_0}{d_2} = \frac{d_1}{d_4} = \frac{d_3}{d_5} = \frac{d_4}{d_6} = \frac{d_5}{d_7} (20 \text{ occurrences})\)
2.15.9 Construction RCC170

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(A, B) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(C, A) \)
   \( D \in p_1 \cap k_3 , E \in k_2 \cap k_3 , F \in k_2 \cap k_3 \)
4. \( k_4 = k(B, C) \)
   \( G \in k_3 \cap k_4 , H \in k_3 \cap k_4 \)
5. \( k_5 = k(G, B) \)
   \( I \in k_2 \cap k_5 , J \in k_3 \cap k_5 , K \in k_4 \cap k_5 , L \in k_4 \cap k_5 \)

Distances

\( d_0 = |IL| \) (red) \ldots 1
\( d_1 = |CL| \) (blue) \ldots 1
\( d_2 = |EJ| \) (green) \ldots 1
\( d_3 = |BE| = |BF| = |DE| = |DF| = |EF| \) (cyan) \ldots 5
\( d_4 = |AG| = |AH| = |CI| = |DJ| \) (magenta) \ldots 4
\( d_5 = |CK| \) (yellow) \ldots 1
\( d_6 = |FJ| \) (grass) \ldots 1
\( d_7 = |IK| \) (sea) \ldots 1

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} = \frac{d_4}{d_6} = \frac{d_5}{d_7} \] (20 occurrences)
2.15.10 Construction RCC179

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(A, B) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(C, A) \)
   \( D \in p_1 \cap k_3 \), \( E \in k_2 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(B, C) \)
   \( G \in k_3 \cap k_4 \), \( H \in k_3 \cap k_4 \)
5. \( k_5 = k(H, B) \)
   \( I \in k_2 \cap k_5 \), \( J \in k_3 \cap k_5 \), \( K \in k_4 \cap k_5 \), \( L \in k_4 \cap k_5 \)

Distances

\[
\begin{align*}
   d_0 &= |IL| \text{ (red) } \ldots 1 \\
   d_1 &= |CL| \text{ (blue) } \ldots 1 \\
   d_2 &= |FJ| \text{ (green) } \ldots 1 \\
   d_3 &= |BE| = |BF| = |DE| = |DF| = |EF| \text{ (cyan) } \ldots 5 \\
   d_4 &= |AG| = |AH| = |CI| = |DJ| \text{ (magenta) } \ldots 4 \\
   d_5 &= |CK| \text{ (yellow) } \ldots 1 \\
   d_6 &= |EJ| \text{ (grass) } \ldots 1 \\
   d_7 &= |IK| \text{ (sea) } \ldots 1 \\
\end{align*}
\]

Occurrences of the Golden Ratio

\[
\phi = \frac{d_6}{d_2} = \frac{d_5}{d_3} = \frac{d_4}{d_1} = \frac{d_5}{d_2} = \frac{d_4}{d_1} = \frac{d_6}{d_2} \quad (20 \text{ occurrences})
\]
2.15 Class of Similar Constructions No. 8

2.15.11 Construction RCC197

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(A, B) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(C, A) \)
   \( D \in p_1 \cap k_3 , E \in k_2 \cap k_3 , F \in k_2 \cap k_3 \)
4. \( k_4 = k(D, A) \)
   \( G \in k_2 \cap k_4 , H \in k_2 \cap k_4 \)
5. \( k_5 = k(G, D) \)
   \( I \in k_2 \cap k_5 , J \in k_3 \cap k_5 , K \in k_4 \cap k_5 , L \in k_4 \cap k_5 \)

Distances

\[
\begin{align*}
    d_0 &= |JK| \quad \text{(red)} \quad \ldots 1 \\
    d_1 &= |AK| \quad \text{(blue)} \quad \ldots 1 \\
    d_2 &= |EI| \quad \text{(green)} \quad \ldots 1 \\
    d_3 &= |BE| = |BF| = |DF| = |EF| \quad \text{(cyan)} \quad \ldots 5 \\
    d_4 &= |AJ| = |BI| = |CG| = |CH| \quad \text{(magenta)} \quad \ldots 4 \\
    d_5 &= |AL| \quad \text{(yellow)} \quad \ldots 1 \\
    d_6 &= |FI| \quad \text{(grass)} \quad \ldots 1 \\
    d_7 &= |JL| \quad \text{(sea)} \quad \ldots 1 \\
\end{align*}
\]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} = \frac{d_4}{d_6} = \frac{d_5}{d_7} \quad (20 \text{ occurrences}) \]
2.15.12 Construction RCC206

Construction Process

A, B given initial points
1. \(p_1 = p(A, B)\)
2. \(k_2 = k(A, B)\)
   \(C \in p_1 \cap k_2\)
3. \(k_3 = k(C, A)\)
   \(D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3\)
4. \(k_4 = k(D, A)\)
   \(G \in k_2 \cap k_4, H \in k_2 \cap k_4\)
5. \(k_5 = k(H, D)\)
   \(I \in k_2 \cap k_5, J \in k_3 \cap k_5, K \in k_4 \cap k_5, L \in k_4 \cap k_5\)

Distances

- \(d_0 = |JK|\) (red) \(\ldots 1\)
- \(d_1 = |AK|\) (blue) \(\ldots 1\)
- \(d_2 = |FI|\) (green) \(\ldots 1\)
- \(d_3 = |BE| = |BF| = |DE| = |DF| = |EF|\) (cyan) \(\ldots 5\)
- \(d_4 = |AJ| = |BI| = |CG| = |CH|\) (magenta) \(\ldots 4\)
- \(d_5 = |AL|\) (yellow) \(\ldots 1\)
- \(d_6 = |EI|\) (grass) \(\ldots 1\)
- \(d_7 = |JL|\) (sea) \(\ldots 1\)

Occurrences of the Golden Ratio

\[\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} = \frac{d_4}{d_6} = \frac{d_5}{d_7} (20\text{ occurrences})\]
2.16 Class of Similar Constructions No. 9

Contained Constructions

RCC10, RCC22, RCC43, RCC55, RCC95, RCC107, RCC135, RCC147, RCC173, RCC185, RCC198, RCC210
2.16.1 Construction RCC10

Construction Process

1. $p_1 = p(A, B)$
2. $k_2 = k(B, A)$
3. $k_3 = k(A, B)$
4. $k_4 = k(C, A)$
5. $p_5 = p(E, G)$

Distances

\[
\begin{align*}
&d_0 = |EK| = |HK| \quad \text{(red)} \quad \ldots 2 \\
&d_1 = |AJ| \quad \text{(blue)} \quad \ldots 1 \\
&d_2 = |GI| = |HI| \quad \text{(green)} \quad \ldots 2 \\
&d_3 = |EI| \quad \text{(cyan)} \quad \ldots 2 \\
&d_4 = |AJ| \quad \text{(magenta)} \quad \ldots 1 \\
&d_5 = |BI| \quad \text{(yellow)} \quad \ldots 1 \\
&d_6 = |DG| = |DH| = |EJ| \quad \text{(grass)} \quad \ldots 3 \\
&d_7 = |IK| \quad \text{(sea)} \quad \ldots 1 \\
&d_8 = |BG| = |BH| \quad \text{(darkorange)} \quad \ldots 2 \\
&d_9 = |AB| = |AD| = |AE| = |AF| = |AG| = |AH| = |BC| = |BE| = |BF| = |BJ| \quad \text{(violet)} \quad \ldots 10 \\
&d_{10} = |CI| \quad \text{(myorange)} \quad \ldots 1 \\
\end{align*}
\]

Occurrences of the Golden Ratio

\[
\phi = \frac{d_8}{d_3} = \frac{d_4}{d_5} = \frac{d_5}{d_6} = \frac{d_6}{d_7} = \frac{d_7}{d_8} = \frac{d_8}{d_9} \quad (35 \text{ occurrences})
\]
2.16.2 Construction RCC22

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(A, B) \)
   \( D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3 \)
4. \( k_4 = k(C, A) \)
   \( G \in k_3 \cap k_4, H \in k_3 \cap k_4 \)
5. \( p_5 = p(F, H) \)
   \( I \in p_1 \cap p_5, J \in p_5 \cap k_2, K \in p_5 \cap k_4 \)

Distances

\[
\begin{align*}
d_0 &= |FK| = |GK| \text{ (red)} \ldots 2 \\
d_1 &= |AJ| \text{ (blue)} \ldots 1 \\
d_2 &= |GI| = |HI| \text{ (green)} \ldots 2 \\
d_3 &= |EI| = |FI| \text{ (cyan)} \ldots 2 \\
d_4 &= |AJ| \text{ (magenta)} \ldots 1 \\
d_5 &= |BI| \text{ (yellow)} \ldots 1 \\
d_6 &= |DG| = |DH| = |FJ| \text{ (grass)} \ldots 3 \\
d_7 &= |IK| \text{ (sea)} \ldots 1 \\
d_8 &= |BG| = |BH| \text{ (darkorange)} \ldots 2 \\
d_9 &= |AB| = |AD| = |AE| = |AF| = |AG| = |AH| = |BC| = |BE| = |BF| = |BJ| \text{ (violet)} \ldots 10 \\
d_{10} &= |CI| \text{ (myorange)} \ldots 1 \\
\end{align*}
\]

Occurrences of the Golden Ratio

\[\phi = \frac{d_0}{d_3} = \frac{d_1}{d_5} = \frac{d_2}{d_6} = \frac{d_4}{d_7} = \frac{d_5}{d_8} = \frac{d_9}{d_{10}} \text{ (35 occurrences)}\]
2.16.3 Construction RCC43

Construction Process

\( A, B \) given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(A, B) \)
   \( D \in p_1 \cap k_3 \), \( E \in k_2 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(D, B) \)
   \( G \in k_2 \cap k_4 \), \( H \in k_2 \cap k_4 \)
5. \( p_5 = p(E, G) \)
   \( I \in p_1 \cap p_5 \), \( J \in p_5 \cap k_3 \), \( K \in p_5 \cap k_4 \)

Distances

\( d_0 = |EK| = |HK| \) (red) \ldots 2
\( d_1 = |BI| \) (blue) \ldots 1
\( d_2 = |GI| = |HI| \) (green) \ldots 2
\( d_3 = |EI| = |FI| \) (cyan) \ldots 2
\( d_4 = |BJ| \) (magenta) \ldots 1
\( d_5 = |AF| \) (yellow) \ldots 1
\( d_6 = |CG| = |CH| = |EJ| \) (grass) \ldots 3
\( d_7 = |IK| \) (sea) \ldots 1
\( d_8 = |AG| = |AH| \) (darkorange) \ldots 2
\( d_9 = |AB| = |AD| = |AE| = |AF| = |AJ| = |BC| = |BE| = |BF| = |BG| = |BH| \) (violet) \ldots 10
\( d_{10} = |DI| \) (myorange) \ldots 1

Occurrences of the Golden Ratio

\[ \phi = \frac{d_6}{d_3} = \frac{d_4}{d_5} = \frac{d_2}{d_4} = \frac{d_3}{d_6} = \frac{d_5}{d_7} = \frac{d_7}{d_{10}} \] (35 occurrences)
2.16.4 Construction RCC55

**Construction Process**

A, B given initial points
1. $p_1 = p(A, B)$
2. $k_2 = k(B, A)$
   $C \in p_1 \cap k_2$
3. $k_3 = k(A, B)$
   $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
4. $k_4 = k(D, B)$
   $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
5. $p_5 = p(F, H)$
   $I \in p_1 \cap p_5, J \in p_5 \cap k_3, K \in p_5 \cap k_4$

**Distances**

$d_0 = |FK| = |GK|$ (red) ... 2
$d_1 = |BI|$ (blue) ... 1
$d_2 = |GI| = |HI|$ (green) ... 2
$d_3 = |EI| = |FI|$ (cyan) ... 2
$d_4 = |BJ|$ (magenta) ... 1
$d_5 = |AF|$ (yellow) ... 1
$d_6 = |CG| = |CH| = |FJ|$ (grass) ... 3
$d_7 = |IK|$ (sea) ... 1
$d_8 = |AG| = |AH|$ (darkorange) ... 2
$d_9 = |AB| = |AD| = |AE| = |AF| = |AJ| = |BC| = |BE| = |BF| = |BG| = |BH|$ (violet) ... 10
$d_{10} = |DI|$ (myorange) ... 1

**Occurrences of the Golden Ratio**

$\phi = \frac{d_0}{d_3} = \frac{d_4}{d_7} = \frac{d_5}{d_8} = \frac{d_6}{d_9} = \frac{d_7}{d_{10}}$ (35 occurrences)
2.16.5 Construction RCC95

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(A, C) \)
4. \( k_4 = k(C, B) \)
   \( D \in p_1 \cap k_4 \)
5. \( p_5 = p(E, G) \)

\( I \in p_1 \cap p_5 \), \( J \in p_5 \cap k_2 \), \( K \in p_5 \cap k_3 \)

Distances

\( d_0 = |EK| = |HK| \) (red) \ldots 2
\( d_1 = |CI| \) (blue) \ldots 1
\( d_2 = |GI| = |HI| \) (green) \ldots 2
\( d_3 = |EI| = |FI| \) (cyan) \ldots 2
\( d_4 = |CJ| \) (magenta) \ldots 1
\( d_5 = |BI| \) (yellow) \ldots 1
\( d_6 = |DG| = |DH| = |EJ| \) (grass) \ldots 3
\( d_7 = |IK| \) (sea) \ldots 1
\( d_8 = |BG| = |BH| \) (darkorange) \ldots 2
\( d_9 = |AB| = |BC| = |BE| = |BF| = |BJ| = |CD| = |CE| = |CF| = |CG| = |CH| \) (violet) \ldots 10
\( d_{10} = |AI| \) (myorange) \ldots 1

Occurrences of the Golden Ratio

\( \phi = \frac{d_5}{d_2} = \frac{d_2}{d_4} = \frac{d_4}{d_6} = \frac{d_6}{d_8} = \frac{d_8}{d_{10}} \) (35 occurrences)
2.16.6 Construction RCC107

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(A, C) \)
4. \( k_4 = k(C, B) \)
   \( D \in p_1 \cap k_4 \)
   \( E \in k_2 \cap k_4 \)
   \( F \in k_2 \cap k_4 \)
   \( G \in k_3 \cap k_4 \)
   \( H \in k_3 \cap k_4 \)
5. \( p_5 = p(F, H) \)
   \( I \in p_1 \cap p_5 \)
   \( J \in p_5 \cap k_2 \)
   \( K \in p_5 \cap k_3 \)

Distances

\[
\begin{align*}
  d_0 &= |FK| = |GK| \ (\text{red}) \ldots 2 \\
  d_1 &= |CI| \ (\text{blue}) \ldots 1 \\
  d_2 &= |GI| = |HI| \ (\text{green}) \ldots 2 \\
  d_3 &= |EI| = |FI| \ (\text{cyan}) \ldots 2 \\
  d_4 &= |CJ| \ (\text{magenta}) \ldots 1 \\
  d_5 &= |BI| \ (\text{yellow}) \ldots 1 \\
  d_6 &= |DG| = |DH| = |FJ| \ (\text{grass}) \ldots 3 \\
  d_7 &= |IK| \ (\text{sea}) \ldots 1 \\
  d_8 &= |BG| = |BH| \ (\text{darkorange}) \ldots 2 \\
  d_9 &= |AB| = |BC| = |BE| = |BF| = |BJ| = |CD| = |CE| = |CF| = |CG| = |CH| \ (\text{violet}) \ldots 10 \\
  d_{10} &= |AI| \ (\text{myorange}) \ldots 1 \\
\end{align*}
\]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_3} = \frac{d_1}{d_2} = \frac{d_3}{d_4} = \frac{d_4}{d_5} = \frac{d_5}{d_8} = \frac{d_7}{d_{10}} \ (35 \text{ occurrences}) \]
2.16.7 Construction RCC135

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(C, B) \)
   \( D \in p_1 \cap k_3 , E \in k_2 \cap k_3 , F \in k_2 \cap k_3 \)
4. \( k_4 = k(D, B) \)
   \( G \in k_2 \cap k_4 , H \in k_2 \cap k_4 \)
5. \( p_5 = p(E, G) \)
   \( I \in p_1 \cap p_5 , J \in p_5 \cap k_3 , K \in p_5 \cap k_4 \)

Distances

\[
\begin{align*}
  d_0 &= |EK| = |HK| \text{ (red)} \ldots 2 \\
  d_1 &= |BI| \text{ (blue)} \ldots 1 \\
  d_2 &= |GI| = |HI| \text{ (green)} \ldots 2 \\
  d_3 &= |EI| = |FI| \text{ (cyan)} \ldots 2 \\
  d_4 &= |BJ| \text{ (magenta)} \ldots 1 \\
  d_5 &= |CI| \text{ (yellow)} \ldots 1 \\
  d_6 &= |AG| = |AH| = |EJ| \text{ (grass)} \ldots 3 \\
  d_7 &= |IK| \text{ (sea)} \ldots 1 \\
  d_8 &= |CG| = |CH| \text{ (darkorange)} \ldots 2 \\
  d_9 &= |AB| = |BC| = |BE| = |BF| = |BG| = |BH| = |CD| = |CE| = |CF| = |CJ| \text{ (violet)} \ldots 10 \\
  d_{10} &= |DI| \text{ (myorange)} \ldots 1 
\end{align*}
\]

Occurrences of the Golden Ratio

\[
\phi = \frac{d_6}{d_3} = \frac{d_4}{d_1} = \frac{d_2}{d_5} = \frac{d_3}{d_4} = \frac{d_5}{d_6} = \frac{d_7}{d_{10}} \quad (35 \text{ occurrences})
\]
Construction RCC147

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(C, B) \)
   \( D \in p_1 \cap k_3, \ E \in k_2 \cap k_3, \ F \in k_2 \cap k_3 \)
4. \( k_4 = k(D, B) \)
   \( G \in k_2 \cap k_4, \ H \in k_2 \cap k_4 \)
5. \( p_5 = p(F, H) \)
   \( I \in p_1 \cap p_5, \ J \in p_5 \cap k_3, \ K \in p_5 \cap k_4 \)

Distances

\[
\begin{align*}
d_0 &= |FK| = |GK| \text{(red)} \ldots 2 \\
d_1 &= |BI| \text{(blue)} \ldots 1 \\
d_2 &= |GI| = |HI| \text{(green)} \ldots 2 \\
d_3 &= |EI| = |FI| \text{(cyan)} \ldots 2 \\
d_4 &= |BJ| \text{(magenta)} \ldots 1 \\
d_5 &= |CI| \text{(yellow)} \ldots 1 \\
d_6 &= |AG| = |AH| = |FJ| \text{(grass)} \ldots 3 \\
d_7 &= |JK| \text{(sea)} \ldots 1 \\
d_8 &= |CG| = |CH| \text{(darkorange)} \ldots 2 \\
d_9 &= |AB| = |BC| = |BE| = |BF| = |BG| = |BH| = |CD| = |CE| = |CF| = |CJ| \text{(violet)} \ldots 10 \\
d_{10} &= |DI| \text{(myorange)} \ldots 1
\end{align*}
\]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_3} = \frac{d_4}{d_7} = \frac{d_5}{d_8} = \frac{d_6}{d_9} = \frac{d_7}{d_{10}} = \frac{d_8}{d_{10}} \text{ (35 occurrences)} \]
2.16.9 Construction RCC173

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(A, B) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(C, A) \)
   \( D \in p_1 \cap k_3 , E \in k_2 \cap k_3 , F \in k_2 \cap k_3 \)
4. \( k_4 = k(B, C) \)
   \( G \in k_3 \cap k_4 , H \in k_3 \cap k_4 \)
5. \( p_5 = p(E, G) \)
   \( I \in p_1 \cap p_5 , J \in p_5 \cap k_2 , K \in p_5 \cap k_4 \)

Distances

\[
\begin{align*}
  d_0 &= |EK| = |HK| \text{ (red)} \ldots 2 \\
  d_1 &= |CI| \text{ (blue)} \ldots 1 \\
  d_2 &= |GI| = |HI| \text{ (green)} \ldots 2 \\
  d_3 &= |EI| = |FI| \text{ (cyan)} \ldots 2 \\
  d_4 &= |CJ| \text{ (magenta)} \ldots 1 \\
  d_5 &= |AF| \text{ (yellow)} \ldots 1 \\
  d_6 &= |DG| = |DH| = |EJ| \text{ (grass)} \ldots 3 \\
  d_7 &= |IK| \text{ (sea)} \ldots 1 \\
  d_8 &= |AG| = |AH| \text{ (darkorange)} \ldots 2 \\
  d_9 &= |AB| = |AC| = |AE| = |AF| = |AJ| = |CD| = |CE| = |CF| = |CG| = |CH| \text{ (violet)} \ldots 10 \\
  d_{10} &= |BI| \text{ (myorange)} \ldots 1
\end{align*}
\]

Occurrences of the Golden Ratio

\[
\phi = \frac{d_6}{d_5} = \frac{d_4}{d_3} = \frac{d_2}{d_1} = \frac{d_0}{d_9} = \frac{d_8}{d_{10}} \text{ (35 occurrences)}
\]
2.16.10 Construction RCC185

Construction Process

\(A, B\) given initial points
1. \(p_1 = p(A, B)\)
2. \(k_2 = k(A, B)\)
   \(C \in p_1 \cap k_2\)
3. \(k_3 = k(C, A)\)
   \(D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3\)
4. \(k_4 = k(B, C)\)
   \(G \in k_3 \cap k_4, H \in k_3 \cap k_4\)
5. \(p_5 = p(F, H)\)
   \(I \in p_1 \cap p_5, J \in p_5 \cap k_2, K \in p_5 \cap k_4\)

Distances

\[d_0 = |FK| = |GK| \text{ (red)} \ldots 2\]
\[d_1 = |CI| \text{ (blue)} \ldots 1\]
\[d_2 = |GI| = |HI| \text{ (green)} \ldots 2\]
\[d_3 = |EI| = |FI| \text{ (cyan)} \ldots 2\]
\[d_4 = |CJ| \text{ (magenta)} \ldots 1\]
\[d_5 = |AH| \text{ (yellow)} \ldots 1\]
\[d_6 = |DG| = |DH| = |FJ| \text{ (grass)} \ldots 3\]
\[d_7 = |IK| \text{ (sea)} \ldots 1\]
\[d_8 = |AH| = |AH| \text{ (darkerange)} \ldots 2\]
\[d_9 = |AB| = |AC| = |AE| = |AF| = |AJ| = |CD| = |CE| = |CF| = |CG| = |CI| \text{ (violet)} \ldots 10\]
\[d_{10} = |BI| \text{ (myorange)} \ldots 1\]

Occurrences of the Golden Ratio

\[\phi = \frac{d_9}{d_5} = \frac{d_4}{d_8} = \frac{d_3}{d_6} = \frac{d_2}{d_7} = \frac{d_1}{d_9} = \frac{d_0}{d_{10}} \text{ (35 occurrences)}\]
2.16.11 Construction RCC198

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(A, B) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(C, A) \)
   \( D \in p_1 \cap k_3 , E \in k_2 \cap k_3 , F \in k_2 \cap k_3 \)
4. \( k_4 = k(D, A) \)
   \( G \in k_2 \cap k_4 , H \in k_2 \cap k_4 \)
5. \( p_5 = p(E, G) \)
   \( I \in p_1 \cap p_5 , J \in p_5 \cap k_3 , K \in p_5 \cap k_4 \)

Distances

\[ d_0 = \|EK\| = \|HK\| \text{ (red)} \ldots 2 \]
\[ d_1 = \|AI\| \text{ (blue)} \ldots 1 \]
\[ d_2 = \|GI\| = \|HI\| \text{ (green)} \ldots 2 \]
\[ d_3 = \|EI\| = \|FI\| \text{ (cyan)} \ldots 2 \]
\[ d_4 = \|AJ\| \text{ (magenta)} \ldots 1 \]
\[ d_5 = \|CI\| \text{ (yellow)} \ldots 1 \]
\[ d_6 = \|BG\| = \|BH\| = \|EJ\| \text{ (grass)} \ldots 3 \]
\[ d_7 = \|IK\| \text{ (sea)} \ldots 1 \]
\[ d_8 = \|CG\| = \|CH\| \text{ (darkorange)} \ldots 2 \]
\[ d_9 = \|AB\| = \|AC\| = \|AE\| = \|AF\| = \|AG\| = \|AH\| = \|CD\| = \|CE\| = \|CF\| = \|CJ\| \text{ (violet)} \ldots 10 \]
\[ d_{10} = \|DI\| \text{ (myorange)} \ldots 1 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_3} = \frac{d_1}{d_5} = \frac{d_2}{d_6} = \frac{d_3}{d_7} = \frac{d_4}{d_8} = \frac{d_5}{d_9} = \frac{d_6}{d_{10}} \text{ (35 occurrences)} \]
2.16.12 Construction RCC210

Construction Process

\[ A, B \text{ given initial points} \]

1. \[ p_1 = p(A, B) \]
2. \[ k_2 = k(A, B) \]
   \[ C \in p_1 \cap k_2 \]
3. \[ k_3 = k(C, A) \]
   \[ D \in p_1 \cap k_3 , \ E \in k_2 \cap k_3 , \ F \in k_2 \cap k_3 \]
4. \[ k_4 = k(D, A) \]
   \[ G \in k_2 \cap k_4 , \ H \in k_2 \cap k_4 \]
5. \[ p_5 = p(F, H) \]
   \[ I \in p_1 \cap p_5 , \ J \in p_5 \cap k_3 , \ K \in p_5 \cap k_4 \]

Distances

\[
\begin{align*}
d_0 &= |FK| = |GK| \text{ (red)} \ldots 2 \\
d_1 &= |AJ| \text{ (blue)} \ldots 1 \\
d_2 &= |GI| = |HI| \text{ (green)} \ldots 2 \\
d_3 &= |EI| = |FI| \text{ (cyan)} \ldots 2 \\
d_4 &= |AJ| \text{ (magenta)} \ldots 1 \\
d_5 &= |CI| \text{ (yellow)} \ldots 1 \\
d_6 &= |BG| = |BH| = |FJ| \text{ (grass)} \ldots 3 \\
d_7 &= |IK| \text{ (sea)} \ldots 1 \\
d_8 &= |CG| = |CH| \text{ (darkorange)} \ldots 2 \\
d_9 &= |AB| = |AC| = |AE| = |AF| = |AG| = |AH| = |CD| = |CE| = |CF| = |CJ| \text{ (violet)} \ldots 10 \\
d_{10} &= |DI| \text{ (myorange)} \ldots 1 \\
\end{align*}
\]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_3} = \frac{d_4}{d_5} = \frac{d_3}{d_4} = \frac{d_4}{d_5} = \frac{d_5}{d_6} = \frac{d_6}{d_{10}} \text{ (35 occurrences)} \]
2.17 Class of Similar Constructions No. 10

Contained Constructions
RCC11, RCC23, RCC44, RCC56, RCC96, RCC108, RCC136, RCC148, RCC174, RCC186, RCC199, RCC211
2.17.1 Construction RCC11

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(A, B) \)
   \( D \in k_2 \cap k_3 \)
4. \( k_4 = k(C, A) \)
   \( E \in k_3 \cap k_4 \), \( F \in k_3 \cap k_4 \)
5. \( k_5 = k(E, D) \)
   \( G \in k_2 \cap k_5 \)

Distances

\( d_0 = |BE| = |BF| \) (red) \ldots 2
\( d_1 = |AG| \) (blue) \ldots 1

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} \) (2 occurrences)
2.17.2 Construction RCC23

Construction Process

\( A, B \) given initial points
1. \( p_1 = p(A,B) \)
2. \( k_2 = k(B,A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(A,B) \)
   \( D \in k_2 \cap k_3 \)
4. \( k_4 = k(C,A) \)
   \( E \in k_3 \cap k_4, F \in k_3 \cap k_4 \)
5. \( k_5 = k(F,D) \)
   \( G \in k_2 \cap k_5 \)

Distances

\[ d_0 = |BE| = |BF| \text{ (red)} \ldots 2 \]
\[ d_1 = |AG| \text{ (blue)} \ldots 1 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} \text{ (2 occurrences)} \]
2.17.3 Construction RCC44

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
3. \( k_3 = k(A, B) \)
   \( C \in p_1 \cap k_3 \), \( D \in k_2 \cap k_3 \)
4. \( k_4 = k(C, B) \)
   \( E \in k_2 \cap k_4 \), \( F \in k_2 \cap k_4 \)
5. \( k_5 = k(E, D) \)
   \( G \in k_3 \cap k_5 \)

Distances

\( d_0 = |AE| = |AF| \) (red) \ldots 2
\( d_1 = |BG| \) (blue) \ldots 1

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} \) (2 occurrences)
2.17.4 Construction RCC56

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
3. \( k_3 = k(A, B) \)
   \( C \in p_1 \cap k_3 \), \( D \in k_2 \cap k_3 \)
4. \( k_4 = k(C, B) \)
   \( E \in k_2 \cap k_4 \), \( F \in k_2 \cap k_4 \)
5. \( k_5 = k(F, D) \)
   \( G \in k_3 \cap k_5 \)

Distances

\( d_0 = |AE| = |AF| \) (red) \( \ldots 2 \)
\( d_1 = |BG| \) (blue) \( \ldots 1 \)

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} \) (2 occurrences)
2.17.5 Construction RCC96

Construction Process

A, B given initial points
1. $p_1 = p(A, B)$
2. $k_2 = k(B, A)$
   $C \in p_1 \cap k_2$
3. $k_3 = k(A, C)$
4. $k_4 = k(C, B)$
   $D \in k_2 \cap k_4$,
   $E \in k_3 \cap k_4$,
   $F \in k_3 \cap k_4$
5. $k_5 = k(E, D)$
   $G \in k_2 \cap k_5$

Distances

$d_0 = |BE| = |BF|$ (red) ... 2
$d_1 = |CG|$ (blue) ... 1

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_1}$ (2 occurrences)
2.17.6 Construction RCC108

Construction Process

- $A, B$ given initial points
- $p_1 = p(A, B)$
- $k_2 = k(B, A)$
- $C \in p_1 \cap k_2$
- $k_3 = k(A, C)$
- $k_4 = k(C, B)$
- $D \in k_2 \cap k_4$, $E \in k_3 \cap k_4$, $F \in k_3 \cap k_4$
- $k_5 = k(F, D)$
- $G \in k_2 \cap k_5$

Distances

$$d_0 = |BE| = |BF| \, \text{(red)} \ldots 2$$
$$d_1 = |CG| \, \text{(blue)} \ldots 1$$

Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \, (2 \text{ occurrences})$$
2.17.7 Construction RCC136

Construction Process

- A, B given initial points
- \( p_1 = p(A, B) \)
- \( k_2 = k(B, A) \)
- \( C \in p_1 \cap k_2 \)
- \( k_3 = k(C, B) \)
- \( D \in p_1 \cap k_3 \), \( E \in k_2 \cap k_3 \)
- \( k_4 = k(D, B) \)
- \( F \in k_2 \cap k_4 \), \( G \in k_2 \cap k_4 \)
- \( k_5 = k(F, E) \)
- \( H \in k_3 \cap k_5 \)

Distances

- \( d_0 = |CF| = |CG| \) (red) \( \ldots 2 \)
- \( d_1 = |BH| \) (blue) \( \ldots 1 \)

Occurrences of the Golden Ratio

- \( \phi = \frac{d_0}{d_1} \) (2 occurrences)
2.17.8 Construction RCC148

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
3. \( k_3 = k(C, B) \)
4. \( k_4 = k(D, B) \)
5. \( k_5 = k(G, E) \)

Distances

\[ d_0 = |CF| = |CG| \text{ (red)} \ldots 2 \]
\[ d_1 = |BH| \text{ (blue)} \ldots 1 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} \text{ (2 occurrences)} \]
2.17.9 Construction RCC174

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(A, B) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(C, A) \)
   \( D \in k_2 \cap k_3 \)
4. \( k_4 = k(B, C) \)
   \( E \in k_3 \cap k_4 \), \( F \in k_3 \cap k_4 \)
5. \( k_5 = k(E, D) \)
   \( G \in k_2 \cap k_5 \)

Distances

\( d_0 = |AE| = |AF| \) (red) \ldots 2
\( d_1 = |CG| \) (blue) \ldots 1

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} \) (2 occurrences)
2.17.10 Construction RCC186

Construction Process

1. $p_1 = p(A, B)$
2. $k_2 = k(A, B)$
   $C \in p_1 \cap k_2$
3. $k_3 = k(C, A)$
   $D \in k_2 \cap k_3$
4. $k_4 = k(B, C)$
   $E \in k_3 \cap k_4$, $F \in k_3 \cap k_4$
5. $k_5 = k(F, D)$
   $G \in k_2 \cap k_5$

Distances

$d_0 = |AE| = |AF|$ (red) … 2
$d_1 = |CG|$ (blue) … 1

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_1}$ (2 occurrences)
2.17.11 Construction RCC199

Construction Process

A, B given initial points

1. \( p_1 = p(A, B) \)
2. \( k_2 = k(A, B) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(C, A) \)
   \( D \in p_1 \cap k_3 , E \in k_2 \cap k_3 \)
4. \( k_4 = k(D, A) \)
   \( F \in k_2 \cap k_4 , G \in k_2 \cap k_4 \)
5. \( k_5 = k(F, E) \)
   \( H \in k_3 \cap k_5 \)

Distances

\[ d_0 = |CF| = |CG| \text{ (red)} \ldots 2 \]
\[ d_1 = |AH| \text{ (blue)} \ldots 1 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} \text{ (2 occurrences)} \]
2.17.12 Construction RCC211

Construction Process

A, B given initial points
1. $p_1 = p(A, B)$
2. $k_2 = k(A, B)$
   $C \in p_1 \cap k_2$
3. $k_3 = k(C, A)$
   $D \in p_1 \cap k_3, E \in k_2 \cap k_3$
4. $k_4 = k(D, A)$
   $F \in k_2 \cap k_4, G \in k_2 \cap k_4$
5. $k_5 = k(G, E)$
   $H \in k_3 \cap k_5$

Distances

$d_0 = |CF| = |CG|$ (red) \ldots 2
$d_1 = |AH|$ (blue) \ldots 1

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_1}$ (2 occurrences)
2.18 Class of Similar Constructions No. 11

Contained Constructions
RCC12, RCC24, RCC45, RCC57, RCC97, RCC109, RCC137, RCC149, RCC175, RCC187, RCC200, RCC212
2.18.1 Construction RCC12

Construction Process

A, B given initial points
1. $p_1 = p(A, B)$
2. $k_2 = k(B, A)$
   $C \in p_1 \cap k_2$
3. $k_3 = k(A, B)$
   $D \in p_1 \cap k_3$, $E \in k_2 \cap k_3$, $F \in k_2 \cap k_3$
4. $k_4 = k(C, A)$
   $G \in k_3 \cap k_4$, $H \in k_3 \cap k_4$
5. $k_5 = k(E, G)$
   $I \in k_2 \cap k_5$, $J \in k_2 \cap k_5$, $K \in k_3 \cap k_5$, $L \in k_4 \cap k_5$

Distances

$d_0 = |CE| = |CF| = |DE| = |DF| = |EF| = |HK|$ (red) ... 6
$d_1 = |AJ| = |BG| = |BH|$ (blue) ... 3
$d_2 = |AL|$ (green) ... 1
$d_3 = |AI| = |BK|$ (cyan) ... 2
$d_4 = |GK| = |IJ|$ (magenta) ... 2
$d_5 = |JL|$ (yellow) ... 1
$d_6 = |IL|$ (grass) ... 1

Occurrences of the Golden Ratio

$\phi = \frac{d_4}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5}$ (16 occurrences)
2.18.2 Construction RCC24

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(A, B) \)
   \( D \in p_1 \cap k_3 \), \( E \in k_2 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(C, A) \)
   \( G \in k_3 \cap k_4 \), \( H \in k_3 \cap k_4 \)
5. \( k_5 = k(F, H) \)
   \( I \in k_2 \cap k_5 \), \( J \in k_2 \cap k_5 \), \( K \in k_3 \cap k_5 \), \( L \in k_4 \cap k_5 \)

Distances

\( d_0 = |CE| = |CF| = |DE| = |DF| = |EF| = |GK| \) (red) \ldots 6
\( d_1 = |AJ| = |BG| = |BH| \) (blue) \ldots 3
\( d_2 = |AL| \) (green) \ldots 1
\( d_3 = |AI| = |BK| \) (cyan) \ldots 2
\( d_4 = |HK| = |IJ| \) (magenta) \ldots 2
\( d_5 = |JE| \) (yellow) \ldots 1
\( d_6 = |IL| \) (grass) \ldots 1

Occurrences of the Golden Ratio

\( \phi = \frac{d_4}{d_2} = \frac{d_5}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \) (16 occurrences)
2.18.3 Construction RCC45

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(A, B) \)
   \( D \in p_1 \cap k_3 \), \( E \in k_2 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(D, B) \)
   \( G \in k_2 \cap k_4 \), \( H \in k_2 \cap k_4 \)
5. \( k_5 = k(E, G) \)
   \( I \in k_2 \cap k_5 \), \( J \in k_3 \cap k_5 \), \( K \in k_3 \cap k_5 \), \( L \in k_4 \cap k_5 \)

Distances

\( d_0 = |CE| = |CF| = |DE| = |DF| = |EF| = |HI| \) (red) \ldots 6
\( d_1 = |AG| = |AH| = |BJ| \) (blue) \ldots 3
\( d_2 = |BL| \) (green) \ldots 1
\( d_3 = |AI| = |BK| \) (cyan) \ldots 2
\( d_4 = |GI| = |JK| \) (magenta) \ldots 2
\( d_5 = |JL| \) (yellow) \ldots 1
\( d_6 = |KL| \) (grass) \ldots 1

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_2} = \frac{d_3}{d_5} = \frac{d_4}{d_6} = \frac{d_5}{d_6} \) (16 occurrences)
2.18.4 Construction RCC57

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(A, B) \)
   \( D \in p_1 \cap k_3, \ E \in k_2 \cap k_3, \ F \in k_2 \cap k_3 \)
4. \( k_4 = k(D, B) \)
   \( G \in k_2 \cap k_4, \ H \in k_2 \cap k_4 \)
5. \( k_5 = k(F, H) \)
   \( I \in k_2 \cap k_5, \ J \in k_3 \cap k_5, \ K \in k_3 \cap k_5, \ L \in k_4 \cap k_5 \)

Distances

\( d_0 = |CE| = |CF| = |DE| = |DF| = |EF| = |GI| \) (red) \( \ldots 6 \)
\( d_1 = |AG| = |AH| = |BJ| \) (blue) \( \ldots 3 \)
\( d_2 = |BL| \) (green) \( \ldots 1 \)
\( d_3 = |AI| = |BK| \) (cyan) \( \ldots 2 \)
\( d_4 = |HI| = |JK| \) (magenta) \( \ldots 2 \)
\( d_5 = |JE| \) (yellow) \( \ldots 1 \)
\( d_6 = |KL| \) (grass) \( \ldots 1 \)

Occurrences of the Golden Ratio

\( \phi = \frac{d_4}{d_3} = \frac{d_3}{d_2} = \frac{d_2}{d_1} = \frac{d_1}{d_0} \) (16 occurrences)
2.18.5 Construction RCC97

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(A, C) \)
4. \( k_4 = k(C, B) \)
   \( D \in p_1 \cap k_4 , E \in k_2 \cap k_4 , F \in k_2 \cap k_4 , G \in k_3 \cap k_4 , H \in k_3 \cap k_4 \)
5. \( k_5 = k(E, G) \)
   \( I \in k_2 \cap k_5 , J \in k_2 \cap k_5 , K \in k_3 \cap k_5 , L \in k_4 \cap k_5 \)

Distances

\[
\begin{align*}
    d_0 &= |AE| = |AF| = |DE| = |DF| = |EF| = |HL| \ (\text{red}) \ldots 6 \\
    d_1 &= |BG| = |BH| = |CI| \ (\text{blue}) \ldots 3 \\
    d_2 &= |CK| \ (\text{green}) \ldots 1 \\
    d_3 &= |BL| = |CJ| \ (\text{cyan}) \ldots 2 \\
    d_4 &= |GL| = |IJ| \ (\text{magenta}) \ldots 2 \\
    d_5 &= |IK| \ (\text{yellow}) \ldots 1 \\
    d_6 &= |JK| \ (\text{grass}) \ldots 1
\end{align*}
\]

Occurrences of the Golden Ratio

\[
\phi = \frac{d_0}{d_2} = \frac{d_2}{d_3} = \frac{d_3}{d_4} = \frac{d_4}{d_5} \ (16 \ \text{occurrences})
\]
2.18.6 Construction RCC109

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(A, C) \)
4. \( k_4 = k(C, B) \)
   \( D \in p_1 \cap k_4 , E \in k_2 \cap k_4 , F \in k_2 \cap k_4 , G \in k_3 \cap k_4 , H \in k_3 \cap k_4 \)
5. \( k_5 = k(F, H) \)
   \( I \in k_2 \cap k_5 , J \in k_2 \cap k_5 , K \in k_3 \cap k_5 , L \in k_4 \cap k_5 \)

Distances

\( d_0 = |AE| = |AF| = |DE| = |DF| = |EF| = |GL| \) (red) \( \ldots 6 \)
\( d_1 = |BG| = |BH| = |CI| \) (blue) \( \ldots 3 \)
\( d_2 = |CK| \) (green) \( \ldots 1 \)
\( d_3 = |BL| = |CJ| \) (cyan) \( \ldots 2 \)
\( d_4 = |HL| = |IJ| \) (magenta) \( \ldots 2 \)
\( d_5 = |IK| \) (yellow) \( \ldots 1 \)
\( d_6 = |JK| \) (grass) \( \ldots 1 \)

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_4}{d_5} = \frac{d_5}{d_6} \) (16 occurrences)
2.18.7 Construction RCC137

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(C, B) \)
   \( D \in p_1 \cap k_3 \), \( E \in k_2 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(D, B) \)
   \( G \in k_2 \cap k_4 \), \( H \in k_2 \cap k_4 \)
5. \( k_5 = k(E, G) \)
   \( I \in k_2 \cap k_5 \), \( J \in k_3 \cap k_5 \), \( K \in k_3 \cap k_5 \), \( L \in k_4 \cap k_5 \)

Distances

\[ d_0 = |AE| = |AF| = |DE| = |DF| = |EF| = |HI| \text{ (red)} \ldots 6 \]
\[ d_1 = |BK| = |CG| = |CH| \text{ (blue)} \ldots 3 \]
\[ d_2 = |BL| \text{ (green)} \ldots 1 \]
\[ d_3 = |BJ| = |CI| \text{ (cyan)} \ldots 2 \]
\[ d_4 = |GI| = |JK| \text{ (magenta)} \ldots 2 \]
\[ d_5 = |KL| \text{ (yellow)} \ldots 1 \]
\[ d_6 = |JL| \text{ (grass)} \ldots 1 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_5}{d_6} \text{ (16 occurrences)} \]
2.18.8 Construction RCC149

Construction Process

$A, B$ given initial points

1. $p_1 = p(A, B)$
2. $k_2 = k(B, A)$
   \[ C \in p_1 \cap k_2 \]
3. $k_3 = k(C, B)$
   \[ D \in p_1 \cap k_3 , \ E \in k_2 \cap k_3 , \ F \in k_2 \cap k_3 \]
4. $k_4 = k(D, B)$
   \[ G \in k_2 \cap k_4 , \ H \in k_2 \cap k_4 \]
5. $k_5 = k(F, H)$
   \[ I \in k_2 \cap k_5 , \ J \in k_3 \cap k_5 , \ K \in k_3 \cap k_5 , \ L \in k_4 \cap k_5 \]

Distances

\[
\begin{align*}
    d_0 &= |AE| = |AF| = |DE| = |DF| = |EF| = |GI| \text{ (red)} \ldots 6 \\
    d_1 &= |BK| = |CG| = |CH| \text{ (blue)} \ldots 3 \\
    d_2 &= |BL| \text{ (green)} \ldots 1 \\
    d_3 &= |BJ| = |CI| \text{ (cyan)} \ldots 2 \\
    d_4 &= |HI| = |JK| \text{ (magenta)} \ldots 2 \\
    d_5 &= |KL| \text{ (yellow)} \ldots 1 \\
    d_6 &= |JL| \text{ (grass)} \ldots 1
\end{align*}
\]

Occurrences of the Golden Ratio

\[
\phi = \frac{d_5}{d_3} = \frac{d_4}{d_2} = \frac{d_4}{d_6} = \frac{d_4}{d_8} \text{ (16 occurrences)}
\]
2.18.9 Construction RCC175

Construction Process

$A, B$ given initial points
1. $p_1 = p(A, B)$
2. $k_2 = k(A, B)$
   $C \in p_1 \cap k_2$
3. $k_3 = k(C, A)$
   $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
4. $k_4 = k(B, C)$
   $G \in k_3 \cap k_4, H \in k_3 \cap k_4$
5. $k_5 = k(E, G)$
   $I \in k_2 \cap k_5, J \in k_2 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5$

Distances

$d_0 = |BE| = |BF| = |DE| = |DF| = |EF| = |HK|$ (red) \ldots 6
$d_1 = |AG| = |AH| = |CJ|$ (blue) \ldots 3
$d_2 = |CL|$ (green) \ldots 1
$d_3 = |AK| = |CI|$ (cyan) \ldots 2
$d_4 = |GK| = |IJ|$ (magenta) \ldots 2
$d_5 = |IL|$ (yellow) \ldots 1
$d_6 = |IJ|$ (grass) \ldots 1

Occurrences of the Golden Ratio

$\phi = \frac{d_4}{d_2} = \frac{d_5}{d_3} = \frac{d_4}{d_5} = \frac{d_4}{d_6}$ (16 occurrences)
Construction RCC187

Construction Process

- \( A, B \) given initial points
- 1. \( p_1 = p(A, B) \)
- 2. \( k_2 = k(A, B) \)
  \( C \in p_1 \cap k_2 \)
- 3. \( k_3 = k(C, A) \)
  \( D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3 \)
- 4. \( k_4 = k(B, C) \)
  \( G \in k_3 \cap k_4, H \in k_3 \cap k_4 \)
- 5. \( k_5 = k(F, H) \)
  \( I \in k_2 \cap k_5, J \in k_2 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5 \)

Distances

- \( d_0 = |BE| = |BF| = |DE| = |DF| = |EF| = |GK| \) (red) ... 6
- \( d_1 = |AG| = |AH| = |CJ| \) (blue) ... 3
- \( d_2 = |CL| \) (green) ... 1
- \( d_3 = |AK| = |CI| \) (cyan) ... 2
- \( d_4 = |HK| = |IJ| \) (magenta) ... 2
- \( d_5 = |JL| \) (yellow) ... 1
- \( d_6 = |IL| \) (grass) ... 1

Occurrences of the Golden Ratio

- \( \phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_4}{d_5} = \frac{d_4}{d_6} \) (16 occurrences)
2.18.11 Construction RCC200

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(A, B) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(C, A) \)
   \( D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3 \)
4. \( k_4 = k(D, A) \)
   \( G \in k_2 \cap k_4, H \in k_2 \cap k_4 \)
5. \( k_5 = k(E, G) \)
   \( I \in k_2 \cap k_5, J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5 \)

Distances

\[ d_0 = |BE| = |BF| = |DE| = |DF| = |EF| = |HI|. \text{ (red)} \ldots 6 \]
\[ d_1 = |AJ| = |CG| = |CH|. \text{ (blue)} \ldots 3 \]
\[ d_2 = |AL|. \text{ (green)} \ldots 1 \]
\[ d_3 = |AK|. \text{ (cyan)} \ldots 2 \]
\[ d_4 = |GI| = |JK|. \text{ (magenta)} \ldots 2 \]
\[ d_5 = |JL|. \text{ (yellow)} \ldots 1 \]
\[ d_6 = |KL|. \text{ (grass)} \ldots 1 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_4}{d_5} = \frac{d_5}{d_6} \] (16 occurrences)
2.18.12 Construction RCC212

Construction Process

\[ A, \ B \ \text{given initial points} \]

1. \[ p_1 = p(A, B) \]
2. \[ k_2 = k(A, B) \]
   \[ C \in p_1 \cap k_2 \]
3. \[ k_3 = k(C, A) \]
   \[ D \in p_1 \cap k_3, \ E \in k_2 \cap k_3, \ F \in k_2 \cap k_3 \]
4. \[ k_4 = k(D, A) \]
   \[ G \in k_2 \cap k_4, \ H \in k_2 \cap k_4 \]
5. \[ k_5 = k(F, H) \]
   \[ I \in k_2 \cap k_5, \ J \in k_3 \cap k_5, \ K \in k_3 \cap k_5, \ L \in k_4 \cap k_5 \]

Distances

\[ d_0 = |BE| = |BF| = |DE| = |DF| = |EF| = |GI| \ (red) \ldots 6 \]
\[ d_1 = |AJ| = |CG| = |CH| \ (blue) \ldots 3 \]
\[ d_2 = |AL| \ (green) \ldots 1 \]
\[ d_3 = |AK| = |CI| \ (cyan) \ldots 2 \]
\[ d_4 = |HI| = |JK| \ (magenta) \ldots 2 \]
\[ d_5 = |JL| \ (yellow) \ldots 1 \]
\[ d_6 = |KL| \ (grass) \ldots 1 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_4}{d_5} = \frac{d_3}{d_6} = \frac{d_6}{d_5} \ (16 \ occurrences) \]
2.19 Class of Similar Constructions No. 12

Contained Constructions

RCC13, RCC19, RCC46, RCC52, RCC98, RCC104, RCC138, RCC144, RCC176, RCC182, RCC201, RCC207
2.19.1 Construction RCC13

Construction Process

\begin{itemize}
\item A, B given initial points
\item \( p_1 = p(A, B) \)
\item \( k_2 = k(B, A) \)
\item \( C \in p_1 \cap k_2 \)
\item \( k_3 = k(A, B) \)
\item \( D \in p_1 \cap k_3 \), \( E \in k_2 \cap k_3 \), \( F \in k_2 \cap k_3 \)
\item \( k_4 = k(C, A) \)
\item \( G \in k_3 \cap k_4 \), \( H \in k_3 \cap k_4 \)
\item \( p_5 = p(F, G) \)
\item \( I \in p_1 \cap p_5 \), \( J \in p_5 \cap k_2 \), \( K \in p_5 \cap k_4 \)
\end{itemize}

Distances

\[
\begin{align*}
  d_0 &= |CI| \quad \text{(red)} \ldots 1 \\
  d_1 &= |DG| = |DH| = |FJ| \quad \text{(blue)} \ldots 3 \\
  d_2 &= |IK| \quad \text{(green)} \ldots 1 \\
  d_3 &= |BG| = |BH| \quad \text{(cyan)} \ldots 2 \\
  d_4 &= |AB| = |AD| = |AE| = |AF| = |AG| = |AH| = |BC| = |BE| = |BF| = |BJ| \quad \text{(magenta)} \ldots 10 \\
  d_5 &= |GI| = |HI| \quad \text{(yellow)} \ldots 2 \\
  d_6 &= |EI| = |FI| \quad \text{(grass)} \ldots 2 \\
  d_7 &= |AJ| \quad \text{(sea)} \ldots 1 \\
  d_8 &= |BI| \quad \text{(darkorange)} \ldots 1 \\
  d_9 &= |FK| = |HK| \quad \text{(violet)} \ldots 2 \\
  d_{10} &= |AI| \quad \text{(myorange)} \ldots 1 \\
\end{align*}
\]

Occurrences of the Golden Ratio

\[
\phi = \frac{d_6}{d_4} = \frac{d_5}{d_3} = \frac{d_6}{d_4} = \frac{d_7}{d_5} = \frac{d_8}{d_6} = \frac{d_9}{d_7} (35 \text{ occurrences})
\]
2.19.2 Construction RCC19

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(A, B) \)
   \( D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3 \)
4. \( k_4 = k(C, A) \)
   \( G \in k_3 \cap k_4, H \in k_3 \cap k_4 \)
5. \( p_5 = p(E, H) \)
   \( I \in p_1 \cap p_5, J \in p_5 \cap k_2, K \in p_5 \cap k_4 \)

Distances

\[ d_0 = |CI| \text{ (red) } \ldots 1 \]
\[ d_1 = |DG| = |DH| = |EJ| \text{ (blue) } \ldots 3 \]
\[ d_2 = |IK| \text{ (green) } \ldots 1 \]
\[ d_3 = |BG| = |BH| \text{ (cyan) } \ldots 2 \]
\[ d_4 = |AB| = |AD| = |AE| = |AF| = |AG| = |AH| = |BC| = |BE| = |BF| = |BJ| \text{ (magenta) } \ldots 10 \]
\[ d_5 = |GI| = |HI| \text{ (yellow) } \ldots 2 \]
\[ d_6 = |EI| = |FI| \text{ (grass) } \ldots 2 \]
\[ d_7 = |AJ| \text{ (sea) } \ldots 1 \]
\[ d_8 = |BI| \text{ (darkorange) } \ldots 1 \]
\[ d_9 = |EK| = |GK| \text{ (violet) } \ldots 2 \]
\[ d_{10} = |AI| \text{ (myorangep) } \ldots 1 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_6}{d_4} = \frac{d_7}{d_5} = \frac{d_8}{d_6} = \frac{d_9}{d_7} = \frac{d_9}{d_5} = \frac{d_9}{d_{10}} \text{ (35 occurrences) } \]
2.19.3 Construction RCC46

Construction Process

A, B given initial points
1. $p_1 = p(A, B)$
2. $k_2 = k(B, A)$
   \[ C \in p_1 \cap k_2 \]
3. $k_3 = k(A, B)$
   \[ D \in p_1 \cap k_3, \quad E \in k_2 \cap k_3, \quad F \in k_2 \cap k_3 \]
4. $k_4 = k(D, B)$
   \[ G \in k_2 \cap k_4, \quad H \in k_2 \cap k_4 \]
5. $p_5 = p(F, G)$
   \[ I \in p_1 \cap p_5, \quad J \in p_5 \cap k_3, \quad K \in p_5 \cap k_4 \]

Distances

\[
\begin{align*}
d_0 &= |DI| \text{ (red)} \ldots 1 \\
d_1 &= |CG| = |CH| = |FJ| \text{ (blue)} \ldots 3 \\
d_2 &= |IK| \text{ (green)} \ldots 1 \\
d_3 &= |AG| = |AH| \text{ (cyan)} \ldots 2 \\
d_4 &= |AB| = |AD| = |AE| = |AF| = |AJ| = |BC| = |BE| = |BF| = \\
&= |BG| = |BH| \text{ (magenta)} \ldots 10 \\
d_5 &= |GI| = |HI| \text{ (yellow)} \ldots 2 \\
d_6 &= |EI| = |FI| \text{ (grass)} \ldots 2 \\
d_7 &= |BJ| \text{ (sea)} \ldots 1 \\
d_8 &= |AI| \text{ (darkorange)} \ldots 1 \\
d_9 &= |FK| = |HK| \text{ (violet)} \ldots 2 \\
d_{10} &= |BI| \text{ (myorange)} \ldots 1
\end{align*}
\]

Occurrences of the Golden Ratio

\[
\phi = \frac{d_0}{d_4} = \frac{d_4}{d_8} = \frac{d_5}{d_9} = \frac{d_7}{d_{10}} = \frac{d_6}{d_5} = \frac{d_7}{d_8} = \frac{d_6}{d_5} = \frac{d_9}{d_8} = \frac{d_7}{d_6} = \frac{d_8}{d_7} (35 \text{ occurrences})
\]
2.19.4 Construction RCC52

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(A, B) \)
   \( D \in p_1 \cap k_3 , E \in k_2 \cap k_3 , F \in k_2 \cap k_3 \)
4. \( k_4 = k(D, B) \)
   \( G \in k_2 \cap k_4 , H \in k_2 \cap k_4 \)
5. \( p_5 = p(E, H) \)
   \( I \in p_1 \cap p_5 , J \in p_5 \cap k_3 , K \in p_5 \cap k_4 \)

Distances

\[
\begin{align*}
d_0 &= |DI| \text{ (red)} \ldots 1 \\
d_1 &= |CG| = |CH| = |EJ| \text{ (blue)} \ldots 3 \\
d_2 &= |IK| \text{ (green)} \ldots 1 \\
d_3 &= |AG| = |AH| \text{ (cyan)} \ldots 2 \\
d_4 &= |AB| = |AD| = |AE| = |AF| = |AJ| = |BC| = |BE| = |BF| = \\
&\quad |BG| = |BH| \text{ (magenta)} \ldots 10 \\
d_5 &= |GI| = |HI| \text{ (yellow)} \ldots 2 \\
d_6 &= |EI| = |FI| \text{ (grass)} \ldots 2 \\
d_7 &= |BJ| \text{ (sea)} \ldots 1 \\
d_8 &= |AI| \text{ (darkorange)} \ldots 1 \\
d_9 &= |EK| = |GK| \text{ (violet)} \ldots 2 \\
d_{10} &= |BI| \text{ (myorange)} \ldots 1
\end{align*}
\]

Occurrences of the Golden Ratio

\[
\phi = \frac{d_0}{d_4} = \frac{d_4}{d_8} = \frac{d_8}{d_2} = \frac{d_2}{d_6} = \frac{d_6}{d_0} = \frac{d_0}{d_{10}} \text{ (35 occurrences)}
\]
2.19.5 Construction RCC98

Construction Process

$A$, $B$ given initial points
1. $p_1 = p(A, B)$
2. $k_2 = k(B, A)$
   $C \in p_1 \cap k_2$
3. $k_3 = k(A, C)$
4. $k_4 = k(C, B)$
   $D \in p_1 \cap k_4$, $E \in k_2 \cap k_4$, $F \in k_2 \cap k_4$, $G \in k_3 \cap k_4$, $H \in k_3 \cap k_4$
5. $p_5 = p(F, G)$
   $I \in p_1 \cap p_5$, $J \in p_5 \cap k_2$, $K \in p_5 \cap k_3$

Distances

$d_0 = |AI|$ (red) \ldots 1
$d_1 = |DG| = |DH| = |FJ|$ (blue) \ldots 3
$d_2 = |IK|$ (green) \ldots 1
$d_3 = |BG| = |BH|$ (cyan) \ldots 2
$d_4 = |AB| = |BC| = |BE| = |BF| = |BJ| = |CD| = |CE| = |CF| = |CG| = |CH|$ (magenta) \ldots 10
$d_5 = |GI| = |HI|$ (yellow) \ldots 2
$d_6 = |EI| = |FI|$ (grass) \ldots 2
$d_7 = |CJ|$ (sea) \ldots 1
$d_8 = |BI|$ (darkorange) \ldots 1
$d_9 = |FK| = |HK|$ (violet) \ldots 2
$d_{10} = |CI|$ (myorange) \ldots 1

Occurrences of the Golden Ratio

$\phi = \frac{d_4}{x_1} = \frac{d_4}{x_2} = \frac{d_4}{x_3} = \frac{d_4}{x_4} = \frac{d_4}{x_5} = \frac{d_4}{x_{10}}$ (35 occurrences)
2.19.6 Construction RCC104

Construction Process

A, B given initial points
1. $p_1 = p(A, B)$
2. $k_2 = k(B, A)$
   $C \in p_1 \cap k_2$
3. $k_3 = k(A, C)$
4. $k_4 = k(C, B)$
   $D \in p_1 \cap k_4$ , $E \in k_2 \cap k_4$ , $F \in k_2 \cap k_4$ , $G \in k_3 \cap k_4$ , $H \in k_3 \cap k_4$
5. $p_5 = p(E, H)$
   $I \in p_1 \cap p_5$ , $J \in p_5 \cap k_2$ , $K \in p_5 \cap k_3$

Distances

\[
\begin{align*}
d_0 &= |AI| \, (red) \ldots 1 \\
d_1 &= |DG| = |DH| = |EJ| \, (blue) \ldots 3 \\
d_2 &= |IK| \, (green) \ldots 1 \\
d_3 &= |BG| = |BH| \, (cyan) \ldots 2 \\
d_4 &= |AB| = |BC| = |BE| = |BF| = |BJ| = |CD| = |CE| = |CF| = |CG| = |CH| \, (magenta) \ldots 10 \\
d_5 &= |GI| = |HI| \, (yellow) \ldots 2 \\
d_6 &= |EI| = |FI| \, (grass) \ldots 2 \\
d_7 &= |CJ| \, (sea) \ldots 1 \\
d_8 &= |BI| \, (darkorange) \ldots 1 \\
d_9 &= |EK| = |GK| \, (violet) \ldots 2 \\
d_{10} &= |CI| \, (myorange) \ldots 1 \\
\end{align*}
\]

Occurrences of the Golden Ratio

\[
\phi = \frac{d_4}{d_3} = \frac{d_5}{d_6} = \frac{d_6}{d_5} = \frac{d_4}{d_7} = \frac{d_5}{d_8} = \frac{d_8}{d_7} = \frac{d_2}{d_0} \quad (35 \text{ occurrences})
\]
2.19.7 Construction RCC138

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(C, B) \)
   \( D \in p_1 \cap k_3 , E \in k_2 \cap k_3 , F \in k_2 \cap k_3 \)
4. \( k_4 = k(D, B) \)
   \( G \in k_2 \cap k_4 , H \in k_2 \cap k_4 \)
5. \( p_5 = p(F, G) \)
   \( I \in p_1 \cap p_5 , J \in p_5 \cap k_3 , K \in p_5 \cap k_4 \)

Distances

\[
\begin{align*}
d_0 & = |DI| \quad \text{(red)} \quad 1 \\
d_1 & = |AG| = |AH| = |FJ| \quad \text{(blue)} \quad 3 \\
d_2 & = |IK| \quad \text{(green)} \quad 1 \\
d_3 & = |CG| = |CH| \quad \text{(cyan)} \quad 2 \\
d_4 & = |AB| = |BC| = |BE| = |BF| = |BG| = |BH| = |CD| = |CE| = |CF| = |CJ| \quad \text{(magenta)} \quad 10 \\
d_5 & = |GI| = |HI| \quad \text{(yellow)} \quad 2 \\
d_6 & = |EI| = |FI| \quad \text{(grass)} \quad 2 \\
d_7 & = |BJ| \quad \text{(sea)} \quad 1 \\
d_8 & = |CI| \quad \text{(darkorange)} \quad 1 \\
d_9 & = |FK| = |HK| \quad \text{(violet)} \quad 2 \\
d_{10} & = |BI| \quad \text{(myorange)} \quad 1
\end{align*}
\]

Occurrences of the Golden Ratio

\[
\phi = \frac{d_7}{d_6} = \frac{d_5}{d_6} = \frac{d_2}{d_6} = \frac{d_4}{d_6} = \frac{d_6}{d_9} = \frac{d_8}{d_{10}} \quad (35 \text{ occurrences})
\]
2.19.8 Construction RCC144

Construction Process

A, B given initial points

1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(C, B) \)
   \( D \in p_1 \cap k_3 , E \in k_2 \cap k_3 , F \in k_2 \cap k_3 \)
4. \( k_4 = k(D, B) \)
   \( G \in k_2 \cap k_4 , H \in k_2 \cap k_4 \)
5. \( p_5 = p(E, H) \)
   \( I \in p_1 \cap p_5 , J \in p_5 \cap k_3 , K \in p_5 \cap k_4 \)

Distances

\[
\begin{align*}
d_0 &= |DI| \quad \text{(red)} \ldots 1 \\
d_1 &= |AG| = |AH| = |EJ| \quad \text{(blue)} \ldots 3 \\
d_2 &= |IK| \quad \text{(green)} \ldots 1 \\
d_3 &= |CG| = |CH| \quad \text{(cyan)} \ldots 2 \\
d_4 &= |AB| = |BC| = |BE| = |BF| = |BG| = |BH| = |CD| = |CE| = |CF| = |CJ| \quad \text{(magenta)} \ldots 10 \\
d_5 &= |GI| = |HI| \quad \text{(yellow)} \ldots 2 \\
d_6 &= |EI| = |FI| \quad \text{(grass)} \ldots 2 \\
d_7 &= |BJ| \quad \text{(sea)} \ldots 1 \\
d_8 &= |CI| \quad \text{(darkorange)} \ldots 1 \\
d_9 &= |EK| = |GK| \quad \text{(violet)} \ldots 2 \\
d_{10} &= |BI| \quad \text{(myorange)} \ldots 1
\end{align*}
\]

Occurrences of the Golden Ratio

\[
\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} = \frac{d_2}{d_3} = \frac{d_3}{d_4} = \frac{d_4}{d_5} = \frac{d_5}{d_6} = \frac{d_8}{d_{10}} \quad (35 \text{ occurrences})
\]
2.19.9 Construction RCC176

Construction Process

A, B given initial points

1. \( p_1 = p(A, B) \)
2. \( k_2 = k(A, B) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(C, A) \)
   \( D \in p_1 \cap k_3 \), \( E \in k_2 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(B, C) \)
   \( G \in k_3 \cap k_4 \), \( H \in k_3 \cap k_4 \)
5. \( p_5 = p(F, G) \)
   \( I \in p_1 \cap p_5 \), \( J \in p_5 \cap k_2 \), \( K \in p_5 \cap k_4 \)

Distances

\[
\begin{align*}
d_0 &= |BI| \text{ (red)} \ldots 1 \\
d_1 &= |DG| = |DH| = |FJ| \text{ (blue)} \ldots 3 \\
d_2 &= |IK| \text{ (green)} \ldots 1 \\
d_3 &= |AG| = |AH| \text{ (cyan)} \ldots 2 \\
d_4 &= |AB| = |AC| = |AE| = |AF| = |AJ| = |CD| = |CE| = |CF| = |CG| = |CH| \text{ (magenta)} \ldots 10 \\
d_5 &= |GI| = |HI| \text{ (yellow)} \ldots 2 \\
d_6 &= |EI| = |FI| \text{ (grass)} \ldots 2 \\
d_7 &= |CJ| \text{ (sea)} \ldots 1 \\
d_8 &= |AI| \text{ (darkorange)} \ldots 1 \\
d_9 &= |FK| = |HK| \text{ (violet)} \ldots 2 \\
d_{10} &= |CI| \text{ (myorangle)} \ldots 1
\end{align*}
\]

Occurrences of the Golden Ratio

\[\phi = \frac{d_6}{d_4} = \frac{d_4}{d_2} = \frac{d_2}{d_4} = \frac{d_4}{d_6} = \frac{d_6}{d_9} = \frac{d_9}{d_{10}} \text{ (35 occurrences)}\]
2.19.10 Construction RCC182

A, B given initial points
1. \(p_1 = p(A, B)\)
2. \(k_2 = k(A, B)\)
   \(C \in p_1 \cap k_2\)
3. \(k_3 = k(C, A)\)
   \(D \in p_1 \cap k_3\), \(E \in k_2 \cap k_3\), \(F \in k_2 \cap k_3\)
4. \(k_4 = k(B, C)\)
   \(G \in k_3 \cap k_4\), \(H \in k_3 \cap k_4\)
5. \(p_5 = p(E, H)\)
   \(I \in p_1 \cap p_5\), \(J \in p_5 \cap k_2\), \(K \in p_5 \cap k_4\)

Distances
\[
\begin{align*}
d_0 &= |BI| \text{ (red)} \ldots 1 \\
d_1 &= |DG| = |DH| = |EJ| \text{ (blue)} \ldots 3 \\
d_2 &= |IK| \text{ (green)} \ldots 1 \\
d_3 &= |AG| = |AH| \text{ (cyan)} \ldots 2 \\
d_4 &= |AB| = |AC| = |AE| = |AF| = |AJ| = |CD| = |CE| = |CF| = |CG| = |CH| \text{ (magenta)} \ldots 10 \\
d_5 &= |GI| = |HI| \text{ (yellow)} \ldots 2 \\
d_6 &= |EI| = |FI| \text{ (grass)} \ldots 2 \\
d_7 &= |CJ| \text{ (sea)} \ldots 1 \\
d_8 &= |AI| \text{ (darkorange)} \ldots 1 \\
d_9 &= |EK| = |GK| \text{ (violet)} \ldots 2 \\
d_{10} &= |CI| \text{ (myorange)} \ldots 1 \\
\end{align*}
\]

Occurrences of the Golden Ratio
\[
\phi = \frac{d_1}{d_4} = \frac{d_1}{d_5} = \frac{d_2}{d_5} = \frac{d_2}{d_7} = \frac{d_5}{d_6} = \frac{d_8}{d_{10}} \text{ (35 occurrences)}
\]

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Construction RCC201

Construction Process

1. \( p_1 = p(A, B) \)
2. \( k_2 = k(A, B) \)
3. \( k_3 = k(C, A) \)
4. \( k_4 = k(D, A) \)
5. \( p_5 = p(F, G) \)

Distances

- \( d_0 = |DI| \) (red) ... 1
- \( d_1 = |BG| = |BH| = |FJ| \) (blue) ... 3
- \( d_2 = |IK| \) (green) ... 1
- \( d_3 = |CG| = |CH| \) (cyan) ... 2
- \( d_4 = |AB| = |AC| = |AE| = |AF| = |AG| = |AH| = |CD| = |CE| = |CF| = |CJ| \) (magenta) ... 10
- \( d_5 = |GI| = |HI| \) (yellow) ... 2
- \( d_6 = |EI| = |FI| \) (grass) ... 2
- \( d_7 = |AJ| \) (sea) ... 1
- \( d_8 = |CI| \) (darkorange) ... 1
- \( d_9 = |FK| = |HK| \) (violet) ... 2
- \( d_{10} = |AI| \) (myorange) ... 1

Occurrences of the Golden Ratio

\[
\phi = \frac{d_6}{d_4} = \frac{d_4}{d_2} = \frac{d_2}{d_0} = \frac{d_4}{d_6} = \frac{d_6}{d_8} = \frac{d_8}{d_{10}} \quad (35 \text{ occurrences})
\]
2.19.12 Construction RCC207

Construction Process

A, B given initial points
1. $p_1 = p(A, B)$
2. $k_2 = k(A, B)$
   $C \in p_1 \cap k_2$
3. $k_3 = k(C, A)$
   $D \in p_1 \cap k_3$, $E \in k_2 \cap k_3, F \in k_2 \cap k_3$
4. $k_4 = k(D, A)$
   $G \in k_2 \cap k_4$, $H \in k_2 \cap k_4$
5. $p_5 = p(E, H)$
   $I \in p_1 \cap p_5$, $J \in p_5 \cap k_3$, $K \in p_5 \cap k_4$

Distances

$d_0 = |DI|$ (red) ... 1
$d_1 = |BG| = |BH| = |EJ|$ (blue) ... 3
$d_2 = |IK|$ (green) ... 1
$d_3 = |CG| = |CH|$ (cyan) ... 2
$d_4 = |AB| = |AC| = |AE| = |AF| = |AG| = |AH| = |CD| = |CE| = |CF| = |CJ|$ (magenta) ... 10
$d_5 = |GI| = |HI|$ (yellow) ... 2
$d_6 = |EI| = |FI|$ (grass) ... 2
$d_7 = |AJ|$ (sea) ... 1
$d_8 = |CI|$ (darkorange) ... 1
$d_9 = |EK| = |GK|$ (violet) ... 2
$d_{10} = |AI|$ (myorange) ... 1

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_4} = \frac{d_1}{d_5} = \frac{d_2}{d_6} = \frac{d_3}{d_7} = \frac{d_4}{d_8} = \frac{d_5}{d_9} = \frac{d_6}{d_{10}}$ (35 occurrences)
2.20 Class of Similar Constructions No. 13

Contained Constructions
RCC14, RCC20, RCC47, RCC53, RCC99, RCC105, RCC139, RCC145, RCC177, RCC183, RCC202, RCC208
2.20.1 Construction RCC14

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(A, B) \)
   \( D \in k_2 \cap k_3 \)
4. \( k_4 = k(C, A) \)
   \( E \in k_3 \cap k_4, F \in k_3 \cap k_4 \)
5. \( k_5 = k(E, D) \)
   \( G \in k_2 \cap k_5 \)

Distances

\( d_0 = |AG| \) (red) ... 1
\( d_1 = |BE| = |BF| \) (blue) ... 2

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} \) (2 occurrences)
2.20.2 Construction RCC20

Construction Process

1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(A, B) \)
   \( D \in k_2 \cap k_3 \)
4. \( k_4 = k(C, A) \)
   \( E \in k_3 \cap k_4 \), \( F \in k_3 \cap k_4 \)
5. \( k_5 = k(F, D) \)
   \( G \in k_2 \cap k_5 \)

Distances

\[ d_0 = |AG| \text{ (red)} \ldots 1 \]
\[ d_1 = |BE| = |BF| \text{ (blue)} \ldots 2 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} \text{ (2 occurrences)} \]
2.20.3 Construction RCC47

Construction Process

\[ \begin{align*}
A, B \text{ given initial points} \\
1. & \quad p_1 = p(A, B) \\
2. & \quad k_2 = k(B, A) \\
3. & \quad k_3 = k(A, B) \\
& \quad C \in p_1 \cap k_3, \ D \in k_2 \cap k_3 \\
4. & \quad k_4 = k(C, B) \\
& \quad E \in k_2 \cap k_4, \ F \in k_2 \cap k_4 \\
5. & \quad k_5 = k(E, D) \\
& \quad G \in k_3 \cap k_5
\end{align*} \]

Distances

\[ \begin{align*}
d_0 &= |BG| \quad \text{(red)} \ldots 1 \\
d_1 &= |AE| = |AF| \quad \text{(blue)} \ldots 2
\end{align*} \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} \quad \text{(2 occurrences)} \]
2.20.4 Construction RCC53

**Construction Process**

1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
3. \( k_3 = k(A, B) \)
   
   \( C \in p_1 \cap k_3, \ D \in k_2 \cap k_3 \)
4. \( k_4 = k(C, B) \)
   
   \( E \in k_2 \cap k_4, \ F \in k_2 \cap k_4 \)
5. \( k_5 = k(F, D) \)
   
   \( G \in k_3 \cap k_5 \)

**Distances**

\[ d_0 = |BG| \text{ (red)} \ldots 1 \]
\[ d_1 = |AE| = |AF| \text{ (blue)} \ldots 2 \]

**Occurrences of the Golden Ratio**

\[ \phi = \frac{d_0}{d_1} \text{ (2 occurrences)} \]
2.20.5 Construction RCC99

Construction Process

A, B given initial points
1. $p_1 = p(A, B)$
2. $k_2 = k(B, A)$
   $C \in p_1 \cap k_2$
3. $k_3 = k(A, C)$
4. $k_4 = k(C, B)$
   $D \in k_2 \cap k_4$, $E \in k_3 \cap k_4$, $F \in k_3 \cap k_4$
5. $k_5 = k(E, D)$
   $G \in k_2 \cap k_5$

Distances

$d_0 = |CG|$ (red) \ldots 1
$d_1 = |BE| = |BF|$ (blue) \ldots 2

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_1}$ (2 occurrences)
2.20.6 Construction RCC105

Construction Process

\(A, B\) given initial points
1. \(p_1 = p(A, B)\)
2. \(k_2 = k(B, A)\)
\(C \in p_1 \cap k_2\)
3. \(k_3 = k(A, C)\)
4. \(k_4 = k(C, B)\)
\(D \in k_2 \cap k_4\ , \ E \in k_3 \cap k_4\ , \ F \in k_3 \cap k_4\)
5. \(k_5 = k(F, D)\)
\(G \in k_2 \cap k_5\)

Distances

\(d_0 = |CG|\) (red) \ldots 1
\(d_1 = |BE| = |BF|\) (blue) \ldots 2

Occurrences of the Golden Ratio

\(\phi = \frac{d_0}{d_1}\) (2 occurrences)
2.20 Class of Similar Constructions No. 13

2.20.7 Construction RCC139

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(C, B) \)
   \( D \in p_1 \cap k_3 \), \( E \in k_2 \cap k_3 \)
4. \( k_4 = k(D, B) \)
   \( F \in k_2 \cap k_4 \), \( G \in k_2 \cap k_4 \)
5. \( k_5 = k(F, E) \)
   \( H \in k_3 \cap k_5 \)

Distances

\( d_0 = |BH| \) (red) \ldots 1
\( d_1 = |CF| = |CG| \) (blue) \ldots 2

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} \) (2 occurrences)
2.20.8 Construction RCC145

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(C, B) \)
   \( D \in p_1 \cap k_3, E \in k_2 \cap k_3 \)
4. \( k_4 = k(D, B) \)
   \( F \in k_2 \cap k_4, G \in k_2 \cap k_4 \)
5. \( k_5 = k(G, E) \)
   \( H \in k_3 \cap k_5 \)

Distances

\( d_0 = |BH| \) (red) \ldots 1
\( d_1 = |CF| = |CG| \) (blue) \ldots 2

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} \) (2 occurrences)
2.20.9 Construction RCC177

Construction Process

\(A, B\) given initial points
1. \(p_1 = p(A,B)\)
2. \(k_2 = k(A,B)\)
   \(C \in p_1 \cap k_2\)
3. \(k_3 = k(C,A)\)
   \(D \in k_2 \cap k_3\)
4. \(k_4 = k(B,C)\)
   \(E \in k_3 \cap k_4, F \in k_3 \cap k_4\)
5. \(k_5 = k(E,D)\)
   \(G \in k_2 \cap k_5\)

Distances

\[d_0 = |CG|\] (red) \ldots 1
\[d_1 = |AE| = |AF|\] (blue) \ldots 2

Occurrences of the Golden Ratio

\[\phi = \frac{d_0}{d_1}\] (2 occurrences)
2.20.10 Construction RCC183

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(A, B) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(C, A) \)
   \( D \in k_2 \cap k_3 \)
4. \( k_4 = k(B, C) \)
   \( E \in k_3 \cap k_4, F \in k_3 \cap k_4 \)
5. \( k_5 = k(F, D) \)
   \( G \in k_2 \cap k_5 \)

Distances

\( d_0 = |CG| \) (red) \( \ldots 1 \)
\( d_1 = |AE| = |AF| \) (blue) \( \ldots 2 \)

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} \) (2 occurrences)
2.20.11 Construction RCC202

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(A, B) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(C, A) \)
   \( D \in p_1 \cap k_3 , E \in k_2 \cap k_3 \)
4. \( k_4 = k(D, A) \)
   \( F \in k_2 \cap k_4 , G \in k_2 \cap k_4 \)
5. \( k_5 = k(F, E) \)
   \( H \in k_3 \cap k_5 \)

Distances

\( d_0 = |AH| \) (red) \dots 1
\( d_1 = |CF| = |CG| \) (blue) \dots 2

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} \) (2 occurrences)
2.20.12 Construction RCC208

Construction Process

\[ A, B \text{ given initial points} \]
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(A, B) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(C, A) \)
   \( D \in p_1 \cap k_3 \), \( E \in k_2 \cap k_3 \)
4. \( k_4 = k(D, A) \)
   \( F \in k_2 \cap k_4 \), \( G \in k_2 \cap k_4 \)
5. \( k_5 = k(G, E) \)
   \( H \in k_3 \cap k_5 \)

Distances
\[ d_0 = |AH| \text{ (red)} \ldots 1 \]
\[ d_1 = |CF| = |CG| \text{ (blue)} \ldots 2 \]

Occurrences of the Golden Ratio
\[ \phi = \frac{d_0}{d_1} \text{ (2 occurrences)} \]
2.21 Class of Similar Constructions No. 14

Contained Constructions
RCC15, RCC21, RCC48, RCC54, RCC100, RCC106, RCC140, RCC146, RCC178, RCC184, RCC203, RCC209
2.21.1 Construction RCC15

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(A, B) \)
   \( D \in p_1 \cap k_3 , E \in k_2 \cap k_3 , F \in k_2 \cap k_3 \)
4. \( k_4 = k(C, A) \)
   \( G \in k_3 \cap k_4 , H \in k_3 \cap k_4 \)
5. \( k_5 = k(F, G) \)
   \( I \in k_2 \cap k_5 , J \in k_2 \cap k_5 , K \in k_3 \cap k_5 , L \in k_4 \cap k_5 \)

Distances

\[
\begin{align*}
   d_0 &= |IL| \text{ (red)} \ldots 1 \\
   d_1 &= |AL| \text{ (blue)} \ldots 1 \\
   d_2 &= |JL| \text{ (green)} \ldots 1 \\
   d_3 &= |AJ| = |BK| \text{ (cyan)} \ldots 2 \\
   d_4 &= |CE| = |CF| = |DE| = |DF| = |EF| = |HK| \text{ (magenta)} \ldots 6 \\
   d_5 &= |JK| = |IJ| \text{ (yellow)} \ldots 2 \\
   d_6 &= |AJ| = |BG| = |BH| \text{ (grass)} \ldots 3 
\end{align*}
\]

Occurrences of the Golden Ratio

\[
\phi = \frac{d_6}{d_5} = \frac{d_5}{d_4} = \frac{d_4}{d_3} = \frac{d_3}{d_2} = \frac{d_2}{d_1} \quad (16 \text{ occurrences})
\]
2.21.2 Construction RCC21

Construction Process

$A, B$ given initial points
1. $p_1 = p(A, B)$
2. $k_2 = k(B, A)$
   $C \in p_1 \cap k_2$
3. $k_3 = k(A, B)$
   $D \in p_1 \cap k_3$, $E \in k_2 \cap k_3$, $F \in k_2 \cap k_3$
4. $k_4 = k(C, A)$
   $G \in k_3 \cap k_4$, $H \in k_3 \cap k_4$
5. $k_5 = k(E, H)$
   $I \in k_2 \cap k_5$, $J \in k_2 \cap k_5$, $K \in k_3 \cap k_5$, $L \in k_4 \cap k_5$

Distances

$d_0 = |IL|$ (red) \ldots 1
$d_1 = |AL|$ (blue) \ldots 1
$d_2 = |AJ|$ (green) \ldots 1
$d_3 = |AJ| = |BK|$ (cyan) \ldots 2
$d_4 = |CE| = |CF| = |DE| = |DF| = |EF| = |GK|$ (magenta) \ldots 6
$d_5 = |HK| = |IJ|$ (yellow) \ldots 2
$d_6 = |AI| = |BG| = |BH|$ (grass) \ldots 3

Occurrences of the Golden Ratio

$\phi = \frac{d_2}{d_1} = \frac{d_3}{d_2} = \frac{d_4}{d_3} = \frac{d_5}{d_4} = \frac{d_6}{d_5}$ (16 occurrences)
2.21.3 Construction RCC48

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(A, B) \)
   \( D \in p_1 \cap k_3 , E \in k_2 \cap k_3 , F \in k_2 \cap k_3 \)
4. \( k_4 = k(D, B) \)
   \( G \in k_2 \cap k_4 , H \in k_2 \cap k_4 \)
5. \( k_5 = k(F, G) \)
   \( I \in k_2 \cap k_5 , J \in k_3 \cap k_5 , K \in k_3 \cap k_5 , L \in k_4 \cap k_5 \)

Distances

\( d_0 = |KL| \) (red) \ldots 1
\( d_1 = |BL| \) (blue) \ldots 1
\( d_2 = |JL| \) (green) \ldots 1
\( d_3 = |AI| = |BJ| \) (cyan) \ldots 2
\( d_4 = |CE| = |CF| = |DE| = |DF| = |EF| = |HI| \) (magenta) \ldots 6
\( d_5 = |GI| = |JK| \) (yellow) \ldots 2
\( d_6 = |AG| = |AH| = |BK| \) (grass) \ldots 3

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} = \frac{d_2}{d_3} = \frac{d_4}{d_5} = \frac{d_6}{d_6} \) (16 occurrences)
2.21.4 Construction RCC54

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(A, B) \)
   \( D \in p_1 \cap k_3 \), \( E \in k_2 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(D, B) \)
   \( G \in k_2 \cap k_4 \), \( H \in k_2 \cap k_4 \)
5. \( k_5 = k(E, H) \)
   \( I \in k_2 \cap k_5 \), \( J \in k_3 \cap k_5 \), \( K \in k_3 \cap k_5 \), \( L \in k_4 \cap k_5 \)

Distances

\( d_0 = |KL| \) (red) ... 1
\( d_1 = |BL| \) (blue) ... 1
\( d_2 = |JL| \) (green) ... 1
\( d_3 = |AI| = |BJ| \) (cyan) ... 2
\( d_4 = |CE| = |CF| = |DE| = |DF| = |EF| = |GI| \) (magenta) ... 6
\( d_5 = |HI| = |JK| \) (yellow) ... 2
\( d_6 = |AG| = |AH| = |BK| \) (grass) ... 3

Occurrences of the Golden Ratio

\( \phi = \frac{d_4}{d_1} = \frac{d_3}{d_2} = \frac{d_5}{d_1} = \frac{d_6}{d_5} \) (16 occurrences)
2.21.5 Construction RCC100

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(A, C) \)
4. \( k_4 = k(C, B) \)
   \( D \in p_1 \cap k_4, E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4 \)
5. \( k_5 = k(F, G) \)
   \( I \in k_2 \cap k_5, J \in k_2 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5 \)

Distances

\[ d_0 = |JK| \text{ (red)} \ldots 1 \]
\[ d_1 = |CK| \text{ (blue)} \ldots 1 \]
\[ d_2 = |IK| \text{ (green)} \ldots 1 \]
\[ d_3 = |BL| = |CI| \text{ (cyan)} \ldots 2 \]
\[ d_4 = |AE| = |AF| = |DE| = |DF| = |EF| = |HL| \text{ (magenta)} \ldots 6 \]
\[ d_5 = |GL| = |IJ| \text{ (yellow)} \ldots 2 \]
\[ d_6 = |BG| = |BH| = |CJ| \text{ (grass)} \ldots 3 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_6}{d_3} = \frac{d_4}{d_4} = \frac{d_2}{d_5} = \frac{d_3}{d_6} \quad (16 \text{ occurrences}) \]
2.21.6 Construction RCC106

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(A, C) \)
4. \( k_4 = k(C, B) \)
   \( D \in p_1 \cap k_4 , E \in k_2 \cap k_4 , F \in k_2 \cap k_4 , G \in k_3 \cap k_4 , H \in k_3 \cap k_4 \)
5. \( k_5 = k(E, H) \)
   \( I \in k_2 \cap k_5 , J \in k_2 \cap k_5 , K \in k_3 \cap k_5 , L \in k_4 \cap k_5 \)

Distances

\[
\begin{align*}
d_0 &= |JK| \text{ (red)} \ldots 1 \\
d_1 &= |CK| \text{ (blue)} \ldots 1 \\
d_2 &= |JK| \text{ (green)} \ldots 1 \\
d_3 &= |BL| = |CI| \text{ (cyan)} \ldots 2 \\
d_4 &= |AE| = |AF| = |DE| = |DF| = |EF| = |GL| \text{ (magenta)} \ldots 6 \\
d_5 &= |HL| = |IJ| \text{ (yellow)} \ldots 2 \\
d_6 &= |BG| = |BH| = |CJ| \text{ (grass)} \ldots 3
\end{align*}
\]

Occurrences of the Golden Ratio

\[
\phi = \frac{d_0}{d_3} = \frac{d_3}{d_4} = \frac{d_4}{d_5} = \frac{d_5}{d_6} \text{ (16 occurrences)}
\]
### Construction Process

A, B given initial points

1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(C, B) \)
   
   \( D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3 \)
4. \( k_4 = k(D, B) \)
   
   \( G \in k_2 \cap k_4, H \in k_2 \cap k_4 \)
5. \( k_5 = k(F, G) \)
   
   \( I \in k_2 \cap k_5, J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5 \)

### Distances

\[
\begin{align*}
d_0 &= |JL| \text{ (red) } \ldots 1 \\
d_1 &= |BL| \text{ (blue) } \ldots 1 \\
d_2 &= |KL| \text{ (green) } \ldots 1 \\
d_3 &= |BK| = |CI| \text{ (cyan) } \ldots 2 \\
d_4 &= |AE| = |AF| = |DF| = |EF| = |HI| \text{ (magenta) } \ldots 6 \\
d_5 &= |GI| = |JK| \text{ (yellow) } \ldots 2 \\
d_6 &= |BJ| = |CG| = |CH| \text{ (grass) } \ldots 3
\end{align*}
\]

### Occurrences of the Golden Ratio

\[
\phi = \frac{d_6}{d_4} = \frac{d_5}{d_3} = \frac{d_2}{d_1} = \frac{d_4}{d_2} \quad (16 \text{ occurrences})
\]
2.21.8 Construction RCC146

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(C, B) \)
   \( D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3 \)
4. \( k_4 = k(D, B) \)
   \( G \in k_2 \cap k_4, H \in k_2 \cap k_4 \)
5. \( k_5 = k(E, H) \)
   \( I \in k_2 \cap k_5, J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5 \)

Distances

\[
\begin{align*}
  d_0 &= |JL| \quad \text{(red)} \ldots 1 \\
  d_1 &= |BL| \quad \text{(blue)} \ldots 1 \\
  d_2 &= |KL| \quad \text{(green)} \ldots 1 \\
  d_3 &= |BK| = |CI| \quad \text{(cyan)} \ldots 2 \\
  d_4 &= |AE| = |AF| = |DE| = |DF| = |EF| = |GI| \quad \text{(magenta)} \ldots 6 \\
  d_5 &= |HI| = |JK| \quad \text{(yellow)} \ldots 2 \\
  d_6 &= |BJ| = |CG| = |CH| \quad \text{(grass)} \ldots 3 \\
\end{align*}
\]

Occurrences of the Golden Ratio

\[
\phi = \frac{d_0}{d_1} = \frac{d_2}{d_3} = \frac{d_4}{d_5} = \frac{d_6}{d_4} \quad (16 \text{ occurrences})
\]


2.21.9 Construction RCC178

Construction Process

\( A, B \) given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(A, B) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(C, A) \)
   \( D \in p_1 \cap k_3 \), \( E \in k_2 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(B, C) \)
   \( G \in k_3 \cap k_4 \), \( H \in k_3 \cap k_4 \)
5. \( k_5 = k(F, G) \)
   \( I \in k_2 \cap k_5 \), \( J \in k_2 \cap k_5 \), \( K \in k_3 \cap k_5 \), \( L \in k_4 \cap k_5 \)

Distances

\( d_0 = |IL| \) (red) \ldots 1  
\( d_1 = |CL| \) (blue) \ldots 1  
\( d_2 = |JL| \) (green) \ldots 1  
\( d_3 = |AK| = |CJ| \) (cyan) \ldots 2  
\( d_4 = |BE| = |BF| = |DF| = |EF| = |HK| \) (magenta) \ldots 6  
\( d_5 = |GK| = |IJ| \) (yellow) \ldots 2  
\( d_6 = |AG| = |AH| = |CI| \) (grass) \ldots 3

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_3} = \frac{d_3}{d_4} = \frac{d_4}{d_5} = \frac{d_5}{d_6} \] (16 occurrences)
2.21 Class of Similar Constructions No. 14

2.21.10 Construction RCC184

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(A, B) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(C, A) \)
   \( D \in p_1 \cap k_3 , E \in k_2 \cap k_3 , F \in k_2 \cap k_3 \)
4. \( k_4 = k(B, C) \)
   \( G \in k_3 \cap k_4 , H \in k_3 \cap k_4 \)
5. \( k_5 = k(E, H) \)
   \( I \in k_2 \cap k_5 , J \in k_2 \cap k_5 , K \in k_3 \cap k_5 , L \in k_4 \cap k_5 \)

Distances

\[
\begin{align*}
  d_0 &= |IL| \text{ (red)} \ldots 1 \\
  d_1 &= |CL| \text{ (blue)} \ldots 1 \\
  d_2 &= |JL| \text{ (green)} \ldots 1 \\
  d_3 &= |AK| = |CJ| \text{ (cyan)} \ldots 2 \\
  d_4 &= |BE| = |BF| = |DE| = |DF| = |EF| = |GK| \text{ (magenta)} \ldots 6 \\
  d_5 &= |HK| = |IJ| \text{ (yellow)} \ldots 2 \\
  d_6 &= |AG| = |AH| = |CI| \text{ (grass)} \ldots 3 \\
\end{align*}
\]

Occurrences of the Golden Ratio

\[
\phi = \frac{d_4}{d_3} = \frac{d_6}{d_5} = \frac{d_2}{d_1} = \frac{d_4}{d_3} \text{ (16 occurrences)}
\]
2.21.11 Construction RCC203

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(A, B) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(C, A) \)
   \( D \in p_1 \cap k_3 \), \( E \in k_2 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(D, A) \)
   \( G \in k_2 \cap k_4 \), \( H \in k_2 \cap k_4 \)
5. \( k_5 = k(F, G) \)
   \( I \in k_2 \cap k_5 \), \( J \in k_3 \cap k_5 \), \( K \in k_3 \cap k_5 \), \( L \in k_4 \cap k_5 \)

Distances

\( d_0 = |KL| \, \text{(red)} \ldots 1 \)
\( d_1 = |AL| \, \text{(blue)} \ldots 1 \)
\( d_2 = |JJ| \, \text{(green)} \ldots 1 \)
\( d_3 = |AJ| = |CI| \, \text{(cyan)} \ldots 2 \)
\( d_4 = |BE| = |BF| = |DE| = |EF| = |HI| \, \text{(magenta)} \ldots 6 \)
\( d_5 = |GI| = |JK| \, \text{(yellow)} \ldots 2 \)
\( d_6 = |AK| = |CG| = |CH| \, \text{(grass)} \ldots 3 \)

Occurrences of the Golden Ratio

\( \phi = \frac{d_6}{d_0} = \frac{d_4}{d_1} = \frac{d_2}{d_3} = \frac{d_4}{d_5} \) (16 occurrences)
2.21.12 Construction RCC209

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(A, B) \)
  \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(C, A) \)
  \( D \in p_1 \cap k_3 \), \( E \in k_2 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(D, A) \)
  \( G \in k_2 \cap k_4 \), \( H \in k_2 \cap k_4 \)
5. \( k_5 = k(E, H) \)
  \( I \in k_2 \cap k_5 \), \( J \in k_3 \cap k_5 \), \( K \in k_3 \cap k_5 \), \( L \in k_4 \cap k_5 \)

Distances

\[
\begin{align*}
d_0 &= |KL| \text{ (red)} \ldots 1 \\
d_1 &= |AL| \text{ (blue)} \ldots 1 \\
d_2 &= |JL| \text{ (green)} \ldots 1 \\
d_3 &= |AJ| = |CL| \text{ (cyan)} \ldots 2 \\
d_4 &= |BE| = |BF| = |DE| = |DF| = |EF| = |GI| \text{ (magenta)} \ldots 6 \\
d_5 &= |HI| = |JK| \text{ (yellow)} \ldots 2 \\
d_6 &= |AK| = |CG| = |CH| \text{ (grass)} \ldots 3
\end{align*}
\]

Occurrences of the Golden Ratio

\[
\phi = \frac{d_2}{d_1} = \frac{d_3}{d_2} = \frac{d_4}{d_3} = \frac{d_5}{d_4} \quad (16 \text{ occurrences})
\]
2.22 Class of Similar Constructions No. 15

-contained Constructions

RCC25, RCC26, RCC58, RCC59, RCC110, RCC111, RCC150, RCC151, RCC188, RCC189, RCC213, RCC214
2.22.1 Construction RCC25

\[ \text{Construction Process} \]

\[ A, B \text{ given initial points} \]
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(A, B) \)
4. \( k_4 = k(C, A) \)
   \( D \in p_1 \cap k_4, E \in k_3 \cap k_4, F \in k_3 \cap k_4 \)
5. \( k_5 = k(F, E) \)
   \( G \in p_1 \cap k_5, H \in p_1 \cap k_5 \)

\[ \text{Distances} \]

\[ d_0 = |DG| \text{ (red)} \ldots 1 \]
\[ d_1 = |GH| \text{ (blue)} \ldots 1 \]
\[ d_2 = |DH| \text{ (green)} \ldots 1 \]

\[ \text{Occurrences of the Golden Ratio} \]

\[ \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (2 occurrences)} \]
2.22.2 Construction RCC26

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(A, B) \)
4. \( k_4 = k(C, A) \)
   \( D \in p_1 \cap k_4 , \ E \in k_3 \cap k_4 , \ F \in k_3 \cap k_4 \)
5. \( k_5 = k(E, F) \)
   \( G \in p_1 \cap k_5 , \ H \in p_1 \cap k_5 \)

Distances

\( d_0 = |DG| \) (red) ... 1
\( d_1 = |GH| \) (blue) ... 1
\( d_2 = |DH| \) (green) ... 1

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \) (2 occurrences)
2.22.3 Construction RCC58

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
3. \( k_3 = k(A, B) \)
   \( C \in p_1 \cap k_3 \)
4. \( k_4 = k(C, B) \)
   \( D \in p_1 \cap k_4 , E \in k_2 \cap k_4 \), \( F \in k_2 \cap k_4 \)
5. \( k_5 = k(F, E) \)
   \( G \in p_1 \cap k_5 , H \in p_1 \cap k_5 \)

Distances

\( d_0 = |DH| \) (red) \ldots 1
\( d_1 = |GH| \) (blue) \ldots 1
\( d_2 = |DG| \) (green) \ldots 1

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \) (2 occurrences)
2.22.4 Construction RCC59

Construction Process

\(A, B\) given initial points
1. \(p_1 = p(A, B)\)
2. \(k_2 = k(B, A)\)
3. \(k_3 = k(A, B)\)
   \(C \in p_1 \cap k_3\)
4. \(k_4 = k(C, B)\)
   \(D \in p_1 \cap k_4, E \in k_2 \cap k_4, F \in k_2 \cap k_4\)
5. \(k_5 = k(E, F)\)
   \(G \in p_1 \cap k_5, H \in p_1 \cap k_5\)

Distances
\[d_0 = |DH| \text{ (red)} \ldots 1\]
\[d_1 = |GH| \text{ (blue)} \ldots 1\]
\[d_2 = |DG| \text{ (green)} \ldots 1\]

Occurrences of the Golden Ratio
\[\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (2 occurrences)}\]
2.22.5 Construction RCC110

Construction Process

A, B given initial points
1. $p_1 = p(A, B)$
2. $k_2 = k(B, A)$
   $C \in p_1 \cap k_2$
3. $k_3 = k(A, C)$
   $D \in p_1 \cap k_3$
4. $k_4 = k(C, B)$
   $E \in k_3 \cap k_4$, $F \in k_3 \cap k_4$
5. $k_5 = k(F, E)$
   $G \in p_1 \cap k_5$, $H \in p_1 \cap k_5$

Distances

$d_0 = |DH|$ (red) \ldots 1
$d_1 = |GH|$ (blue) \ldots 1
$d_2 = |DG|$ (green) \ldots 1

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2}$ (2 occurrences)
2.22.6 Construction RCC111

Construction Process

\( A, B \) given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(A, C) \)
   \( D \in p_1 \cap k_3 \)
4. \( k_4 = k(C, B) \)
   \( E \in k_3 \cap k_4, F \in k_3 \cap k_4 \)
5. \( k_5 = k(E, F) \)
   \( G \in p_1 \cap k_5, H \in p_1 \cap k_5 \)

Distances
\( d_0 = |DH| \) (red) \ldots 1
\( d_1 = |GH| \) (blue) \ldots 1
\( d_2 = |DG| \) (green) \ldots 1

Occurrences of the Golden Ratio
\[ \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \] (2 occurrences)
2.22.7 Construction RCC150

Construction Process

A, B given initial points
1. $p_1 = p(A, B)$
2. $k_2 = k(B, A)$
   $C \in p_1 \cap k_2$
3. $k_3 = k(C, B)$
   $D \in p_1 \cap k_3$
4. $k_4 = k(D, B)$
   $E \in p_1 \cap k_4$, $F \in k_2 \cap k_4$, $G \in k_2 \cap k_4$
5. $k_5 = k(G, F)$
   $H \in p_1 \cap k_5$, $I \in p_1 \cap k_5$

Distances

$d_0 = |EH|$ (red) \ldots 1
$d_1 = |HI|$ (blue) \ldots 1
$d_2 = |EI|$ (green) \ldots 1

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_4} = \frac{d_1}{d_2}$ (2 occurrences)
2.22.8 Construction RCC151

Construction Process

Given initial points $A, B$

1. $p_1 = p(A, B)$
2. $k_2 = k(B, A)$
   $C \in p_1 \cap k_2$
3. $k_3 = k(C, B)$
   $D \in p_1 \cap k_3$
4. $k_4 = k(D, B)$
   $E \in p_1 \cap k_4$, $F \in k_2 \cap k_4$, $G \in k_2 \cap k_4$
5. $k_5 = k(F, G)$
   $H \in p_1 \cap k_5$, $I \in p_1 \cap k_5$

Distances

$d_0 = |EH|$ (red) ... 1
$d_1 = |HI|$ (blue) ... 1
$d_2 = |EI|$ (green) ... 1

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2}$ (2 occurrences)
2.22.9 Construction RCC188

Construction Process

\( A, B \) given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(A, B) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(C, A) \)
4. \( k_4 = k(B, C) \)
   \( D \in p_1 \cap k_4, E \in k_3 \cap k_4, F \in k_3 \cap k_4 \)
5. \( k_5 = k(F, E) \)
   \( G \in p_1 \cap k_5, H \in p_1 \cap k_5 \)

Distances

\( d_0 = |DG| \) (red) \ldots 1
\( d_1 = |GH| \) (blue) \ldots 1
\( d_2 = |DH| \) (green) \ldots 1

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \) (2 occurrences)
2.22.10 Construction RCC189

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(A, B) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(C, A) \)
4. \( k_4 = k(B, C) \)
   \( D \in p_1 \cap k_4 \), \( E \in k_3 \cap k_4 \), \( F \in k_3 \cap k_4 \)
5. \( k_5 = k(E, F) \)
   \( G \in p_1 \cap k_5 \), \( H \in p_1 \cap k_5 \)

Distances

\[ d_0 = |DG| \text{ (red)} \ldots 1 \]
\[ d_1 = |GH| \text{ (blue)} \ldots 1 \]
\[ d_2 = |DH| \text{ (green)} \ldots 1 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (2 occurrences)} \]
2.22.11 Construction RCC213

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(A, B) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(C, A) \)
   \( D \in p_1 \cap k_3 \)
4. \( k_4 = k(D, A) \)
   \( E \in p_1 \cap k_4 \), \( F \in k_2 \cap k_4 \), \( G \in k_2 \cap k_4 \)
5. \( k_5 = k(G, F) \)
   \( H \in p_1 \cap k_5 \), \( I \in p_1 \cap k_5 \)

Distances

\( d_0 = |EI| \) (red) \ldots 1
\( d_1 = |HI| \) (blue) \ldots 1
\( d_2 = |EH| \) (green) \ldots 1

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \) (2 occurrences)
2.22.12 Construction RCC214

![Construction Diagram]

Construction Process

1. \( p_1 = p(A, B) \)
2. \( k_2 = k(A, B) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(C, A) \)
   \( D \in p_1 \cap k_3 \)
4. \( k_4 = k(D, A) \)
   \( E \in p_1 \cap k_4, F \in k_2 \cap k_4, G \in k_2 \cap k_4 \)
5. \( k_5 = k(F, G) \)
   \( H \in p_1 \cap k_5, I \in p_1 \cap k_5 \)

Distances

\[
\begin{align*}
d_0 &= |EI| \text{ (red)} \ldots 1 \\
d_1 &= |HI| \text{ (blue)} \ldots 1 \\
d_2 &= |EH| \text{ (green)} \ldots 1 
\end{align*}
\]

Occurrences of the Golden Ratio

\[
\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (2 occurrences)}
\]
2.23 Class of Similar Constructions No. 16

Contained Constructions

RCC27, RCC68, RCC160
2.23 Class of Similar Constructions No. 16

2.23.1 Construction RCC27

Construction Process

\[ A, B \text{ given initial points} \]

1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(A, B) \)
   \( D \in p_1 \cap k_3, \ E \in k_2 \cap k_3, \ F \in k_2 \cap k_3 \)
4. \( k_4 = k(A, C) \)
5. \( k_5 = k(B, D) \)
   \( G \in k_4 \cap k_5, \ H \in k_4 \cap k_5 \)

Distances

\[ d_0 = |EH| = |FG| \text{ (red)} \ldots 2 \]
\[ d_1 = |CE| = |CF| = |DE| = |DF| = |EF| \text{ (blue)} \ldots 5 \]
\[ d_2 = |EG| = |FH| \text{ (green)} \ldots 2 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} = \frac{d_4}{d_2} \text{ (20 occurrences)} \]
2.23.2 Construction RCC68

Construction Process

A, B given initial points
1. $p_1 = p(A, B)$
2. $k_2 = k(B, A)$
   $C \in p_1 \cap k_2$
3. $k_3 = k(C, A)$
4. $k_4 = k(C, B)$
   $D \in p_1 \cap k_4, E \in k_2 \cap k_4, F \in k_2 \cap k_4$
5. $k_5 = k(B, D)$
   $G \in k_3 \cap k_5, H \in k_3 \cap k_5$

Distances

\[
d_0 = |EH| = |FG| \quad \text{(red)} \quad \cdots 2
\]
\[
d_1 = |AE| = |AF| = |DE| = |DF| = |EF| \quad \text{(blue)} \quad \cdots 5
\]
\[
d_2 = |EG| = |FH| \quad \text{(green)} \quad \cdots 2
\]

Occurrences of the Golden Ratio

\[
\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \quad \text{(20 occurrences)}
\]
2.23.3 Construction RCC160

Construction Process

- A, B given initial points
- $p_1 = p(A, B)$
- $k_2 = k(A, B)$
  - $C \in p_1 \cap k_2$
- $k_3 = k(C, A)$
  - $D \in p_1 \cap k_3$, $E \in k_2 \cap k_3$, $F \in k_2 \cap k_3$
- $k_4 = k(C, B)$
- $k_5 = k(A, D)$
  - $G \in k_4 \cap k_5$, $H \in k_4 \cap k_5$

Distances

- $d_0 = |EH| = |FG|$ (red) ... 2
- $d_1 = |BE| = |BF| = |DE| = |DF| = |EF|$ (blue) ... 5
- $d_2 = |EG| = |FH|$ (green) ... 2

Occurrences of the Golden Ratio

- $\phi = \frac{d_0}{d_1} = \frac{d_4}{d_2}$ (20 occurrences)
2.24 Class of Similar Constructions No. 17

Contained Constructions

RCC28, RCC29, RCC60, RCC61, RCC69, RCC70, RCC152, RCC153, RCC161, RCC162, RCC215, RCC216
Construction RCC28

Construction Process

- A, B given initial points
- $p_1 = p(A, B)$
- $k_2 = k(B, A)$
- $C \in p_1 \cap k_2$
- $k_3 = k(A, B)$
- $D \in p_1 \cap k_3$, $E \in k_2 \cap k_3$, $F \in k_2 \cap k_3$
- $k_4 = k(A, C)$
- $p_5 = p(D, E)$
- $G \in p_5 \cap k_4$, $H \in p_5 \cap k_4$

Distances

- $d_0 = |CG|$ (red) \ldots 1
- $d_1 = |DH| = |EG|$ (blue) \ldots 2
- $d_2 = |FG| = |FH|$ (green) \ldots 2
- $d_3 = |CE| = |CF| = |DE| = |DF| = |EF|$ (cyan) \ldots 5
- $d_4 = |CH|$ (magenta) \ldots 1
- $d_5 = |DG| = |EH|$ (yellow) \ldots 2

Occurrences of the Golden Ratio

- $\phi = \frac{d_0}{d_2} = \frac{d_3}{d_4} = \frac{d_5}{d_6} = \frac{d_6}{d_5}$ (24 occurrences)
2.24.2 Construction RCC29

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(A, B) \)
   \( D \in p_1 \cap k_3 \), \( E \in k_2 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(A, C) \)
5. \( p_5 = p(D, F) \)
   \( G \in p_5 \cap k_4 \), \( H \in p_5 \cap k_4 \)

Distances

\[ d_0 = |CG| \text{ (red) } \ldots 1 \]
\[ d_1 = |DH| = |FG| \text{ (blue) } \ldots 2 \]
\[ d_2 = |EG| = |EH| \text{ (green) } \ldots 2 \]
\[ d_3 = |CE| = |CF| = |DE| = |DF| = |EF| \text{ (cyan) } \ldots 5 \]
\[ d_4 = |CH| \text{ (magenta) } \ldots 1 \]
\[ d_5 = |DG| = |FH| \text{ (yellow) } \ldots 2 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_3}{d_4} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (24 occurrences) } \]
2.24.3 Construction RCC60

Construction Process

$A, B$ given initial points
1. $p_1 = p(A, B)$
2. $k_2 = k(B, A)$
   $C \in p_1 \cap k_2$
3. $k_3 = k(A, B)$
   $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
4. $k_4 = k(B, D)$
5. $p_5 = p(C, E)$
   $G \in p_5 \cap k_4, H \in p_5 \cap k_4$

Distances

$d_0 = |DH|$ (red) ... 1
$d_1 = |CG| = |EH|$ (blue) ... 2
$d_2 = |FG| = |FH|$ (green) ... 2
$d_3 = |CE| = |CF| = |DE| = |DF| = |EF|$ (cyan) ... 5
$d_4 = |DG|$ (magenta) ... 1
$d_5 = |CH| = |EG|$ (yellow) ... 2

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_2} = \frac{d_2}{d_4} = \frac{d_4}{d_5} = \frac{d_5}{d_6}$ (24 occurrences)
2.24.4 Construction RCC61

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(A, B) \)
   \( D \in p_1 \cap k_3 , E \in k_2 \cap k_3 , F \in k_2 \cap k_3 \)
4. \( k_4 = k(B, D) \)
5. \( p_5 = p(C, F) \)
   \( G \in p_5 \cap k_4 , H \in p_5 \cap k_4 \)

Distances

\( d_0 = |DH| \) (red) \ldots 1
\( d_1 = |CG| = |FH| \) (blue) \ldots 2
\( d_2 = |EG| = |EH| \) (green) \ldots 2
\( d_3 = |CE| \) = \( |CF| \) = \( |DE| \) = \( |DF| \) = \( |EF| \) (cyan) \ldots 5
\( d_4 = |DG| \) (magenta) \ldots 1
\( d_5 = |CH| \) = \( |FG| \) (yellow) \ldots 2

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_2} = \frac{d_2}{d_4} = \frac{d_4}{d_6} \) (24 occurrences)
2.24.5 Construction RCC69

Construction Process

- A, B given initial points
- $p_1 = p(A, B)$
- $k_2 = k(B, A)$
- $C \in p_1 \cap k_2$
- $k_3 = k(C, A)$
- $k_4 = k(C, B)$
- $D \in p_1 \cap k_4$, $E \in k_2 \cap k_4$, $F \in k_2 \cap k_4$
- $p_5 = p(D, E)$
- $G \in p_5 \cap k_3$, $H \in p_5 \cap k_3$

Distances

- $d_0 = |AH|$ (red) ... 1
- $d_1 = |DG| = |EH|$ (blue) ... 2
- $d_2 = |FG| = |FH|$ (green) ... 2
- $d_3 = |AE| = |AF| = |DE| = |DF| = |EF|$ (cyan) ... 5
- $d_4 = |AG|$ (magenta) ... 1
- $d_5 = |DH| = |EG|$ (yellow) ... 2

Occurrences of the Golden Ratio

- $\phi = \frac{d_0}{d_2} = \frac{d_2}{d_4} = \frac{d_3}{d_5} = \frac{d_5}{d_6}$ (24 occurrences)
2.24.6 Construction RCC70

Construction Process

1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(C, A) \)
4. \( k_4 = k(C, B) \)
   \( D \in p_1 \cap k_4 \), \( E \in k_2 \cap k_4 \), \( F \in k_2 \cap k_4 \)
5. \( p_5 = p(D, F) \)
   \( G \in p_5 \cap k_3 \), \( H \in p_5 \cap k_3 \)

Distances

\[ d_0 = |AH| \text{ (red)} \ldots 1 \]
\[ d_1 = |DG| = |FH| \text{ (blue)} \ldots 2 \]
\[ d_2 = |EG| = |EH| \text{ (green)} \ldots 2 \]
\[ d_3 = |AE| = |AF| = |DE| = |DF| = |EF| \text{ (cyan)} \ldots 5 \]
\[ d_4 = |AG| \text{ (magenta)} \ldots 1 \]
\[ d_5 = |DH| = |FG| \text{ (yellow)} \ldots 2 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_2}{d_4} = \frac{d_4}{d_5} = \frac{d_5}{d_6} \text{ (24 occurrences)} \]
2.24.7 Construction RCC152

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(C, B) \)
   \( D \in p_1 \cap k_3 \), \( E \in k_2 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(B, D) \)
5. \( p_5 = p(A, E) \)
   \( G \in p_5 \cap k_4 \), \( H \in p_5 \cap k_4 \)

Distances

\( d_0 = |DG| \) (red) \ldots 1
\( d_1 = |AH| = |EG| \) (blue) \ldots 2
\( d_2 = |FG| = |FH| \) (green) \ldots 2
\( d_3 = |AE| = |AF| = |DE| = |DF| = |EF| \) (cyan) \ldots 5
\( d_4 = |DH| \) (magenta) \ldots 1
\( d_5 = |AG| = |EH| \) (yellow) \ldots 2

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_2} = \frac{d_3}{d_4} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \) (24 occurrences)
### 2.24.8 Construction RCC153

**Construction Process**

1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(C, B) \)
   
   \( D \in p_1 \cap k_3 \), \( E \in k_2 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(B, D) \)
5. \( p_5 = p(A, F) \)
   
   \( G \in p_5 \cap k_4 \), \( H \in p_5 \cap k_4 \)

**Distances**

\( d_0 = |DG| \) (red) \( \ldots 1 \)
\( d_1 = |AH| = |FG| \) (blue) \( \ldots 2 \)
\( d_2 = |EG| = |EH| \) (green) \( \ldots 2 \)
\( d_3 = |AE| = |AF| = |DE| = |DF| = |EF| \) (cyan) \( \ldots 5 \)
\( d_4 = |DH| \) (magenta) \( \ldots 1 \)
\( d_5 = |AG| = |FH| \) (yellow) \( \ldots 2 \)

**Occurrences of the Golden Ratio**

\( \phi = \frac{d_0}{d_2} = \frac{d_2}{d_4} = \frac{d_4}{d_5} = \frac{d_5}{d_6} \) (24 occurrences)
2.24.9 Construction RCC161

Construction Process

$A, B$ given initial points
1. $p_1 = p(A, B)$
2. $k_2 = k(A, B)$
   $C \in p_1 \cap k_2$
3. $k_3 = k(C, A)$
   $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
4. $k_4 = k(C, B)$
5. $p_5 = p(D, E)$
   $G \in p_5 \cap k_4, H \in p_5 \cap k_4$

Distances

$d_0 = |BG|$ (red) \ldots 1
$d_1 = |DH| = |EG|$ (blue) \ldots 2
$d_2 = |FG| = |FH|$ (green) \ldots 2
$d_3 = |BE| = |BF| = |DE| = |DF| = |EF|$ (cyan) \ldots 5
$d_4 = |BH|$ (magenta) \ldots 1
$d_5 = |DG| = |EH|$ (yellow) \ldots 2

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_2} = \frac{d_2}{d_4} = \frac{d_4}{d_5} = \frac{d_5}{d_6}$ (24 occurrences)
2.24.10 Construction RCC162

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(A, B) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(C, A) \)
   \( D \in p_1 \cap k_3 , E \in k_2 \cap k_3 , F \in k_2 \cap k_3 \)
4. \( k_4 = k(C, B) \)
5. \( p_5 = p(D, F) \)
   \( G \in p_5 \cap k_4 , H \in p_5 \cap k_4 \)

Distances

\( d_0 = |BG| \) (red) \( \ldots 1 \)
\( d_1 = |DH| = |FG| \) (blue) \( \ldots 2 \)
\( d_2 = |EG| = |EH| \) (green) \( \ldots 2 \)
\( d_3 = |BE| = |BF| = |DE| = |DF| = |EF| \) (cyan) \( \ldots 5 \)
\( d_4 = |BH| \) (magenta) \( \ldots 1 \)
\( d_5 = |DG| = |FH| \) (yellow) \( \ldots 2 \)

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_2} = \frac{d_2}{d_4} = \frac{d_4}{d_6} = \frac{d_6}{d_8} \) (24 occurrences)
2.2411 Construction RCC215

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(A, B) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(C, A) \)
   \( D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3 \)
4. \( k_4 = k(A, D) \)
5. \( p_5 = p(B, E) \)
   \( G \in p_5 \cap k_4, H \in p_5 \cap k_4 \)

Distances

\[ d_0 = |DH| \text{ (red)} \ldots 1 \]
\[ d_1 = |BG| = |EH| \text{ (blue)} \ldots 2 \]
\[ d_2 = |FG| = |FH| \text{ (green)} \ldots 2 \]
\[ d_3 = |BE| = |BF| = |DE| = |DF| = |EF| \text{ (cyan)} \ldots 5 \]
\[ d_4 = |DG| \text{ (magenta)} \ldots 1 \]
\[ d_5 = |BH| = |EG| \text{ (yellow)} \ldots 2 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_2}{d_4} = \frac{d_4}{d_6} = \frac{d_6}{d_8} \text{ (24 occurrences)} \]
2.24.12 Construction RCC216

Construction Process

1. \( p_1 = p(A, B) \)
2. \( k_2 = k(A, B) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(C, A) \)
   \( D \in p_1 \cap k_3 , E \in k_2 \cap k_3 , F \in k_2 \cap k_3 \)
4. \( k_4 = k(A, D) \)
5. \( p_5 = p(B, F) \)
   \( G \in p_5 \cap k_4 , H \in p_5 \cap k_4 \)

Distances

\[ d_0 = |DH| \text{ (red)} \ldots 1 \]
\[ d_1 = |BG| = |FH| \text{ (blue)} \ldots 2 \]
\[ d_2 = |EG| = |EH| \text{ (green)} \ldots 2 \]
\[ d_3 = |BE| = |BF| = |DE| = |DF| = |EF| \text{ (cyan)} \ldots 5 \]
\[ d_4 = |DG| \text{ (magenta)} \ldots 1 \]
\[ d_5 = |BH| = |FG| \text{ (yellow)} \ldots 2 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_2}{d_4} = \frac{d_4}{d_6} = \frac{d_6}{d_8} \text{ (24 occurrences)} \]
2.25 Class of Similar Constructions No. 18

Contained Constructions
RCC30, RCC62, RCC71, RCC154, RCC163, RCC217
2.25.1 Construction RCC30

Construction Process

$A, B$ given initial points
1. $p_1 = p(A, B)$
2. $k_2 = k(B, A)$
   $C \in p_1 \cap k_2$
3. $k_3 = k(A, B)$
   $D \in p_1 \cap k_3$, $E \in k_2 \cap k_3$, $F \in k_2 \cap k_3$
4. $k_4 = k(A, C)$
5. $p_5 = p(E, F)$
   $G \in p_5 \cap k_4$, $H \in p_5 \cap k_4$

Distances

$d_0 = |EH| = |FG|$ (red) \ldots 2
$d_1 = |CE| = |CF| = |DE| = |DF| = |EF|$ (blue) \ldots 5
$d_2 = |EG| = |FH|$ (green) \ldots 2

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2}$ (20 occurrences)
2.25.2 Construction RCC62

Construction Process

\(A, B\) given initial points

1. \(p_1 = p(A, B)\)
2. \(k_2 = k(B, A)\)
   \(C \in p_1 \cap k_2\)
3. \(k_3 = k(A, B)\)
   \(D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3\)
4. \(k_4 = k(B, D)\)
5. \(p_5 = p(E, F)\)
   \(G \in p_5 \cap k_4, H \in p_5 \cap k_4\)

Distances

\[d_0 = |EH| = |FG| \text{ (red)} \ldots 2\]
\[d_1 = |CE| = |CF| = |DE| = |DF| = |EF| \text{ (blue)} \ldots 5\]
\[d_2 = |EG| = |FH| \text{ (green)} \ldots 2\]

Occurrences of the Golden Ratio

\[\phi = \frac{d_0}{d_1} = \frac{d_4}{d_2} \text{ (20 occurrences)}\]
2.25.3 Construction RCC71

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(C, A) \)
4. \( k_4 = k(C, B) \)
   \( D \in p_1 \cap k_4 \), \( E \in k_2 \cap k_4 \), \( F \in k_2 \cap k_4 \)
5. \( p_5 = p(E, F) \)
   \( G \in p_5 \cap k_3 \), \( H \in p_5 \cap k_3 \)

Distances

\( d_0 = |EH| = |FG| \) (red) \ldots 2
\( d_1 = |AE| = |AF| = |DE| = |DF| = |EF| \) (blue) \ldots 5
\( d_2 = |EG| = |FH| \) (green) \ldots 2

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \) (20 occurrences)
2.25.4 Construction RCC154

**Construction Process**

\(A, B\) given initial points

1. \(p_1 = p(A, B)\)
2. \(k_2 = k(B, A)\)
   \(C \in p_1 \cap k_2\)
3. \(k_3 = k(C, B)\)
   \(D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3\)
4. \(k_4 = k(B, D)\)
5. \(p_5 = p(E, F)\)
   \(G \in p_5 \cap k_4, H \in p_5 \cap k_4\)

**Distances**

\[d_0 = |EH| = |FG| \text{ (red) } \ldots 2\]
\[d_1 = |AE| = |AF| = |DE| = |DF| = |EF| \text{ (blue) } \ldots 5\]
\[d_2 = |EG| = |FH| \text{ (green) } \ldots 2\]

**Occurrences of the Golden Ratio**

\[\phi = \frac{d_0}{d_1} = \frac{d_4}{d_2} \text{ (20 occurrences)}\]
2.25.5 Construction RCC163

Construction Process

A, B given initial points
1. $p_1 = p(A,B)$
2. $k_2 = k(A,B)$
   $C \in p_1 \cap k_2$
3. $k_3 = k(C,A)$
   $D \in p_1 \cap k_3$, $E \in k_2 \cap k_3$, $F \in k_2 \cap k_3$
4. $k_4 = k(C,B)$
5. $p_5 = p(E,F)$
   $G \in p_5 \cap k_4$, $H \in p_5 \cap k_4$

Distances

$d_0 = |EH| = |FG|$ (red) \ ... 2
$d_1 = |BE| = |BF| = |DE| = |DF| = |EF|$ (blue) \ ... 5
$d_2 = |EG| = |FH|$ (green) \ ... 2

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2}$ (20 occurrences)
2.25.6 Construction RCC217

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(A, B) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(C, A) \)
   \( D \in p_1 \cap k_3 , E \in k_2 \cap k_3 , F \in k_2 \cap k_3 \)
4. \( k_4 = k(A, D) \)
5. \( p_5 = p(E, F) \)
   \( G \in p_5 \cap k_4 , H \in p_5 \cap k_4 \)

Distances
\[
\begin{align*}
d_0 &= |EH| = |FG| \text{ (red) } \ldots 2 \\
d_1 &= |BE| = |BF| = |DF| = |EF| \text{ (blue) } \ldots 5 \\
d_2 &= |EG| = |FH| \text{ (green) } \ldots 2
\end{align*}
\]

Occurrences of the Golden Ratio
\[
\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (20 occurrences)}
\]
2.26 Class of Similar Constructions No. 19

Contained Constructions
RCC31, RCC63, RCC67, RCC155, RCC164, RCC218
2.26.1 Construction RCC31

Construction Process

A, B given initial points
1. $p_1 = p(A, B)$
2. $k_2 = k(B, A)$
   $C \in p_1 \cap k_2$
3. $k_3 = k(A, B)$
   $D \in p_1 \cap k_3$, $E \in k_2 \cap k_3$, $F \in k_2 \cap k_3$
4. $k_4 = k(A, C)$
   $G \in p_1 \cap k_4$
5. $k_5 = k(G, A)$
   $H \in k_3 \cap k_5$, $I \in k_3 \cap k_5$, $J \in k_4 \cap k_5$, $K \in k_4 \cap k_5$

Distances

\[ d_0 = |HK| = |IJ| \text{ (red)} \ldots 2 \]
\[ d_1 = |EI| = |FH| \text{ (blue)} \ldots 2 \]
\[ d_2 = |CE| = |CF| = |DE| = |DF| = |DJ| = |DK| = |EF| = |EJ| = |FK| \text{ (green)} \ldots 9 \]
\[ d_3 = |DH| = |DI| \text{ (cyan)} \ldots 2 \]
\[ d_4 = |HJ| = |IK| \text{ (magenta)} \ldots 2 \]
\[ d_5 = |EH| = |FI| \text{ (yellow)} \ldots 2 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_1}{d_1} = \frac{d_2}{d_4} = \frac{d_4}{d_5} \text{ (44 occurrences)} \]
2.26.2 Construction RCC63

Construction Process

\[ A, B \text{ given initial points} \]

1. \( p_1 = p(A,B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(A, B) \)
   \( D \in p_1 \cap k_3 , E \in k_2 \cap k_3 , F \in k_2 \cap k_3 \)
4. \( k_4 = k(B, D) \)
   \( G \in p_1 \cap k_4 \)
5. \( k_5 = k(G, B) \)
   \( H \in k_2 \cap k_5 , I \in k_2 \cap k_5 , J \in k_4 \cap k_5 , K \in k_4 \cap k_5 \)

Distances

\[ d_0 = |HK| = |IJ| \text{ (red) } \ldots 2 \]
\[ d_1 = |EI| = |FH| \text{ (blue) } \ldots 2 \]
\[ d_2 = |CE| = |CF| = |CJ| = |CK| = |DE| = |DF| = |EF| = |EJ| = |FK| \text{ (green) } \ldots 9 \]
\[ d_3 = |CH| = |CI| \text{ (cyan) } \ldots 2 \]
\[ d_4 = |HJ| = |IK| \text{ (magenta) } \ldots 2 \]
\[ d_5 = |EH| = |FI| \text{ (yellow) } \ldots 2 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (44 occurrences)} \]
2.26.3 Construction RCC67

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(C, A) \)
   \( D \in p_1 \cap k_3 \)
4. \( k_4 = k(C, B) \)
   \( E \in p_1 \cap k_4 \), \( F \in k_2 \cap k_4 \), \( G \in k_2 \cap k_4 \)
5. \( k_5 = k(D, C) \)
   \( H \in k_3 \cap k_5 \), \( I \in k_3 \cap k_5 \), \( J \in k_4 \cap k_5 \), \( K \in k_4 \cap k_5 \)

Distances

\[ d_0 = |HK| = |IJ| \text{ (red)} \ldots 2 \]
\[ d_1 = |FK| = |GJ| \text{ (blue)} \ldots 2 \]
\[ d_2 = |AF| = |AG| = |EF| = |EG| = |EH| = |EI| = |FG| = |FH| = |GI| \text{ (green)} \ldots 9 \]
\[ d_3 = |EJ| = |EK| \text{ (cyan)} \ldots 2 \]
\[ d_4 = |HJ| = |IK| \text{ (magenta)} \ldots 2 \]
\[ d_5 = |FJ| = |GK| \text{ (yellow)} \ldots 2 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (44 occurrences)} \]
2.26.4 Construction RCC155

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(C, B) \)
   \( D \in p_1 \cap k_3 , E \in k_2 \cap k_3 , F \in k_2 \cap k_3 \)
4. \( k_4 = k(B, D) \)
   \( G \in p_1 \cap k_4 \)
5. \( k_5 = k(G, B) \)
   \( H \in k_2 \cap k_5 , I \in k_2 \cap k_5 , J \in k_4 \cap k_5 , K \in k_4 \cap k_5 \)

Distances

\[ d_0 = |HK| = |IJ| \text{ (red) ... 2} \]
\[ d_1 = |EI| = |FH| \text{ (blue) ... 2} \]
\[ d_2 = |AE| = |AF| = |AJ| = |AK| = |DE| = |DF| = |EF| = |EJ| = |FK| \text{ (green) ... 9} \]
\[ d_3 = |AH| = |AI| \text{ (cyan) ... 2} \]
\[ d_4 = |HJ| = |IK| \text{ (magenta) ... 2} \]
\[ d_5 = |EH| = |FI| \text{ (yellow) ... 2} \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (44 occurrences)} \]
Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(A, B) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(C, A) \)
   \( D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3 \)
4. \( k_4 = k(C, B) \)
   \( G \in p_1 \cap k_4 \)
5. \( k_5 = k(G, C) \)
   \( H \in k_3 \cap k_5, I \in k_3 \cap k_5, J \in k_4 \cap k_5, K \in k_4 \cap k_5 \)

Distances

\[
\begin{align*}
 d_0 &= |HK| = |IJ| \text{ (red)} \ldots 2 \\
 d_1 &= |EI| = |FH| \text{ (blue)} \ldots 2 \\
 d_2 &= |BE| = |BF| = |DE| = |DF| = |DJ| = |DK| = |EF| = |EJ| = |FK| \text{ (green)} \ldots 9 \\
 d_3 &= |DH| = |DI| \text{ (cyan)} \ldots 2 \\
 d_4 &= |HJ| = |IK| \text{ (magenta)} \ldots 2 \\
 d_5 &= |EH| = |FI| \text{ (yellow)} \ldots 2
\end{align*}
\]

Occurrences of the Golden Ratio

\[
\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_4}{d_5} = \frac{d_5}{d_5} \text{ (44 occurrences)}
\]
2.26.6 Construction RCC218

Construction Process

A, B given initial points
1. $p_1 = p(A, B)$
2. $k_2 = k(A, B)$
   $C \in p_1 \cap k_2$
3. $k_3 = k(C, A)$
   $D \in p_1 \cap k_3$, $E \in k_2 \cap k_3$, $F \in k_2 \cap k_3$
4. $k_4 = k(A, D)$
   $G \in p_1 \cap k_4$
5. $k_5 = k(G, A)$
   $H \in k_2 \cap k_5$, $I \in k_2 \cap k_5$, $J \in k_4 \cap k_5$, $K \in k_4 \cap k_5$

Distances

\[d_0 = |HK| = |IJ| \text{ (red)} \ldots 2\]
\[d_1 = |EI| = |FH| \text{ (blue)} \ldots 2\]
\[d_2 = |BE| = |BF| = |BJ| = |BK| = |DE| = |DF| = |EF| = |EJ| = |FK| \text{ (green)} \ldots 9\]
\[d_3 = |BH| = |BI| \text{ (cyan)} \ldots 2\]
\[d_4 = |HJ| = |IK| \text{ (magenta)} \ldots 2\]
\[d_5 = |EH| = |FI| \text{ (yellow)} \ldots 2\]

Occurrences of the Golden Ratio

\[\phi = \frac{d_1}{d_2} = \frac{d_2}{d_3} = \frac{d_3}{d_4} = \frac{d_4}{d_5} \text{ (44 occurrences)}\]
2.27 Class of Similar Constructions No. 20

Contained Constructions

RCC64, RCC65, RCC157, RCC158, RCC220, RCC221
2.27.1 Construction RCC64

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
3. \( k_3 = k(A, B) \)
   \( C \in k_2 \cap k_3 \), \( D \in k_2 \cap k_3 \)
4. \( p_4 = p(C, D) \)
   \( E \in p_1 \cap p_4 \)
5. \( k_5 = k(A, E) \)
   \( F \in k_2 \cap k_5 \), \( G \in k_2 \cap k_5 \)

Distances

\[ d_0 = |CG| = |DF| \text{ (red)} \ldots 2 \]
\[ d_1 = |CE| = |DE| \text{ (blue)} \ldots 2 \]
\[ d_2 = |CF| = |DG| \text{ (green)} \ldots 2 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)} \]
2.27.2 Construction RCC65

Construction Process

\[ A, B \text{ given initial points} \]
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
3. \( k_3 = k(A, B) \)
   \[ C \in k_2 \cap k_3, \quad D \in k_2 \cap k_3 \]
4. \( p_4 = p(C, D) \)
   \[ E \in p_1 \cap p_4 \]
5. \( k_5 = k(B, E) \)
   \[ F \in k_3 \cap k_5, \quad G \in k_3 \cap k_5 \]

Distances

\[ d_0 = |CG| = |DF| \quad (\text{red}) \quad \ldots \quad 2 \]
\[ d_1 = |CE| = |DE| \quad (\text{blue}) \quad \ldots \quad 2 \]
\[ d_2 = |CF| = |DG| \quad (\text{green}) \quad \ldots \quad 2 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \quad (8 \text{ occurrences}) \]
2.27.3 Construction RCC157

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(C, B) \)
   \( D \in k_2 \cap k_3, E \in k_2 \cap k_3 \)
4. \( p_4 = p(D, E) \)
   \( F \in p_1 \cap p_4 \)
5. \( k_5 = k(B, F) \)
   \( G \in k_3 \cap k_5, H \in k_3 \cap k_5 \)

Distances

\( d_0 = |DH| = |EG| \) (red) \( \ldots 2 \)
\( d_1 = |DF| = |EF| \) (blue) \( \ldots 2 \)
\( d_2 = |DG| = |EH| \) (green) \( \ldots 2 \)

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \) (8 occurrences)
2.27.4 Construction RCC158

Construction Process

\[ A, B \text{ given initial points} \]
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(C, B) \)
   \( D \in k_2 \cap k_3, \ E \in k_2 \cap k_3 \)
4. \( p_4 = p(D, E) \)
   \( F \in p_1 \cap p_4 \)
5. \( k_5 = k(C, F) \)
   \( G \in k_2 \cap k_5, \ H \in k_2 \cap k_5 \)

Distances

\[ d_0 = |DH| = |EG| \text{(red)} \ldots 2 \]
\[ d_1 = |DF| = |EF| \text{(blue)} \ldots 2 \]
\[ d_2 = |DG| = |EH| \text{(green)} \ldots 2 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_5} = \frac{d_1}{d_2} \text{ (8 occurrences)} \]
2.27.5 Construction RCC220

Construction Process

A, B given initial points
1. $p_1 = p(A, B)$
2. $k_2 = k(A, B)$
   \[ C \in p_1 \cap k_2 \]
3. $k_3 = k(C, A)$
   \[ D \in k_2 \cap k_3, \ E \in k_2 \cap k_3 \]
4. $p_4 = p(D, E)$
   \[ F \in p_1 \cap p_4 \]
5. $k_5 = k(A, F)$
   \[ G \in k_3 \cap k_5, \ H \in k_3 \cap k_5 \]

Distances

\[ d_0 = |DH| = |EG| \text{ (red) } ... 2 \]
\[ d_1 = |DF| = |EF| \text{ (blue) } ... 2 \]
\[ d_2 = |DG| = |EH| \text{ (green) } ... 2 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)} \]
2.27.6 Construction RCC221

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(A, B) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(C, A) \)
   \( D \in k_2 \cap k_3, E \in k_2 \cap k_3 \)
4. \( p_4 = p(D, E) \)
   \( F \in p_1 \cap p_4 \)
5. \( k_5 = k(C, F) \)
   \( G \in k_2 \cap k_5, H \in k_2 \cap k_5 \)

Distances

\( d_0 = |DH| = |EG| \) (red) ... 2
\( d_1 = |DF| = |EF| \) (blue) ... 2
\( d_2 = |DG| = |EH| \) (green) ... 2

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \) (8 occurrences)
2.28 Class of Similar Constructions No. 21

Contained Constructions
RCC72, RCC79, RCC113, RCC120, RCC222, RCC229, RCC238, RCC245
2.28.1 Construction RCC72

**Construction Process**

1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
3. \( C \in p_1 \cap k_2 \)
4. \( k_3 = k(C, A) \)
5. \( D \in p_1 \cap k_3 \)
6. \( k_4 = k(D, B) \)

\( E \in k_2 \cap k_4 \), \( F \in k_2 \cap k_4 \), \( G \in k_3 \cap k_4 \), \( H \in k_3 \cap k_4 \)
7. \( k_5 = k(G, A) \)
8. \( I \in p_1 \cap k_5 \), \( J \in k_2 \cap k_5 \)

**Distances**

\( d_0 = |EH| = |FG| \) (red) . . . 2
\( d_1 = |AJ| = |IJ| \) (blue) . . . 2
\( d_2 = |EG| = |FH| \) (green) . . . 2

**Occurrences of the Golden Ratio**

\( \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \) (8 occurrences)
2.28.2 Construction RCC79

Construction Process

\(A, B\) given initial points
1. \(p_1 = p(A, B)\)
2. \(k_2 = k(B, A)\)
   \(C \in p_1 \cap k_2\)
3. \(k_3 = k(C, A)\)
   \(D \in p_1 \cap k_3\)
4. \(k_4 = k(D, B)\)
   \(E \in k_2 \cap k_4\), \(F \in k_2 \cap k_4\), \(G \in k_3 \cap k_4\), \(H \in k_3 \cap k_4\)
5. \(k_5 = k(H, A)\)
   \(I \in p_1 \cap k_5\), \(J \in k_2 \cap k_5\)

Distances

\[d_0 = |EH| = |FG| \text{ (red)} \ldots 2\]
\[d_1 = |AJ| = |IJ| \text{ (blue)} \ldots 2\]
\[d_2 = |EG| = |FH| \text{ (green)} \ldots 2\]

Occurrences of the Golden Ratio

\[\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)}\]
2.28.3 Construction RCC113

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(A, C) \)
   \( D \in p_1 \cap k_3 \)
4. \( k_4 = k(D, B) \)
   \( E \in k_2 \cap k_4 \), \( F \in k_2 \cap k_4 \), \( G \in k_3 \cap k_4 \), \( H \in k_3 \cap k_4 \)
5. \( k_5 = k(G, C) \)
   \( I \in p_1 \cap k_5 \), \( J \in k_2 \cap k_5 \)

Distances

\[ d_0 = |EH| = |FG| \text{ (red)} \ldots 2 \]
\[ d_1 = |CJ| = |IJ| \text{ (blue)} \ldots 2 \]
\[ d_2 = |EG| = |FH| \text{ (green)} \ldots 2 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)} \]
2.28.4 Construction RCC120

Construction Process

1. $p_1 = p(A, B)$
2. $k_2 = k(B, A)$
   - $C \in p_1 \cap k_2$
3. $k_3 = k(A, C)$
   - $D \in p_1 \cap k_3$
4. $k_4 = k(D, B)$
   - $E \in k_2 \cap k_4$ , $F \in k_2 \cap k_4$ , $G \in k_3 \cap k_4$ , $H \in k_3 \cap k_4$
5. $k_5 = k(H, C)$
   - $I \in p_1 \cap k_5$ , $J \in k_2 \cap k_5$

Distances

- $d_0 = |EH| = |FG|$ (red) \dots 2
- $d_1 = |CJ| = |IJ|$ (blue) \dots 2
- $d_2 = |EG| = |FH|$ (green) \dots 2

Occurrences of the Golden Ratio

- $\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2}$ (8 occurrences)
2.28.5 Construction RCC222

Construction Process

1. \( p_1 = p(A, B) \)
2. \( k_2 = k(A, B) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(C, B) \)
   \( D \in p_1 \cap k_3 \)
4. \( k_4 = k(D, A) \)
   \( E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4 \)
5. \( k_5 = k(G, B) \)
   \( I \in p_1 \cap k_5, J \in k_2 \cap k_5 \)

Distances

\( d_0 = |EH| = |FG| \) (red) \( \ldots 2 \)
\( d_1 = |BJ| = |IJ| \) (blue) \( \ldots 2 \)
\( d_2 = |EG| = |FH| \) (green) \( \ldots 2 \)

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \) (8 occurrences)
2.28.6 Construction RCC229

Construction Process

$A, B$ given initial points
1. $p_1 = p(A, B)$
2. $k_2 = k(A, B)$
   $C \in p_1 \cap k_2$
3. $k_3 = k(C, B)$
   $D \in p_1 \cap k_3$
4. $k_4 = k(D, A)$
   $E \in k_2 \cap k_4$, $F \in k_2 \cap k_4$, $G \in k_3 \cap k_4$, $H \in k_3 \cap k_4$
5. $k_5 = k(H, B)$
   $I \in p_1 \cap k_5$, $J \in k_2 \cap k_5$

Distances

$d_0 = |EH| = |FG|$ (red) \ldots 2
$d_1 = |BJ| = |IJ|$ (blue) \ldots 2
$d_2 = |EG| = |FH|$ (green) \ldots 2

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2}$ (8 occurrences)
2.28.7 Construction RCC238

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(A, B) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(B, C) \)
   \( D \in p_1 \cap k_3 \)
4. \( k_4 = k(D, A) \)
   \( E \in k_2 \cap k_4 , F \in k_2 \cap k_4 , G \in k_3 \cap k_4 , H \in k_3 \cap k_4 \)
5. \( k_5 = k(G, C) \)
   \( I \in p_1 \cap k_5 , J \in k_2 \cap k_5 \)

Distances

\[ d_0 = |EH| = |FG| \text{ (red)} \ldots 2 \]
\[ d_1 = |CJ| = |IJ| \text{ (blue)} \ldots 2 \]
\[ d_2 = |EG| = |FH| \text{ (green)} \ldots 2 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)} \]
2.28.8 Construction RCC245

Construction Process

- $A, B$ given initial points
- $p_1 = p(A, B)$
- $k_2 = k(A, B)$
- $C \in p_1 \cap k_2$
- $k_3 = k(B, C)$
- $D \in p_1 \cap k_3$
- $k_4 = k(D, A)$
  - $E \in k_2 \cap k_4$, $F \in k_2 \cap k_4$, $G \in k_3 \cap k_4$, $H \in k_3 \cap k_4$
- $k_5 = k(H, C)$
  - $I \in p_1 \cap k_5$, $J \in k_2 \cap k_5$

Distances

- $d_0 = |EH| = |FG|$ (red) . . . 2
- $d_1 = |CJ| = |IJ|$ (blue) . . . 2
- $d_2 = |EG| = |FH|$ (green) . . . 2

Occurrences of the Golden Ratio

- $\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2}$ (8 occurrences)
2.29 Class of Similar Constructions No. 22

Contained Constructions
RCC73, RCC80, RCC112, RCC119, RCC223, RCC230, RCC237, RCC244
2.29 Class of Similar Constructions No. 22

2.29.1 Construction RCC73

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(C, A) \)
   \( D \in p_1 \cap k_3 \)
4. \( k_4 = k(D, B) \)
   \( E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4 \)
5. \( k_5 = k(G, C) \)
   \( I \in k_2 \cap k_5 \)

Distances

\[ d_0 = |EH| = |FG| \text{ (red)} \ldots 2 \]
\[ d_1 = |CI| \text{ (blue)} \ldots 1 \]
\[ d_2 = |EG| = |FH| \text{ (green)} \ldots 2 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (4 occurrences)} \]
2.29.2 Construction RCC80

Construction Process

\( A, B \) given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(C, A) \)
   \( D \in p_1 \cap k_3 \)
4. \( k_4 = k(D, B) \)
   \( E \in k_2 \cap k_4 \) , \( F \in k_2 \cap k_4 \) , \( G \in k_3 \cap k_4 \) , \( H \in k_3 \cap k_4 \)
5. \( k_5 = k(H, C) \)
   \( I \in k_2 \cap k_5 \)

Distances

\( d_0 = |EH| = |FG| \) (red) \ldots 2
\( d_1 = |CI| \) (blue) \ldots 1
\( d_2 = |EG| = |FH| \) (green) \ldots 2

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \) (4 occurrences)
2.29.3 Construction RCC112

Construction Process

\(A, B\) given initial points

1. \(p_1 = p(A, B)\)
2. \(k_2 = k(B, A)\)
   \(C \in p_1 \cap k_2\)
3. \(k_3 = k(A, C)\)
   \(D \in p_1 \cap k_3\)
4. \(k_4 = k(D, B)\)
   \(E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4\)
5. \(k_5 = k(G, A)\)
   \(I \in k_2 \cap k_5\)

Distances

\(d_0 = |EH| = |FG|\) (red) \(\ldots 2\)
\(d_1 = |AI|\) (blue) \(\ldots 1\)
\(d_2 = |EG| = |FH|\) (green) \(\ldots 2\)

Occurrences of the Golden Ratio

\(\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2}\) (4 occurrences)
2.29.4 Construction RCC119

Construction Process

\( A, B \) given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(A, C) \)
   \( D \in p_1 \cap k_3 \)
4. \( k_4 = k(D, B) \)
   \( E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4 \)
5. \( k_5 = k(H, A) \)
   \( I \in k_2 \cap k_5 \)

Distances

\( d_0 = |EH| = |FG| \) (red) \ldots 2
\( d_1 = |AI| \) (blue) \ldots 1
\( d_2 = |EG| = |FH| \) (green) \ldots 2

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \) (4 occurrences)
2.29.5 Construction RCC223

Construction Process

$A$, $B$ given initial points
1. $p_1 = p(A, B)$
2. $k_2 = k(A, B)$
   $C \in p_1 \cap k_2$
3. $k_3 = k(C, B)$
   $D \in p_1 \cap k_3$
4. $k_4 = k(D, A)$
   $E \in k_2 \cap k_4$, $F \in k_2 \cap k_4$, $G \in k_3 \cap k_4$, $H \in k_3 \cap k_4$
5. $k_5 = k(G, C)$
   $I \in k_2 \cap k_5$

Distances

$d_0 = |EH| = |FG|$ (red) \ldots 2
$d_1 = |CI|$ (blue) \ldots 1
$d_2 = |EG| = |FH|$ (green) \ldots 2

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2}$ (4 occurrences)
2.29.6 Construction RCC230

Construction Process

$A, B$ given initial points
1. $p_1 = p(A, B)$
2. $k_2 = k(A, B)$
   $C \in p_1 \cap k_2$
3. $k_3 = k(C, B)$
   $D \in p_1 \cap k_3$
4. $k_4 = k(D, A)$
   $E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4$
5. $k_5 = k(H, C)$
   $I \in k_2 \cap k_5$

Distances

$d_0 = |EH| = |FG|$ (red) ... 2
$d_1 = |CI|$ (blue) ... 1
$d_2 = |EG| = |FH|$ (green) ... 2

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2}$ (4 occurrences)
2.29.7 Construction RCC237

Construction Process

\(A, B\) given initial points
1. \(p_1 = p(A, B)\)
2. \(k_2 = k(A, B)\)
   \(C \in p_1 \cap k_2\)
3. \(k_3 = k(B, C)\)
   \(D \in p_1 \cap k_3\)
4. \(k_4 = k(D, A)\)
   \(E \in k_2 \cap k_4\), \(F \in k_2 \cap k_4\), \(G \in k_3 \cap k_4\), \(H \in k_3 \cap k_4\)
5. \(k_5 = k(G, B)\)
   \(I \in k_2 \cap k_5\)

Distances

\(d_0 = |EH| = |FG| \) (red) \ldots 2
\(d_1 = |BI| \) (blue) \ldots 1
\(d_2 = |EG| = |FH| \) (green) \ldots 2

Occurrences of the Golden Ratio

\(\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \) (4 occurrences)
2.29 Class of Similar Constructions No. 22

2.29.8 Construction RCC244

Construction Process

\( A, B \) given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(A, B) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(B, C) \)
   \( D \in p_1 \cap k_3 \)
4. \( k_4 = k(D, A) \)
   \( E \in k_2 \cap k_4 \), \( F \in k_2 \cap k_4 \), \( G \in k_3 \cap k_4 \), \( H \in k_3 \cap k_4 \)
5. \( k_5 = k(H, B) \)
   \( I \in k_2 \cap k_5 \)

Distances

\( d_0 = |EH| = |FG| \) (red) \ldots 2
\( d_1 = |BI| \) (blue) \ldots 1
\( d_2 = |EG| = |FH| \) (green) \ldots 2

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \) (4 occurrences)
2.30 Class of Similar Constructions No. 23

Contained Constructions
RCC74, RCC81, RCC114, RCC121, RCC224, RCC231, RCC239, RCC246
2.30.1 Construction RCC74

Construction Process

$A, B$ given initial points
1. $p_1 = p(A, B)$
2. $k_2 = k(B, A)$
   $C \in p_1 \cap k_2$
3. $k_3 = k(C, A)$
   $D \in p_1 \cap k_3$
4. $k_4 = k(D, B)$
   $E \in p_1 \cap k_4$, $F \in k_2 \cap k_4$, $G \in k_2 \cap k_4$, $H \in k_3 \cap k_4$, $I \in k_3 \cap k_4$
5. $p_5 = p(E, H)$
   $J \in p_5 \cap k_3$

Distances

$d_0 = |FI| = |GH|$ (red) . . . 2
$d_1 = |HJ|$ (blue) . . . 1
$d_2 = |FH| = |GI|$ (green) . . . 2

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_1} = \frac{d_3}{d_2}$ (4 occurrences)
2.30.2 Construction RCC81

Construction Process

A, B given initial points
1. $p_1 = p(A, B)$
2. $k_2 = k(B, A)$
   \[ C \in p_1 \cap k_2 \]
3. $k_3 = k(C, A)$
   \[ D \in p_1 \cap k_3 \]
4. $k_4 = k(D, B)$
   \[ E \in p_1 \cap k_4 , F \in k_2 \cap k_4 , G \in k_2 \cap k_4 , H \in k_3 \cap k_4 , I \in k_3 \cap k_4 \]
5. $p_5 = p(E, I)$
   \[ J \in p_5 \cap k_3 \]

Distances

\[ d_0 = |FI| = |GH| \text{ (red)} \ldots 2 \]
\[ d_1 = |IJ| \text{ (blue)} \ldots 1 \]
\[ d_2 = |FH| = |GI| \text{ (green)} \ldots 2 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (4 occurrences)} \]
2.30.3 Construction RCC114

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(A, C) \)
   \( D \in p_1 \cap k_3 \)
4. \( k_4 = k(D, B) \)
   \( E \in p_1 \cap k_4 \), \( F \in k_2 \cap k_4 \), \( G \in k_2 \cap k_4 \), \( H \in k_3 \cap k_4 \), \( I \in k_3 \cap k_4 \)
5. \( p_5 = p(E, H) \)
   \( J \in p_5 \cap k_3 \)

Distances

\( d_0 = |FI| = |GH| \) (red) ... 2
\( d_1 = |HJ| \) (blue) ... 1
\( d_2 = |FH| = |GI| \) (green) ... 2

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} = \frac{d_2}{d_1} \) (4 occurrences)
2.30.4 Construction RCC121

Construction Process

$A, B$ given initial points
1. $p_1 = p(A, B)$
2. $k_2 = k(B, A)$
   $C \in p_1 \cap k_2$
3. $k_3 = k(A, C)$
   $D \in p_1 \cap k_3$
4. $k_4 = k(D, B)$
   $E \in p_1 \cap k_4$, $F \in k_2 \cap k_4$, $G \in k_2 \cap k_4$, $H \in k_3 \cap k_4$, $I \in k_3 \cap k_4$
5. $p_5 = p(E, I)$
   $J \in p_5 \cap k_3$

Distances

$d_0 = |FI| = |GH|$ (red) \ldots 2
$d_1 = |IJ|$ (blue) \ldots 1
$d_2 = |FH| = |GI|$ (green) \ldots 2

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_1} = \frac{d_2}{d_2}$ (4 occurrences)
2.30.5 Construction RCC224

Construction Process

A, B given initial points
1. $p_1 = p(A, B)$
2. $k_2 = k(A, B)$
   $C \in p_1 \cap k_2$
3. $k_3 = k(C, B)$
   $D \in p_1 \cap k_3$
4. $k_4 = k(D, A)$
   $E \in p_1 \cap k_4$ , $F \in k_2 \cap k_4$ , $G \in k_2 \cap k_4$ , $H \in k_3 \cap k_4$ , $I \in k_3 \cap k_4$
5. $p_5 = p(E, H)$
   $J \in p_5 \cap k_3$

Distances

\[ d_0 = |FI| = |GH| \ (\text{red}) \ldots 2 \]
\[ d_1 = |HJ| \ (\text{blue}) \ldots 1 \]
\[ d_2 = |FH| = |GI| \ (\text{green}) \ldots 2 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \ (4 \ \text{occurrences}) \]
2.30.6 Construction RCC231

Construction Process

\[ A, B \text{ given initial points} \]
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(A, B) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(C, B) \)
   \( D \in p_1 \cap k_3 \)
4. \( k_4 = k(D, A) \)
   \( E \in p_1 \cap k_4 \), \( F \in k_2 \cap k_4 \), \( G \in k_2 \cap k_4 \), \( H \in k_3 \cap k_4 \), \( I \in k_3 \cap k_4 \)
5. \( p_5 = p(E, I) \)
   \( J \in p_5 \cap k_3 \)

Distances

\[ d_0 = |FI| = |GH| \text{ (red)} \ldots 2 \]
\[ d_1 = |IJ| \text{ (blue)} \ldots 1 \]
\[ d_2 = |FH| = |GI| \text{ (green)} \ldots 2 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} = \frac{d_3}{d_2} \text{ (4 occurrences)} \]
2.30.7 Construction RCC239

Construction Process

1. \( p_1 = p(A, B) \)
2. \( k_2 = k(A, B) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(B, C) \)
   \( D \in p_1 \cap k_3 \)
4. \( k_4 = k(D, A) \)
   \( E \in p_1 \cap k_4 , F \in k_2 \cap k_4 , G \in k_2 \cap k_4 , H \in k_3 \cap k_4 , I \in k_3 \cap k_4 \)
5. \( p_5 = p(E, H) \)
   \( J \in p_5 \cap k_3 \)

Distances

\[ d_0 = |FI| = |GH| \text{ (red)} \ldots 2 \]
\[ d_1 = |HJ| \text{ (blue)} \ldots 1 \]
\[ d_2 = |FH| = |GI| \text{ (green)} \ldots 2 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_4} = \frac{d_4}{d_2} \text{ (4 occurrences)} \]
2.30.8 Construction RCC246

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(A, B) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(B, C) \)
   \( D \in p_1 \cap k_3 \)
4. \( k_4 = k(D, A) \)
   \( E \in p_1 \cap k_4 , F \in k_2 \cap k_4 , G \in k_2 \cap k_4 , H \in k_3 \cap k_4 , I \in k_3 \cap k_4 \)
5. \( p_5 = p(E, I) \)
   \( J \in p_5 \cap k_3 \)

Distances

\( d_0 = |FI| = |GH| \) (red) \( \ldots 2 \)
\( d_1 = |IJ| \) (blue) \( \ldots 1 \)
\( d_2 = |FH| = |GI| \) (green) \( \ldots 2 \)

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \) (4 occurrences)
2.31 Class of Similar Constructions No. 24

Contained Constructions
RCC75, RCC84, RCC115, RCC124, RCC225, RCC234, RCC240, RCC249
2.31.1 Construction RCC75

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(C, A) \)
   \( D \in p_1 \cap k_3 \)
4. \( k_4 = k(D, B) \)
   \( E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4 \)
5. \( p_5 = p(E, G) \)
   \( I \in p_5 \cap k_2 \)

Distances

\[ d_0 = |EH| = |FG| = |GI| \text{ (red)} \ldots 3 \]
\[ d_1 = |EI| \text{ (blue)} \ldots 1 \]
\[ d_2 = |EG| = |FH| \text{ (green)} \ldots 2 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} = \frac{d_3}{d_2} \text{ (5 occurrences)} \]
2.31.2 Construction RCC84

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(C, A) \)
   \( D \in p_1 \cap k_3 \)
4. \( k_4 = k(D, B) \)
   \( E \in k_2 \cap k_4 \), \( F \in k_2 \cap k_4 \), \( G \in k_3 \cap k_4 \), \( H \in k_3 \cap k_4 \)
5. \( p_5 = p(F, H) \)
   \( I \in p_5 \cap k_2 \)

Distances

\( d_0 = |EH| = |FG| = |HI| \) (red) \ldots 3
\( d_1 = |FI| \) (blue) \ldots 1
\( d_2 = |EG| = |FH| \) (green) \ldots 2

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \) (5 occurrences)
2.31.3 Construction RCC115

Construction Process

\[ A, B \text{ given initial points} \]
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(A, C) \)
   \( D \in p_1 \cap k_3 \)
4. \( k_4 = k(D, B) \)
   \( E \in k_2 \cap k_4 , F \in k_2 \cap k_4 , G \in k_3 \cap k_4 , H \in k_3 \cap k_4 \)
5. \( p_5 = p(E, G) \)
   \( I \in p_5 \cap k_2 \)

Distances

\[ d_0 = |EH| = |FG| = |GI| \text{ (red)} \ldots 3 \]
\[ d_1 = |EI| \text{ (blue)} \ldots 1 \]
\[ d_2 = |EG| = |FH| \text{ (green)} \ldots 2 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} = \frac{d_3}{d_2} \text{ (5 occurrences)} \]
2.31.4 Construction RCC124

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(A, C) \)
   \( D \in p_1 \cap k_3 \)
4. \( k_4 = k(D, B) \)
   \( E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4 \)
5. \( p_5 = p(F, H) \)
   \( I \in p_5 \cap k_2 \)

Distances

\( d_0 = |EH| = |FG| = |HI| \) (red) \ldots 3
\( d_1 = |FI| \) (blue) \ldots 1
\( d_2 = |EG| = |FH| \) (green) \ldots 2

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} = \frac{d_3}{d_2} \) (5 occurrences)
2.31.5 Construction RCC225

Construction Process

$A, B$ given initial points
1. $p_1 = p(A, B)$
2. $k_2 = k(A, B)$
   $C \in p_1 \cap k_2$
3. $k_3 = k(C, B)$
   $D \in p_1 \cap k_3$
4. $k_4 = k(D, A)$
   $E \in k_2 \cap k_4$, $F \in k_2 \cap k_4$, $G \in k_3 \cap k_4$, $H \in k_3 \cap k_4$
5. $p_5 = p(E, G)$
   $I \in p_5 \cap k_2$

Distances

$d_0 = |EH| = |FG| = |GI|$ (red) ... 3
$d_1 = |EI|$ (blue) ... 1
$d_2 = |EG| = |FH|$ (green) ... 2

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_2}$ (5 occurrences)
Construction RCC234

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(A, B) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(C, B) \)
   \( D \in p_1 \cap k_3 \)
4. \( k_4 = k(D, A) \)
   \( E \in k_2 \cap k_4 , F \in k_2 \cap k_4 , G \in k_3 \cap k_4 , H \in k_3 \cap k_4 \)
5. \( p_5 = p(F, H) \)
   \( I \in p_5 \cap k_2 \)

Distances

\[ d_0 = |EH| = |FG| = |HI| \text{ (red) } \ldots 3 \]
\[ d_1 = |FI| \text{ (blue) } \ldots 1 \]
\[ d_2 = |EG| = |FH| \text{ (green) } \ldots 2 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (5 occurrences) } \]
2.31.7 Construction RCC240

Construction Process

1. \(p_1 = p(A, B)\)
2. \(k_2 = k(A, B)\)
   \(C \in p_1 \cap k_2\)
3. \(k_3 = k(B, C)\)
   \(D \in p_1 \cap k_3\)
4. \(k_4 = k(D, A)\)
   \(E \in k_2 \cap k_4\), \(F \in k_2 \cap k_4\), \(G \in k_3 \cap k_4\), \(H \in k_3 \cap k_4\)
5. \(p_5 = p(E, G)\)
   \(I \in p_5 \cap k_2\)

Distances

\[d_0 = |EH| = |FG| = |GI| \quad \text{(red)} \quad 3\]
\[d_1 = |EI| \quad \text{(blue)} \quad 1\]
\[d_2 = |EG| = |FH| \quad \text{(green)} \quad 2\]

Occurrences of the Golden Ratio

\[\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \quad \text{(5 occurrences)}\]
2.31.8 Construction RCC249

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(A, B) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(B, C) \)
   \( D \in p_1 \cap k_3 \)
4. \( k_4 = k(D, A) \)
   \( E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4 \)
5. \( p_5 = p(F, H) \)
   \( I \in p_5 \cap k_2 \)

Distances

\( d_0 = |EH| = |FG| = |HI| \) (red) \ldots 3
\( d_1 = |FI| \) (blue) \ldots 1
\( d_2 = |EG| = |FH| \) (green) \ldots 2

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \) (5 occurrences)
2.32 Class of Similar Constructions No. 25

Contained Constructions
RCC76, RCC85, RCC116, RCC125, RCC226, RCC235, RCC241, RCC250
2.32.1 Construction RCC76

Construction Process

1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(C, A) \)
   \( D \in p_1 \cap k_3 \)
4. \( k_4 = k(D, B) \)
   \( E \in k_2 \cap k_4 \), \( F \in k_2 \cap k_4 \), \( G \in k_3 \cap k_4 \), \( H \in k_3 \cap k_4 \)
5. \( k_5 = k(G, E) \)
   \( I \in k_2 \cap k_5 \), \( J \in k_3 \cap k_5 \), \( K \in k_3 \cap k_5 \), \( L \in k_4 \cap k_5 \)

Distances

\( d_0 = |EH| = |FG| \) (red) \ldots 2
\( d_1 = |FI| \) (blue) \ldots 1
\( d_2 = |EG| = |FH| = |GI| = |GJ| = |GK| = |GL| \) (green) \ldots 6

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_2} = \frac{d_1}{d_2} \) (8 occurrences)
2.32.2 Construction RCC85

Construction Process

1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(C, A) \)
   \( D \in p_1 \cap k_3 \)
4. \( k_4 = k(D, B) \)
   \( E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4 \)
5. \( k_5 = k(H, F) \)
   \( I \in k_2 \cap k_5, J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5 \)

Distances

\( d_0 = |EH| = |FG| \) (red) \( \ldots 2 \)
\( d_1 = |EI| \) (blue) \( \ldots 1 \)
\( d_2 = |EG| = |FH| = |HI| = |HJ| = |HK| = |HL| \) (green) \( \ldots 6 \)

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} = \frac{d_2}{d_2} \) (8 occurrences)
2.32.3 Construction RCC116

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(A, C) \)
   \( D \in p_1 \cap k_3 \)
4. \( k_4 = k(D, B) \)
   \( E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4 \)
5. \( k_5 = k(G, E) \)
   \( I \in k_2 \cap k_5, J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5 \)

Distances

\[ d_0 = |EH| = |FG| \text{ (red)} \ldots 2 \]
\[ d_1 = |FI| \text{ (blue)} \ldots 1 \]
\[ d_2 = |EG| = |FH| = |GI| = |GJ| = |GK| = |GL| \text{ (green)} \ldots 6 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} = \frac{d_3}{d_4} \text{ (8 occurrences)} \]
2.32 Class of Similar Constructions No. 25

2.32.4 Construction RCC125

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(A, C) \)
   \( D \in p_1 \cap k_3 \)
4. \( k_4 = k(D, B) \)
   \( E \in k_2 \cap k_4 \), \( F \in k_2 \cap k_4 \), \( G \in k_3 \cap k_4 \), \( H \in k_3 \cap k_4 \)
5. \( k_5 = k(H, F) \)
   \( I \in k_2 \cap k_5 \), \( J \in k_3 \cap k_5 \), \( K \in k_3 \cap k_5 \), \( L \in k_4 \cap k_5 \)

Distances

\( d_0 = |EH| = |FG| \) (red) \ldots 2
\( d_1 = |EI| \) (blue) \ldots 1
\( d_2 = |EG| = |FH| = |HI| = |HJ| = |HK| = |HL| \) (green) \ldots 6

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} = \frac{d_3}{d_4} \) (8 occurrences)
2.32.5 Construction RCC226

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(A, B) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(C, B) \)
   \( D \in p_1 \cap k_3 \)
4. \( k_4 = k(D, A) \)
   \( E \in k_2 \cap k_4 \), \( F \in k_2 \cap k_4 \), \( G \in k_3 \cap k_4 \), \( H \in k_3 \cap k_4 \)
5. \( k_5 = k(G, E) \)
   \( I \in k_2 \cap k_5 \), \( J \in k_3 \cap k_5 \), \( K \in k_3 \cap k_5 \), \( L \in k_4 \cap k_5 \)

Distances

\( d_0 = |EH| = |FG| \) (red) \( \ldots 2 \)
\( d_1 = |FI| \) (blue) \( \ldots 1 \)
\( d_2 = |EG| = |FH| = |GI| = |GJ| = |GK| = |GL| \) (green) \( \ldots 6 \)

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} = \frac{d_3}{d_2} \) (8 occurrences)
2.32.6 Construction RCC235

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(A, B) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(C, B) \)
   \( D \in p_1 \cap k_3 \)
4. \( k_4 = k(D, A) \)
   \( E \in k_2 \cap k_4 \), \( F \in k_2 \cap k_4 \), \( G \in k_3 \cap k_4 \), \( H \in k_3 \cap k_4 \)
5. \( k_5 = k(H, F) \)
   \( I \in k_2 \cap k_5 \), \( J \in k_3 \cap k_5 \), \( K \in k_3 \cap k_5 \), \( L \in k_4 \cap k_5 \)

Distances

\( d_0 = |EH| = |FG| \) (red) \( \ldots 2 \)
\( d_1 = |EI| \) (blue) \( \ldots 1 \)
\( d_2 = |EG| = |FH| = |HI| = |HJ| = |HK| = |HL| \) (green) \( \ldots 6 \)

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \) (8 occurrences)
2.32.7 Construction RCC241

Construction Process

\( A, B \) given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(A, B) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(B, C) \)
   \( D \in p_1 \cap k_3 \)
4. \( k_4 = k(D, A) \)
   \( E \in k_2 \cap k_4 , F \in k_2 \cap k_4 , G \in k_3 \cap k_4 , H \in k_3 \cap k_4 \)
5. \( k_5 = k(G, E) \)
   \( I \in k_2 \cap k_5 , J \in k_3 \cap k_5 , K \in k_3 \cap k_5 , L \in k_4 \cap k_5 \)

Distances

\( d_0 = |EH| = |FG| \) (red) \ldots 2
\( d_1 = |FI| \) (blue) \ldots 1
\( d_2 = |EG| = |FH| = |GI| = |GJ| = |GK| = |GL| \) (green) \ldots 6

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \) (8 occurrences)
2.32.8 Construction RCC250

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(A, B) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(B, C) \)
   \( D \in p_1 \cap k_3 \)
4. \( k_4 = k(D, A) \)
   \( E \in k_2 \cap k_4 \), \( F \in k_2 \cap k_4 \), \( G \in k_3 \cap k_4 \), \( H \in k_3 \cap k_4 \)
5. \( k_5 = k(H, F) \)
   \( I \in k_2 \cap k_5 \), \( J \in k_3 \cap k_5 \), \( K \in k_3 \cap k_5 \), \( L \in k_4 \cap k_5 \)

Distances

\[ d_0 = |EH| = |FG| \text{ (red)} \ldots 2 \]
\[ d_1 = |EI| \text{ (blue)} \ldots 1 \]
\[ d_2 = |EG| = |FH| = |HI| = |HJ| = |HK| = |HL| \text{ (green)} \ldots 6 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)} \]
2.33 Class of Similar Constructions No. 26

Contained Constructions
RCC77, RCC82, RCC117, RCC122, RCC227, RCC232, RCC242, RCC247
2.33.1 Construction RCC77

Construction Process

1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(C, A) \)
   \( D \in p_1 \cap k_3 \)
4. \( k_4 = k(D, B) \)
   \( E \in k_2 \cap k_4 , F \in k_2 \cap k_4 , G \in k_3 \cap k_4 , H \in k_3 \cap k_4 \)
5. \( p_5 = p(F, G) \)
   \( I \in p_5 \cap k_2 \)

Distances

\[ d_0 = |EH| = |FG| \text{ (red)} \ldots 2 \]
\[ d_1 = |FI| \text{ (blue)} \ldots 1 \]
\[ d_2 = |EG| = |FH| = |GI| \text{ (green)} \ldots 3 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} = \frac{d_3}{d_2} \text{ (5 occurrences)} \]
2.33.2 Construction RCC82

**Construction Process**

A, B given initial points
1. $p_1 = p(A, B)$
2. $k_2 = k(B, A)$
   $C \in p_1 \cap k_2$
3. $k_3 = k(C, A)$
   $D \in p_1 \cap k_3$
4. $k_4 = k(D, B)$
   $E \in k_2 \cap k_4$ , $F \in k_2 \cap k_4$ , $G \in k_3 \cap k_4$ , $H \in k_3 \cap k_4$
5. $p_5 = p(E, H)$
   $I \in p_5 \cap k_2$

**Distances**

\[ d_0 = |EH| = |FG| \text{ (red)} \ldots 2 \]
\[ d_1 = |EI| \text{ (blue)} \ldots 1 \]
\[ d_2 = |EG| = |FH| = |HI| \text{ (green)} \ldots 3 \]

**Occurrences of the Golden Ratio**

\[ \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (5 occurrences)} \]
2.33.3 Construction RCC117

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(A, C) \)
   \( D \in p_1 \cap k_3 \)
4. \( k_4 = k(D, B) \)
   \( E \in k_2 \cap k_4 , F \in k_2 \cap k_4 , G \in k_3 \cap k_4 , H \in k_3 \cap k_4 \)
5. \( p_5 = p(F, G) \)
   \( I \in p_5 \cap k_2 \)

Distances

\( d_0 = |EH| = |FG| \) (red) \ldots 2
\( d_1 = |FI| \) (blue) \ldots 1
\( d_2 = |EG| = |FH| = |GI| \) (green) \ldots 3

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \) (5 occurrences)
2.33.4 Construction RCC122

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(A, C) \)
   \( D \in p_1 \cap k_3 \)
4. \( k_4 = k(D, B) \)
   \( E \in k_2 \cap k_4 , F \in k_2 \cap k_4 , G \in k_3 \cap k_4 , H \in k_3 \cap k_4 \)
5. \( p_5 = p(E, H) \)
   \( I \in p_5 \cap k_2 \)

Distances

\( d_0 = |EH| = |FG| \) (red) \( \ldots 2 \)
\( d_1 = |EI| \) (blue) \( \ldots 1 \)
\( d_2 = |EG| = |FH| = |HI| \) (green) \( \ldots 3 \)

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \) (5 occurrences)
2.33.5 Construction RCC227

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(A, B) \)
\( C \in p_1 \cap k_2 \)
3. \( k_3 = k(C, B) \)
\( D \in p_1 \cap k_3 \)
4. \( k_4 = k(D, A) \)
\( E \in k_2 \cap k_4 \), \( F \in k_2 \cap k_4 \), \( G \in k_3 \cap k_4 \), \( H \in k_3 \cap k_4 \)
5. \( p_5 = p(F, G) \)
\( I \in p_5 \cap k_2 \)

Distances

\[ d_0 = |EH| = |FG| \] (red) \( \ldots 2 \)
\[ d_1 = |FI| \] (blue) \( \ldots 1 \)
\[ d_2 = |EG| = |FH| = |GI| \] (green) \( \ldots 3 \)

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} = \frac{d_3}{d_2} \] (5 occurrences)
2.33.6 Construction RCC232

**Construction Process**

- **A, B given initial points**
  1. \( p_1 = p(A, B) \)
  2. \( k_2 = k(A, B) \)
     - \( C \in p_1 \cap k_2 \)
  3. \( k_3 = k(C, B) \)
     - \( D \in p_1 \cap k_3 \)
  4. \( k_4 = k(D, A) \)
     - \( E \in k_2 \cap k_4 \), \( F \in k_2 \cap k_4 \), \( G \in k_3 \cap k_4 \), \( H \in k_3 \cap k_4 \)
  5. \( p_5 = p(E, H) \)
     - \( I \in p_5 \cap k_2 \)

**Distances**

- \( d_0 = |EH| = |FG| \) (red) ... 2
- \( d_1 = |EI| \) (blue) ... 1
- \( d_2 = |EG| = |FH| = |HI| \) (green) ... 3

**Occurrences of the Golden Ratio**

- \( \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \) (5 occurrences)
2.33.7 Construction RCC242

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(A, B) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(B, C) \)
   \( D \in p_1 \cap k_3 \)
4. \( k_4 = k(D, A) \)
   \( E \in k_2 \cap k_4 , F \in k_2 \cap k_4 , G \in k_3 \cap k_4 , H \in k_3 \cap k_4 \)
5. \( p_5 = p(F, G) \)
   \( I \in p_5 \cap k_2 \)

Distances

\( d_0 = |EH| = |FG| \) (red) \( \ldots 2 \)
\( d_1 = |FI| \) (blue) \( \ldots 1 \)
\( d_2 = |EG| = |FH| = |GI| \) (green) \( \ldots 3 \)

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \) (5 occurrences)
2.33.8 Construction RCC247

Construction Process

\[ A, B \text{ given initial points} \]
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(A, B) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(B, C) \)
   \( D \in p_1 \cap k_3 \)
4. \( k_4 = k(D, A) \)
   \( E \in k_2 \cap k_4 , F \in k_2 \cap k_4 , G \in k_3 \cap k_4 , H \in k_3 \cap k_4 \)
5. \( p_5 = p(E, H) \)
   \( I \in p_5 \cap k_2 \)

Distances

\[ d_0 = |EH| = |FG| \text{ (red)} \ldots 2 \]
\[ d_1 = |EI| \text{ (blue)} \ldots 1 \]
\[ d_2 = |EG| = |FH| = |HI| \text{ (green)} \ldots 3 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} = \frac{d_3}{d_2} \text{ (5 occurrences)} \]
2.34 Class of Similar Constructions No. 27

Contained Constructions
RCC78, RCC83, RCC118, RCC123, RCC228, RCC233, RCC243, RCC248
2.34.1 Construction RCC78

Construction Process

A, B given initial points
1. $p_1 = p(A, B)$
2. $k_2 = k(B, A)$
   $C \in p_1 \cap k_2$
3. $k_3 = k(C, A)$
   $D \in p_1 \cap k_3$
4. $k_4 = k(D, B)$
   $E \in k_2 \cap k_4$, $F \in k_2 \cap k_4$, $G \in k_3 \cap k_4$, $H \in k_3 \cap k_4$
5. $k_5 = k(G, F)$
   $I \in p_1 \cap k_5$, $J \in p_1 \cap k_5$, $K \in k_2 \cap k_5$, $L \in k_3 \cap k_5$, $M \in k_3 \cap k_5$

Distances

$d_0 = |EH| = |FG| = |GI| = |GJ| = |GK| = |GL| = |GM| = |GN| = |HI| = |HJ| \text{ (red)} \ldots 10$
$d_1 = |EK| \text{ (blue)} \ldots 1$
$d_2 = |EG| = |FH| \text{ (green)} \ldots 2$

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_1} = \frac{d_2}{d_1}$ (12 occurrences)
2.34.2 Construction RCC83

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(C, A) \)
   \( D \in p_1 \cap k_3 \)
4. \( k_4 = k(D, B) \)
   \( E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4 \)
5. \( k_5 = k(H, E) \)
   \( I \in p_1 \cap k_5, J \in p_1 \cap k_5, K \in k_2 \cap k_5, L \in k_3 \cap k_5, M \in k_3 \cap k_5 \)
   \( N \in k_4 \cap k_5 \)

Distances

\[ d_0 = |EH| = |FG| = |GI| = |GJ| = |HI| = |HJ| = |HK| = |HL| = |HM| = |HN| \text{ (red) } \ldots 10 \]
\[ d_1 = |FK| \text{ (blue) } \ldots 1 \]
\[ d_2 = |EG| = |FH| \text{ (green) } \ldots 2 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} = \frac{d_2}{d_4} \text{ (12 occurrences)} \]
2.34.3 Construction RCC118

Construction Process

$A, B$ given initial points
1. $p_1 = p(A, B)$
2. $k_2 = k(B, A)$
   $C \in p_1 \cap k_2$
3. $k_3 = k(A, C)$
   $D \in p_1 \cap k_3$
4. $k_4 = k(D, B)$
   $E \in k_2 \cap k_4$ , $F \in k_2 \cap k_4$ , $G \in k_3 \cap k_4$ , $H \in k_3 \cap k_4$
5. $k_5 = k(G, F)$
   $I \in p_1 \cap k_5$ , $J \in p_1 \cap k_5$ , $K \in k_2 \cap k_5$ , $L \in k_3 \cap k_5$ , $M \in k_3 \cap k_5$
   $N \in k_4 \cap k_5$

Distances

\[d_0 = |EH| = |FG| = |GI| = |GJ| = |GL| = |GM| = |GN| = \]
\[|HI| = |HJ| \text{ (red)} \ldots 10\]
\[d_1 = |EK| \text{ (blue)} \ldots 1\]
\[d_2 = |EG| = |FH| \text{ (green)} \ldots 2\]

Occurrences of the Golden Ratio

\[\phi = \frac{d_0}{d_4} = \frac{d_4}{d_2} \text{ (12 occurrences)}\]
2.34.4 Construction RCC123

Construction Process

A, B given initial points

1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(A, C) \)
   \( D \in p_1 \cap k_3 \)
4. \( k_4 = k(D, B) \)
   \( E \in k_2 \cap k_4 \), \( F \in k_2 \cap k_4 \), \( G \in k_3 \cap k_4 \), \( H \in k_3 \cap k_4 \)
5. \( k_5 = k(H, E) \)
   \( I \in p_1 \cap k_5 \), \( J \in p_1 \cap k_5 \), \( K \in k_2 \cap k_5 \), \( L \in k_3 \cap k_5 \), \( M \in k_3 \cap k_5 \), \( N \in k_4 \cap k_5 \)

Distances

\[
\begin{align*}
d_0 &= |EH| = |FG| = |GI| = |GJ| = |HI| = |HJ| = |HK| = |HL| = |HM| = |HN| \quad \text{(red)} \ldots 10 \\
d_1 &= |FK| \quad \text{(blue)} \ldots 1 \\
d_2 &= |EG| = |FH| \quad \text{(green)} \ldots 2
\end{align*}
\]

Occurrences of the Golden Ratio

\[
\phi = \frac{d_0}{d_1} = \frac{d_2}{d_3} \quad \text{(12 occurrences)}
\]
2.34.5 Construction RCC228

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(A, B) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(C, B) \)
   \( D \in p_1 \cap k_3 \)
4. \( k_4 = k(D, A) \)
   \( E \in k_2 \cap k_4 , F \in k_2 \cap k_4 , G \in k_3 \cap k_4 , H \in k_3 \cap k_4 \)
5. \( k_5 = k(G, F) \)
   \( I \in p_1 \cap k_5 , J \in p_1 \cap k_5 , K \in k_2 \cap k_5 , L \in k_3 \cap k_5 , M \in k_3 \cap k_5 \),
   \( N \in k_4 \cap k_5 \)

Distances

\[ d_0 = |EH| = |FG| = |GI| = |GK| = |GL| = |GM| = |GN| = |HI| = |HJ| \quad \text{red} \quad \ldots \quad 10 \]
\[ d_1 = |EK| \quad \text{blue} \quad \ldots \quad 1 \]
\[ d_2 = |EG| = |FH| \quad \text{green} \quad \ldots \quad 2 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} = \frac{d_2}{d_2} \quad (12 \text{ occurrences}) \]
2.34.6 Construction RCC233

Construction Process

\begin{itemize}
\item $A, B$ given initial points
\item $p_1 = p(A, B)$
\item $k_2 = k(A, B)$
\item $C \in p_1 \cap k_2$
\item $k_3 = k(C, B)$
\item $D \in p_1 \cap k_3$
\item $k_4 = k(D, A)$
\item $E \in k_2 \cap k_4$, $F \in k_2 \cap k_4$, $G \in k_3 \cap k_4$, $H \in k_3 \cap k_4$
\item $k_5 = k(H, E)$
\item $I \in p_1 \cap k_5$, $J \in p_1 \cap k_5$, $K \in k_2 \cap k_5$, $L \in k_3 \cap k_5$, $M \in k_3 \cap k_5$
\item $N \in k_4 \cap k_5$
\end{itemize}

Distances

\begin{itemize}
\item $d_0 = |EH| = |FG| = |GI| = |GI| = |HI| = |HK| = |HL| = |HM| = |HN|$ (red) \ldots 10
\item $d_1 = |FK|$ (blue) \ldots 1
\item $d_2 = |EG|$ = $|FH|$ (green) \ldots 2
\end{itemize}

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} = \frac{d_2}{d_1} \] (12 occurrences)
2.34.7 Construction RCC243

Construction Process

A, B given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(A, B) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(B, C) \)
   \( D \in p_1 \cap k_3 \)
4. \( k_4 = k(D, A) \)
   \( E \in k_2 \cap k_4,\ F \in k_2 \cap k_4,\ G \in k_3 \cap k_4,\ H \in k_3 \cap k_4 \)
5. \( k_5 = k(G, F) \)
   \( I \in p_1 \cap k_5,\ J \in p_1 \cap k_5,\ K \in k_2 \cap k_5,\ L \in k_3 \cap k_5,\ M \in k_3 \cap k_5,\ N \in k_4 \cap k_5 \)

Distances

\( d_0 = |EH| = |FG| = |GI| = |GJ| = |GK| = |GL| = |GM| = |GN| = |HI| = |HJ| \) (red) ... 10
\( d_1 = |EK| \) (blue) ... 1
\( d_2 = |EG| = |FH| \) (green) ... 2

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \) (12 occurrences)
Construction RCC248

Construction Process

A, B given initial points
1. $p_1 = p(A, B)$
2. $k_2 = k(A, B)$
   \[ C \in p_1 \cap k_2 \]
3. $k_3 = k(B, C)$
   \[ D \in p_1 \cap k_3 \]
4. $k_4 = k(D, A)$
   \[ E \in k_2 \cap k_4 \], \[ F \in k_2 \cap k_4 \], \[ G \in k_3 \cap k_4 \], \[ H \in k_3 \cap k_4 \]
5. $k_5 = k(H, E)$
   \[ I \in p_1 \cap k_5 \], \[ J \in p_1 \cap k_5 \], \[ K \in k_2 \cap k_5 \], \[ L \in k_3 \cap k_5 \], \[ M \in k_3 \cap k_5 \], \[ N \in k_4 \cap k_5 \]

Distances

\[ d_0 = |EH| = |FG| = |GI| = |GJ| = |HI| = |HJ| = |HK| = |HL| = |HM| = |HN| \quad \text{(red)} \ldots 10 \]
\[ d_1 = |FK| \quad \text{(blue)} \ldots 1 \]
\[ d_2 = |EG| = |FH| \quad \text{(green)} \ldots 2 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} = \frac{d_2}{d_1} \quad (12 \text{ occurrences}) \]
2.35 Class of Similar Constructions No. 28

Contained Constructions

RCC86, RCC126, RCC236, RCC251
2.35.1 Construction RCC86

Construction Process

\[ A, B \text{ given initial points} \]
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(C, A) \)
   \( D \in p_1 \cap k_3 \)
4. \( k_4 = k(D, B) \)
   \( E \in k_2 \cap k_4 \), \( F \in k_2 \cap k_4 \), \( G \in k_3 \cap k_4 \), \( H \in k_3 \cap k_4 \)
5. \( p_5 = p(G, H) \)
   \( I \in p_5 \cap k_2 \), \( J \in p_5 \cap k_2 \)

Distances

\[ d_0 = |EH| = |FG| \text{ (red)} \ldots 2 \]
\[ d_1 = |AI| = |AJ| \text{ (blue)} \ldots 2 \]
\[ d_2 = |EG| = |FH| \text{ (green)} \ldots 2 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)} \]
2.35.2 Construction RCC126

Construction Process

1. \( p_1 = p(A, B) \)
2. \( k_2 = k(B, A) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(A, C) \)
   \( D \in p_1 \cap k_3 \)
4. \( k_4 = k(D, B) \)
   \( E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4 \)
5. \( p_5 = p(G, H) \)
   \( I \in p_5 \cap k_2, J \in p_5 \cap k_2 \)

Distances

\( d_0 = |EH| = |FG| \) (red) \( \ldots 2 \)
\( d_1 = |CI| = |CJ| \) (blue) \( \ldots 2 \)
\( d_2 = |EG| = |FH| \) (green) \( \ldots 2 \)

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \) (8 occurrences)
2.35.3 Construction RCC236

Construction Process

\(A, B\) given initial points
1. \(p_1 = p(A, B)\)
2. \(k_2 = k(A, B)\)
   \(� \in p_1 \cap k_2\)
3. \(k_3 = k(C, B)\)
   \(D \in p_1 \cap k_3\)
4. \(k_4 = k(D, A)\)
   \(E \in k_2 \cap k_4\), \(F \in k_2 \cap k_4\), \(G \in k_3 \cap k_4\), \(H \in k_3 \cap k_4\)
5. \(p_5 = p(G, H)\)
   \(I \in p_5 \cap k_2\), \(J \in p_5 \cap k_2\)

Distances
\[d_0 = |EH| = |FG| \text{ (red)} \ldots 2\]
\[d_1 = |BI| = |BJ| \text{ (blue)} \ldots 2\]
\[d_2 = |EG| = |FH| \text{ (green)} \ldots 2\]

Occurrences of the Golden Ratio
\[φ = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)}\]
2.35.4 Construction RCC251

Construction Process

\( A, B \) given initial points
1. \( p_1 = p(A, B) \)
2. \( k_2 = k(A, B) \)
   \( C \in p_1 \cap k_2 \)
3. \( k_3 = k(B, C) \)
   \( D \in p_1 \cap k_3 \)
4. \( k_4 = k(D, A) \)
   \( E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4 \)
5. \( p_5 = p(G, H) \)
   \( I \in p_5 \cap k_2, J \in p_5 \cap k_2 \)

Distances
\( d_0 = |EH| = |FG| \) (red) \( \ldots 2 \)
\( d_1 = |CI| = |CJ| \) (blue) \( \ldots 2 \)
\( d_2 = |EG| = |FH| \) (green) \( \ldots 2 \)

Occurrences of the Golden Ratio
\( \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \) (8 occurrences)
2.36 Class of Similar Constructions No. 29

Contained Constructions
RCC254, RCC290, RCC304, RCC336
### 2.36.1 Construction RCC254

**Construction Process**

- **A, B** given initial points
- 1. $k_1 = k(B, A)$
- 2. $k_2 = k(A, B)$
  - $C \in k_1 \cap k_2$, $D \in k_1 \cap k_2$
- 3. $p_3 = p(A, C)$
  - $E \in p_3 \cap k_2$
- 4. $k_4 = k(D, C)$
  - $F \in k_1 \cap k_4$, $G \in k_2 \cap k_4$, $H \in p_3 \cap k_4$
- 5. $k_5 = k(F, A)$
  - $I \in k_2 \cap k_5$, $J \in k_2 \cap k_5$, $K \in p_3 \cap k_5$, $L \in k_4 \cap k_5$, $M \in k_4 \cap k_5$

**Distances**

- $d_0 = |JK|$ (red) \ldots 1
- $d_1 = |EI|$ (blue) \ldots 1
- $d_2 = |BE| = |BK| = |CD| = |CF| = |CG| = |DF| = |DG| = |DH| = |DL| = |DM| = |GH|$ (green) \ldots 11
- $d_3 = |BI| = |BJ|$ (cyan) \ldots 2
- $d_4 = |IK|$ (magenta) \ldots 1
- $d_5 = |EJ|$ (yellow) \ldots 1

**Occurrences of the Golden Ratio**

\[
\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \quad (26 \text{ occurrences})
\]
2.36.2 Construction RCC290

Construction Process

\( A, B \) given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2, D \in k_1 \cap k_2 \)
3. \( p_3 = p(B, C) \)
   \( E \in p_3 \cap k_1 \)
4. \( k_4 = k(D, C) \)
   \( F \in k_1 \cap k_4, G \in k_2 \cap k_4, H \in p_3 \cap k_4 \)
5. \( k_5 = k(G, B) \)
   \( I \in k_1 \cap k_5, J \in k_1 \cap k_5, K \in p_3 \cap k_5, L \in k_4 \cap k_5, M \in k_4 \cap k_5 \)

Distances
\[
\begin{align*}
d_0 &= |JK| \text{ (red)} \ldots 1 \\
d_1 &= |EI| \text{ (blue)} \ldots 1 \\
d_2 &= |AE| = |AK| = |CD| = |CF| = |CG| = |DF| = |DG| = |DH| = |DL| = |DM| = |FH| \text{ (green)} \ldots 11 \\
d_3 &= |AI| = |AJ| \text{ (cyan)} \ldots 2 \\
d_4 &= |IK| \text{ (magenta)} \ldots 1 \\
d_5 &= |EJ| \text{ (yellow)} \ldots 1
\end{align*}
\]

Occurrences of the Golden Ratio
\[
\phi = \frac{d_4}{d_2} = \frac{d_4}{d_3} = \frac{d_5}{d_4} = \frac{d_5}{d_3} \text{ (26 occurrences)}
\]
2.36.3 Construction RCC304

Construction Process

A, B given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2 \) , \( D \in k_1 \cap k_2 \)
3. \( p_3 = p(A, D) \)
   \( E \in p_3 \cap k_2 \)
4. \( k_4 = k(C, D) \)
   \( F \in k_1 \cap k_4 \) , \( G \in k_2 \cap k_4 \) , \( H \in p_3 \cap k_4 \)
5. \( k_5 = k(F, A) \)
   \( I \in k_2 \cap k_5 \) , \( J \in k_2 \cap k_5 \) , \( K \in p_3 \cap k_5 \) , \( L \in k_4 \cap k_5 \) , \( M \in k_4 \cap k_5 \)

Distances

\( d_0 = |IK| \) (red) . . . 1
\( d_1 = |EJ| \) (blue) . . . 1
\( d_2 = |BE| = |BK| = |CD| = |CF| = |CG| = |CH| = |CL| = |CM| = |DF| = |DG| = |GH| \) (green) . . . 11
\( d_3 = |BI| = |BJ| \) (cyan) . . . 2
\( d_4 = |JK| \) (magenta) . . . 1
\( d_5 = |EI| \) (yellow) . . . 1

Occurrences of the Golden Ratio

\( \phi = \frac{d_5}{d_3} = \frac{d_4}{d_2} = \frac{d_3}{d_1} = \frac{d_2}{d_1} \) (26 occurrences)
2.36.4 Construction RCC336

Construction Process

A, B given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( p_3 = p(B, D) \)
   \( E \in p_3 \cap k_1 \)
4. \( k_4 = k(C, D) \)
   \( F \in k_1 \cap k_4 \), \( G \in k_2 \cap k_4 \), \( H \in p_3 \cap k_4 \)
5. \( k_5 = k(G, B) \)
   \( I \in k_1 \cap k_5 \), \( J \in k_1 \cap k_5 \), \( K \in p_3 \cap k_5 \), \( L \in k_4 \cap k_5 \), \( M \in k_4 \cap k_5 \)

Distances

\[ d_0 = |IK| \ (red) \ldots 1 \]
\[ d_1 = |EJ| \ (blue) \ldots 1 \]
\[ d_2 = |AE| = |AK| = |CD| = |CF| = |CG| = |CH| = |CL| = |CM| = |DF| = |DG| = |FH| \ (green) \ldots 11 \]
\[ d_3 = |AI| = |AJ| \ (cyan) \ldots 2 \]
\[ d_4 = |JK| \ (magenta) \ldots 1 \]
\[ d_5 = |EI| \ (yellow) \ldots 1 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \ (26 \ occurrences) \]
2.37 Class of Similar Constructions No. 30

Contained Constructions
RCC255, RCC291, RCC305, RCC337
2.37.1 Construction RCC255

Construction Process

A, B given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( p_3 = p(A, C) \)
   \( E \in p_3 \cap k_2 \)
4. \( k_4 = k(D, C) \)
   \( F \in p_3 \cap k_4 \)
5. \( k_5 = k(F, A) \)
   \( G \in k_2 \cap k_5 \), \( H \in k_2 \cap k_5 \)

Distances

\[ d_0 = |BG| \text{ (red) } \ldots 1 \]
\[ d_1 = |EG| = |EH| \text{ (blue) } \ldots 2 \]
\[ d_2 = |BH| \text{ (green) } \ldots 1 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (4 occurrences) } \]
2.37 Class of Similar Constructions No. 30

2.37.2 Construction RCC291

Construction Process

1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2 , D \in k_1 \cap k_2 \)
3. \( p_3 = p(B, C) \)
   \( E \in p_3 \cap k_1 \)
4. \( k_4 = k(D, C) \)
   \( F \in p_3 \cap k_4 \)
5. \( k_5 = k(F, B) \)
   \( G \in k_1 \cap k_5 , H \in k_1 \cap k_5 \)

Distances

\[ d_0 = |AH| \text{ (red)} \ldots 1 \]
\[ d_1 = |EG| = |EH| \text{ (blue)} \ldots 2 \]
\[ d_2 = |AG| \text{ (green)} \ldots 1 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} = \frac{d_4}{d_2} \text{ (4 occurrences)} \]
2.37.3 Construction RCC305

Construction Process

1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2, D \in k_1 \cap k_2 \)
3. \( p_3 = p(A, D) \)
   \( E \in p_3 \cap k_2 \)
4. \( k_4 = k(C, D) \)
   \( F \in p_3 \cap k_4 \)
5. \( k_5 = k(F, A) \)
   \( G \in k_2 \cap k_5, H \in k_2 \cap k_5 \)

Distances

\[ d_0 = |BG| \text{ (red)} \ldots 1 \]
\[ d_1 = |EG| = |EH| \text{ (blue)} \ldots 2 \]
\[ d_2 = |BH| \text{ (green)} \ldots 1 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (4 occurrences)} \]
2.37.4 Construction RCC337

Construction Process

A, B given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( p_3 = p(B, D) \)
   \( E \in p_3 \cap k_1 \)
4. \( k_4 = k(C, D) \)
   \( F \in p_3 \cap k_4 \)
5. \( k_5 = k(F, B) \)
   \( G \in k_1 \cap k_5 \), \( H \in k_1 \cap k_5 \)

Distances

\( d_0 = |AH| \) (red) \ldots 1
\( d_1 = |EG| = |EH| \) (blue) \ldots 2
\( d_2 = |AG| \) (green) \ldots 1

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \) (4 occurrences)
2.38 Class of Similar Constructions No. 31

Contained Constructions
RCC256, RCC292, RCC302, RCC334
2.38 Class of Similar Constructions No. 31

2.38.1 Construction RCC256

Construction Process

A, B given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( p_3 = p(A, C) \)
   \( E \in p_3 \cap k_2 \)
4. \( k_4 = k(C, D) \)
   \( F \in k_1 \cap k_4 \), \( G \in k_2 \cap k_4 \), \( H \in p_3 \cap k_4 \), \( I \in p_3 \cap k_4 \)
5. \( k_5 = k(F, A) \)
   \( J \in k_2 \cap k_5 \), \( K \in k_2 \cap k_5 \), \( L \in p_3 \cap k_5 \), \( M \in k_4 \cap k_5 \), \( N \in k_4 \cap k_5 \)

Distances

\( d_0 = |KL| \) (red) \( \ldots 1 \)
\( d_1 = |EJ| \) (blue) \( \ldots 1 \)
\( d_2 = |BE| = |BL| = |CD| = |CF| = |CG| = |CH| = |CI| = |CM| = |CN| = |DF| = |DG| \) (green) \( \ldots 11 \)
\( d_3 = |BJ| = |BK| \) (cyan) \( \ldots 2 \)
\( d_4 = |JL| \) (magenta) \( \ldots 1 \)
\( d_5 = |EK| \) (yellow) \( \ldots 1 \)

Occurrences of the Golden Ratio

\( \phi = \frac{d_4}{d_2} = \frac{d_3}{d_5} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \) (26 occurrences)
2.38.2 Construction RCC292

Construction Process

\( A, B \) given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( p_3 = p(B, C) \)
   \( E \in p_3 \cap k_1 \)
4. \( k_4 = k(C, D) \)
   \( F \in k_1 \cap k_4 \), \( G \in k_2 \cap k_4 \), \( H \in p_3 \cap k_4 \), \( I \in p_3 \cap k_4 \)
5. \( k_5 = k(G, B) \)
   \( J \in k_1 \cap k_5 \), \( K \in k_1 \cap k_5 \), \( L \in p_3 \cap k_5 \), \( M \in k_4 \cap k_5 \), \( N \in k_4 \cap k_5 \)

Distances

\[ \begin{align*}
  d_0 &= |KL| \quad (\text{red}) \ldots 1 \\
  d_1 &= |EJ| \quad (\text{blue}) \ldots 1 \\
  d_2 &= |AE| = |AL| = |CD| = |CF| = |CG| = |CH| = |CI| = |CM| = |CN| = |DF| = |DG| \quad (\text{green}) \ldots 11 \\
  d_3 &= |AJ| = |AK| \quad (\text{cyan}) \ldots 2 \\
  d_4 &= |JL| \quad (\text{magenta}) \ldots 1 \\
  d_5 &= |EK| \quad (\text{yellow}) \ldots 1
\end{align*} \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \quad (26 \text{ occurrences}) \]
2.38.3 Construction RCC302

Construction Process

A, B given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( p_3 = p(A, D) \)
   \( E \in p_3 \cap k_2 \)
4. \( k_4 = k(D, C) \)
   \( F \in k_1 \cap k_4 \), \( G \in k_2 \cap k_4 \), \( H \in p_3 \cap k_4 \), \( I \in p_3 \cap k_4 \)
5. \( k_5 = k(F, A) \)
   \( J \in k_2 \cap k_5 \), \( K \in k_2 \cap k_5 \), \( L \in p_3 \cap k_5 \), \( M \in k_4 \cap k_5 \), \( N \in k_4 \cap k_5 \)

Distances

\( d_0 = |JL| \) (red) \ldots 1
\( d_1 = |EK| \) (blue) \ldots 1
\( d_2 = |BE| = |BL| = |CD| = |CF| = |CG| = |DF| = |DG| = |DH| = |DI| = |DM| = |DN| \) (green) \ldots 11
\( d_3 = |BJ| = |BK| \) (cyan) \ldots 2
\( d_4 = |KL| \) (magenta) \ldots 1
\( d_5 = |EJ| \) (yellow) \ldots 1

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \) (26 occurrences)
2.38.4 Construction RCC334

Construction Process

A, B given initial points
1. \( k_1 = k(B, A) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
2. \( k_2 = k(A, B) \)
3. \( p_3 = p(B, D) \)
   \( E \in p_3 \cap k_1 \)
4. \( k_4 = k(D, C) \)
   \( F \in k_1 \cap k_4 \), \( G \in k_2 \cap k_4 \), \( H \in p_3 \cap k_4 \), \( I \in p_3 \cap k_4 \)
5. \( k_5 = k(G, B) \)
   \( J \in k_1 \cap k_5 \), \( K \in k_1 \cap k_5 \), \( L \in p_3 \cap k_5 \), \( M \in k_4 \cap k_5 \), \( N \in k_4 \cap k_5 \)

Distances

\( d_0 = |JL| \) (red) \ldots 1
\( d_1 = |EK| \) (blue) \ldots 1
\( d_2 = |AE| = |AL| = |CD| = |CF| = |CG| = |DF| = |DG| = |DH| = |DI| = |DM| = |DN| \) (green) \ldots 11
\( d_3 = |AJ| = |AK| \) (cyan) \ldots 2
\( d_4 = |KL| \) (magenta) \ldots 1
\( d_5 = |EJ| \) (yellow) \ldots 1

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \) (26 occurrences)
2.39 Class of Similar Constructions No. 32

Contained Constructions
RCC257, RCC293, RCC303, RCC335
2.39 Class of Similar Constructions No. 32

2.39.1 Construction RCC257

Construction Process

A, B given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2, \quad D \in k_1 \cap k_2 \)
3. \( p_3 = p(A, C) \)
4. \( k_4 = k(C, D) \)
   \( E \in k_1 \cap k_4, \quad F \in k_2 \cap k_4, \quad G \in p_3 \cap k_4 \)
5. \( k_5 = k(E, G) \)
   \( H \in k_2 \cap k_5, \quad I \in k_2 \cap k_5 \)

Distances

\[ d_0 = |CI| = |DH| \quad \text{(red)} \ldots 2 \]
\[ d_1 = |FH| = |FI| \quad \text{(blue)} \ldots 2 \]
\[ d_2 = |CH| = |DI| \quad \text{(green)} \ldots 2 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \quad \text{(8 occurrences)} \]
2.39.2 Construction RCC293

Construction Process

A, B given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2, \ D \in k_1 \cap k_2 \)
3. \( p_3 = p(B, C) \)
4. \( k_4 = k(C, D) \)
   \( E \in k_1 \cap k_4, \ F \in k_2 \cap k_4, \ G \in p_3 \cap k_4 \)
5. \( k_5 = k(F, G) \)
   \( H \in k_1 \cap k_5, \ I \in k_1 \cap k_5 \)

Distances

\[ d_0 = |CI| = |DH| \text{ (red) } \ldots 2 \]
\[ d_1 = |EH| = |EI| \text{ (blue) } \ldots 2 \]
\[ d_2 = |CH| = |DI| \text{ (green) } \ldots 2 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences) } \]
2.39.3 Construction RCC303

Construction Process

1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
3. \( p_3 = p(A, D) \)
4. \( k_4 = k(D, C) \)
5. \( k_5 = k(E, G) \)

Distances

\[
\begin{align*}
d_0 &= |CI| = |DH| \quad \text{(red)} \quad \ldots 2 \\
d_1 &= |FH| = |FI| \quad \text{(blue)} \quad \ldots 2 \\
d_2 &= |CH| = |DI| \quad \text{(green)} \quad \ldots 2
\end{align*}
\]

Occurrences of the Golden Ratio

\[
\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \quad \text{(8 occurrences)}
\]
2.39.4 Construction RCC335

Construction Process

A, B given initial points
1. $k_1 = k(B, A)$
2. $k_2 = k(A, B)$
   $C \in k_1 \cap k_2 , D \in k_1 \cap k_2$
3. $p_3 = p(B, D)$
4. $k_4 = k(D, C)$
   $E \in k_1 \cap k_4 , F \in k_2 \cap k_4 , G \in p_3 \cap k_4$
5. $k_5 = k(F, G)$
   $H \in k_1 \cap k_5 , I \in k_1 \cap k_5$

Distances

$d_0 = |CI| = |DH|$ (red) ... 2
$d_1 = |EH| = |EI|$ (blue) ... 2
$d_2 = |CH| = |DI|$ (green) ... 2

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2}$ (8 occurrences)
2.40 Class of Similar Constructions No. 33

Contained Constructions
RCC258, RCC295, RCC306, RCC339
2.40.1 Construction RCC258

Construction Process

\( A, B \) given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2, D \in k_1 \cap k_2 \)
3. \( p_3 = p(A, C) \)
   \( E \in p_3 \cap k_2 \)
4. \( k_4 = k(E, A) \)
   \( F \in k_2 \cap k_4, G \in p_3 \cap k_4 \)
5. \( k_5 = k(C, E) \)
   \( H \in k_1 \cap k_5, I \in k_4 \cap k_5, J \in k_4 \cap k_5 \)

Distances

\[ d_0 = |HI| \text{ (red)} \ldots 1 \]
\[ d_1 = |AH| = |BE| = |CD| = |CF| = |DF| = |DG| = |FG| \text{ (blue)} \ldots 7 \]
\[ d_2 = |HJ| \text{ (green)} \ldots 1 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_4} = \frac{d_4}{d_2} \text{ (14 occurrences)} \]
2.40 Class of Similar Constructions No. 33

2.40.2 Construction RCC295

**Construction Process**

1. $k_1 = k(B, A)$
2. $k_2 = k(A, B)$
   - $C \in k_1 \cap k_2$, $D \in k_1 \cap k_2$
3. $p_3 = p(B, C)$
   - $E \in p_3 \cap k_1$
4. $k_4 = k(E, B)$
   - $F \in k_1 \cap k_4$, $G \in p_3 \cap k_4$
5. $k_5 = k(C, E)$
   - $H \in k_2 \cap k_5$, $I \in k_4 \cap k_5$, $J \in k_4 \cap k_5$

**Distances**

\[
\begin{align*}
d_0 &= |HJ| \text{ (red)} \ldots 1 \\
d_1 &= |AE| = |BH| = |CD| = |CF| = |DF| = |DG| = |FG| \text{ (blue)} \ldots 7 \\
d_2 &= |HI| \text{ (green)} \ldots 1
\end{align*}
\]

**Occurrences of the Golden Ratio**

\[
\phi = \frac{d_0}{d_1} = \frac{d_4}{d_5} \text{ (14 occurrences)}
\]
2.40.3 Construction RCC306

Construction Process

1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   
   \( C \in k_1 \cap k_2, \ D \in k_1 \cap k_2 \)
3. \( p_3 = p(A, D) \)
   
   \( E \in p_3 \cap k_2 \)
4. \( k_4 = k(E, A) \)
   
   \( F \in k_2 \cap k_4, \ G \in p_3 \cap k_4 \)
5. \( k_5 = k(D, E) \)
   
   \( H \in k_1 \cap k_5, \ I \in k_4 \cap k_5, \ J \in k_4 \cap k_5 \)

Distances

\[
\begin{align*}
    d_0 &= |HI| \quad \text{(red)} \\
    d_1 &= |AH| = |BE| = |CD| = |CF| = |CG| = |DF| = |FG| \quad \text{(blue)} \\
    d_2 &= |HJ| \quad \text{(green)}
\end{align*}
\]

Occurrences of the Golden Ratio

\[
\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \quad (14 \text{ occurrences})
\]
2.40.4 Construction RCC339

Construction Process

A, B given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( p_3 = p(B, D) \)
   \( E \in p_3 \cap k_1 \)
4. \( k_4 = k(E, B) \)
   \( F \in k_1 \cap k_4 \), \( G \in p_3 \cap k_4 \)
5. \( k_5 = k(D, E) \)
   \( H \in k_2 \cap k_5 \), \( I \in k_4 \cap k_5 \), \( J \in k_4 \cap k_5 \)

Distances

\( d_0 = |HJ| \) (red) \( \ldots 1 \)
\( d_1 = |AE| = |BH| = |CD| = |CF| = |CG| = |DF| = |FG| \) (blue) \( \ldots 7 \)
\( d_2 = |HI| \) (green) \( \ldots 1 \)

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_4} = \frac{d_4}{d_2} \) (14 occurrences)
2.41 Class of Similar Constructions No. 34

Contained Constructions
RCC259, RCC296, RCC307, RCC340
2.41.1 Construction RCC259

Construction Process

1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2, D \in k_1 \cap k_2 \)
3. \( p_3 = p(A, C) \)
   \( E \in p_3 \cap k_2 \)
4. \( k_4 = k(E, A) \)
   \( F \in k_2 \cap k_4, G \in p_3 \cap k_4 \)
5. \( k_5 = k(F, B) \)
   \( H \in k_1 \cap k_5, I \in k_1 \cap k_5, J \in k_4 \cap k_5 \)

Distances

\[ d_0 = |HJ| \quad \text{(red)} \quad 1 \]
\[ d_1 = |AJ| = |BE| = |CD| = |CF| = |DF| = |DG| = |FG| \quad \text{(blue)} \quad 7 \]
\[ d_2 = |IJ| \quad \text{(green)} \quad 1 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} = \frac{d_2}{d_1} \quad \text{(14 occurrences)} \]
2.41.2 Construction RCC296

Construction Process

$A, B$ given initial points
1. $k_1 = k(B, A)$
2. $k_2 = k(A, B)$
   $C \in k_1 \cap k_2$, $D \in k_1 \cap k_2$
3. $p_3 = p(B, C)$
   $E \in p_3 \cap k_1$
4. $k_4 = k(E, B)$
   $F \in k_1 \cap k_4$, $G \in p_3 \cap k_4$
5. $k_5 = k(F, A)$
   $H \in k_2 \cap k_5$, $I \in k_2 \cap k_5$, $J \in k_4 \cap k_5$

Distances

$d_0 = |HJ|$ (red) \ldots 1
$d_1 = |AE| = |BJ| = |CD| = |CF| = |DF| = |DG| = |FG|$ (blue) \ldots 7
$d_2 = |IJ|$ (green) \ldots 1

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2}$ (14 occurrences)
2.41.3 Construction RCC307

Construction Process

\[ \begin{align*}
A, B \text{ given initial points} \\
1. & \quad k_1 = k(B, A) \\
2. & \quad k_2 = k(A, B) \\
\quad & \quad C \in k_1 \cap k_2, D \in k_1 \cap k_2 \\
3. & \quad p_3 = p(A, D) \\
\quad & \quad E \in p_3 \cap k_2 \\
4. & \quad k_4 = k(E, A) \\
\quad & \quad F \in k_2 \cap k_4, G \in p_3 \cap k_4 \\
5. & \quad k_5 = k(F, B) \\
\quad & \quad H \in k_1 \cap k_5, I \in k_1 \cap k_5, J \in k_4 \cap k_5
\end{align*} \]

Distances

\[ \begin{align*}
d_0 &= |IJ| \text{ (red)} \ldots 1 \\
d_1 &= |AJ| = |BE| = |CD| = |CF| = |CG| = |DF| = |FG| \text{ (blue)} \ldots 7 \\
d_2 &= |HJ| \text{ (green)} \ldots 1
\end{align*} \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} = \frac{d_4}{d_7} \text{ (14 occurrences)} \]
2.41.4 Construction RCC340

Construction Process

A, B given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( p_3 = p(B, D) \)
   \( E \in p_3 \cap k_1 \)
4. \( k_4 = k(E, B) \)
   \( F \in k_1 \cap k_4 \), \( G \in p_3 \cap k_4 \)
5. \( k_5 = k(F, A) \)
   \( H \in k_2 \cap k_5 \), \( I \in k_2 \cap k_5 \), \( J \in k_4 \cap k_5 \)

Distances

\( d_0 = |IJ| \) (red) \ldots 1
\( d_1 = |AE| = |BJ| = |CD| = |CF| = |CG| = |DF| = |FG| \) (blue) \ldots 7
\( d_2 = |HJ| \) (green) \ldots 1

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} = \frac{d_2}{d_4} \) (14 occurrences)
2.42 Class of Similar Constructions No. 35

Contained Constructions
RCC260, RCC297, RCC308, RCC341
2.42.1 Construction RCC260

Construction Process

A, B given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2 \)
3. \( p_3 = p(A, C) \)
   \( D \in p_3 \cap k_2 \)
4. \( k_4 = k(D, A) \)
   \( E \in p_3 \cap k_4 \)
5. \( k_5 = k(E, A) \)
   \( F \in k_2 \cap k_5, G \in k_2 \cap k_5 \)

Distances

\[ d_0 = |BF| \text{ (red)} \ldots 1 \]
\[ d_1 = |DF| = |DG| \text{ (blue)} \ldots 2 \]
\[ d_2 = |BG| \text{ (green)} \ldots 1 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (4 occurrences)} \]
2.42.2 Construction RCC297

Construction Process

1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2 \)
3. \( p_3 = p(B, C) \)
   \( D \in p_3 \cap k_1 \)
4. \( k_4 = k(D, B) \)
   \( E \in p_3 \cap k_4 \)
5. \( k_5 = k(E, B) \)
   \( F \in k_1 \cap k_5 \), \( G \in k_1 \cap k_5 \)

Distances

\[ d_0 = |AG| \] (red) \dots 1
\[ d_1 = |DF| = |DG| \] (blue) \dots 2
\[ d_2 = |AF| \] (green) \dots 1

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} = \frac{d_2}{d_2} \] (4 occurrences)
2.42.3 Construction RCC308

Construction Process

A, B given initial points
1. $k_1 = k(B, A)$
2. $k_2 = k(A, B)$
   $C \in k_1 \cap k_2$
3. $p_3 = p(A, C)$
   $D \in p_3 \cap k_2$
4. $k_4 = k(D, A)$
   $E \in p_3 \cap k_4$
5. $k_5 = k(E, A)$
   $F \in k_2 \cap k_5$, $G \in k_2 \cap k_5$

Distances

$d_0 = |BF|$ (red) ... 1
$d_1 = |DF| = |DG|$ (blue) ... 2
$d_2 = |BG|$ (green) ... 1

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2}$ (4 occurrences)
2.42.4 Construction RCC341

Construction Process

A, B given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2 \)
3. \( p_3 = p(B, C) \)
   \( D \in p_3 \cap k_1 \)
4. \( k_4 = k(D, B) \)
   \( E \in p_3 \cap k_4 \)
5. \( k_5 = k(E, B) \)
   \( F \in k_1 \cap k_5 \), \( G \in k_1 \cap k_5 \)

Distances

\( d_0 = |AG| \) (red) \ldots 1
\( d_1 = |DF| = |DG| \) (blue) \ldots 2
\( d_2 = |AF| \) (green) \ldots 1

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \] (4 occurrences)
2.43 Class of Similar Constructions No. 36

Contained Constructions
RCC261, RCC294, RCC309, RCC338
2.43.1 Construction RCC261

Construction Process

A, B given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( p_3 = p(A, C) \)
   \( E \in p_3 \cap k_2 \)
4. \( k_4 = k(B, E) \)
   \( F \in k_2 \cap k_4 \), \( G \in p_3 \cap k_4 \)
5. \( k_5 = k(G, A) \)
   \( H \in k_1 \cap k_5 \), \( I \in k_2 \cap k_5 \), \( J \in k_2 \cap k_5 \), \( K \in k_4 \cap k_5 \), \( L \in k_4 \cap k_5 \)

Distances

\( d_0 = |HI| \) (red) \( \ldots 1 \)
\( d_1 = |DI| \) (blue) \( \ldots 1 \)
\( d_2 = |BE| = |BF| = |BG| = |BK| = |BL| = |CD| = |CH| = |DH| = |EF| = |FG| \) (green) \( \ldots 10 \)
\( d_3 = |CI| = |CJ| \) (cyan) \( \ldots 2 \)
\( d_4 = |HJ| \) (magenta) \( \ldots 1 \)
\( d_5 = |DJ| \) (yellow) \( \ldots 1 \)

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_2} = \frac{d_2}{d_4} = \frac{d_2}{d_4} = \frac{d_4}{d_5} \) (24 occurrences)
2.43.2 Construction RCC294

Construction Process

\begin{itemize}
  \item A, B given initial points
  \item \( k_1 = k(B, A) \)
  \item \( k_2 = k(A, B) \)
  \item \( C \in k_1 \cap k_2 , \ D \in k_1 \cap k_2 \)
  \item \( p_3 = p(B, C) \)
  \item \( E \in p_3 \cap k_1 \)
  \item \( k_4 = k(A, E) \)
  \item \( F \in k_1 \cap k_4 , \ G \in p_3 \cap k_4 \)
  \item \( k_5 = k(G, B) \)
  \item \( H \in k_1 \cap k_5 , \ I \in k_1 \cap k_5 , \ J \in k_2 \cap k_5 , \ K \in k_4 \cap k_5 , \ L \in k_4 \cap k_5 \)
\end{itemize}

Distances

\begin{itemize}
  \item \( d_0 = |IJ| \text{ (red)} \ldots 1 \)
  \item \( d_1 = |DI| \text{ (blue)} \ldots 1 \)
  \item \( d_2 = |AE| = |AF| = |AG| = |AK| = |AL| = |CD| = |CJ| = |DJ| = |EF| = |FG| \text{ (green)} \ldots 10 \)
  \item \( d_3 = |CH| = |CI| \text{ (cyan)} \ldots 2 \)
  \item \( d_4 = |HJ| \text{ (magenta)} \ldots 1 \)
  \item \( d_5 = |DH| \text{ (yellow)} \ldots 1 \)
\end{itemize}

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_2}{d_4} = \frac{d_4}{d_5} = \frac{d_5}{d_6} \text{ (24 occurrences)} \]
2.43.3 Construction RCC309

**Construction Process**

A, B given initial points

1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( p_3 = p(A, D) \)
   
   \( E \in p_3 \cap k_2 \)
4. \( k_4 = k(B, E) \)
   
   \( F \in k_2 \cap k_4 \), \( G \in p_3 \cap k_4 \)
5. \( k_5 = k(G, A) \)
   
   \( H \in k_1 \cap k_5 \), \( I \in k_2 \cap k_5 \), \( J \in k_2 \cap k_5 \), \( K \in k_4 \cap k_5 \), \( L \in k_4 \cap k_5 \)

**Distances**

\[
\begin{align*}
  d_0 &= |HI| \text{ (red)} \ldots 1 \\
  d_1 &= |CI| \text{ (blue)} \ldots 1 \\
  d_2 &= |BE| = |BF| = |BG| = |BK| = |BL| = |CD| = |CH| = |DH| = |EF| = |FG| \text{ (green)} \ldots 10 \\
  d_3 &= |DI| = |DJ| \text{ (cyan)} \ldots 2 \\
  d_4 &= |HJ| \text{ (magenta)} \ldots 1 \\
  d_5 &= |CJ| \text{ (yellow)} \ldots 1
\end{align*}
\]

**Occurrences of the Golden Ratio**

\[
\phi = \frac{d_0}{d_2} = \frac{d_2}{d_4} = \frac{d_4}{d_6} = \frac{d_3}{d_5} \quad (24 \text{ occurrences})
\]
2.43.4 Construction RCC338

Construction Process

A, B given initial points
1. $k_1 = k(B, A)$
2. $k_2 = k(A, B)$
   $C \in k_1 \cap k_2$ , $D \in k_1 \cap k_2$
3. $p_3 = p(B, D)$
   $E \in p_3 \cap k_1$
4. $k_4 = k(A, E)$
   $F \in k_1 \cap k_4$ , $G \in p_3 \cap k_4$
5. $k_5 = k(G, B)$
   $H \in k_1 \cap k_5$ , $I \in k_1 \cap k_5$, $J \in k_2 \cap k_5$ , $K \in k_4 \cap k_5$ , $L \in k_4 \cap k_5$

Distances

$d_0 = |IJ|$ (red) ... 1
$d_1 = |CI|$ (blue) ... 1
$d_2 = |AE| = |AF| = |AG| = |AK| = |AL| = |CD| = | CJ | = |DJ| = |EF| = |FG|$ (green) ... 10
$d_3 = |DH| = |DI|$ (cyan) ... 2
$d_4 = |HJ|$ (magenta) ... 1
$d_5 = |CH|$ (yellow) ... 1

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_2} = \frac{d_4}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5}$ (24 occurrences)
2.44 Class of Similar Constructions No. 37

Contained Constructions
RCC262, RCC298, RCC310, RCC342
2.44.1 Construction RCC262

Construction Process

1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   
   \( C \in k_1 \cap k_2, D \in k_1 \cap k_2 \)
3. \( p_3 = p(A, C) \)
   
   \( E \in p_3 \cap k_2 \)
4. \( k_4 = k(E, C) \)
   
   \( F \in k_1 \cap k_4 \)
5. \( k_5 = k(F, B) \)
   
   \( G \in k_1 \cap k_5, H \in k_4 \cap k_5, I \in k_4 \cap k_5 \)

Distances

\[
\begin{align*}
d_0 &= |CI| \quad \text{(red)} \ldots 1 \\
d_1 &= |GI| \quad \text{(blue)} \ldots 1 \\
d_2 &= |AF| = |BE| = |CD| = |CG| = |DG| \quad \text{(green)} \ldots 5 \\
d_3 &= |DH| = |DI| \quad \text{(cyan)} \ldots 2 \\
d_4 &= |CH| \quad \text{(magenta)} \ldots 1 \\
d_5 &= |GH| \quad \text{(yellow)} \ldots 1
\end{align*}
\]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_4}{d_5} \quad \text{(14 occurrences)} \]
2.44.2 Construction RCC298

Construction Process

A, B given initial points
1. $k_1 = k(B, A)$
2. $k_2 = k(A, B)$
   $C \in k_1 \cap k_2$, $D \in k_1 \cap k_2$
3. $p_3 = p(B, C)$
   $E \in p_3 \cap k_1$
4. $k_4 = k(E, C)$
   $F \in k_2 \cap k_1$
5. $k_5 = k(F, A)$
   $G \in k_2 \cap k_5$, $H \in k_4 \cap k_5$, $I \in k_4 \cap k_5$

Distances

\[ d_0 = |CI| \text{ (red)} \ldots 1 \]
\[ d_1 = |GI| \text{ (blue)} \ldots 1 \]
\[ d_2 = |AE| = |BF| = |CD| = |CG| = |DG| \text{ (green)} \ldots 5 \]
\[ d_3 = |DH| = |DI| \text{ (cyan)} \ldots 2 \]
\[ d_4 = |CH| \text{ (magenta)} \ldots 1 \]
\[ d_5 = |GH| \text{ (yellow)} \ldots 1 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (14 occurrences)} \]
2.44.3 Construction RCC310

Construction Process

A, B given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2, \ D \in k_1 \cap k_2 \)
3. \( p_3 = p(A, D) \)
   \( E \in p_3 \cap k_2 \)
4. \( k_4 = k(E, D) \)
   \( F \in k_1 \cap k_4 \)
5. \( k_5 = k(F, B) \)
   \( G \in k_1 \cap k_5, \ H \in k_4 \cap k_5, \ I \in k_4 \cap k_5 \)

Distances

- \( d_0 = |DH| \) (red) \dots 1
- \( d_1 = |GH| \) (blue) \dots 1
- \( d_2 = |AF| = |BE| = |CD| = |CG| = |DG| \) (green) \dots 5
- \( d_3 = |CH| = |CI| \) (cyan) \dots 2
- \( d_4 = |DI| \) (magenta) \dots 1
- \( d_5 = |GI| \) (yellow) \dots 1

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_2} = \frac{d_2}{d_3} = \frac{d_3}{d_4} = \frac{d_4}{d_5} \) (14 occurrences)
2.44.4 Construction RCC342

Construction Process

- A, B given initial points
- \( k_1 = k(B, A) \)
- \( k_2 = k(A, B) \)
  - \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
- \( p_3 = p(B, D) \)
  - \( E \in p_3 \cap k_1 \)
- \( k_4 = k(E, D) \)
  - \( F \in k_2 \cap k_4 \)
- \( k_5 = k(F, A) \)
  - \( G \in k_2 \cap k_5 \), \( H \in k_4 \cap k_5 \), \( I \in k_4 \cap k_5 \)

Distances

\[
\begin{align*}
d_0 &= |DH| \text{ (red)} \ldots 1 \\
d_1 &= |GH| \text{ (blue)} \ldots 1 \\
d_2 &= |AE| = |BF| = |CD| = |CG| = |DG| \text{ (green)} \ldots 5 \\
d_3 &= |CH| = |CI| \text{ (cyan)} \ldots 2 \\
d_4 &= |DI| \text{ (magenta)} \ldots 1 \\
d_5 &= |GI| \text{ (yellow)} \ldots 1 \\
\end{align*}
\]

Occurrences of the Golden Ratio

\[
\phi = \frac{d_0}{d_2} = \frac{d_3}{d_4} = \frac{d_4}{d_5} = \frac{d_5}{d_6} \text{ (14 occurrences)}
\]
2.45 Class of Similar Constructions No. 38

Contained Constructions
RCC263, RCC299, RCC311, RCC343
2.45.1 Construction RCC263

Construction Process

1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
3. \( p_3 = p(A, C) \)
4. \( k_4 = k(C, E) \)
5. \( k_5 = k(F, B) \)

Distances

- \( d_0 = |EI| \) (red) ... 1
- \( d_1 = |AF| = |BE| = |CD| = |CG| = |DG| \) (blue) ... 5
- \( d_2 = |EH| \) (green) ... 1

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \] (10 occurrences)
2.45.2 Construction RCC299

Construction Process

\[ A, B \text{ given initial points} \]
\[ 1. \ k_1 = k(B, A) \]
\[ 2. \ k_2 = k(A, B) \]
\[ C \in k_1 \cap k_2, \ D \in k_1 \cap k_2 \]
\[ 3. \ p_3 = p(B, C) \]
\[ E \in p_3 \cap k_1 \]
\[ 4. \ k_4 = k(C, E) \]
\[ F \in k_2 \cap k_4 \]
\[ 5. \ k_5 = k(F, A) \]
\[ G \in k_2 \cap k_5, \ H \in k_4 \cap k_5, \ I \in k_4 \cap k_5 \]

Distances

\[ d_0 = |EH| \text{ (red)} \ldots 1 \]
\[ d_1 = |AE| = |BF| = |CD| = |CG| = |DG| \text{ (blue)} \ldots 5 \]
\[ d_2 = |EI| \text{ (green)} \ldots 1 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} = \frac{d_2}{d_5} \text{ (10 occurrences)} \]
2.45.3 Construction RCC311

Construction Process

1. Given initial points $A, B$
2. $k_1 = k(B, A)$
3. $p_3 = p(A, D)$
4. $k_2 = k(A, B)$
5. $k_3 = k(B, A)$

Distances

$d_0 = |EI|$ (red) ... 1
$d_1 = |AF| = |BE| = |CD| = |CG| = |DG|$ (blue) ... 5
$d_2 = |EH|$ (green) ... 1

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2}$ (10 occurrences)
2.45.4 Construction RCC343

Construction Process

A, B given initial points
1. $k_1 = k(B, A)$
2. $k_2 = k(A, B)$
   $C \in k_1 \cap k_2$, $D \in k_1 \cap k_2$
3. $p_3 = p(B, D)$
   $E \in p_3 \cap k_1$
4. $k_4 = k(D, E)$
   $F \in k_2 \cap k_4$
5. $k_5 = k(F, A)$
   $G \in k_2 \cap k_5$, $H \in k_4 \cap k_5$, $I \in k_4 \cap k_5$

Distances

$d_0 = |EH|$ (red) \ldots 1
$d_1 = |AE| = |BF| = |CD| = |CG| = |DG|$ (blue) \ldots 5
$d_2 = |EI|$ (green) \ldots 1

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_1} = \frac{d_4}{d_2}$ (10 occurrences)
2.46 Class of Similar Constructions No. 39

Contained Constructions
RCC344, RCC345, RCC346, RCC347
2.46.1 Construction RCC344

Construction Process

A, B given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2, \ D \in k_1 \cap k_2 \)
3. \( p_3 = p(C, D) \)
4. \( k_4 = k(D, C) \)
   \( E \in k_1 \cap k_4, \ F \in k_2 \cap k_4, \ G \in p_3 \cap k_4 \)
5. \( k_5 = k(A, E) \)
   \( H \in p_3 \cap k_5, \ I \in p_3 \cap k_5, \ J \in k_4 \cap k_5 \)

Distances

\[
\begin{align*}
d_0 &= |GH| \text{ (red)} \ldots 1 \\
d_1 &= |HJ| \text{ (blue)} \ldots 1 \\
d_2 &= |CI| = |DH| \text{ (green)} \ldots 2 \\
d_3 &= |EH| = |EI| = |FH| = |FI| \text{ (cyan)} \ldots 4 \\
d_4 &= |CD| = |CE| = |CF| = |DE| = |DF| = |DG| = |DJ| = |FJ| = |GJ| \text{ (magenta)} \ldots 9 \\
d_5 &= |IJ| \text{ (yellow)} \ldots 1 \\
d_6 &= |CH| = |DI| \text{ (grass)} \ldots 2 \\
d_7 &= |GI| \text{ (sea)} \ldots 1
\end{align*}
\]

Occurrences of the Golden Ratio

\[
\phi = \frac{d_6}{d_7} = \frac{d_5}{d_6} = \frac{d_4}{d_5} = \frac{d_3}{d_4} = \frac{d_2}{d_3} = \frac{d_1}{d_2} \quad (48 \text{ occurrences})
\]
2.46.2 Construction RCC345

Construction Process

A, B given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
    \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( p_3 = p(C, D) \)
4. \( k_4 = k(D, C) \)
    \( E \in k_1 \cap k_4 \), \( F \in k_2 \cap k_4 \), \( G \in p_3 \cap k_4 \)
5. \( k_5 = k(B, F) \)
    \( H \in p_3 \cap k_5 \), \( I \in p_3 \cap k_5 \), \( J \in k_4 \cap k_5 \)

Distances

\[ d_0 = |GH| \] (red) \ldots 1
\[ d_1 = |HJ| \] (blue) \ldots 1
\[ d_2 = |CI| = |DH| \] (green) \ldots 2
\[ d_3 = |EH| = |EI| = |FH| = |FI| \] (cyan) \ldots 4
\[ d_4 = |CD| = |CE| = |CF| = |DE| = |DF| = |DG| = |DJ| = |EF| = |GJ| \] (magenta) \ldots 9
\[ d_5 = |IJ| \] (yellow) \ldots 1
\[ d_6 = |CH| = |DI| \] (grass) \ldots 2
\[ d_7 = |GI| \] (sea) \ldots 1

Occurrences of the Golden Ratio

\[ \phi = \frac{d_6}{d_7} = \frac{d_4}{d_5} = \frac{d_4}{d_3} = \frac{d_5}{d_4} = \frac{d_5}{d_3} = \frac{d_6}{d_5} \] (48 occurrences)
2.46.3 Construction RCC346

Construction Process

A, B given initial points

1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2, \ D \in k_1 \cap k_2 \)
3. \( p_3 = p(C, D) \)
4. \( k_4 = k(C, D) \)
   \( E \in k_1 \cap k_4, \ F \in k_2 \cap k_4, \ G \in p_3 \cap k_4 \)
5. \( k_5 = k(A, E) \)
   \( H \in p_3 \cap k_5, \ I \in p_3 \cap k_5, \ J \in k_4 \cap k_5 \)

Distances

\( d_0 = |GI| \) (red) \ldots 1
\( d_1 = |IJ| \) (blue) \ldots 1
\( d_2 = |CI| = |DH| \) (green) \ldots 2
\( d_3 = |EH| = |EI| = |FH| = |FI| \) (cyan) \ldots 4
\( d_4 = |CD| = |CE| = |CF| = |CG| = |CJ| = |DE| = |DF| = |FJ| = |GJ| \) (magenta) \ldots 9
\( d_5 = |HJ| \) (yellow) \ldots 1
\( d_6 = |CH| = |DI| \) (grass) \ldots 2
\( d_7 = |GH| \) (sea) \ldots 1

Occurrences of the Golden Ratio

\( \phi = \frac{d_6}{d_2} = \frac{d_4}{d_4} = \frac{d_2}{d_2} = \frac{d_5}{d_6} = \frac{d_6}{d_7} \) (48 occurrences)
2.46.4 Construction RCC347

Construction Process

A, B given initial points
1. $k_1 = k(B, A)$
2. $k_2 = k(A, B)$
   
   $C \in k_1 \cap k_2$, $D \in k_1 \cap k_2$
3. $p_3 = p(C, D)$
4. $k_4 = k(C, D)$
   
   $E \in k_1 \cap k_4$, $F \in k_2 \cap k_4$, $G \in p_3 \cap k_4$
5. $k_5 = k(B, F)$
   
   $H \in p_3 \cap k_5$, $I \in p_3 \cap k_5$, $J \in k_4 \cap k_5$

Distances

$d_0 = |GI|$ (red) ... 1
$d_1 = |IJ|$ (blue) ... 1
$d_2 = |CI| = |DH|$ (green) ... 2
$d_3 = |EH| = |EI| = |FH| = |FI|$ (cyan) ... 4
$d_4 = |CD| = |CE| = |CF| = |CG| = |CJ| = |DE| = |DF| = |EJ| = |GJ|$ (magenta) ... 9
$d_5 = |HJ|$ (yellow) ... 1
$d_6 = |CH| = |DI|$ (grass) ... 2
$d_7 = |GH|$ (sea) ... 1

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} = \frac{d_4}{d_6} = \frac{d_5}{d_7}$ (48 occurrences)
2.47 Class of Similar Constructions No. 40

Contained Constructions
RCC350, RCC380, RCC416, RCC446
2.47.1 Construction RCC350

Construction Process

A, B given initial points
1. \(k_1 = k(B, A)\)
2. \(k_2 = k(A, B)\)
   \(C \in k_1 \cap k_2\), \(D \in k_1 \cap k_2\)
3. \(k_3 = k(D, C)\)
   \(E \in k_1 \cap k_3\), \(F \in k_2 \cap k_3\)
4. \(k_4 = k(E, A)\)
   \(G \in k_2 \cap k_4\), \(H \in k_2 \cap k_4\)
5. \(p_5 = p(A, G)\)
   \(I \in p_5 \cap k_1\), \(J \in p_5 \cap k_2\)

Distances

\(d_0 = |CJ|\) (red) \(\ldots 1\)
\(d_1 = |BG| = |BH| = |FI| = |FJ|\) (blue) \(\ldots 4\)
\(d_2 = |DJ|\) (green) \(\ldots 1\)

Occurrences of the Golden Ratio

\(\phi = \frac{d_0}{d_1} = \frac{d_4}{d_2}\) (8 occurrences)
2.47.2 Construction RCC380

Construction Process

A, B given initial points
1. $k_1 = k(B, A)$
2. $k_2 = k(A, B)$
   $C \in k_1 \cap k_2$, $D \in k_1 \cap k_2$
3. $k_3 = k(D, C)$
   $E \in k_1 \cap k_3$, $F \in k_2 \cap k_3$
4. $k_4 = k(F, B)$
   $G \in k_1 \cap k_4$, $H \in k_1 \cap k_4$
5. $p_5 = p(B, G)$
   $I \in p_5 \cap k_1$, $J \in p_5 \cap k_2$

Distances

\[ d_0 = |CI| \text{ (red)} \ldots 1 \]
\[ d_1 = |AG| = |AH| = |EI| = |EJ| \text{ (blue)} \ldots 4 \]
\[ d_2 = |DI| \text{ (green)} \ldots 1 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} = \frac{d_4}{d_2} \text{ (8 occurrences)} \]
2.47.3 Construction RCC416

Construction Process

A, B given initial points

1. \(k_1 = k(B, A)\)
2. \(k_2 = k(A, B)\)
   \(C \in k_1 \cap k_2, \ D \in k_1 \cap k_2\)
3. \(k_3 = k(C, D)\)
   \(E \in k_1 \cap k_3, \ F \in k_2 \cap k_3\)
4. \(k_4 = k(E, A)\)
   \(G \in k_2 \cap k_4, \ H \in k_2 \cap k_4\)
5. \(p_5 = p(A, H)\)
   \(I \in p_5 \cap k_1, \ J \in p_5 \cap k_2\)

Distances

\(d_0 = |DJ| \) (red) ... 1
\(d_1 = |BG| = |BH| = |FI| = |FJ| \) (blue) ... 4
\(d_2 = |CJ| \) (green) ... 1

Occurrences of the Golden Ratio

\(\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \) (8 occurrences)
2.47.4 Construction RCC446

Construction Process

- A, B given initial points
- 1. \( k_1 = k(B, A) \)
- 2. \( k_2 = k(A, B) \)
  - \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
- 3. \( k_3 = k(C, D) \)
  - \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
- 4. \( k_4 = k(F, B) \)
  - \( G \in k_1 \cap k_4 \), \( H \in k_1 \cap k_4 \)
- 5. \( p_5 = p(B, H) \)
  - \( I \in p_5 \cap k_1 \), \( J \in p_5 \cap k_2 \)

Distances

- \( d_0 = |DI| \) (red) \( \cdots 1 \)
- \( d_1 = |AG| = |AH| = |EI| = |EJ| \) (blue) \( \cdots 4 \)
- \( d_2 = |CI| \) (green) \( \cdots 1 \)

Occurrences of the Golden Ratio

- \( \phi = \frac{d_0}{d_1} = \frac{d_4}{d_2} \) (8 occurrences)
2.48 Class of Similar Constructions No. 41

Contained Constructions

RCC352, RCC382, RCC421, RCC451
2.48.1 Construction RCC352

Construction Process

A, B given initial points
1. $k_1 = k(B, A)$
2. $k_2 = k(A, B)$
   $C \in k_1 \cap k_2$, $D \in k_1 \cap k_2$
3. $k_3 = k(D, C)$
   $E \in k_1 \cap k_3$
4. $k_4 = k(E, A)$
   $F \in k_2 \cap k_4$, $G \in k_2 \cap k_4$
5. $p_5 = p(C, F)$
   $H \in p_5 \cap k_1$, $I \in p_5 \cap k_3$, $J \in p_5 \cap k_4$

Distances

$d_0 = |HJ|$ (red) ... 1
$d_1 = |AH|$ (blue) ... 1
$d_2 = |CI|$ (green) ... 1
$d_3 = |BF| = |BG|$ (cyan) ... 2

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_2} = \frac{d_2}{d_3}$ (3 occurrences)
2.48.2 Construction RCC382

Construction Process

1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
3. \( C \in k_1 \cap k_2 \) , \( D \in k_1 \cap k_2 \)
4. \( k_3 = k(D, C) \)
5. \( p_5 = p(C, F) \)
6. \( E \in k_2 \cap k_3 \)
7. \( F \in k_1 \cap k_2 \) , \( G \in k_1 \cap k_4 \)
8. \( H \in p_5 \cap k_2 \) , \( I \in p_5 \cap k_3 \) , \( J \in p_6 \cap k_4 \)

Distances

\[
\begin{align*}
    d_0 &= |HJ| \quad \text{(red)} \quad \ldots 1 \\
    d_1 &= |BH| \quad \text{(blue)} \quad \ldots 1 \\
    d_2 &= |CI| \quad \text{(green)} \quad \ldots 1 \\
    d_3 &= |AF| = |AG| \quad \text{(cyan)} \quad \ldots 2
\end{align*}
\]

Occurrences of the Golden Ratio

\[
\phi = \frac{d_0}{d_2} = \frac{d_2}{d_5} \quad (3 \text{ occurrences})
\]
2.48.3 Construction RCC421

Construction Process

A, B given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2, D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, D) \)
   \( E \in k_1 \cap k_3 \)
4. \( k_4 = k(E, A) \)
   \( F \in k_2 \cap k_4, G \in k_2 \cap k_4 \)
5. \( p_5 = p(D, G) \)
   \( H \in p_5 \cap k_1, I \in p_5 \cap k_3, J \in p_5 \cap k_4 \)

Distances

\[ d_0 = |HJ| \text{ (red)} \ldots 1 \]
\[ d_1 = |AH| \text{ (blue)} \ldots 1 \]
\[ d_2 = |DF| \text{ (green)} \ldots 1 \]
\[ d_3 = |BF| = |BG| \text{ (cyan)} \ldots 2 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_3}{d_5} \text{ (3 occurrences)} \]
2.48.4 Construction RCC451

Construction Process

A, B given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, D) \)
   \( E \in k_2 \cap k_3 \)
4. \( k_4 = k(E, B) \)
   \( F \in k_1 \cap k_4 \), \( G \in k_1 \cap k_4 \)
5. \( p_5 = p(D, G) \)
   \( H \in p_5 \cap k_2 \), \( I \in p_5 \cap k_3 \), \( J \in p_5 \cap k_4 \)

Distances

\( d_0 = |HJ| \) (red) \( \ldots 1 \)
\( d_1 = |BH| \) (blue) \( \ldots 1 \)
\( d_2 = |DI| \) (green) \( \ldots 1 \)
\( d_3 = |AF| = |AG| \) (cyan) \( \ldots 2 \)

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_2} = \frac{d_2}{d_5} \) (3 occurrences)
2.49 Class of Similar Constructions No. 42

Contained Constructions
RCC355, RCC385, RCC418, RCC448
2.49.1 Construction RCC355

Construction Process

A, B given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2 \) , \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, C) \)
   \( E \in k_1 \cap k_3 \)
4. \( k_4 = k(E, A) \)
   \( F \in k_2 \cap k_4 \) , \( G \in k_2 \cap k_4 \)
5. \( p_5 = p(D, F) \)
   \( H \in p_5 \cap k_1 \)

Distances

\( d_0 = |BF| = |BG| \) (red) \ldots 2
\( d_1 = |AH| \) (blue) \ldots 1

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} \) (2 occurrences)
2.49.2 Construction RCC385

Construction Process

1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, C) \)
   \( E \in k_2 \cap k_3 \)
4. \( k_4 = k(E, B) \)
   \( F \in k_1 \cap k_4 \), \( G \in k_1 \cap k_4 \)
5. \( p_5 = p(D, F) \)
   \( H \in p_5 \cap k_2 \)

Distances

\[ d_0 = |AF| = |AG| \text{ (red)} \ldots 2 \]
\[ d_1 = |BH| \text{ (blue)} \ldots 1 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} \text{ (2 occurrences)} \]
2.49.3 Construction RCC418

Construction Process

A, B given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2, \ D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, D) \)
   \( E \in k_1 \cap k_3 \)
4. \( k_4 = k(E, A) \)
   \( F \in k_2 \cap k_4, \ G \in k_2 \cap k_4 \)
5. \( p_5 = p(C, G) \)
   \( H \in p_5 \cap k_1 \)

Distances

\( d_0 = |BF| = |BG| \) (red) \ldots 2
\( d_1 = |AH| \) (blue) \ldots 1

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} \) (2 occurrences)
2.49.4 Construction RCC448

Construction Process

\( A, B \) given initial points

1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2, \ D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, D) \)
   \( E \in k_2 \cap k_3 \)
4. \( k_4 = k(E, B) \)
   \( F \in k_1 \cap k_4, \ G \in k_1 \cap k_4 \)
5. \( p_5 = p(C, G) \)
   \( H \in p_5 \cap k_2 \)

Distances

\[ d_0 = |AF| = |AG| \text{ (red)} \ldots 2 \]
\[ d_1 = |BH| \text{ (blue)} \ldots 1 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} \text{ (2 occurrences)} \]
2.50 Class of Similar Constructions No. 43

Contained Constructions

RCC359, RCC388, RCC425, RCC454
2.50.1 Construction RCC359

Construction Process

1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
3. \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
4. \( k_3 = k(D, C) \)
5. \( p_5 = p(F, G) \)

Distances

\( d_0 = |EH| \) (red) \ldots 1
\( d_1 = |FH| \) (blue) \ldots 1

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} \) (1 occurrence)
2.50.2 Construction RCC388

Construction Process

- \( A, B \) given initial points
- 1. \( k_1 = k(B, A) \)
- 2. \( k_2 = k(A, B) \)
  - \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
- 3. \( k_3 = k(D, C) \)
  - \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
- 4. \( k_4 = k(F, B) \)
  - \( G \in k_1 \cap k_4 \)
- 5. \( p_5 = p(E, G) \)
  - \( H \in p_5 \cap k_3 \)

Distances

- \( d_0 = |FH| \) (red) \( \ldots 1 \)
- \( d_1 = |EH| \) (blue) \( \ldots 1 \)

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} \) (1 occurrence)
2.50.3 Construction RCC425

Construction Process

1. \( k_1 = k(B,A) \)
2. \( k_2 = k(A,B) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(C,D) \)
   \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(E,A) \)
   \( G \in k_2 \cap k_4 \)
5. \( p_5 = p(F,G) \)
   \( H \in p_5 \cap k_3 \)

Distances

\( d_0 = |EH| \) (red) \( \ldots 1 \)
\( d_1 = |FH| \) (blue) \( \ldots 1 \)

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} \) (1 occurrence)
2.50.4 Construction RCC454

Construction Process

A, B given initial points
1. $k_1 = k(B, A)$
2. $k_2 = k(A, B)$
   $C \in k_1 \cap k_2 \ , \ D \in k_1 \cap k_2$
3. $k_3 = k(C, D)$
   $E \in k_1 \cap k_3 \ , \ F \in k_2 \cap k_3$
4. $k_4 = k(F, B)$
   $G \in k_1 \cap k_4$
5. $p_5 = p(E, G)$
   $H \in p_5 \cap k_3$

Distances

$d_0 = |FH|$ (red) \ldots 1
$d_1 = |EH|$ (blue) \ldots 1

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_1}$ (1 occurrence)
2.51 Class of Similar Constructions No. 44

Contained Constructions
RCC361, RCC391, RCC405, RCC435
2.51.1 Construction RCC361

Construction Process

A, B given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, C) \)
   \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(E, A) \)
   \( G \in k_2 \cap k_4 \), \( H \in k_2 \cap k_4 \)
5. \( p_5 = p(A, H) \)
   \( I \in p_5 \cap k_1 \), \( J \in p_5 \cap k_2 \)

Distances

\[ d_0 = |DJ| \text{ (red)} \ldots 1 \]
\[ d_1 = |BG| = |BH| = |FI| = |FJ| \text{ (blue)} \ldots 4 \]
\[ d_2 = |CJ| \text{ (green)} \ldots 1 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} = \frac{d_2}{d_2} \text{ (8 occurrences)} \]
2.51.2 Construction RCC391

Construction Process

A, B given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2, D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, C) \)
   \( E \in k_1 \cap k_3, F \in k_2 \cap k_3 \)
4. \( k_4 = k(F, B) \)
   \( G \in k_1 \cap k_4, H \in k_1 \cap k_4 \)
5. \( p_5 = p(B, H) \)
   \( I \in p_5 \cap k_1, J \in p_5 \cap k_2 \)

Distances

\[ d_0 = |DI| \text{ (red)} \ldots 1 \]
\[ d_1 = |AG| = |AH| = |EI| = |EJ| \text{ (blue)} \ldots 4 \]
\[ d_2 = |CI| \text{ (green)} \ldots 1 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} = \frac{d_4}{d_2} \text{ (8 occurrences)} \]
2.51.3 Construction RCC405

Construction Process

A, B given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, D) \)
   \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(E, A) \)
   \( G \in k_2 \cap k_4 \), \( H \in k_2 \cap k_4 \)
5. \( p_5 = p(A, G) \)
   \( I \in p_5 \cap k_1 \), \( J \in p_5 \cap k_2 \)

Distances

\( d_0 = |CJ| \) (red) \( \ldots 1 \)
\( d_1 = |BG| = |BH| = |FI| = |FJ| \) (blue) \( \ldots 4 \)
\( d_2 = |DJ| \) (green) \( \ldots 1 \)

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} = \frac{d_4}{d_2} \] (8 occurrences)
2.51.4 Construction RCC435

Construction Process

A, B given initial points
1. $k_1 = k(B, A)$
2. $k_2 = k(A, B)$
   $C \in k_1 \cap k_2$, $D \in k_1 \cap k_2$
3. $k_3 = k(C, D)$
   $E \in k_1 \cap k_3$, $F \in k_2 \cap k_3$
4. $k_4 = k(F, B)$
   $G \in k_1 \cap k_4$, $H \in k_1 \cap k_4$
5. $p_5 = p(B, G)$
   $I \in p_5 \cap k_1$, $J \in p_5 \cap k_2$

Distances

$d_0 = |CI|$ (red) \ldots 1
$d_1 = |AG| = |AH| = |EI| = |EJ|$ (blue) \ldots 4
$d_2 = |DI|$ (green) \ldots 1

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_1} = \frac{d_4}{d_2}$ (8 occurrences)
2.52 Class of Similar Constructions No. 45

Contained Constructions

RCC363, RCC393, RCC410, RCC440
2.52.1 Construction RCC363

Construction Process

A, B given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2 , D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, C) \)
   \( E \in k_1 \cap k_3 \)
4. \( k_4 = k(E, A) \)
   \( F \in k_2 \cap k_4 , G \in k_2 \cap k_4 \)
5. \( p_5 = p(C, G) \)
   \( H \in p_5 \cap k_1 , I \in p_5 \cap k_3 , J \in p_5 \cap k_4 \)

Distances
\[ d_0 = |CI| \text{ (red)} \ldots 1 \]
\[ d_1 = |HJ| \text{ (blue)} \ldots 1 \]
\[ d_2 = |BF| = |BG| \text{ (green)} \ldots 2 \]
\[ d_3 = |AH| \text{ (cyan)} \ldots 1 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_4} = \frac{d_2}{d_5} \text{ (3 occurrences)} \]
2.52.2 Construction RCC393

Construction Process

\( A, B \) given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2 , D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, C) \)
   \( E \in k_2 \cap k_3 \)
4. \( k_4 = k(E, B) \)
   \( F \in k_1 \cap k_4 , G \in k_1 \cap k_4 \)
5. \( p_5 = p(C, G) \)
   \( H \in p_5 \cap k_2 , I \in p_5 \cap k_3 , J \in p_5 \cap k_4 \)

Distances

\( d_0 = |CI| \) (red) \( \ldots 1 \)
\( d_1 = |HJ| \) (blue) \( \ldots 1 \)
\( d_2 = |AF| = |AG| \) (green) \( \ldots 2 \)
\( d_3 = |BH| \) (cyan) \( \ldots 1 \)

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_5} = \frac{d_2}{d_5} \) (3 occurrences)
2.52.3 Construction RCC410

Construction Process

\[ A, B \] given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
\( C \in k_1 \cap k_2, D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, D) \)
\( E \in k_1 \cap k_3 \)
4. \( k_4 = k(E, A) \)
\( F \in k_2 \cap k_4, G \in k_2 \cap k_4 \)
5. \( p_5 = p(D, F) \)
\( H \in p_5 \cap k_1, I \in p_5 \cap k_3, J \in p_5 \cap k_4 \)

Distances

\[ d_0 = |DI| \text{ (red)} \ldots 1 \]
\[ d_1 = |HJ| \text{ (blue)} \ldots 1 \]
\[ d_2 = |BF| = |BG| \text{ (green)} \ldots 2 \]
\[ d_3 = |AH| \text{ (cyan)} \ldots 1 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} = \frac{d_2}{d_3} \text{ (3 occurrences)} \]
2.52.4 Construction RCC440

Construction Process

A, B given initial points
1. $k_1 = k(B, A)$
2. $k_2 = k(A, B)$
   $C \in k_1 \cap k_2 , D \in k_1 \cap k_2$
3. $k_3 = k(C, D)$
   $E \in k_2 \cap k_3$
4. $k_4 = k(E, B)$
   $F \in k_1 \cap k_4 , G \in k_1 \cap k_4$
5. $p_5 = p(D, F)$
   $H \in p_5 \cap k_2 , I \in p_5 \cap k_3 , J \in p_5 \cap k_4$

Distances

$d_0 = |DI|$ (red) ... 1
$d_1 = |HJ|$ (blue) ... 1
$d_2 = |AF| = |AG|$ (green) ... 2
$d_3 = |BH|$ (cyan) ... 1

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_5} = \frac{d_2}{d_5}$ (3 occurrences)
2.53 Class of Similar Constructions No. 46

Contained Constructions
RCC366, RCC396, RCC407, RCC437
2.53.1 Construction RCC366

Construction Process

A, B given initial points

1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2 \) ,  \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, C) \)
   \( E \in k_1 \cap k_3 \)
4. \( k_4 = k(E, A) \)
   \( F \in k_2 \cap k_4 \) ,  \( G \in k_2 \cap k_4 \)
5. \( p_5 = p(D, G) \)
   \( H \in p_5 \cap k_1 \)

Distances

\( d_0 = |AH| \) (red) \ldots 1
\( d_1 = |BF| = |BG| \) (blue) \ldots 2

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} \) (2 occurrences)
2.53.2 Construction RCC396

Construction Process

A, B given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, C) \)
   \( E \in k_2 \cap k_3 \)
4. \( k_4 = k(E, B) \)
   \( F \in k_1 \cap k_4 \), \( G \in k_1 \cap k_4 \)
5. \( p_5 = p(D, G) \)
   \( H \in p_5 \cap k_2 \)

Distances

\( d_0 = |BH| \) (red) \ldots 1
\( d_1 = |AF| = |AG| \) (blue) \ldots 2

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} \) (2 occurrences)
2.53.3 Construction RCC407

Construction Process

A, B given initial points
1. $k_1 = k(B, A)$
2. $k_2 = k(A, B)$
   $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
3. $k_3 = k(C, D)$
   $E \in k_1 \cap k_3$
4. $k_4 = k(E, A)$
   $F \in k_2 \cap k_4, G \in k_2 \cap k_4$
5. $p_5 = p(C, F)$
   $H \in p_5 \cap k_1$

Distances

$d_0 = |AH| \text{ (red)} \ldots 1$
$d_1 = |BF| = |BG| \text{ (blue)} \ldots 2$

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_1} \text{ (2 occurrences)}$
2.53.4 Construction RCC437

Construction Process

1. $k_1 = k(B, A)$
2. $k_2 = k(A, B)$
   - $C \in k_1 \cap k_2$, $D \in k_1 \cap k_2$
3. $k_3 = k(C, D)$
   - $E \in k_2 \cap k_3$
4. $k_4 = k(E, B)$
   - $F \in k_1 \cap k_4$, $G \in k_1 \cap k_4$
5. $p_5 = p(C, F)$
   - $H \in p_5 \cap k_2$

Distances

- $d_0 = |BH|$ (red) ... 1
- $d_1 = |AF| = |AG|$ (blue) ... 2

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_1}$ (2 occurrences)
2.54 Class of Similar Constructions No. 47

Contained Constructions
RCC370, RCC399, RCC414, RCC443
2.54.1 Construction RCC370

Construction Process

A, B given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, C) \)
   \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(E, A) \)
   \( G \in k_2 \cap k_4 \)
5. \( p_5 = p(F, G) \)
   \( H \in p_5 \cap k_3 \)

Distances

\[ d_0 = |FH| \text{ (red)} \ldots 1 \]
\[ d_1 = |EH| \text{ (blue)} \ldots 1 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} \text{ (1 occurrence)} \]
2.54.2 Construction RCC399

Construction Process

\[ A, B \text{ given initial points} \]

1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
3. \( C \in k_1 \cap k_2, D \in k_1 \cap k_2 \)
4. \( k_3 = k(D, C) \)
5. \( E \in k_1 \cap k_3, F \in k_2 \cap k_3 \)

Distances

\[ d_0 = |EH| \text{ (red)} \ldots 1 \]
\[ d_1 = |FH| \text{ (blue)} \ldots 1 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} \text{ (1 occurrence)} \]
2.54.3 Construction RCC414

Construction Process

A, B given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2 , \ D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, D) \)
   \( E \in k_1 \cap k_3 , \ F \in k_2 \cap k_3 \)
4. \( k_4 = k(E, A) \)
   \( G \in k_2 \cap k_4 \)
5. \( p_5 = p(F, G) \)
   \( H \in p_5 \cap k_3 \)

Distances

\( d_0 = |FH| \) (red) \( \ldots 1 \)
\( d_1 = |EH| \) (blue) \( \ldots 1 \)

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} \) (1 occurrence)
2.54.4 Construction RCC443

Construction Process

1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2, D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, D) \)
   \( E \in k_1 \cap k_3, F \in k_2 \cap k_3 \)
4. \( k_4 = k(F, B) \)
   \( G \in k_1 \cap k_4 \)
5. \( p_5 = p(E, G) \)
   \( H \in p_5 \cap k_3 \)

Distances

\[ d_0 = |EH| \text{ (red)} \ldots 1 \]
\[ d_1 = |FH| \text{ (blue)} \ldots 1 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} \text{ (1 occurrence)} \]
2.55 Class of Similar Constructions No. 48

Contained Constructions
RCC373, RCC377, RCC429, RCC433
2.55.1 Construction RCC373

Construction Process

1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
3. \( C \in k_1 \cap k_2 \) \( , D \in k_1 \cap k_2 \)
4. \( k_3 = k(D, C) \)
5. \( E \in k_1 \cap k_3 \) \( , F \in k_2 \cap k_3 \)
6. \( k_4 = k(A, E) \)
7. \( G \in k_3 \cap k_4 \)
8. \( p_5 = p(C, F) \)
9. \( H \in p_5 \cap k_4 \) \( , I \in p_5 \cap k_4 \)

Distances

\[ d_0 = |EH| = |GI| \quad \text{(red)} \quad \ldots \quad 2 \]
\[ d_1 = |CH| = |FI| \quad \text{(blue)} \quad \ldots \quad 2 \]
\[ d_2 = |DH| = |DI| \quad \text{(green)} \quad \ldots \quad 2 \]
\[ d_3 = |CD| = |CE| = |CF| = |DE| = |DF| = |DG| = |FG| \quad \text{(cyan)} \quad \ldots \quad 7 \]
\[ d_4 = |EI| = |GH| \quad \text{(magenta)} \quad \ldots \quad 2 \]
\[ d_5 = |CI| = |FH| \quad \text{(yellow)} \quad \ldots \quad 2 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_4}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \quad \text{(36 occurrences)} \]
2.55.2 Construction RCC377

Construction Process

A, B given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, C) \)
   \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( p_4 = p(C, E) \)
5. \( k_5 = k(B, F) \)
   \( G \in k_3 \cap k_5 \), \( H \in p_4 \cap k_5 \), \( I \in p_4 \cap k_5 \)

Distances

\[
\begin{align*}
  d_0 &= |FI| = |GH| \quad \text{(red)} \ldots 2 \\
  d_1 &= |CI| = |EH| \quad \text{(blue)} \ldots 2 \\
  d_2 &= |DH| = |DI| \quad \text{(green)} \ldots 2 \\
  d_3 &= |CD| = |CF| = |DE| = |DF| = |DG| = |EG| \quad \text{(cyan)} \ldots 7 \\
  d_4 &= |FH| = |GI| \quad \text{(magenta)} \ldots 2 \\
  d_5 &= |CH| = |EI| \quad \text{(yellow)} \ldots 2
\end{align*}
\]

Occurrences of the Golden Ratio

\[
\phi = \frac{d_0}{d_2} = \frac{d_3}{d_5} = \frac{d_2}{d_4} = \frac{d_4}{d_6} \quad \text{(36 occurrences)}
\]
2.55.3 Construction RCC429

Construction Process

\[ A, B \text{ given initial points} \]

1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \[ C \in k_1 \cap k_2, \; D \in k_1 \cap k_2 \]
3. \( k_3 = k(C, D) \)
   \[ E \in k_1 \cap k_3, \; F \in k_2 \cap k_3 \]
4. \( k_4 = k(A, E) \)
   \[ G \in k_3 \cap k_4 \]
5. \( p_5 = p(D, F) \)
   \[ H \in p_5 \cap k_4, \; I \in p_5 \cap k_4 \]

Distances

\[ d_0 = \|EH\| = \|GI\| \text{ (red)} \ldots 2 \]
\[ d_1 = \|DH\| = \|FI\| \text{ (blue)} \ldots 2 \]
\[ d_2 = \|CH\| = \|CI\| \text{ (green)} \ldots 2 \]
\[ d_3 = \|CD\| = \|CE\| = \|CF\| = \|CG\| = \|DE\| = \|DF\| = \|FG\| \text{ (cyan)} \ldots 7 \]
\[ d_4 = \|EI\| = \|GH\| \text{ (magenta)} \ldots 2 \]
\[ d_5 = \|DI\| = \|FH\| \text{ (yellow)} \ldots 2 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_3}{d_4} = \frac{d_4}{d_5} = \frac{d_5}{d_6} \text{ (36 occurrences)} \]
2.55.4 Construction RCC433

Construction Process

A, B given initial points
1. $k_1 = k(B,A)$
2. $k_2 = k(A,B)$
   $C \in k_1 \cap k_2$, $D \in k_1 \cap k_2$
3. $k_3 = k(C,D)$
   $E \in k_1 \cap k_3$, $F \in k_2 \cap k_3$
4. $p_4 = p(D,E)$
5. $k_5 = k(B,F)$
   $G \in k_3 \cap k_5$, $H \in p_4 \cap k_5$, $I \in p_4 \cap k_5$

Distances

\[ d_0 = |FI| = |GH| \text{ (red) } \ldots 2 \]
\[ d_1 = |DI| = |EH| \text{ (blue) } \ldots 2 \]
\[ d_2 = |CH| = |CI| \text{ (green) } \ldots 2 \]
\[ d_3 = |CD| = |CE| = |CF| = |CG| = |DE| = |DF| = |EG| \text{ (cyan) } \ldots 7 \]
\[ d_4 = |FH| = |GI| \text{ (magenta) } \ldots 2 \]
\[ d_5 = |DH| = |EI| \text{ (yellow) } \ldots 2 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_4}{d_5} = \frac{d_2}{d_4} = \frac{d_4}{d_5} \text{ (36 occurrences) } \]
2.56 Class of Similar Constructions No. 49

Contained Constructions
RCC374, RCC378, RCC428, RCC432
2.56.1 Construction RCC374

Construction Process

A, B given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2, \ D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, C) \)
   \( E \in k_1 \cap k_3, \ F \in k_2 \cap k_3 \)
4. \( k_4 = k(A, E) \)
   \( G \in k_3 \cap k_4 \)
5. \( p_5 = p(D, F) \)
   \( H \in p_5 \cap k_3, \ I \in p_5 \cap k_4, \ J \in p_5 \cap k_4 \)

Distances

\[
\begin{align*}
&d_0 = |HI| \text{ (red)} \ldots 1 \\
&d_1 = |EI| \text{ (blue)} \ldots 1 \\
&d_2 = |DI| = |FJ| \text{ (green)} \ldots 2 \\
&d_3 = |CI| = |CJ| = |GI| = |GJ| \text{ (cyan)} \ldots 4 \\
&d_4 = |CD| = |CE| = |CF| = |DE| = |DF| = |DG| = |DH| = |EH| = |FG| \text{ (magenta)} \ldots 9 \\
&d_5 = |EJ| \text{ (yellow)} \ldots 1 \\
&d_6 = |DJ| = |FI| \text{ (grass)} \ldots 2 \\
&d_7 = |HJ| \text{ (sea)} \ldots 1
\end{align*}
\]

Occurrences of the Golden Ratio

\[
\phi = \frac{d_6}{d_7} = \frac{d_6}{d_5} = \frac{d_7}{d_6} = \frac{d_4}{d_5} = \frac{d_5}{d_6} = \frac{d_7}{d_8} \text{ (48 occurrences)}
\]
2.56.2 Construction RCC378

Construction Process

A, B given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2, \ D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, C) \)
   \( E \in k_1 \cap k_3, \ F \in k_2 \cap k_3 \)
4. \( p_4 = p(D, E) \)
   \( G \in p_4 \cap k_3 \)
5. \( k_5 = k(B, F) \)
   \( H \in k_3 \cap k_5, \ I \in p_4 \cap k_5, \ J \in p_4 \cap k_5 \)

Distances

\[ d_0 = |GJ| \] \( \text{red} \ldots 1 \)
\[ d_1 = |FJ| \] \( \text{blue} \ldots 1 \)
\[ d_2 = |DJ| = |EI| \] \( \text{green} \ldots 2 \)
\[ d_3 = |CI| = |CJ| = |HI| = |HJ| \] \( \text{cyan} \ldots 4 \)
\[ d_4 = |CD| = |CE| = |CF| = |DE| = |DF| = |DG| = |DH| = |EH| = |FG| \] \( \text{magenta} \ldots 9 \)
\[ d_5 = |FI| \] \( \text{yellow} \ldots 1 \)
\[ d_6 = |DI| = |EJ| \] \( \text{grass} \ldots 2 \)
\[ d_7 = |GI| \] \( \text{sea} \ldots 1 \)

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} = \frac{d_4}{d_6} = \frac{d_6}{d_7} \] \( \text{48 occurrences} \)
2.56.3 Construction RCC428

Construction Process

A, B given initial points
1. $k_1 = k(B, A)$
2. $k_2 = k(A, B)$
   $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
3. $k_3 = k(C, D)$
   $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
4. $k_4 = k(A, E)$
   $G \in k_3 \cap k_4$
5. $p_5 = p(C, F)$
   $H \in p_5 \cap k_3, I \in p_5 \cap k_4, J \in p_5 \cap k_4$

Distances

\[
\begin{align*}
d_0 &= |HI| \text{ (red)} \ldots 1 \\
d_1 &= |EI| \text{ (blue)} \ldots 1 \\
d_2 &= |CI| = |FJ| \text{ (green)} \ldots 2 \\
d_3 &= |DI| = |DJ| = |GI| = |GJ| \text{ (cyan)} \ldots 4 \\
d_4 &= |CD| = |CE| = |CF| = |CG| = |CH| = |DE| = |DF| = |EH| = |FG| \text{ (magenta)} \ldots 9 \\
d_5 &= |EJ| \text{ (yellow)} \ldots 1 \\
d_6 &= |CJ| = |FI| \text{ (grass)} \ldots 2 \\
d_7 &= |HJ| \text{ (sea)} \ldots 1
\end{align*}
\]

Occurrences of the Golden Ratio

\[
\phi = \frac{d_0}{d_2} = \frac{d_1}{d_4} = \frac{d_4}{d_6} = \frac{d_4}{d_8} = \frac{d_6}{d_8} \text{ (48 occurrences)}
\]
2.56.4 Construction RCC432

Construction Process

A, B given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2, \ D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, D) \)
   \( E \in k_1 \cap k_3, \ F \in k_2 \cap k_3 \)
4. \( p_4 = p(C, E) \)
   \( G \in p_4 \cap k_3 \)
5. \( k_5 = k(B, F) \)
   \( H \in k_3 \cap k_5, \ I \in p_4 \cap k_5, \ J \in p_4 \cap k_5 \)

Distances

\[
\begin{align*}
  d_0 &= |GJ| \text{ (red)} \ldots 1 \\
  d_1 &= |FJ| \text{ (blue)} \ldots 1 \\
  d_2 &= |CJ| = |EI| \text{ (green)} \ldots 2 \\
  d_3 &= |DJ| = |HI| = |HJ| \text{ (cyan)} \ldots 4 \\
  d_4 &= |CD| = |CE| = |CF| = |CG| = |CH| = |DE| = |DF| = |EH| = |FG| \text{ (magenta)} \ldots 9 \\
  d_5 &= |FI| \text{ (yellow)} \ldots 1 \\
  d_6 &= |CI| = |EJ| \text{ (grass)} \ldots 2 \\
  d_7 &= |GI| \text{ (sea)} \ldots 1
\end{align*}
\]

Occurrences of the Golden Ratio

\( \phi = \frac{d_6}{d_5} = \frac{d_4}{d_3} = \frac{d_2}{d_1} = \frac{d_1}{d_0} = \frac{d_6}{d_7} \) (48 occurrences)
2.57 MM

The constructions described in this section consist only of circles; the processes use a compass to measure the distance between two points for the radius (a circle with a given center and radius).

These constructions are sometimes called Mascheroni’s or Mohr-Mascheroni’s constructions. Lorenzo Mascheroni described the constructions in [2] in 1797, and Georg Mohr described the identical constructions one hundred years prior in [3]. For the names of these constructions, we selected the prefix MM.

The shortest Mohr-Mascheroni construction of the golden ratio requires 5 steps. In total, 368 of these constructions resulting from 988 different processes have been noted. These constructions form 80 classes of mutually similar constructions.

The constructions are labeled MM1 to MM368.

From this list (the numbering remains unaltered), we omit 114 constructions that contain only circles with a radius equal to the distance from the center to the other point; these constructions are mentioned in the list of CCO constructions.

Remarks

Several MM constructions are analogous to RCC constructions, in which a straight line is replaced by a circle.

When construction RCC361 is compared with construction MM331, straight line $p_5$ is replaced by circle $k_5$. The resulting points of intersection are identical. In MM331, more usable points of intersection are noted; therefore, the golden ratio can be found more frequently.

In the analysis of RCC constructions, we compared RC3 and RCC35. Here we can look for comparison with those constructions, where we would replace the straight line $p_1$ by a circle. Such construction can really be found. It contains five lines, but measuring or transferring the distance to the radius of the circle is not required; therefore, we will find it between the constructions of CCO.
2.58 Constructions Omitted Due to Identity

MM1 ( RCC267 CCO2), MM2 ( RCC266 CCO1), MM3 ( RCC269 CCO4), MM4 ( RCC268 CCO3), MM5 ( RCC276 CCO11), MM6 ( RCC277 CCO12), MM7 ( RCC288 CCO22), MM8 ( RCC289 CCO24), MM9 ( RCC273 CCO8), MM10 ( RCC272 CCO7), MM11 ( RCC271 CCO6), MM12 ( RCC270 CCO5), MM13 ( RCC283 CCO18), MM14 ( RCC282 CCO17), MM15 ( RCC284 CCO19), MM16 ( RCC285 CCO20), MM17 ( RCC286 CCO21), MM18 ( RCC287 CCO22), MM19 ( RCC278 CCO13), MM20 ( RCC279 CCO14), MM21 ( RCC280 CCO15), MM22 ( RCC281 CCO16), MM23 ( RCC274 CCO9), MM24 ( RCC275 CCO10), MM26 ( RCC289 CCO43), MM27 ( RCC320 CCO31), MM28 ( RCC321 CCO32), MM29 ( RCC322 CCO42), MM30 ( RCC323 CCO34), MM31 ( RCC324 CCO35), MM32 ( RCC325 CCO36), MM33 ( RCC318 CCO29), MM34 ( RCC319 CCO30), MM35 ( RCC404 CCO81), MM36 ( RCC431 CCO98), MM37 ( RCC427 CCO96), MM38 ( RCC403 CCO80), MM39 ( RCC430 CCO97), MM40 ( RCC434 CCO99), MM41 ( RCC457 CCO114), MM42 ( RCC436 CCO100), MM43 ( RCC445 CCO106), MM44 ( RCC447 CCO107), MM45 ( RCC456 CCO113), MM46 ( RCC439 CCO102), MM47 ( RCC453 CCO111), MM48 ( RCC452 CCO110), MM49 ( RCC450 CCO109), MM50 ( RCC444 CCO104), MM51 ( RCC441 CCO103), MM52 ( RCC449 CCO108), MM53 ( RCC445 CCO112), MM54 ( RCC406 CCO82), MM55 ( RCC413 CCO87), MM56 ( RCC417 CCO89), MM57 ( RCC414 CCO88), MM58 ( RCC425 CCO93), MM59 ( RCC408 CCO83), MM60 ( RCC422 CCO92), MM61 ( RCC420 CCO91), MM62 ( RCC412 CCO86), MM63 ( RCC411 CCO85), MM64 ( RCC419 CCO90), MM65 ( RCC415 CCO88), MM66 ( RCC426 CCO95), MM67 ( RCC434 CCO46), MM68 ( RCC437 CCO63), MM69 ( RCC432 CCO61), MM70 ( RCC438 CCO45), MM71 ( RCC439 CCO45), MM72 ( RCC402 CCO79), MM73 ( RCC381 CCO65), MM74 ( RCC390 CCO71), MM75 ( RCC392 CCO72), MM76 ( RCC401 CCO78), MM77 ( RCC384 CCO67), MM78 ( RCC398 CCO76), MM79 ( RCC383 CCO66), MM80 ( RCC397 CCO75), MM81 ( RCC395 CCO74), MM82 ( RCC387 CCO69), MM83 ( RCC386 CCO68), MM84 ( RCC394 CCO73), MM85 ( RCC389 CCO70), MM86 ( RCC400 CCO77), MM87 ( RCC351 CCO47), MM88 ( RCC358 CCO52), MM89 ( RCC362 CCO54), MM90 ( RCC369 CCO59), MM91 ( RCC354 CCO49), MM92 ( RCC368 CCO38), MM93 ( RCC353 CCO48), MM94 ( RCC367 CCO57), MM95 ( RCC365 CCO56), MM96 ( RCC357 CCO51), MM97 ( RCC356 CCO50), MM98 ( RCC364 CCO55), MM99 ( RCC360 CCO53), MM100 ( RCC371 CCO60),

Omitted 114 constructions.
Contains 254 constructions.
2.59 Class of Similar Constructions No. 50

Contained Constructions
MM25, MM26, MM33, MM66, MM68, MM75
2.59.1 Construction MM25

Construction Process

\(A, B\) given initial points
1. \(k_1 = k(A, |AB|)\)
2. \(k_2 = k(B, |AB|)\)
   \(C \in k_1 \cap k_2, D \in k_1 \cap k_2\)
3. \(k_3 = k(C, |AB|)\)
   \(E \in k_1 \cap k_3, F \in k_2 \cap k_3\)
4. \(k_4 = k(A, |DE|)\)
   \(G \in k_2 \cap k_4, H \in k_3 \cap k_4\)
5. \(k_5 = k(B, |DE|)\)
   \(I \in k_1 \cap k_5, J \in k_3 \cap k_5, K \in k_4 \cap k_5, L \in k_4 \cap k_5\)

Distances

\(d_0 = |CL| = |DK| \) (red) \dots 2
\(d_1 = |AF| = |AJ| = |BE| = |BH| = |CD| = |CG| = |CI| = |DG| = |DI| = |EH| = |FJ| \) (blue) \dots 11
\(d_2 = |CK| = |DL| \) (green) \dots 2

Occurrences of the Golden Ratio

\(\phi = \frac{d_0}{d_1} = \frac{d_2}{d_2} \) (44 occurrences)
2.59.2 Construction MM26

Construction Process

A, B given initial points
1. $k_1 = k(A, |AB|)$
2. $k_2 = k(B, |AB|)$
   $C \in k_1 \cap k_2$, $D \in k_1 \cap k_2$
3. $k_3 = k(C, |AB|)$
   $E \in k_1 \cap k_3$, $F \in k_2 \cap k_3$
4. $k_4 = k(A, |DE|)$
   $G \in k_2 \cap k_4$, $H \in k_3 \cap k_4$
5. $k_5 = k(C, |DE|)$
   $I \in k_1 \cap k_5$, $J \in k_2 \cap k_5$, $K \in k_4 \cap k_5$, $L \in k_4 \cap k_5$

Distances

$d_0 = |BK| = |EL|$ (red) \ldots 2
$d_1 = |AF| = |AJ| = |BE| = |BH| = |BI| = |CD| = |CG| = |DG| = |EH| = |EI| = |FJ|$ (blue) \ldots 11
$d_2 = |BL| = |EK|$ (green) \ldots 2

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_1} = \frac{d_4}{d_5}$ (44 occurrences)
2.59.3 Construction MM33

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2, D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, |AB|) \)
   \( E \in k_1 \cap k_3, F \in k_2 \cap k_3 \)
4. \( k_4 = k(B, |DE|) \)
   \( G \in k_1 \cap k_4, H \in k_3 \cap k_4 \)
5. \( k_5 = k(C, |DE|) \)
   \( I \in k_1 \cap k_5, J \in k_2 \cap k_5, K \in k_4 \cap k_5, L \in k_4 \cap k_5 \)

Distances
\[
\begin{align*}
d_0 &= |AL| = |FK| & \text{(red)} \quad \ldots 2 \\
d_1 &= |AF| = |AH| = |AJ| = |BE| = |BI| = |CD| = |CG| = |DG| = |EI| = |FH| = |FJ| & \text{(blue)} \quad \ldots 11 \\
d_2 &= |AK| = |FL| & \text{(green)} \quad \ldots 2
\end{align*}
\]

Occurrences of the Golden Ratio
\[
\phi = \frac{d_0}{d_1} = \frac{d_2}{d_2} \quad (44 \text{ occurrences})
\]
2.59.4 Construction MM66

Construction Process

A, B given initial points
1. $k_1 = k(A, |AB|)$
2. $k_2 = k(B, |AB|)$
   $C \in k_1 \cap k_2$, $D \in k_1 \cap k_2$
3. $k_3 = k(D, |AB|)$
   $E \in k_1 \cap k_3$, $F \in k_2 \cap k_3$
4. $k_4 = k(A, |CE|)$
   $G \in k_2 \cap k_4$, $H \in k_3 \cap k_4$
5. $k_5 = k(B, |CE|)$
   $I \in k_1 \cap k_5$, $J \in k_3 \cap k_5$, $K \in k_4 \cap k_5$, $L \in k_4 \cap k_5$

Distances

$d_0 = |CL| = |DK|$ (red) ... 2
$d_1 = |AF| = |AJ| = |BE| = |BH| = |CD| = |CG| = |CI| = |DG| = |DI| = |EH| = |FJ|$ (blue) ... 11
$d_2 = |CK| = |DL|$ (green) ... 2

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_1} = \frac{d_2}{d_2}$ (44 occurrences)
2.59.5 Construction MM68

Construction Process

1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, |AB|) \)
   \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(A, |CE|) \)
   \( G \in k_2 \cap k_4 \), \( H \in k_3 \cap k_4 \)
5. \( k_5 = k(D, |CE|) \)
   \( I \in k_1 \cap k_5 \), \( J \in k_2 \cap k_5 \), \( K \in k_4 \cap k_5 \), \( L \in k_4 \cap k_5 \)

Distances

\( d_0 = |BK| = |EL| \) (red) \( \ldots 2 \)
\( d_1 = |AF| = |AJ| = |BE| = |BH| = |BI| = |CD| = |CG| = |DG| = |EH| = |EI| = |FJ| \) (blue) \( \ldots 11 \)
\( d_2 = |BL| = |EK| \) (green) \( \ldots 2 \)

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} = \frac{d_2}{d_1} \) (44 occurrences)
2.59.6 Construction MM75

Construction Process

\( A, B \) given initial points

1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2, D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, |AB|) \)
   \( E \in k_1 \cap k_3, F \in k_2 \cap k_3 \)
4. \( k_4 = k(B, |CE|) \)
   \( G \in k_1 \cap k_4, H \in k_3 \cap k_4 \)
5. \( k_5 = k(D, |CE|) \)
   \( I \in k_1 \cap k_5, J \in k_2 \cap k_5, K \in k_4 \cap k_5, L \in k_4 \cap k_5 \)

Distances

\[ d_0 = |AL| = |FK| \quad \text{(red)} \quad \ldots 2 \]
\[ d_1 = |AF| = |AH| = |AJ| = |BE| = |B1| = |CD| = |CG| = |DG| = |EI| = |FH| = |FJ| \quad \text{(blue)} \quad \ldots 11 \]
\[ d_2 = |AK| = |FL| \quad \text{(green)} \quad \ldots 2 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} = \frac{d_2}{d_2} \quad \text{(44 occurrences)} \]
2.60 Class of Similar Constructions No. 51

Contained Constructions
MM27, MM28, MM34, MM35, MM40, MM41, MM67, MM69, MM74, MM76, MM81, MM82
2.60.1 Construction MM27

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, |AB|) \)
   \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(A, |DE|) \)
   \( G \in k_2 \cap k_4 \), \( H \in k_3 \cap k_4 \)
5. \( k_5 = k(D, |DE|) \)
   \( I \in k_4 \cap k_5 \), \( J \in k_4 \cap k_5 \)

Distances

\( d_0 = |HI| \) (red) . . . 1
\( d_1 = |BI| \) (blue) . . . 1
\( d_2 = |EI| = |EJ| \) (green) . . . 2
\( d_3 = |AF| = |BE| = |BH| = |CD| = |CG| = |DG| = |EH| \) (cyan) . . . 7
\( d_4 = |HJ| \) (magenta) . . . 1
\( d_5 = |BJ| \) (yellow) . . . 1

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_3}{d_4} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \] (18 occurrences)
2.60 Class of Similar Constructions No. 51

2.60.2 Construction MM28

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, |AB|) \)
   \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(A, |DE|) \)
   \( G \in k_2 \cap k_4 \), \( H \in k_3 \cap k_4 \)
5. \( k_5 = k(E, |DE|) \)
   \( I \in k_4 \cap k_5 \), \( J \in k_4 \cap k_5 \)

Distances

\( d_0 = |GI| \) (red) ... 1
\( d_1 = |CI| \) (blue) ... 1
\( d_2 = |DJ| = |DJ| \) (green) ... 2
\( d_3 = |AF| = |BE| = |BH| = |CD| = |CG| = |DG| = |EH| \) (cyan) ... 7
\( d_4 = |GJ| \) (magenta) ... 1
\( d_5 = |CJ| \) (yellow) ... 1

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \] (18 occurrences)
2.60.3 Construction MM34

Construction Process

\[ A, B \text{ given initial points} \]
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \[ C \in k_1 \cap k_2, \, D \in k_1 \cap k_2 \]
3. \( k_3 = k(C, |AB|) \)
   \[ E \in k_1 \cap k_3, \, F \in k_2 \cap k_3 \]
4. \( k_4 = k(B, |DE|) \)
   \[ G \in k_1 \cap k_4, \, H \in k_3 \cap k_4 \]
5. \( k_5 = k(D, |DE|) \)
   \[ I \in k_4 \cap k_5, \, J \in k_4 \cap k_5 \]

Distances

\[ d_0 = |HJ| \text{ (red)} \ldots 1 \]
\[ d_1 = |AJ| \text{ (blue)} \ldots 1 \]
\[ d_2 = |FI| = |FJ| \text{ (green)} \ldots 2 \]
\[ d_3 = |AF| = |AH| = |BE| = |CD| = |CG| = |DG| = |FH| \text{ (cyan)} \ldots 7 \]
\[ d_4 = |HI| \text{ (magenta)} \ldots 1 \]
\[ d_5 = |AI| \text{ (yellow)} \ldots 1 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_5}{d_6} \text{ (18 occurrences)} \]
2.60.4 Construction MM35

Construction Process

- \( A, B \) given initial points
- 1. \( k_1 = k(A, |AB|) \)
- 2. \( k_2 = k(B, |AB|) \)
- \( C \in k_1 \cap k_2, D \in k_1 \cap k_2 \)
- 3. \( k_3 = k(C, |AB|) \)
- \( E \in k_1 \cap k_3, F \in k_2 \cap k_3 \)
- 4. \( k_4 = k(B, |DE|) \)
- \( G \in k_1 \cap k_4, H \in k_3 \cap k_4 \)
- 5. \( k_5 = k(F, |DE|) \)
- \( I \in k_4 \cap k_5, J \in k_4 \cap k_5 \)

Distances

- \( d_0 = |GJ| \) (red) \ldots 1
- \( d_1 = |CJ| \) (blue) \ldots 1
- \( d_2 = |DJ| = |DJ| \) (green) \ldots 2
- \( d_3 = |AF| = |AH| = |BE| = |CD| = |CG| = |DG| = |FH| \) (cyan) \ldots 7
- \( d_4 = |GI| \) (magenta) \ldots 1
- \( d_5 = |CI| \) (yellow) \ldots 1

Occurrences of the Golden Ratio

- \( \phi = \frac{d_0}{d_2} = \frac{d_3}{d_4} = \frac{d_4}{d_5} = \frac{d_5}{d_6} \) (18 occurrences)
2.60.5 Construction MM40

Construction Process

\begin{align*}
A, B \text{ given initial points} \\
1. \quad & k_1 = k(A, |AB|) \\
2. \quad & k_2 = k(B, |AB|) \\
& C \in k_1 \cap k_2, \quad D \in k_1 \cap k_2 \\
3. \quad & k_3 = k(C, |AB|) \\
& E \in k_1 \cap k_3, \quad F \in k_2 \cap k_3 \\
4. \quad & k_4 = k(C, |DE|) \\
& G \in k_1 \cap k_4, \quad H \in k_2 \cap k_4 \\
5. \quad & k_5 = k(E, |DE|) \\
& I \in k_4 \cap k_5, \quad J \in k_4 \cap k_5
\end{align*}

Distances

\begin{align*}
& d_0 = |HI| \quad (\text{red}) \quad \ldots 1 \\
& d_1 = |AI| \quad (\text{blue}) \quad \ldots 1 \\
& d_2 = |FI| = |FJ| \quad (\text{green}) \quad \ldots 2 \\
& d_3 = |AF| = |AH| = |BE| = |BG| = |CD| = |EG| = |FH| \quad (\text{cyan}) \quad \ldots 7 \\
& d_4 = |HJ| \quad (\text{magenta}) \quad \ldots 1 \\
& d_5 = |AJ| \quad (\text{yellow}) \quad \ldots 1
\end{align*}

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \quad (\text{18 occurrences}) \]
2.60.6 Construction MM41

Construction Process

$A, B$ given initial points
1. $k_1 = k(A, |AB|)$
2. $k_2 = k(B, |AB|)$
   $C \in k_1 \cap k_2$, $D \in k_1 \cap k_2$
3. $k_3 = k(C, |AB|)$
   $E \in k_1 \cap k_3$, $F \in k_2 \cap k_3$
4. $k_4 = k(C, |DE|)$
   $G \in k_1 \cap k_4$, $H \in k_2 \cap k_4$
5. $k_5 = k(F, |DE|)$
   $I \in k_4 \cap k_5$, $J \in k_4 \cap k_5$

Distances

$d_0 = |GI|$ (red) ... 1
$d_1 = |BI|$ (blue) ... 1
$d_2 = |EI| = |EJ|$ (green) ... 2
$d_3 = |AF| = |AH| = |BE| = |BG| = |CD| = |EG| = |FH|$ (cyan) ... 7
$d_4 = |GJ|$ (magenta) ... 1
$d_5 = |BJ|$ (yellow) ... 1

Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5}$$ (18 occurrences)
2.60.7 Construction MM67

Construction Process

A, B given initial points
1. $k_1 = k(A, |AB|)$
2. $k_2 = k(B, |AB|)$
   $C \in k_1 \cap k_2$, $D \in k_1 \cap k_2$
3. $k_3 = k(D, |AB|)$
   $E \in k_1 \cap k_3$, $F \in k_2 \cap k_3$
4. $k_4 = k(A, |CE|)$
   $G \in k_2 \cap k_4$, $H \in k_3 \cap k_4$
5. $k_5 = k(C, |CE|)$
   $I \in k_4 \cap k_5$, $J \in k_4 \cap k_5$

Distances

- $d_0 = |HI|$ (red) \ldots 1
- $d_1 = |BI|$ (blue) \ldots 1
- $d_2 = |EI| = |EJ|$ (green) \ldots 2
- $d_3 = |AF| = |BE| = |BH| = |CD| = |CG| = |DG| = |EH|$ (cyan) \ldots 7
- $d_4 = |HJ|$ (magenta) \ldots 1
- $d_5 = |BJ|$ (yellow) \ldots 1

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_3}{d_4} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \quad (18 \text{ occurrences}) \]
2.60.8 Construction MM69

Construction Process

\[ A, B \text{ given initial points} \]
\[ 1. k_1 = k(A, |AB|) \]
\[ 2. k_2 = k(B, |AB|) \]
\[ C \in k_1 \cap k_2, D \in k_1 \cap k_2 \]
\[ 3. k_3 = k(D, |AB|) \]
\[ E \in k_1 \cap k_3, F \in k_2 \cap k_3 \]
\[ 4. k_4 = k(A, |CE|) \]
\[ G \in k_2 \cap k_4, H \in k_3 \cap k_4 \]
\[ 5. k_5 = k(E, |CE|) \]
\[ I \in k_4 \cap k_5, J \in k_4 \cap k_5 \]

Distances

\[ d_0 = |GI| \text{ (red)} \ldots 1 \]
\[ d_1 = |DI| \text{ (blue)} \ldots 1 \]
\[ d_2 = |CI| = |CJ| \text{ (green)} \ldots 2 \]
\[ d_3 = |AF| = |BE| = |BH| = |CD| = |CG| = |DG| = |EH| \text{ (cyan)} \ldots 7 \]
\[ d_4 = |GJ| \text{ (magenta)} \ldots 1 \]
\[ d_5 = |DJ| \text{ (yellow)} \ldots 1 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_2}{d_3} = \frac{d_4}{d_5} = \frac{d_5}{d_6} \text{ (18 occurrences)} \]
2.60.9 Construction MM74

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2, D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, |AB|) \)
   \( E \in k_1 \cap k_3, F \in k_2 \cap k_3 \)
4. \( k_4 = k(B, |CE|) \)
   \( G \in k_1 \cap k_4, H \in k_3 \cap k_4 \)
5. \( k_5 = k(C, |CE|) \)
   \( I \in k_4 \cap k_5, J \in k_4 \cap k_5 \)

Distances

\( d_0 = |HJ| \) (red) \ldots 1
\( d_1 = |AJ| \) (blue) \ldots 1
\( d_2 = |FI| = |FJ| \) (green) \ldots 2
\( d_3 = |AF| = |AH| = |BE| = |CD| = |CG| = |DG| = |FH| \) (cyan) \ldots 7
\( d_4 = |HI| \) (magenta) \ldots 1
\( d_5 = |AI| \) (yellow) \ldots 1

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_2} = \frac{d_3}{d_4} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \) (18 occurrences)
2.60.10 Construction MM76

Construction Process

A, B given initial points
1. \(k_1 = k(A, |AB|)\)
2. \(k_2 = k(B, |AB|)\)
   
   \(C \in k_1 \cap k_2\), \(D \in k_1 \cap k_2\)
3. \(k_3 = k(D, |AB|)\)
   
   \(E \in k_1 \cap k_3\), \(F \in k_2 \cap k_3\)
4. \(k_4 = k(B, |CE|)\)
   
   \(G \in k_1 \cap k_4\), \(H \in k_3 \cap k_4\)
5. \(k_5 = k(F, |CE|)\)
   
   \(I \in k_4 \cap k_5\), \(J \in k_4 \cap k_5\)

Distances

\[d_0 = |GJ| \text{ (red)} \ldots 1\]
\[d_1 = |DJ| \text{ (blue)} \ldots 1\]
\[d_2 = |CI| = |CJ| \text{ (green)} \ldots 2\]
\[d_3 = |AF| = |AH| = |BE| = |CD| = |CG| = |DG| = |FH| \text{ (cyan)} \ldots 7\]
\[d_4 = |GI| \text{ (magenta)} \ldots 1\]
\[d_5 = |DI| \text{ (yellow)} \ldots 1\]

Occurrences of the Golden Ratio

\[\phi = \frac{d_0}{d_2} = \frac{d_4}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (18 occurrences)}\]
2.60.11 Construction MM81

Construction Process

1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2, D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, |AB|) \)
   \( E \in k_1 \cap k_3, F \in k_2 \cap k_3 \)
4. \( k_4 = k(D, |CE|) \)
   \( G \in k_1 \cap k_4, H \in k_2 \cap k_4 \)
5. \( k_5 = k(E, |CE|) \)
   \( I \in k_4 \cap k_5, J \in k_4 \cap k_5 \)

Distances

\[ d_0 = |HJ| \) (red) \ldots 1 \]
\[ d_1 = |AJ| \) (blue) \ldots 1 \]
\[ d_2 = |FI| = |FJ| \) (green) \ldots 2 \]
\[ d_3 = |AF| = |AH| = |BE| = |BG| = |CD| = |EG| = |FH| \) (cyan) \ldots 7 \]
\[ d_4 = |HI| \) (magenta) \ldots 1 \]
\[ d_5 = |AI| \) (yellow) \ldots 1 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_4}{d_3} = \frac{d_5}{d_4} = \frac{d_3}{d_2} \) (18 occurrences)
2.60.12 Construction MM82

Construction Process

A, B given initial points
1. \(k_1 = k(A, |AB|)\)
2. \(k_2 = k(B, |AB|)\)
   \[C \in k_1 \cap k_2, \ D \in k_1 \cap k_2\]
3. \(k_3 = k(D, |AB|)\)
   \[E \in k_1 \cap k_3, \ F \in k_2 \cap k_3\]
4. \(k_4 = k(D, |CE|)\)
   \[G \in k_1 \cap k_4, \ H \in k_2 \cap k_4\]
5. \(k_5 = k(F, |CE|)\)
   \[I \in k_4 \cap k_5, \ J \in k_4 \cap k_5\]

Distances

\[d_0 = |GJ| \text{ (red)} \ldots 1\]
\[d_1 = |BJ| \text{ (blue)} \ldots 1\]
\[d_2 = |EI| = |EJ| \text{ (green)} \ldots 2\]
\[d_3 = |AF| = |AH| = |BE| = |BG| = |CD| = |EG| = |FH| \text{ (cyan)} \ldots 7\]
\[d_4 = |GI| \text{ (magenta)} \ldots 1\]
\[d_5 = |BI| \text{ (yellow)} \ldots 1\]

Occurrences of the Golden Ratio

\[\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_4}{d_5} \text{ (18 occurrences)}\]
2.61 Class of Similar Constructions No. 52

Contained Constructions

MM29, MM31, MM36, MM38, MM42, MM44, MM70, MM72, MM77, MM79, MM83, MM85
2.61.1 Construction MM29

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2, D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, |AB|) \)
   \( E \in k_1 \cap k_3, F \in k_2 \cap k_3 \)
4. \( k_4 = k(A, |DE|) \)
   \( G \in k_2 \cap k_4, H \in k_3 \cap k_4 \)
5. \( k_5 = k(G, |AB|) \)
   \( I \in k_2 \cap k_5, J \in k_4 \cap k_5, K \in k_4 \cap k_5 \)

Distances

\[ d_0 = |HK| \text{ (red)} \ldots 1 \]
\[ d_1 = |AF| = |AI| = |BE| = |BH| = |CD| = |CG| = |DG| = |EH| = |FI| \text{ (blue)} \ldots 9 \]
\[ d_2 = |HJ| \text{ (green)} \ldots 1 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} = \frac{d_2}{d_1} \text{ (18 occurrences)} \]
2.61.2 Construction MM31

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2, D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, |AB|) \)
   \( E \in k_1 \cap k_3, F \in k_2 \cap k_3 \)
4. \( k_4 = k(A, |DE|) \)
   \( G \in k_2 \cap k_4, H \in k_3 \cap k_4 \)
5. \( k_5 = k(H, |AB|) \)
   \( I \in k_3 \cap k_5, J \in k_4 \cap k_5, K \in k_4 \cap k_5 \)

Distances

\( d_0 = |GJ| \) (red) . . . 1
\( d_1 = |AF| = |AI| = |BE| = |BH| = |CD| = |CG| = |DG| = |EH| = |FI| \) (blue) . . . 9
\( d_2 = |GK| \) (green) . . . 1

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} = \frac{d_2}{d_5} \) (18 occurrences)
2.61.3 Construction MM36

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2, \ D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, |AB|) \)
   \( E \in k_1 \cap k_3, \ F \in k_2 \cap k_3 \)
4. \( k_4 = k(B, |DE|) \)
   \( G \in k_1 \cap k_4, \ H \in k_3 \cap k_4 \)
5. \( k_5 = k(G, |AB|) \)
   \( I \in k_1 \cap k_5, \ J \in k_4 \cap k_5, \ K \in k_4 \cap k_5 \)

Distances

\[ d_0 = |HK| \text{ (red)} \ldots 1 \]
\[ d_1 = |AF| = |AH| = |BE| = |BI| = |CD| = |CG| = |DG| = |EI| = |FH| \text{ (blue)} \ldots 9 \]
\[ d_2 = |HJ| \text{ (green)} \ldots 1 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} = \frac{d_2}{d_5} \text{ (18 occurrences)} \]
2.61.4 Construction MM38

Construction Process

A, B given initial points
1. $k_1 = k(A, |AB|)$
2. $k_2 = k(B, |AB|)$
   $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
3. $k_3 = k(C, |AB|)$
   $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
4. $k_4 = k(B, |DE|)$
   $G \in k_1 \cap k_4, H \in k_3 \cap k_4$
5. $k_5 = k(H, |AB|)$
   $I \in k_3 \cap k_5, J \in k_4 \cap k_5, K \in k_4 \cap k_5$

Distances

$d_0 = |GK|$ (red) ... 1
$d_1 = |AF| = |AH| = |BE| = |BI| = |CD| = |CG| = |DG| = |EI| = |FH|$ (blue) ... 9
$d_2 = |GJ|$ (green) ... 1

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_4} = \frac{d_2}{d_5}$ (18 occurrences)
2.61.5 Construction MM42

Construction Process

A, B given initial points
1. $k_1 = k(A, |AB|)$
2. $k_2 = k(B, |AB|)$
   $C \in k_1 \cap k_2 \ , \ D \in k_1 \cap k_2$
3. $k_3 = k(C, |AB|)$
   $E \in k_1 \cap k_3 \ , \ F \in k_2 \cap k_3$
4. $k_4 = k(C, |DE|)$
   $G \in k_1 \cap k_4 \ , \ H \in k_2 \cap k_4$
5. $k_5 = k(G, |AB|)$
   $I \in k_1 \cap k_5 \ , \ J \in k_4 \cap k_5 \ , \ K \in k_4 \cap k_5$

Distances

$d_0 = |HJ|$ (red) ... 1
$d_1 = |AF| = |AH| = |BE| = |BG| = |CD| = |CI| = |DI| = |EG| = |FH|$ (blue) ... 9
$d_2 = |HK|$ (green) ... 1

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_1} = \frac{d_2}{d_5}$ (18 occurrences)
2.61.6 Construction MM44

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, |AB|) \)
   \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(C, |DE|) \)
   \( G \in k_1 \cap k_4 \), \( H \in k_2 \cap k_4 \)
5. \( k_5 = k(H, |AB|) \)
   \( I \in k_2 \cap k_5 \), \( J \in k_4 \cap k_5 \), \( K \in k_4 \cap k_5 \)

Distances

\( d_0 = |GK| \) (red) \ldots 1
\( d_1 = |AF| = |AH| = |BE| = |BG| = |CD| = |CI| = |DI| = |EG| = |FH| \)
(blue) \ldots 9
\( d_2 = |GJ| \) (green) \ldots 1

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} = \frac{d_3}{d_2} \) (18 occurrences)
2.61.7 Construction MM70

Construction Process

\[ A, B \text{ given initial points} \]
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2, \ D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, |AB|) \)
   \( E \in k_1 \cap k_3, \ F \in k_2 \cap k_3 \)
4. \( k_4 = k(A, |CE|) \)
   \( G \in k_2 \cap k_4, \ H \in k_3 \cap k_4 \)
5. \( k_5 = k(G, |AB|) \)
   \( I \in k_2 \cap k_5, \ J \in k_4 \cap k_5, \ K \in k_4 \cap k_5 \)

Distances
\[ d_0 = |HJ| \text{ (red)} \ldots 1 \]
\[ d_1 = |AF| = |AI| = |BE| = |BH| = |CD| = |CG| = |DG| = |EH| = |FI| \text{ (blue)} \ldots 9 \]
\[ d_2 = |HK| \text{ (green)} \ldots 1 \]

Occurrences of the Golden Ratio
\[ \phi = \frac{d_0}{d_1} = \frac{d_2}{d_2} \text{ (18 occurrences)} \]
2.61.8 Construction MM72

Construction Process

$A, B$ given initial points
1. $k_1 = k(A, |AB|)$
2. $k_2 = k(B, |AB|)$
   \[ C \in k_1 \cap k_2, \ D \in k_1 \cap k_2 \]
3. $k_3 = k(D, |AB|)$
   \[ E \in k_1 \cap k_3, \ F \in k_2 \cap k_3 \]
4. $k_4 = k(A, |CE|)$
   \[ G \in k_2 \cap k_4, \ H \in k_3 \cap k_4 \]
5. $k_5 = k(H, |AB|)$
   \[ I \in k_3 \cap k_5, \ J \in k_4 \cap k_5, \ K \in k_4 \cap k_5 \]

Distances

\[ d_0 = |GJ| \ (\text{red}) \ldots 1 \]
\[ d_1 = |AF| = |AI| = |BE| = |BH| = |CD| = |CG| = |DG| = |EH| = |FI| \]
   \[ (\text{blue}) \ldots 9 \]
\[ d_2 = |GK| \ (\text{green}) \ldots 1 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} = \frac{d_2}{d_5} \ (18 \text{ occurrences}) \]
2.61.9 Construction MM77

Construction Process

1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2, \ D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, |AB|) \)
   \( E \in k_1 \cap k_3, \ F \in k_2 \cap k_3 \)
4. \( k_4 = k(B, |CE|) \)
   \( G \in k_1 \cap k_4, \ H \in k_3 \cap k_4 \)
5. \( k_5 = k(G, |AB|) \)
   \( I \in k_1 \cap k_5, \ J \in k_4 \cap k_5, \ K \in k_4 \cap k_5 \)

Distances

- \( d_0 = |HJ| \) (red) \( \ldots 1 \)
- \( d_1 = |AF| = |AH| = |BE| = |BI| = |CD| = |CG| = |DG| = |EI| = |FH| \) (blue) \( \ldots 9 \)
- \( d_2 = |HK| \) (green) \( \ldots 1 \)

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} = \frac{d_2}{d_5} \) (18 occurrences)
2.61.10 Construction MM79

Construction Process

A, B given initial points
1. $k_1 = k(A, |AB|)$
2. $k_2 = k(B, |AB|)$
   $C \in k_1 \cap k_2$, $D \in k_1 \cap k_2$
3. $k_3 = k(D, |AB|)$
   $E \in k_1 \cap k_3$, $F \in k_2 \cap k_3$
4. $k_4 = k(B, |CE|)$
   $G \in k_1 \cap k_4$, $H \in k_3 \cap k_4$
5. $k_5 = k(H, |AB|)$
   $I \in k_3 \cap k_5$, $J \in k_4 \cap k_5$, $K \in k_4 \cap k_5$

Distances

$d_0 = |GK|$ (red) ... 1
$d_1 = |AF| = |AH| = |BE| = |BI| = |CD| = |CG| = |DG| = |EI| = |FH|$ (blue) ... 9
$d_2 = |GJ|$ (green) ... 1

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_1} = \frac{d_4}{d_5}$ (18 occurrences)
2.61 Construction MM83

Construction Process

A, B given initial points
1. \( k_1 = k(A, \|AB\|) \)
2. \( k_2 = k(B, \|AB\|) \)
   \( C \in k_1 \cap k_2, \quad D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, \|AB\|) \)
   \( E \in k_1 \cap k_3, \quad F \in k_2 \cap k_3 \)
4. \( k_4 = k(D, \|CE\|) \)
   \( G \in k_1 \cap k_4, \quad H \in k_2 \cap k_4 \)
5. \( k_5 = k(G, \|AB\|) \)
   \( I \in k_1 \cap k_5, \quad J \in k_4 \cap k_5, \quad K \in k_4 \cap k_5 \)

Distances

\[ d_0 = |HJ| \quad \text{(red)} \quad \ldots 1 \]
\[ d_1 = |AF| = |AH| = |BE| = |BG| = |CD| = |CI| = |DI| = |EG| = |FH| \]
\quad \text{(blue)} \quad \ldots 9
\[ d_2 = |HK| \quad \text{(green)} \quad \ldots 1 \]

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} = \frac{d_2}{d_1} \) (18 occurrences)
2.61.12 Construction MM85

Construction Process

$A, B$ given initial points
1. $k_1 = k(A, |AB|)$
2. $k_2 = k(B, |AB|)$
   $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
3. $k_3 = k(D, |AB|)$
   $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
4. $k_4 = k(D, |CE|)$
   $G \in k_1 \cap k_4, H \in k_2 \cap k_4$
5. $k_5 = k(H, |AB|)$
   $I \in k_2 \cap k_5, J \in k_4 \cap k_5, K \in k_4 \cap k_5$

Distances

$d_0 = |GK|$ (red) \ldots 1
$d_1 = |AF| = |AH| = |BE| = |BG| = |CD| = |CI| = |DI| = |EG| = |FH|$ (blue) \ldots 9
$d_2 = |GJ|$ (green) \ldots 1

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2}$ (18 occurrences)
2.62 Class of Similar Constructions No. 53

Contained Constructions

MM30, MM32, MM37, MM39, MM43, MM45, MM71, MM73, MM78, MM80, MM84, MM86
2.62.1 Construction MM30

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2, D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, |AB|) \)
   \( E \in k_1 \cap k_3, F \in k_2 \cap k_3 \)
4. \( k_4 = k(A, |DE|) \)
   \( G \in k_2 \cap k_4, H \in k_3 \cap k_4 \)
5. \( k_5 = k(G, |DE|) \)
   \( I \in k_1 \cap k_5, J \in k_1 \cap k_5, K \in k_4 \cap k_5 \)

Distances

\( d_0 = |HJ| = |IK| \) (red) \ldots 2
\( d_1 = |EJ| \) (blue) \ldots 1
\( d_2 = |AF| = |BE| = |BH| = |BK| = |CD| = |CG| = |DC| = |EH| \)
   (green) \ldots 8
\( d_3 = |BI| = |BJ| \) (cyan) \ldots 2
\( d_4 = |HI| = |JK| \) (magenta) \ldots 2
\( d_5 = |EI| \) (yellow) \ldots 1

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \] (36 occurrences)
2.62.2 Construction MM32

Construction Process

$A, B$ given initial points
1. $k_1 = k(A, |AB|)$
2. $k_2 = k(B, |AB|)$
   $C \in k_1 \cap k_2, \ D \in k_1 \cap k_2$
3. $k_3 = k(C, |AB|)$
   $E \in k_1 \cap k_3, \ F \in k_2 \cap k_3$
4. $k_4 = k(A, |DE|)$
   $G \in k_2 \cap k_4, \ H \in k_3 \cap k_4$
5. $k_5 = k(H, |DE|)$
   $I \in k_1 \cap k_5, \ J \in k_1 \cap k_5, \ K \in k_4 \cap k_5$

Distances

\[
d_0 = |GI| = |JK| \quad \text{(red)} \ldots 2
\]
\[
d_1 = |DI| \quad \text{(blue)} \ldots 1
\]
\[
d_2 = |AF| = |BE| = |BH| = |CD| = |CG| = |CK| = |DG| = |EH| \quad \text{(green)} \ldots 8
\]
\[
d_3 = |CI| = |CJ| \quad \text{(cyan)} \ldots 2
\]
\[
d_4 = |GJ| = |IK| \quad \text{(magenta)} \ldots 2
\]
\[
d_5 = |DJ| \quad \text{(yellow)} \ldots 1
\]

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_2} = \frac{d_2}{d_4} = \frac{d_4}{d_5} = \frac{d_1}{d_5}$ (36 occurrences)
2.62.3 Construction MM37

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2 \) , \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, |AB|) \)
   \( E \in k_1 \cap k_3 \) , \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(B, |DE|) \)
   \( G \in k_1 \cap k_4 \) , \( H \in k_3 \cap k_4 \)
5. \( k_5 = k(G, |DE|) \)
   \( I \in k_2 \cap k_5 \) , \( J \in k_2 \cap k_5 \) , \( K \in k_4 \cap k_5 \)

Distances

\( d_0 = |HJ| = |IK| \) (red) ... 2
\( d_1 = |FJ| \) (blue) ... 1
\( d_2 = |AF| = |AH| = |AK| = |BE| = |CD| = |CG| = |DG| = |FH| \)
   (green) ... 8
\( d_3 = |AI| = |AJ| \) (cyan) ... 2
\( d_4 = |HI| = |JK| \) (magenta) ... 2
\( d_5 = |FI| \) (yellow) ... 1

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \) (36 occurrences)
2.62.4 Construction MM39

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, |AB|) \)
   \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(B, |DE|) \)
   \( G \in k_1 \cap k_4 \), \( H \in k_3 \cap k_4 \)
5. \( k_5 = k(H, |DE|) \)
   \( I \in k_2 \cap k_5 \), \( J \in k_2 \cap k_5 \), \( K \in k_4 \cap k_5 \)

Distances

\( d_0 = |GJ| = |IK| \) (red) \( \ldots 2 \)
\( d_1 = |DJ| \) (blue) \( \ldots 1 \)
\( d_2 = |AF| = |AH| = |BE| = |CD| = |CG| = |CK| = |DG| = |FH| \) (green) \( \ldots 8 \)
\( d_3 = |CI| = |CJ| \) (cyan) \( \ldots 2 \)
\( d_4 = |GI| = |JK| \) (magenta) \( \ldots 2 \)
\( d_5 = |DI| \) (yellow) \( \ldots 1 \)

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_2} = \frac{d_4}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \) (36 occurrences)
2.62.5 Construction MM43

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, |AB|) \)
   \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(C, |DE|) \)
   \( G \in k_1 \cap k_4 \), \( H \in k_2 \cap k_4 \)
5. \( k_5 = k(G, |DE|) \)
   \( I \in k_3 \cap k_5 \), \( J \in k_3 \cap k_5 \), \( K \in k_4 \cap k_5 \)

Distances

\( d_0 = |HI| = |JK| \) (red) ... 2
\( d_1 = |FI| \) (blue) ... 1
\( d_2 = |AF| = |AH| = |AK| = |BE| = |BG| = |CD| = |EG| = |FH| \) (green) ... 8
\( d_3 = |AI| = |AJ| \) (cyan) ... 2
\( d_4 = |HJ| = |IK| \) (magenta) ... 2
\( d_5 = |FJ| \) (yellow) ... 1

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_2} = \frac{d_4}{d_1} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \) (36 occurrences)
2.62.6 Construction MM45

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
3. \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
4. \( k_3 = k(C, |AB|) \)
5. \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
6. \( k_4 = k(C, |DE|) \)
7. \( G \in k_1 \cap k_4 \), \( H \in k_2 \cap k_4 \)
8. \( I \in k_3 \cap k_5 \), \( J \in k_3 \cap k_5 \)
9. \( K \in k_4 \cap k_5 \)

Distances
\[ d_0 = |GJ| = |IK| \quad \text{(red)} \quad \ldots \quad 2 \]
\[ d_1 = |EJ| \quad \text{(blue)} \quad \ldots \quad 1 \]
\[ d_2 = |AF| = |AH| = |BE| = |BG| = |BK| = |CD| = |EG| = |FH| \quad \text{(green)} \quad \ldots \quad 8 \]
\[ d_3 = |BI| = |BJ| \quad \text{(cyan)} \quad \ldots \quad 2 \]
\[ d_4 = |GI| = |JK| \quad \text{(magenta)} \quad \ldots \quad 2 \]
\[ d_5 = |EI| \quad \text{(yellow)} \quad \ldots \quad 1 \]

Occurrences of the Golden Ratio
\[ \phi = \frac{d_0}{d_2} = \frac{d_4}{d_2} = \frac{d_2}{d_4} = \frac{d_4}{d_6} \quad \text{(36 occurrences)} \]
2.62.7 Construction MM71

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, |AB|) \)
   \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(A, |CE|) \)
   \( G \in k_2 \cap k_4 \), \( H \in k_3 \cap k_4 \)
5. \( k_5 = k(G, |CE|) \)
   \( I \in k_1 \cap k_5 \), \( J \in k_1 \cap k_5 \), \( K \in k_4 \cap k_5 \)

Distances

\[ d_0 = |HI| = |JK| \ (\text{red}) \ ... 2 \]
\[ d_1 = |EI| \ (\text{blue}) \ ... 1 \]
\[ d_2 = |AF| = |BE| = |BH| = |BK| = |CD| = |CG| = |DG| = |EH| \ (\text{green}) \ ... 8 \]
\[ d_3 = |BI| = |BJ| \ (\text{cyan}) \ ... 2 \]
\[ d_4 = |HJ| = |IK| \ (\text{magenta}) \ ... 2 \]
\[ d_5 = |EJ| \ (\text{yellow}) \ ... 1 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_4}{d_5} = \frac{d_2}{d_4} = \frac{d_4}{d_5} \ (36 \text{ occurrences}) \]
2.62.8 Construction MM73

Construction Process

A, B given initial points
1. $k_1 = k(A, |AB|)$
2. $k_2 = k(B, |AB|)$
   $C \in k_1 \cap k_2$, $D \in k_1 \cap k_2$
3. $k_3 = k(D, |AB|)$
   $E \in k_1 \cap k_3$, $F \in k_2 \cap k_3$
4. $k_4 = k(A, |CE|)$
   $G \in k_2 \cap k_4$, $H \in k_3 \cap k_4$
5. $k_5 = k(H, |CE|)$
   $I \in k_1 \cap k_5$, $J \in k_1 \cap k_5$, $K \in k_4 \cap k_5$

Distances

$d_0 = |GI| = |JK|$ (red) \ldots 2
$d_1 = |CI|$ (blue) \ldots 1
$d_2 = |AF| = |BE| = |BH| = |CD| = |CG| = |DG| = |DK| = |EH|$ (green) \ldots 8
$d_3 = |DI| = |DJ|$ (cyan) \ldots 2
$d_4 = |GJ| = |IK|$ (magenta) \ldots 2
$d_5 = |CJ|$ (yellow) \ldots 1

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_2} = \frac{d_2}{d_3} = \frac{d_3}{d_4} = \frac{d_4}{d_5}$ (36 occurrences)
2.62.9 Construction MM78

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2, \; D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, |AB|) \)
   \( E \in k_1 \cap k_3, \; F \in k_2 \cap k_3 \)
4. \( k_4 = k(B, |CE|) \)
   \( G \in k_1 \cap k_4, \; H \in k_3 \cap k_4 \)
5. \( k_5 = k(G, |CE|) \)
   \( I \in k_2 \cap k_5, \; J \in k_2 \cap k_5, \; K \in k_4 \cap k_5 \)

Distances

\[
d_0 = |HI| = |JK| \quad \text{(red)} \quad 2 \]
\[
d_1 = |FI| \quad \text{(blue)} \quad 1 \]
\[
d_2 = |AF| = |AH| = |AK| = |BE| = |CD| = |CG| = |DG| = |FH| \quad \text{(green)} \quad 8 \]
\[
d_3 = |AJ| = |AJ| \quad \text{(cyan)} \quad 2 \]
\[
d_4 = |HJ| = |IK| \quad \text{(magenta)} \quad 2 \]
\[
d_5 = |FJ| \quad \text{(yellow)} \quad 1 \]

Occurrences of the Golden Ratio

\[
\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_3}{d_4} = \frac{d_5}{d_5} \quad \text{(36 occurrences)}
\]
2.62.10 Construction MM80

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   C ∈ \( k_1 \cap k_2 \), D ∈ \( k_1 \cap k_2 \)
3. \( k_3 = k(D, |AB|) \)
   E ∈ \( k_1 \cap k_3 \), F ∈ \( k_2 \cap k_3 \)
4. \( k_4 = k(B, |CE|) \)
   G ∈ \( k_1 \cap k_4 \), H ∈ \( k_3 \cap k_4 \)
5. \( k_5 = k(H, |CE|) \)
   I ∈ \( k_2 \cap k_5 \), J ∈ \( k_2 \cap k_5 \), K ∈ \( k_4 \cap k_5 \)

Distances

\[
\begin{align*}
d_0 &= |GJ| = |IK| \text{ (red)} \ldots 2 \\
d_1 &= |CJ| \text{ (blue)} \ldots 1 \\
d_2 &= |AF| = |AH| = |BE| = |CD| = |CG| = |DG| = |DK| = |FH| \text{ (green)} \ldots 8 \\
d_3 &= |DI| = |DJ| \text{ (cyan)} \ldots 2 \\
d_4 &= |GI| = |JK| \text{ (magenta)} \ldots 2 \\
d_5 &= |CI| \text{ (yellow)} \ldots 1
\end{align*}
\]

Occurrences of the Golden Ratio

\[
\phi = \frac{d_0}{d_2} = \frac{d_4}{d_4} = \frac{d_2}{d_4} = \frac{d_4}{d_5} \text{ (36 occurrences)}
\]
2.62.11 Construction MM84

Construction Process

1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, |AB|) \)
   \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(D, |CE|) \)
   \( G \in k_1 \cap k_4 \), \( H \in k_2 \cap k_4 \)
5. \( k_5 = k(G, |CE|) \)
   \( I \in k_3 \cap k_5 \), \( J \in k_3 \cap k_5 \), \( K \in k_4 \cap k_5 \)

Distances

\( d_0 = |HI| = |JK| \) (red) \( \ldots 2 \)
\( d_1 = |FI| \) (blue) \( \ldots 1 \)
\( d_2 = |AF| = |AH| = |AK| = |BE| = |BG| = |CD| = |EG| = |FH| \) (green) \( \ldots 8 \)
\( d_3 = |AI| = |AJ| \) (cyan) \( \ldots 2 \)
\( d_4 = |HJ| = |IK| \) (magenta) \( \ldots 2 \)
\( d_5 = |FJ| \) (yellow) \( \ldots 1 \)

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_2} = \frac{d_4}{d_1} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \) (36 occurrences)
2.62.12 Construction MM86

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2, \) \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, |AB|) \)
   \( E \in k_1 \cap k_3, \) \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(D, |CE|) \)
   \( G \in k_1 \cap k_4, \) \( H \in k_2 \cap k_4 \)
5. \( k_5 = k(H, |CE|) \)
   \( I \in k_3 \cap k_5, \) \( J \in k_3 \cap k_5, \) \( K \in k_4 \cap k_5 \)

Distances

\[
\begin{align*}
  d_0 &= |GJ| = |IK| \quad \text{(red)} \quad \ldots 2 \\
  d_1 &= |EJ| \quad \text{(blue)} \quad \ldots 1 \\
  d_2 &= |AF| = |AH| = |BE| = |BG| = |BK| = |CD| = |EG| = |FH| \\
        &\quad \text{(green)} \quad \ldots 8 \\
  d_3 &= |BI| = |BJ| \quad \text{(cyan)} \quad \ldots 2 \\
  d_4 &= |GI| = |JK| \quad \text{(magenta)} \quad \ldots 2 \\
  d_5 &= |EI| \quad \text{(yellow)} \quad \ldots 1 
\end{align*}
\]

Occurrences of the Golden Ratio

\[
\phi = \frac{d_0}{d_2} = \frac{d_4}{d_1} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \quad \text{(36 occurrences)}
\]
2.63 Class of Similar Constructions No. 54

Contained Constructions
MM87, MM88, MM117, MM118
2.63.1 Construction MM87

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2, D \in k_1 \cap k_2 \)
3. \( k_3 = k(A, |CD|) \)
   \( E \in k_2 \cap k_3, F \in k_2 \cap k_3 \)
4. \( k_4 = k(C, |CD|) \)
   \( G \in k_1 \cap k_4, H \in k_2 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4 \)
5. \( k_5 = k(G, |DE|) \)
   \( K \in k_2 \cap k_5, L \in k_2 \cap k_5, M \in k_3 \cap k_5, N \in k_3 \cap k_5, O \in k_4 \cap k_5 \)
\( P \in k_4 \cap k_5 \)

Distances

\( d_0 = |KN| = |LM| \) (red) \ldots 2
\( d_1 = |EL| = |FK| \) (blue) \ldots 2
\( d_2 = |AE| = |AF| = |AI| = |AM| = |AN| = |CD| = |CG| =
|CH| = |CI| = |CJ| = |CO| = |CP| = |DG| = |DH| = |EF| = |EM| =
|FN| \) (green) \ldots 18
\( d_3 = |AK| = |AL| \) (cyan) \ldots 2
\( d_4 = |KM| = |LN| \) (magenta) \ldots 2
\( d_5 = |EK| = |FL| \) (yellow) \ldots 2

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \) (80 occurrences)
2.63.2 Construction MM88

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(A, |CD|) \)
   \( E \in k_2 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(D, |CD|) \)
   \( G \in k_1 \cap k_4 \), \( H \in k_2 \cap k_4 \), \( I \in k_3 \cap k_4 \), \( J \in k_3 \cap k_4 \)
5. \( k_5 = k(G, |DE|) \)
   \( K \in k_2 \cap k_5 \), \( L \in k_2 \cap k_5 \), \( M \in k_3 \cap k_5 \), \( N \in k_3 \cap k_5 \), \( O \in k_4 \cap k_5 \), \( P \in k_4 \cap k_5 \)

Distances

\[
\begin{align*}
d_0 &= |KN| = |LM| \text{ (red)} \ldots 2 \\
d_1 &= |EL| = |FK| \text{ (blue)} \ldots 2 \\
d_2 &= |AE| = |AF| = |AI| = |AM| = |AN| = |CD| = |CG| = |CH| = |DG| = |DH| = |DI| = |DO| = |DP| = |EF| = |EM| = |FN| \text{ (green)} \ldots 18 \\
d_3 &= |AK| = |AL| \text{ (cyan)} \ldots 2 \\
d_4 &= |KM| = |LN| \text{ (magenta)} \ldots 2 \\
d_5 &= |EK| = |FL| \text{ (yellow)} \ldots 2
\end{align*}
\]

Occurrences of the Golden Ratio

\[
\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (80 occurrences)}
\]
2.63.3 Construction MM117

Construction Process

A, B given initial points
1. $k_1 = k(A, |AB|)$
2. $k_2 = k(B, |AB|)$
   $C \in k_1 \cap k_2$, $D \in k_1 \cap k_2$
3. $k_3 = k(B, |CD|)$
   $E \in k_1 \cap k_3$, $F \in k_1 \cap k_3$
4. $k_4 = k(C, |CD|)$
   $G \in k_1 \cap k_4$, $H \in k_2 \cap k_4$, $I \in k_3 \cap k_4$, $J \in k_3 \cap k_4$
5. $k_5 = k(H, |DE|)$
   $K \in k_1 \cap k_5$, $L \in k_1 \cap k_5$, $M \in k_3 \cap k_5$, $N \in k_3 \cap k_5$, $O \in k_4 \cap k_5$, $P \in k_4 \cap k_5$

Distances

$d_0 = |KN| = |LM|$ (red) ... 2
$d_1 = |EL| = |FK|$ (blue) ... 2
$d_2 = |BE| = |BF| = |BI| = |BM| = |BN| = |CD| = |CG| = |CH| = |CI| = |CJ| = |CO| = |CP| = |DG| = |DH| = |EF| = |EM| = |FN|$ (green) ... 18
$d_3 = |BK| = |BL|$ (cyan) ... 2
$d_4 = |KM| = |LN|$ (magenta) ... 2
$d_5 = |EK| = |FL|$ (yellow) ... 2

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_2} = \frac{d_2}{d_4} = \frac{d_5}{d_5}$ (80 occurrences)
2.63.4 Construction MM118

Construction Process

\[ A, B \text{ given initial points} \]
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2, D \in k_1 \cap k_2 \)
3. \( k_3 = k(B, |CD|) \)
   \( E \in k_1 \cap k_3, F \in k_1 \cap k_3 \)
4. \( k_4 = k(D, |CD|) \)
   \( G \in k_1 \cap k_4, H \in k_2 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4 \)
5. \( k_5 = k(H, |DE|) \)
   \( K \in k_1 \cap k_5, L \in k_1 \cap k_5, M \in k_3 \cap k_5, N \in k_3 \cap k_5, O \in k_4 \cap k_5, P \in k_4 \cap k_5 \)

Distances

\[ d_0 = |KN| = |LM| \text{ (red)} \ldots 2 \]
\[ d_1 = |EL| = |FK| \text{ (blue)} \ldots 2 \]
\[ d_2 = |BE| = |BF| = |BI| = |BM| = |BN| = |CD| = |CG| = |CH| = |DG| = |DH| = |DI| = |DO| = |DP| = |EF| = |EM| = |FN| \text{ (green)} \ldots 18 \]
\[ d_3 = |BK| = |BL| \text{ (cyan)} \ldots 2 \]
\[ d_4 = |KM| = |LN| \text{ (magenta)} \ldots 2 \]
\[ d_5 = |EK| = |FL| \text{ (yellow)} \ldots 2 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (80 occurrences)} \]
2.64 Class of Similar Constructions No. 55

Contained Constructions

MM89, MM92, MM119, MM122
2.64.1 Construction MM89

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(A, |CD|) \)
   \( E \in k_2 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(E, |AB|) \)
   \( G \in k_2 \cap k_4 \), \( H \in k_3 \cap k_4 \), \( I \in k_3 \cap k_4 \)
5. \( k_5 = k(D, |DE|) \)
   \( J \in k_1 \cap k_5 \), \( K \in k_3 \cap k_5 \), \( L \in k_4 \cap k_5 \), \( M \in k_4 \cap k_5 \)

Distances

\[ d_0 = |JM| \text{ (red)} \ldots 1 \]
\[ d_1 = |AE| = |AF| = |AH| = |AI| = |AK| = |BJ| = |CD| = |CG| = |DG| = |EF| \text{ (blue)} \ldots 10 \]
\[ d_2 = |JL| \text{ (green)} \ldots 1 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (20 occurrences)} \]
2.64.2 Construction MM92

Construction Process

A, B given initial points
1. $k_1 = k(A, |AB|)$
2. $k_2 = k(B, |AB|)$
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. $k_3 = k(A, |CD|)$
   \( E \in k_2 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. $k_4 = k(F, |AB|)$
   \( G \in k_2 \cap k_4 \), \( H \in k_3 \cap k_4 \), \( I \in k_3 \cap k_4 \)
5. $k_5 = k(C, |DE|)$
   \( J \in k_1 \cap k_5 \), \( K \in k_3 \cap k_5 \), \( L \in k_4 \cap k_5 \), \( M \in k_4 \cap k_5 \)

Distances

\[
\begin{align*}
d_0 &= |JM| \quad \text{(red)} \ldots 1 \\
d_1 &= |AE| = |AF| = |AH| = |AI| = |AK| = |BJ| = |CD| = |CG| = |DG| = |EF| \quad \text{(blue)} \ldots 10 \\
d_2 &= |JL| \quad \text{(green)} \ldots 1 \\
\end{align*}
\]

Occurrences of the Golden Ratio

\[
\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \quad \text{(20 occurrences)}
\]
2.64.3 Construction MM119

Construction Process

A, B given initial points

1. \(k_1 = k(A, |AB|)\)
2. \(k_2 = k(B, |AB|)\)
   \(C \in k_1 \cap k_2\), \(D \in k_1 \cap k_2\)
3. \(k_3 = k(B, |CD|)\)
   \(E \in k_1 \cap k_3\), \(F \in k_1 \cap k_3\)
4. \(k_4 = k(E, |AB|)\)
   \(G \in k_1 \cap k_4\), \(H \in k_3 \cap k_4\), \(I \in k_3 \cap k_4\)
5. \(k_5 = k(D, |DE|)\)
   \(J \in k_2 \cap k_5\), \(K \in k_3 \cap k_5\), \(L \in k_4 \cap k_5\), \(M \in k_4 \cap k_5\)

Distances

\[d_0 = |JL| \text{ (red)} \ldots 1\]
\[d_1 = |AJ| = |BE| = |BF| = |BH| = |BI| = |BK| = |CD| = |CG| = |DG| = |EF| \text{ (blue)} \ldots 10\]
\[d_2 = |JM| \text{ (green)} \ldots 1\]

Occurrences of the Golden Ratio

\[\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (20 occurrences)}\]
2.64.4 Construction MM122

Construction Process

A, B given initial points
1. \(k_1 = k(A, |AB|)\)
2. \(k_2 = k(B, |AB|)\)
   \(C \in k_1 \cap k_2\), \(D \in k_1 \cap k_2\)
3. \(k_3 = k(B, |CD|)\)
   \(E \in k_1 \cap k_3\), \(F \in k_1 \cap k_3\)
4. \(k_4 = k(F, |AB|)\)
   \(G \in k_1 \cap k_4\), \(H \in k_3 \cap k_4\), \(I \in k_3 \cap k_4\)
5. \(k_5 = k(C, |DE|)\)
   \(J \in k_2 \cap k_5\), \(K \in k_3 \cap k_5\), \(L \in k_4 \cap k_5\), \(M \in k_4 \cap k_5\)

Distances

\(d_0 = |JL|\) (red) \ldots 1
\(d_1 = |AJ| = |BE| = |BF| = |BH| = |BI| = |BK| = |CD| = |CG| = |DG| = |EF|\) (blue) \ldots 10
\(d_2 = |JM|\) (green) \ldots 1

Occurrences of the Golden Ratio

\(\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2}\) (20 occurrences)
2.65 Class of Similar Constructions No. 56

Contained Constructions

MM90, MM93, MM120, MM123
2.65.1 Construction MM90

Construction Process

1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2, D \in k_1 \cap k_2 \)
3. \( k_3 = k(A, |CD|) \)
   \( E \in k_2 \cap k_3, F \in k_2 \cap k_3 \)
4. \( k_4 = k(E, |AB|) \)
   \( G \in k_2 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4 \)
5. \( k_5 = k(G, |DE|) \)
   \( J \in k_1 \cap k_5, K \in k_1 \cap k_5, L \in k_3 \cap k_5, M \in k_3 \cap k_5, N \in k_4 \cap k_5 \)

Distances

\[
\begin{align*}
d_0 &= |KN| \quad \text{(red)} \quad \ldots 1 \\
d_1 &= |AE| = |AF| = |AH| = |AI| = |AL| = |AM| = |BN| = |CD| = |CG| = |DG| = |EF| \quad \text{(blue)} \quad \ldots 11 \\
d_2 &= |JN| \quad \text{(green)} \quad \ldots 1
\end{align*}
\]

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \) (22 occurrences)
2.65.2 Construction MM93

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2, D \in k_1 \cap k_2 \)
3. \( k_3 = k(A, |CD|) \)
   \( E \in k_2 \cap k_3, F \in k_2 \cap k_3 \)
4. \( k_4 = k(F, |AB|) \)
   \( G \in k_2 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4 \)
5. \( k_5 = k(G, |DE|) \)
   \( J \in k_1 \cap k_5, K \in k_1 \cap k_5, L \in k_3 \cap k_5, M \in k_3 \cap k_5, N \in k_4 \cap k_5 \)

Distances

\( d_0 = |JN| \) (red) \( \ldots 1 \)
\( d_1 = |AE| = |AF| = |AH| = |AI| = |AL| = |AM| = |BN| = |CD| = |CG| = |DG| = |EF| \) (blue) \( \ldots 11 \)
\( d_2 = |KN| \) (green) \( \ldots 1 \)

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} = \frac{d_2}{d_2} \) (22 occurrences)
2.65.3 Construction MM120

Construction Process

1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2, D \in k_1 \cap k_2 \)
3. \( k_3 = k(B, |CD|) \)
   \( E \in k_1 \cap k_3, F \in k_1 \cap k_3 \)
4. \( k_4 = k(E, |AB|) \)
   \( G \in k_1 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4 \)
5. \( k_5 = k(G, |DE|) \)
   \( J \in k_2 \cap k_5, K \in k_2 \cap k_5, L \in k_3 \cap k_5, M \in k_3 \cap k_5, N \in k_4 \cap k_5 \)

Distances

\[
\begin{align*}
d_0 &= |KN| \text{ (red)} \ldots 1 \\
d_1 &= |AN| = |BE| = |BF| = |BH| = |BI| = |BL| = |BM| = |CD| = |CG| = |DG| = |EF| \text{ (blue)} \ldots 11 \\
d_2 &= |JN| \text{ (green)} \ldots 1
\end{align*}
\]

Occurrences of the Golden Ratio

\[
\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (22 occurrences)}
\]
2.65.4 Construction MM123

Construction Process

\[ A, B \text{ given initial points} \]

1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2, D \in k_1 \cap k_2 \)
3. \( k_3 = k(B, |CD|) \)
   \( E \in k_1 \cap k_3, F \in k_1 \cap k_3 \)
4. \( k_4 = k(F, |AB|) \)
   \( G \in k_1 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4 \)
5. \( k_5 = k(G, |DE|) \)
   \( J \in k_2 \cap k_5, K \in k_2 \cap k_5, L \in k_3 \cap k_5, M \in k_3 \cap k_5, N \in k_4 \cap k_5 \)

Distances

\[ d_0 = |JN| \text{ (red)} \ldots 1 \]
\[ d_1 = |AN| = |BE| = |BF| = |BH| = |BI| = |BL| = |BM| = |CD| = |CG| = |DG| = |EF| \text{ (blue)} \ldots 11 \]
\[ d_2 = |KN| \text{ (green)} \ldots 1 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} = \frac{d_2}{d_1} \text{ (22 occurrences)} \]
2.66 Class of Similar Constructions No. 57

Contained Constructions

MM91, MM94, MM121, MM124
2.66.1 Construction MM91

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2, D \in k_1 \cap k_2 \)
3. \( k_3 = k(A, |CD|) \)
   \( E \in k_2 \cap k_3, F \in k_2 \cap k_3 \)
4. \( k_4 = k(E, |CD|) \)
   \( G \in k_1 \cap k_4, H \in k_1 \cap k_4, I \in k_3 \cap k_4 \)
5. \( k_5 = k(I, |DE|) \)
   \( J \in k_1 \cap k_5, K \in k_2 \cap k_5, L \in k_2 \cap k_5, M \in k_3 \cap k_5, N \in k_3 \cap k_5, O \in k_4 \cap k_5, P \in k_4 \cap k_5 \)

Distances

\( d_0 = |JL| \) (red) ... 1
\( d_1 = |DL| \) (blue) ... 1
\( d_2 = |AE| = |AF| = |AI| = |AM| = |AN| = |CD| = |CJ| = |DJ| = |EF| = |EG| = |EH| = |EI| = |EO| = |EP| \) (green) ... 14
\( d_3 = |CK| = |CL| \) (cyan) ... 2
\( d_4 = |JK| \) (magenta) ... 1
\( d_5 = |DK| \) (yellow) ... 1

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \) (32 occurrences)
Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2, \ D \in k_1 \cap k_2 \)
3. \( k_3 = k(A, |CD|) \)
   \( E \in k_2 \cap k_3, \ F \in k_2 \cap k_3 \)
4. \( k_4 = k(F, |CD|) \)
   \( G \in k_1 \cap k_4, \ H \in k_1 \cap k_4, \ I \in k_3 \cap k_4 \)
5. \( k_5 = k(I, |DE|) \)
   \( J \in k_1 \cap k_5, \ K \in k_2 \cap k_5, \ L \in k_2 \cap k_5, \ M \in k_3 \cap k_5, \ N \in k_3 \cap k_5, \)
   \( O \in k_4 \cap k_5, \ P \in k_4 \cap k_5 \)

Distances

\[ d_0 = |JL| \quad (\text{red}) \quad 1 \]
\[ d_1 = |CL| \quad (\text{blue}) \quad 1 \]
\[ d_2 = |AE| = |AF| = |AI| = |AM| = |AN| = |CD| = |CJ| = |DJ| = |EF| = |FG| = |FH| = |FI| = |FO| = |FP| \quad (\text{green}) \quad 14 \]
\[ d_3 = |DK| = |DL| \quad (\text{cyan}) \quad 2 \]
\[ d_4 = |JK| \quad (\text{magenta}) \quad 1 \]
\[ d_5 = |CK| \quad (\text{yellow}) \quad 1 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_4}{d_5} \quad (32 \text{ occurrences}) \]
2.66.3 Construction MM121

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(B, |CD|) \)
   \( E \in k_1 \cap k_3 \), \( F \in k_1 \cap k_3 \)
4. \( k_4 = k(E, |CD|) \)
   \( G \in k_2 \cap k_4 \), \( H \in k_2 \cap k_4 \), \( I \in k_2 \cap k_4 \)
5. \( k_5 = k(I, |DE|) \)
   \( J \in k_1 \cap k_5 \), \( K \in k_1 \cap k_5 \), \( L \in k_2 \cap k_5 \), \( M \in k_3 \cap k_5 \), \( N \in k_3 \cap k_5 \), \( O \in k_4 \cap k_5 \), \( P \in k_4 \cap k_5 \)

Distances

\( d_0 = |JL| \) (red) ... 1
\( d_1 = |DJ| \) (blue) ... 1
\( d_2 = |BE| = |BF| = |BI| = |BM| = |BN| = |CD| = |CL| = |DL| = |EF| = |EG| = |EH| = |EI| = |EO| = |EP| \) (green) ... 14
\( d_3 = |CJ| = |CK| \) (cyan) ... 2
\( d_4 = |KL| \) (magenta) ... 1
\( d_5 = |DK| \) (yellow) ... 1

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_4}{d_5} = \frac{d_5}{d_2} \] (32 occurrences)
2.66.4 Construction MM124

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(B, |CD|) \)
   \( E \in k_1 \cap k_3 \), \( F \in k_1 \cap k_3 \)
4. \( k_4 = k(F, |CD|) \)
   \( G \in k_2 \cap k_4 \), \( H \in k_2 \cap k_4 \), \( I \in k_3 \cap k_4 \)
5. \( k_5 = k(I, |DE|) \)
   \( J \in k_1 \cap k_5 \), \( K \in k_1 \cap k_5 \), \( L \in k_2 \cap k_5 \), \( M \in k_3 \cap k_5 \), \( N \in k_3 \cap k_5 \), \( O \in k_4 \cap k_5 \), \( P \in k_4 \cap k_5 \)

Distances

\( d_0 = |JL| \) (red) ... 1
\( d_1 = |CJ| \) (blue) ... 1
\( d_2 = |BE| = |BF| = |BI| = |BM| = |BN| = |CD| = |CL| = |DL| = |EF| = |FG| = |FH| = |FI| = |FO| = |FP| \) (green) ... 14
\( d_3 = |DJ| = |DK| \) (cyan) ... 2
\( d_4 = |KL| \) (magenta) ... 1
\( d_5 = |CK| \) (yellow) ... 1

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_2} = \frac{d_4}{d_5} = \frac{d_5}{d_4} = \frac{d_1}{d_3} \) (32 occurrences)
2.67 Class of Similar Constructions No. 58

Contained Constructions

MM95, MM125
2.67.1 Construction MM95

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2, \ D \in k_1 \cap k_2 \)
3. \( k_3 = k(A, |CD|) \)
   \( E \in k_2 \cap k_3, \ F \in k_2 \cap k_3 \)
4. \( k_4 = k(A, |DE|) \)
   \( G \in k_2 \cap k_4 \)
5. \( k_5 = k(B, |DE|) \)
   \( H \in k_1 \cap k_5, \ I \in k_3 \cap k_5, \ J \in k_3 \cap k_5, \ K \in k_4 \cap k_5, \ L \in k_4 \cap k_5 \)

Distances

\( d_0 = |CL| = |DK| \) (red) ... 2
\( d_1 = |AE| = |AF| = |AJ| = |CD| = |CG| = |CH| = |DG| = |DH| = |EF| = |EI| = |FJ| \) (blue) ... 12
\( d_2 = |CK| = |DL| \) (green) ... 2

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} = \frac{d_2}{d_5} \) (48 occurrences)
2.67.2 Construction MM125

Construction Process

1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2, D \in k_1 \cap k_2 \)
3. \( k_3 = k(B, |CD|) \)
   \( E \in k_1 \cap k_3, F \in k_1 \cap k_3 \)
4. \( k_4 = k(A, |DE|) \)
   \( G \in k_2 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4 \)
5. \( k_5 = k(B, |DE|) \)
   \( J \in k_1 \cap k_5, K \in k_4 \cap k_5, L \in k_4 \cap k_5 \)

Distances

\[
d_0 = |CL| = |DK| \quad \text{(red)} \ldots 2
\]
\[
d_1 = |BE| = |BF| = |BH| = |BI| = |CD| = |CG| = |CJ| = |DG| =
|DJ| = |EF| = |EH| = |FI| \quad \text{(blue)} \ldots 12
\]
\[
d_2 = |CK| = |DL| \quad \text{(green)} \ldots 2
\]

Occurrences of the Golden Ratio

\[
\phi = \frac{d_0}{d_1} = \frac{d_4}{d_2} \quad \text{(48 occurrences)}
\]
2.68 Class of Similar Constructions No. 59

Contained Constructions

MM96, MM97, MM136, MM137
2.68.1 Construction MM96

Construction Process

A, B given initial points

1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(A, |CD|) \)
   \( E \in k_2 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(A, |DE|) \)
   \( G \in k_2 \cap k_4 \)
5. \( k_5 = k(C, |DE|) \)
   \( H \in k_1 \cap k_5 \), \( I \in k_3 \cap k_5 \), \( J \in k_4 \cap k_5 \), \( K \in k_4 \cap k_5 \)

Distances

\[
d_0 = |BJ| \quad \text{(red)} \quad 1
\]
\[
d_1 = |AE| = |AF| = |AI| = |BH| = |CD| = |CG| = |DG| = |EF| \quad \text{(blue)}
\]
\[
\ldots 8
\]
\[
d_2 = |BK| \quad \text{(green)} \quad 1
\]

Occurrences of the Golden Ratio

\[
\phi = \frac{d_0}{d_1} = \frac{d_2}{d_5} \quad \text{(16 occurrences)}
\]
2.68.2 Construction MM97

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2, D \in k_1 \cap k_2 \)
3. \( k_3 = k(A, |CD|) \)
   \( E \in k_2 \cap k_3, F \in k_2 \cap k_3 \)
4. \( k_4 = k(A, |DE|) \)
   \( G \in k_2 \cap k_4 \)
5. \( k_5 = k(D, |DE|) \)
   \( H \in k_1 \cap k_5, I \in k_3 \cap k_5, J \in k_4 \cap k_5, K \in k_4 \cap k_5 \)

Distances

\( d_0 = |BJ| \) (red) \ldots 1
\( d_1 = |AE| = |AF| = |AI| = |BH| = |CD| = |CG| = |DG| = |EF| \) (blue)
\ldots 8
\( d_2 = |BK| \) (green) \ldots 1

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \) (16 occurrences)
2.68.3 Construction MM136

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(B, |CD|) \)
   \( E \in k_1 \cap k_3 \), \( F \in k_1 \cap k_3 \)
4. \( k_4 = k(B, |DE|) \)
   \( G \in k_1 \cap k_4 \)
5. \( k_5 = k(C, |DE|) \)
   \( H \in k_2 \cap k_5 \), \( I \in k_3 \cap k_5 \), \( J \in k_4 \cap k_5 \), \( K \in k_4 \cap k_5 \)

Distances

\[
\begin{align*}
d_0 &= |AK| \quad \text{(red)} \ldots 1 \\
d_1 &= |AH| = |BE| = |BF| = |BI| = |CD| = |CG| = |DG| = |EF| \quad \text{(blue)} \\
&\ldots 8 \\
d_2 &= |AJ| \quad \text{(green)} \ldots 1
\end{align*}
\]

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} = \frac{d_2}{d_5} \) (16 occurrences)
2.68.4 Construction MM137

Construction Process

- A, B given initial points
- 1. \( k_1 = k(A, |AB|) \)
- 2. \( k_2 = k(B, |AB|) \)
- \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
- 3. \( k_3 = k(B, |CD|) \)
- \( E \in k_1 \cap k_3 \), \( F \in k_1 \cap k_3 \)
- 4. \( k_4 = k(B, |DE|) \)
- \( G \in k_1 \cap k_4 \)
- 5. \( k_5 = k(D, |DE|) \)
- \( H \in k_2 \cap k_5 \), \( I \in k_3 \cap k_5 \), \( J \in k_4 \cap k_5 \), \( K \in k_4 \cap k_5 \)

Distances

- \( d_0 = |AK| \) (red) \( \ldots 1 \)
- \( d_1 = |AH| = |BE| = |BF| = |BI| = |CD| = |CG| = |DG| = |EF| \) (blue)
- \( \ldots 8 \)
- \( d_2 = |AJ| \) (green) \( \ldots 1 \)

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} = \frac{d_2}{d_5} \] (16 occurrences)
2.69 Class of Similar Constructions No. 60

Contained Constructions

MM98, MM138
2.69.1 Construction MM98

Construction Process

$A, B$ given initial points
1. $k_1 = k(A, |AB|)$
2. $k_2 = k(B, |AB|)$
   $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
3. $k_3 = k(A, |CD|)$
   $E \in k_2 \cap k_3, F \in k_2 \cap k_3$
4. $k_4 = k(A, |DE|)$
   $G \in k_2 \cap k_4$
5. $k_5 = k(G, |DE|)$
   $H \in k_1 \cap k_5, I \in k_1 \cap k_5, J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5, M \in k_4 \cap k_5$

Distances

$d_0 = |HM| = |IL| \ (red) \ldots 2$
$d_1 = |AE| = |AF| = |AJ| = |AK| = |BL| = |BM| = |CD| = |CG| = |DG| = |EF| \ (blue) \ldots 10$
$d_2 = |HL| = |IM| \ (green) \ldots 2$

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_1} = \frac{d_0}{d_2} \ (40 \ occurrences)$
2.69.2 Construction MM138

Construction Process

- A, B given initial points
- 1. \( k_1 = k(A, |AB|) \)
- 2. \( k_2 = k(B, |AB|) \)
  - \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
- 3. \( k_3 = k(B, |CD|) \)
  - \( E \in k_1 \cap k_3 \), \( F \in k_1 \cap k_3 \)
- 4. \( k_4 = k(B, |DE|) \)
  - \( G \in k_1 \cap k_4 \)
- 5. \( k_5 = k(G, |DE|) \)
  - \( H \in k_2 \cap k_5 \), \( I \in k_2 \cap k_5 \), \( J \in k_3 \cap k_5 \), \( K \in k_3 \cap k_5 \), \( L \in k_4 \cap k_5 \)
  - \( M \in k_4 \cap k_5 \)

Distances

- \( d_0 = |HM| = |IL| \) (red) ... 2
- \( d_1 = |AL| = |AM| = |BE| = |BF| = |BJ| = |BK| = |CD| = |CG| = |DG| = |EF| \) (blue) ... 10
- \( d_2 = |HL| = |IM| \) (green) ... 2

Occurrences of the Golden Ratio

- \( \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \) (40 occurrences)
2.70 Class of Similar Constructions No. 61

Contained Constructions

MM99, MM100, MM126, MM127
2.70.1 Construction MM99

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2 , D \in k_1 \cap k_2 \)
3. \( k_3 = k(A, |CD|) \)
   \( E \in k_2 \cap k_3 , F \in k_2 \cap k_3 \)
4. \( k_4 = k(B, |DE|) \)
   \( G \in k_1 \cap k_4 , H \in k_3 \cap k_4 , I \in k_3 \cap k_4 \)
5. \( k_5 = k(C, |DE|) \)
   \( J \in k_1 \cap k_5 , K \in k_3 \cap k_5 , L \in k_4 \cap k_5 , M \in k_4 \cap k_5 \)

Distances

\[ d_0 = |IM| = |KM| \quad (\text{red}) \quad 2 \]
\[ d_1 = |AM| = |EL| \quad (\text{blue}) \quad 2 \]
\[ d_2 = |FL| = |FM| = |HL| = |HM| \quad (\text{green}) \quad 4 \]
\[ d_3 = |AE| = |AF| = |AH| = |AI| = |AK| = |BJ| = |CD| = |CG| = |DG| = |EF| = |EH| = |FI| = |HK| \quad (\text{cyan}) \quad 13 \]
\[ d_4 = |IL| = |KL| \quad (\text{magenta}) \quad 2 \]
\[ d_5 = |AL| = |EM| \quad (\text{yellow}) \quad 2 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \quad (68 \text{ occurrences}) \]
2.70.2 Construction MM100

Construction Process

\( A, B \) given initial points

1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2, D \in k_1 \cap k_2 \)
3. \( k_3 = k(A, |CD|) \)
   \( E \in k_2 \cap k_3, F \in k_2 \cap k_3 \)
4. \( k_4 = k(B, |DE|) \)
   \( G \in k_1 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4 \)
5. \( k_5 = k(D, |DE|) \)
   \( J \in k_1 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5, M \in k_4 \cap k_5 \)

Distances

\[ d_0 = |HM| = |KM| \quad \text{red} \ldots 2 \]
\[ d_1 = |AM| = |FL| \quad \text{blue} \ldots 2 \]
\[ d_2 = |EL| = |EM| = |IL| = |IM| \quad \text{green} \ldots 4 \]
\[ d_3 = |AE| = |AF| = |AH| = |AI| = |AK| = |BJ| = |CD| = |CG| = \]
\[ |DG| = |EF| = |EH| = |FI| = |IK| \quad \text{cyan} \ldots 13 \]
\[ d_4 = |HL| = |KL| \quad \text{magenta} \ldots 2 \]
\[ d_5 = |AL| = |FM| \quad \text{yellow} \ldots 2 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_4}{d_5} \quad \text{(68 occurrences)} \]
2.70.3 Construction MM126

Construction Process

\(A, B\) given initial points

1. \(k_1 = k(A, |AB|)\)
2. \(k_2 = k(B, |AB|)\)
   \(C \in k_1 \cap k_2\), \(D \in k_1 \cap k_2\)
3. \(k_3 = k(B, |CD|)\)
   \(E \in k_1 \cap k_3\), \(F \in k_1 \cap k_3\)
4. \(k_4 = k(A, |DE|)\)
   \(G \in k_2 \cap k_4\), \(H \in k_3 \cap k_4\), \(I \in k_3 \cap k_4\)
5. \(k_5 = k(C, |DE|)\)
   \(J \in k_2 \cap k_5\), \(K \in k_3 \cap k_5\), \(L \in k_4 \cap k_5\), \(M \in k_4 \cap k_5\)

Distances

\(d_0 = |IL| = |KL|\) (red) \ldots 2
\(d_1 = |BL| = |EM|\) (blue) \ldots 2
\(d_2 = |FL| = |FM| = |HL| = |HM|\) (green) \ldots 4
\(d_3 = |AJ| = |BE| = |BF| = |BH| = |BI| = |BK| = |CD| = |CG| =
|DG| = |EF| = |EH| = |FI| = |HK|\) (cyan) \ldots 13
\(d_4 = |IM| = |KM|\) (magenta) \ldots 2
\(d_5 = |BM| = |EL|\) (yellow) \ldots 2

Occurrences of the Golden Ratio

\(\phi = \frac{d_4}{d_5} = \frac{d_3}{d_4} = \frac{d_2}{d_3} = \frac{d_1}{d_2}\) (68 occurrences)
2.70.4 Construction MM127

Construction Process

A, B given initial points
1. \(k_1 = k(A, |AB|)\)
2. \(k_2 = k(B, |AB|)\)
   \(C \in k_1 \cap k_2, D \in k_1 \cap k_2\)
3. \(k_3 = k(B, |CD|)\)
   \(E \in k_1 \cap k_3, F \in k_1 \cap k_3\)
4. \(k_4 = k(A, |DE|)\)
   \(G \in k_2 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4\)
5. \(k_5 = k(D, |DE|)\)
   \(J \in k_2 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5, M \in k_4 \cap k_5\)

Distances

\(d_0 = |HL| = |KL| \) (red) \ldots 2
\(d_1 = |BL| = |FM| \) (blue) \ldots 2
\(d_2 = |EL| = |EM| = |IL| = |IM| \) (green) \ldots 4
\(d_3 = |AJ| = |BE| = |BF| = |BH| = |BI| = |BK| = |CD| = |CG| = |DG| = |EF| = |EH| = |FI| = |IK| \) (cyan) \ldots 13
\(d_4 = |HM| = |KM| \) (magenta) \ldots 2
\(d_5 = |BM| = |FL| \) (yellow) \ldots 2

Occurrences of the Golden Ratio

\(\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \) (68 occurrences)
2.71 Class of Similar Constructions No. 62

Contained Constructions

MM101, MM102, MM128, MM129
2.71.1 Construction MM101

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2, \ D \in k_1 \cap k_2 \)
3. \( k_3 = k(A, |CD|) \)
   \( E \in k_2 \cap k_3, \ F \in k_2 \cap k_3 \)
4. \( k_4 = k(B, |DE|) \)
   \( G \in k_1 \cap k_4, \ H \in k_3 \cap k_4, \ I \in k_3 \cap k_4 \)
5. \( k_5 = k(E, |DE|) \)
   \( J \in k_1 \cap k_5, K \in k_3 \cap k_5, L \in k_3 \cap k_5, M \in k_4 \cap k_5, N \in k_4 \cap k_5 \)

Distances

\[
\begin{align*}
d_0 &= |GN| \text{ (red)} \ldots 1 \\
d_1 &= |CN| \text{ (blue)} \ldots 1 \\
d_2 &= |DM| = |DN| \text{ (green)} \ldots 2 \\
d_3 &= |AE| = |AF| = |AH| = |AI| = |AK| = |AL| = |BJ| = |CD| = |CG| = |DG| = |EF| = |EH| = |FI| \text{ (cyan)} \ldots 13 \\
d_4 &= |GM| \text{ (magenta)} \ldots 1 \\
d_5 &= |CM| \text{ (yellow)} \ldots 1
\end{align*}
\]

Occurrences of the Golden Ratio

\[
\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (30 occurrences)}
\]
2.71.2 Construction MM102

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2, \ D \in k_1 \cap k_2 \)
3. \( k_3 = k(A, |CD|) \)
   \( E \in k_2 \cap k_3, \ F \in k_2 \cap k_3 \)
4. \( k_4 = k(B, |DE|) \)
   \( G \in k_1 \cap k_4, \ H \in k_3 \cap k_4, \ I \in k_3 \cap k_4 \)
5. \( k_5 = k(F, |DE|) \)
   \( J \in k_1 \cap k_5, \ K \in k_3 \cap k_5, \ L \in k_3 \cap k_5, \ M \in k_4 \cap k_5, \ N \in k_4 \cap k_5 \)

Distances

\( d_0 = |GN| \) (red) \ldots 1
\( d_1 = |DN| \) (blue) \ldots 1
\( d_2 = |CM| = |CN| \) (green) \ldots 2
\( d_3 = |AE| = |AF| = |AH| = |AI| = |AK| = |AL| = |BJ| = |CD| = |CG| = |DG| = |EF| = |EH| = |FI| \) (cyan) \ldots 13
\( d_4 = |GM| \) (magenta) \ldots 1
\( d_5 = |DM| \) (yellow) \ldots 1

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \) (30 occurrences)
2.71.3 Construction MM128

Construction Process

- A, B given initial points
- 1. \( k_1 = k(A, |AB|) \)
- 2. \( k_2 = k(B, |AB|) \)
- \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
- 3. \( k_3 = k(B, |CD|) \)
- \( E \in k_1 \cap k_3 \), \( F \in k_1 \cap k_3 \)
- 4. \( k_4 = k(A, |DE|) \)
- \( G \in k_2 \cap k_4 \), \( H \in k_3 \cap k_4 \), \( I \in k_3 \cap k_4 \)
- 5. \( k_5 = k(E, |DE|) \)
- \( J \in k_2 \cap k_5 \), \( K \in k_3 \cap k_5 \), \( L \in k_3 \cap k_5 \), \( M \in k_4 \cap k_5 \), \( N \in k_4 \cap k_5 \)

Distances

- \( d_0 = |GM| \) (red) \ldots 1
- \( d_1 = |CM| \) (blue) \ldots 1
- \( d_2 = |DM| = |DN| \) (green) \ldots 2
- \( d_3 = |AJ| = |BE| = |BF| = |BH| = |BI| = |BK| = |BL| = |CD| = |CG| = |DG| = |EF| = |EH| = |FI| \) (cyan) \ldots 13
- \( d_4 = |GN| \) (magenta) \ldots 1
- \( d_5 = |CN| \) (yellow) \ldots 1

Occurrences of the Golden Ratio

- \( \phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \) (30 occurrences)
2.71.4 Construction MM129

Construction Process

A, B given initial points
1. $k_1 = k(A, |AB|)$
2. $k_2 = k(B, |AB|)$
   $C \in k_1 \cap k_2$, $D \in k_1 \cap k_2$
3. $k_3 = k(B, |CD|)$
   $E \in k_1 \cap k_3$, $F \in k_1 \cap k_3$
4. $k_4 = k(A, |DE|)$
   $G \in k_2 \cap k_4$, $H \in k_3 \cap k_4$, $I \in k_3 \cap k_4$
5. $k_5 = k(F, |DE|)$
   $J \in k_2 \cap k_5$, $K \in k_3 \cap k_5$, $L \in k_3 \cap k_5$, $M \in k_4 \cap k_5$, $N \in k_4 \cap k_5$

Distances

$d_0 = |GM|$ (red) ... 1
$d_1 = |DM|$ (blue) ... 1
$d_2 = |CM| = |CN|$ (green) ... 2
$d_3 = |AJ| = |BE| = |BF| = |BH| = |BI| = |BK| = |BL| = |CD| = |CG| = |DG| = |EF| = |EH| = |FI|$ (cyan) ... 13
$d_4 = |GN|$ (magenta) ... 1
$d_5 = |DN|$ (yellow) ... 1

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5}$ (30 occurrences)
2.72 Class of Similar Constructions No. 63

Contained Constructions

MM103, MM130
2.72.1 Construction MM103

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(A, [CD]) \)
   \( E \in k_2 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(B, [DE]) \)
   \( G \in k_1 \cap k_4 \), \( H \in k_3 \cap k_4 \), \( I \in k_3 \cap k_4 \)
5. \( k_5 = k(G, |AB|) \)
   \( J \in k_1 \cap k_5 \), \( K \in k_1 \cap k_5 \), \( L \in k_3 \cap k_5 \), \( M \in k_3 \cap k_5 \), \( N \in k_4 \cap k_5 \), \( O \in k_4 \cap k_5 \)

Distances

\[ d_0 = |HO| = |IN| \quad \text{(red)} \quad \ldots 2 \]
\[ d_1 = |LO| = |MN| \quad \text{(blue)} \quad \ldots 2 \]
\[ d_2 = |AE| = |AF| = |AH| = |AI| = |AL| = |AM| = |BJ| = |BK| = |CD| = |CG| = |DG| = |EF| = |EH| = |FI| = |HL| = |IM| = |JK| = |LM| \quad \text{(green)} \quad \ldots 18 \]
\[ d_3 = |AN| = |AO| \quad \text{(cyan)} \quad \ldots 2 \]
\[ d_4 = |HN| = |IO| \quad \text{(magenta)} \quad \ldots 2 \]
\[ d_5 = |LN| = |MO| \quad \text{(yellow)} \quad \ldots 2 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_1}{d_2} = \frac{d_2}{d_4} = \frac{d_4}{d_5} \quad \text{(80 occurrences)} \]
2.72.2 Construction MM130

Construction Process

\[ A, B \text{ given initial points} \]

1. \[ k_1 = k(A, |AB|) \]
2. \[ k_2 = k(B, |AB|) \]
   \[ C \in k_1 \cap k_2 , D \in k_1 \cap k_2 \]
3. \[ k_3 = k(B, |CD|) \]
   \[ E \in k_1 \cap k_3 , F \in k_1 \cap k_3 \]
4. \[ k_4 = k(A, |DE|) \]
   \[ G \in k_2 \cap k_4 , H \in k_3 \cap k_4 , I \in k_3 \cap k_4 \]
5. \[ k_5 = k(G, |AB|) \]
   \[ J \in k_2 \cap k_5 , K \in k_2 \cap k_5 , L \in k_3 \cap k_5 , M \in k_3 \cap k_5 , N \in k_4 \cap k_5 , O \in k_4 \cap k_5 \]

Distances

\[ d_0 = |HO| = |IN| \text{ (red)} \ldots 2 \]
\[ d_1 = |LO| = |MN| \text{ (blue)} \ldots 2 \]
\[ d_2 = |AJ| = |AK| = |BE| = |BF| = |BH| = |BI| = |BL| = |BM| = |CD| = |CG| = |DG| = |EF| = |EH| = |FI| = |HL| = |IM| = |JK| = |LM| \text{ (green)} \ldots 18 \]
\[ d_3 = |BN| = |BO| \text{ (cyan)} \ldots 2 \]
\[ d_4 = |HN| = |IO| \text{ (magenta)} \ldots 2 \]
\[ d_5 = |LN| = |MO| \text{ (yellow)} \ldots 2 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (80 occurrences)} \]
2.73 Class of Similar Constructions No. 64

Contained Constructions

MM104, MM131
2.73.1 Construction MM104

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2, \ D \in k_1 \cap k_2 \)
3. \( k_3 = k(A, |CD|) \)
   \( E \in k_2 \cap k_3, \ F \in k_2 \cap k_3 \)
4. \( k_4 = k(B, |DE|) \)
   \( G \in k_1 \cap k_4, \ H \in k_3 \cap k_4, \ I \in k_3 \cap k_4 \)
5. \( k_5 = k(G, |DE|) \)
   \( J \in k_2 \cap k_5, \ K \in k_2 \cap k_5 \)

Distances

\( d_0 = |HK| = |IJ| \) (red) \ldots 2
\( d_1 = |EK| = |FJ| \) (blue) \ldots 2
\( d_2 = |AE| = |AF| = |AH| = |AI| = |CD| = |CG| = |DG| = |EF| = |EH| = |FI| \) (green) \ldots 10
\( d_3 = |AJ| = |AK| \) (cyan) \ldots 2
\( d_4 = |HJ| = |IK| \) (magenta) \ldots 2
\( d_5 = |EJ| = |FK| \) (yellow) \ldots 2

Occurrences of the Golden Ratio

\( \phi = \frac{d_1}{d_2} = \frac{d_3}{d_4} = \frac{d_4}{d_3} = \frac{d_5}{d_5} \) (48 occurrences)
2.73.2 Construction MM131

Construction Process

A, B given initial points
1. $k_1 = k(A, |AB|)$
2. $k_2 = k(B, |AB|)$
   $C \in k_1 \cap k_2$, $D \in k_1 \cap k_2$
3. $k_3 = k(B, |CD|)$
   $E \in k_1 \cap k_3$, $F \in k_1 \cap k_3$
4. $k_4 = k(A, |DE|)$
   $G \in k_2 \cap k_4$, $H \in k_3 \cap k_4$, $I \in k_3 \cap k_4$
5. $k_5 = k(G, |DE|)$
   $J \in k_1 \cap k_5$, $K \in k_1 \cap k_5$

Distances

$d_0 = |HK| = |IJ|$ (red) ... 2
$d_1 = |EK| = |FJ|$ (blue) ... 2
$d_2 = |BE| = |BF| = |BH| = |BI| = |CD| = |CG| = |DG| = |EF| = |EH| = |FI|$ (green) ... 10
$d_3 = |BJ| = |BK|$ (cyan) ... 2
$d_4 = |HJ| = |IK|$ (magenta) ... 2
$d_5 = |EJ| = |FK|$ (yellow) ... 2

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5}$ (48 occurrences)
2.74 Class of Similar Constructions No. 65

Contained Constructions

MM105, MM107, MM132, MM134
2.74.1 Construction MM105

**Construction Process**

A, B given initial points

1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   - \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(A, |CD|) \)
   - \( E \in k_2 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(B, |DE|) \)
   - \( G \in k_1 \cap k_4 \), \( H \in k_3 \cap k_4 \), \( I \in k_3 \cap k_4 \)
5. \( k_5 = k(H, |AB|) \)
   - \( J \in k_1 \cap k_5 \), \( K \in k_3 \cap k_5 \), \( L \in k_3 \cap k_5 \), \( M \in k_4 \cap k_5 \), \( N \in k_4 \cap k_5 \)

**Distances**

\[
\begin{align*}
d_0 &= |GN| \quad (\text{red}) \quad \ldots \quad 1 \\
d_1 &= |AE| = |AF| = |AH| = |AI| = |AK| = |AL| = |BJ| = |CD| = |CG| = |DG| = |EF| = |EH| = |FI| \quad (\text{blue}) \quad \ldots \quad 13 \\
d_2 &= |GM| \quad (\text{green}) \quad \ldots \quad 1
\end{align*}
\]

**Occurrences of the Golden Ratio**

\[
\phi = \frac{d_0}{d_1} = \frac{d_2}{d_2} \quad (26 \text{ occurrences})
\]
2.74.2 Construction MM107

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2, D \in k_1 \cap k_2 \)
3. \( k_3 = k(A, |CD|) \)
   \( E \in k_2 \cap k_3, F \in k_2 \cap k_3 \)
4. \( k_4 = k(B, |DE|) \)
   \( G \in k_1 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4 \)
5. \( k_5 = k(I, |AB|) \)
   \( J \in k_1 \cap k_5, K \in k_3 \cap k_5, L \in k_3 \cap k_5, M \in k_4 \cap k_5, N \in k_4 \cap k_5 \)

Distances

\( d_0 = |GN| \) (red) \ldots 1
\( d_1 = |AE| = |AF| = |AH| = |AI| = |AK| = |BJ| = |CD| = |CG| = |DG| = |EF| = |EH| = |FI| \) (blue) \ldots 13
\( d_2 = |GM| \) (green) \ldots 1

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \) (26 occurrences)
2.74.3 Construction MM132

Construction Process

\( A, B \) given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2, D \in k_1 \cap k_2 \)
3. \( k_3 = k(B, |CD|) \)
   \( E \in k_1 \cap k_3, F \in k_1 \cap k_3 \)
4. \( k_4 = k(A, |DE|) \)
   \( G \in k_2 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4 \)
5. \( k_5 = k(H, |AB|) \)
   \( J \in k_2 \cap k_5, K \in k_3 \cap k_5, L \in k_3 \cap k_5, M \in k_4 \cap k_5, N \in k_4 \cap k_5 \)

Distances

\[ d_0 = |GM| \quad \text{(red)} \quad \ldots \quad 1 \]
\[ d_1 = |AJ| = |BE| = |BF| = |BH| = |BI| = |BK| = |BL| = |CD| = |CG| = |DG| = |EF| = |EH| = |FI| \quad \text{(blue)} \quad \ldots \quad 13 \]
\[ d_2 = |GN| \quad \text{(green)} \quad \ldots \quad 1 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} = \frac{d_2}{d_1} \quad \text{(26 occurrences)} \]
2.74.4 Construction MM134

Construction Process

\(A, B\) given initial points

1. \(k_1 = k(A, |AB|)\)
2. \(k_2 = k(B, |AB|)\)
   \(C \in k_1 \cap k_2\), \(D \in k_1 \cap k_2\)
3. \(k_3 = k(B, |CD|)\)
   \(E \in k_1 \cap k_3\), \(F \in k_1 \cap k_3\)
4. \(k_4 = k(A, |DE|)\)
   \(G \in k_2 \cap k_4\), \(H \in k_3 \cap k_4\), \(I \in k_3 \cap k_4\)
5. \(k_5 = k(I, |AB|)\)
   \(J \in k_2 \cap k_5\), \(K \in k_3 \cap k_5\), \(L \in k_3 \cap k_5\), \(M \in k_4 \cap k_5\), \(N \in k_4 \cap k_5\)

Distances

\(d_0 = |GM|\) (red) ... 1
\(d_1 = |AJ| = |BE| = |BF| = |BH| = |BI| = |BK| = |BL| = |CD| = |CG| = |DG| = |EF| = |EH| = |FI|\) (blue) ... 13
\(d_2 = |GN|\) (green) ... 1

Occurrences of the Golden Ratio

\(\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2}\) (26 occurrences)
2.75 Class of Similar Constructions No. 66

Contained Constructions

MM106, MM108, MM133, MM135
2.75.1 Construction MM106

Construction Process

A, B given initial points
1. $k_1 = k(A, |AB|)$
2. $k_2 = k(B, |AB|)$
   $C \in k_1 \cap k_2$, $D \in k_1 \cap k_2$
3. $k_3 = k(A, |CD|)$
   $E \in k_2 \cap k_3$, $F \in k_2 \cap k_3$
4. $k_4 = k(B, |DE|)$
   $G \in k_1 \cap k_4$, $H \in k_3 \cap k_4$, $I \in k_3 \cap k_4$
5. $k_5 = k(H, |DE|)$
   $J \in k_2 \cap k_5$, $K \in k_2 \cap k_5$, $L \in k_3 \cap k_5$, $M \in k_3 \cap k_5$, $N \in k_4 \cap k_5$

Distances

$d_0 = |GK| = |JN|$ (red) \ldots 2
$d_1 = |DK|$ (blue) \ldots 1
$d_2 = |AE| = |AF| = |AH| = |AI| = |AL| = |AM| = |CD| = |CG| = |CN| = |DG| = |EF| = |EH| = |FI|$ (green) \ldots 13
$d_3 = |CJ| = |CK|$ (cyan) \ldots 2
$d_4 = |GJ| = |KN|$ (magenta) \ldots 2
$d_5 = |DJ|$ (yellow) \ldots 1

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_2} = \frac{d_2}{d_4} = \frac{d_4}{d_5} = \frac{d_3}{d_6}$ (56 occurrences)
2.75.2 Construction MM108

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(A, |CD|) \)
   \( E \in k_2 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(B, |DE|) \)
   \( G \in k_1 \cap k_4 \), \( H \in k_3 \cap k_4 \), \( I \in k_3 \cap k_4 \)
5. \( k_5 = k(I, |DE|) \)
   \( J \in k_2 \cap k_5 \), \( K \in k_3 \cap k_5 \), \( L \in k_3 \cap k_5 \), \( M \in k_4 \cap k_5 \), \( N \in k_4 \cap k_5 \)

Distances

\( d_0 = |GK| = |JN| \) (red) \ldots 2
\( d_1 = |CK| \) (blue) \ldots 1
\( d_2 = |AE| = |AF| = |AH| = |AI| = |AL| = |AM| = |CD| = |CG| = |DG| = |DN| = |EF| = |EH| = |FI| \) (green) \ldots 13
\( d_3 = |DJ| = |DK| \) (cyan) \ldots 2
\( d_4 = |GJ| = |KN| \) (magenta) \ldots 2
\( d_5 = |CJ| \) (yellow) \ldots 1

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_3}{d_4} = \frac{d_5}{d_3} \] (56 occurrences)
2.75.3 Construction MM133

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2, \; D \in k_1 \cap k_2 \)
3. \( k_3 = k(B, |CD|) \)
   \( E \in k_1 \cap k_3, \; F \in k_1 \cap k_3 \)
4. \( k_4 = k(A, |DE|) \)
   \( G \in k_2 \cap k_4, \; H \in k_3 \cap k_4, \; I \in k_3 \cap k_4 \)
5. \( k_5 = k(H, |DE|) \)
   \( J \in k_1 \cap k_5, \; K \in k_1 \cap k_5, \; L \in k_3 \cap k_5, \; M \in k_3 \cap k_5, \; N \in k_4 \cap k_5 \)

Distances

\[ d_0 = |GJ| = |KN| \text{ (red) } \ldots 2 \]
\[ d_1 = |DJ| \text{ (blue) } \ldots 1 \]
\[ d_2 = |BE| = |BF| = |BH| = |BI| = |BL| = |BM| = |CD| = |CG| = |CN| = |DG| = |EF| = |EH| = |FI| \text{ (green) } \ldots 13 \]
\[ d_3 = |CJ| = |CK| \text{ (cyan) } \ldots 2 \]
\[ d_4 = |GK| = |JN| \text{ (magenta) } \ldots 2 \]
\[ d_5 = |DK| \text{ (yellow) } \ldots 1 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_4}{d_3} = \frac{d_2}{d_4} = \frac{d_4}{d_5} \text{ (56 occurrences)} \]
2.75.4 Construction MM135

![Construction Diagram](image)

**Construction Process**

1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   - \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(B, |CD|) \)
   - \( E \in k_1 \cap k_3 \), \( F \in k_1 \cap k_3 \)
4. \( k_4 = k(A, |DE|) \)
   - \( G \in k_2 \cap k_4 \), \( H \in k_3 \cap k_4 \), \( I \in k_3 \cap k_4 \)
5. \( k_5 = k(I, |DE|) \)
   - \( J \in k_1 \cap k_5 \), \( K \in k_1 \cap k_5 \), \( L \in k_3 \cap k_5 \), \( M \in k_3 \cap k_5 \), \( N \in k_4 \cap k_5 \)

**Distances**

\[
\begin{align*}
d_0 &= |GJ| = |KN| \quad \text{(red)} \quad 2 \\
d_1 &= |CJ| \quad \text{(blue)} \quad 1 \\
d_2 &= |BE| = |BF| = |BH| = |BI| = |BL| = |BM| = |CD| = |CG| = \\
d_3 &= |DG| = |DN| = |EF| = |EH| = |FI| \quad \text{(green)} \quad 13 \\
d_4 &= |DJ| = |DK| \quad \text{(cyan)} \quad 2 \\
d_5 &= |GK| = |JN| \quad \text{(magenta)} \quad 2 \\
\end{align*}
\]

**Occurrences of the Golden Ratio**

\[
\phi = \frac{d_0}{d_2} = \frac{d_3}{d_4} = \frac{d_3}{d_5} = \frac{d_4}{d_6} \quad \text{(56 occurrences)}
\]
2.76 Class of Similar Constructions No. 67

Contained Constructions

MM109, MM112, MM139, MM142
2.76.1 Construction MM109

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2, D \in k_1 \cap k_2 \)
3. \( k_3 = k(A, |CD|) \)
   \( E \in k_2 \cap k_3, F \in k_2 \cap k_3 \)
4. \( k_4 = k(C, |DE|) \)
   \( G \in k_1 \cap k_4, H \in k_3 \cap k_4 \)
5. \( k_5 = k(E, |DE|) \)
   \( I \in k_1 \cap k_5, J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5, M \in k_4 \cap k_5 \)

Distances

\( d_0 = |GL| \) (red) \ldots 1
\( d_1 = |BL| \) (blue) \ldots 1
\( d_2 = |IL| = |IM| \) (green) \ldots 2
\( d_3 = |AE| = |AF| = |AH| = |AJ| = |AK| = |BG| = |BI| = |CD| = |EF| = |GI| \) (cyan) \ldots 10
\( d_4 = |GM| \) (magenta) \ldots 1
\( d_5 = |BM| \) (yellow) \ldots 1

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_3}{d_4} = \frac{d_4}{d_5} \) (24 occurrences)
2.76.2 Construction MM112

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2, D \in k_1 \cap k_2 \)
3. \( k_3 = k(A, |CD|) \)
   \( E \in k_2 \cap k_3, F \in k_2 \cap k_3 \)
4. \( k_4 = k(D, |DE|) \)
   \( G \in k_1 \cap k_4, H \in k_3 \cap k_4 \)
5. \( k_5 = k(F, |DE|) \)
   \( I \in k_1 \cap k_5, J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5, M \in k_4 \cap k_5 \)

Distances

\[
\begin{align*}
d_0 &= |GM| \text{ (red)} \ldots 1 \\
d_1 &= |BM| \text{ (blue)} \ldots 1 \\
d_2 &= |IL| = |IM| \text{ (green)} \ldots 2 \\
d_3 &= |AE| = |AF| = |AH| = |AK| = |BG| = |BI| = |CD| = |EF| = |GL| \text{ (cyan)} \ldots 10 \\
d_4 &= |GL| \text{ (magenta)} \ldots 1 \\
d_5 &= |BL| \text{ (yellow)} \ldots 1
\end{align*}
\]

Occurrences of the Golden Ratio

\[
\phi = \frac{d_0}{d_2} = \frac{d_3}{d_4} = \frac{d_4}{d_5} = \frac{d_5}{d_2} \text{ (24 occurrences)}
\]
2.76.3 Construction MM139

Construction Process

A, B given initial points
1. $k_1 = k(A, |AB|)$
2. $k_2 = k(B, |AB|)$
   $C \in k_1 \cap k_2$, $D \in k_1 \cap k_2$
3. $k_3 = k(B, |CD|)$
   $E \in k_1 \cap k_3$, $F \in k_1 \cap k_3$
4. $k_4 = k(C, |DE|)$
   $G \in k_2 \cap k_4$, $H \in k_3 \cap k_4$
5. $k_5 = k(E, |DE|)$
   $I \in k_2 \cap k_5$, $J \in k_3 \cap k_5$, $K \in k_3 \cap k_5$, $L \in k_4 \cap k_5$, $M \in k_4 \cap k_5$

Distances

$d_0 = |GL|$ (red) \ldots 1
$d_1 = |AL|$ (blue) \ldots 1
$d_2 = |IL| = |IM|$ (green) \ldots 2
$d_3 = |AG| = |AI| = |BE| = |BF| = |BH| = |BK| = |CD| = |EF| = |GI|$ (cyan) \ldots 10
$d_4 = |GM|$ (magenta) \ldots 1
$d_5 = |AM|$ (yellow) \ldots 1

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_3}{d_4} = \frac{d_4}{d_5}$ (24 occurrences)
2.76.4 Construction MM142

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(B, |CD|) \)
   \( E \in k_1 \cap k_3 \), \( F \in k_1 \cap k_3 \)
4. \( k_4 = k(D, |DE|) \)
   \( G \in k_2 \cap k_4 \), \( H \in k_3 \cap k_4 \)
5. \( k_5 = k(F, |DE|) \)
   \( I \in k_2 \cap k_5 \), \( J \in k_3 \cap k_5 \), \( K \in k_3 \cap k_5 \), \( L \in k_4 \cap k_5 \), \( M \in k_4 \cap k_5 \)

Distances

\( d_0 = |GM| \) (red) \( \ldots 1 \)
\( d_1 = |AM| \) (blue) \( \ldots 1 \)
\( d_2 = |IL| = |IM| \) (green) \( \ldots 2 \)
\( d_3 = |AG| = |AI| = |BE| = |BF| = |BH| = |BK| = |CD| = |EF| = |GI| \) (cyan) \( \ldots 10 \)
\( d_4 = |GL| \) (magenta) \( \ldots 1 \)
\( d_5 = |AL| \) (yellow) \( \ldots 1 \)

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_2} = \frac{d_1}{d_2} = \frac{d_2}{d_4} = \frac{d_4}{d_5} \) (24 occurrences)
2.77 Class of Similar Constructions No. 68

Contained Constructions

MM110, MM113, MM140, MM143
2.77.1 Construction MM110

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(A, [CD]) \)
   \( E \in k_2 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(C, [DE]) \)
   \( G \in k_1 \cap k_4 \), \( H \in k_3 \cap k_4 \)
5. \( k_5 = k(G, |AB|) \)
   \( I \in k_1 \cap k_5 \), \( J \in k_3 \cap k_5 \), \( K \in k_3 \cap k_5 \), \( L \in k_4 \cap k_5 \), \( M \in k_4 \cap k_5 \)

Distances

\( d_0 = |FL| = |HM| \) (red) \ldots 2
\( d_1 = |JM| = |KL| \) (blue) \ldots 2
\( d_2 = |AE| = |AF| = |AH| = |AJ| = |AK| = |BG| = |CD| = |CI| = |DI| = |EF| = |FK| = |HJ| = |JK| \) (green) \ldots 13
\( d_3 = |AL| = |AM| \) (cyan) \ldots 2
\( d_4 = |FM| = |HL| \) (magenta) \ldots 2
\( d_5 = |JL| = |KM| \) (yellow) \ldots 2

Occurrences of the Golden Ratio

\( \phi \approx \frac{d_0}{d_2} \approx \frac{d_1}{d_3} \approx \frac{d_2}{d_4} \approx \frac{d_3}{d_5} \) (60 occurrences)
2.77.2 Construction MM113

Construction Process

A, B given initial points
1. $k_1 = k(A, |AB|)$
2. $k_2 = k(B, |AB|)$
   $C \in k_1 \cap k_2$, $D \in k_1 \cap k_2$
3. $k_3 = k(A, |CD|)$
   $E \in k_2 \cap k_3$, $F \in k_2 \cap k_3$
4. $k_4 = k(D, |DE|)$
   $G \in k_1 \cap k_4$, $H \in k_3 \cap k_4$
5. $k_5 = k(G, |AB|)$
   $I \in k_1 \cap k_5$, $J \in k_3 \cap k_5$, $K \in k_3 \cap k_5$, $L \in k_4 \cap k_5$, $M \in k_4 \cap k_5$

Distances

$d_0 = |EL| = |HM|$ (red) \ldots 2
$d_1 = |JM| = |KL|$ (blue) \ldots 2
$d_2 = |AE| = |AF| = |AH| = |AJ| = |AK| = |BG| = |CD| = |CI| = |DI| = |EF| = |EK| = |HJ| = |JK|$ (green) \ldots 13
$d_3 = |AL| = |AM|$ (cyan) \ldots 2
$d_4 = |EM| = |HL|$ (magenta) \ldots 2
$d_5 = |JL| = |KM|$ (yellow) \ldots 2

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5}$ (60 occurrences)
2.77.3 Construction MM140

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(B, |CD|) \)
   \( E \in k_1 \cap k_3 \), \( F \in k_1 \cap k_3 \)
4. \( k_4 = k(C, |DE|) \)
   \( G \in k_2 \cap k_4 \), \( H \in k_3 \cap k_4 \)
5. \( k_5 = k(G, |AB|) \)
   \( I \in k_2 \cap k_5 \), \( J \in k_3 \cap k_5 \), \( K \in k_3 \cap k_5 \), \( L \in k_4 \cap k_5 \), \( M \in k_4 \cap k_5 \)

Distances

\[ d_0 = |FM| = |HL| \text{ (red)} \ldots 2 \]
\[ d_1 = |JM| = |KL| \text{ (blue)} \ldots 2 \]
\[ d_2 = |AG| = |BE| = |BF| = |BH| = |BJ| = |BK| = |CD| = |CI| = |DI| = |EF| = |FJ| = |HK| = |JK| \text{ (green)} \ldots 13 \]
\[ d_3 = |BL| = |BM| \text{ (cyan)} \ldots 2 \]
\[ d_4 = |FL| = |HM| \text{ (magenta)} \ldots 2 \]
\[ d_5 = |JL| = |KM| \text{ (yellow)} \ldots 2 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (60 occurrences)} \]
2.77.4 Construction MM143

Construction Process

\[ A, B \text{ given initial points} \]

1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
\[ C \in k_1 \cap k_2 , \ D \in k_1 \cap k_2 \]
3. \( k_3 = k(B, |CD|) \)
\[ E \in k_1 \cap k_3 , \ F \in k_1 \cap k_3 \]
4. \( k_4 = k(D, |DE|) \)
\[ G \in k_2 \cap k_4 , \ H \in k_3 \cap k_4 \]
5. \( k_5 = k(G, |AB|) \)
\[ I \in k_2 \cap k_5 , \ J \in k_3 \cap k_5 , \ K \in k_3 \cap k_5 , \ L \in k_4 \cap k_5 , \ M \in k_4 \cap k_5 \]

Distances

\[ d_0 = |EM| = |HL| \text{ (red) } \ldots 2 \]
\[ d_1 = |JM| = |KL| \text{ (blue) } \ldots 2 \]
\[ d_2 = |AG| = |BE| = |BF| = |BH| = |BJ| = |BK| = |CD| = |CI| = |DI| = |EF| = |EJ| = |HK| = |JK| \text{ (green) } \ldots 13 \]
\[ d_3 = |BL| = |BM| \text{ (cyan) } \ldots 2 \]
\[ d_4 = |EL| = |HM| \text{ (magenta) } \ldots 2 \]
\[ d_5 = |JL| = |KM| \text{ (yellow) } \ldots 2 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (60 occurrences)} \]
2.78 Class of Similar Constructions No. 69

Contained Constructions

MM111, MM114, MM141, MM144
2.78.1 Construction MM111

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(A, |CD|) \)
   \( E \in k_2 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(C, |DE|) \)
   \( G \in k_1 \cap k_4 \), \( H \in k_3 \cap k_4 \)
5. \( k_5 = k(H, |AB|) \)
   \( I \in k_1 \cap k_5 \), \( J \in k_1 \cap k_5 \), \( K \in k_3 \cap k_5 \), \( L \in k_3 \cap k_5 \), \( M \in k_4 \cap k_5 \)
   \( N \in k_4 \cap k_5 \)

Distances

\( d_0 = |GM| \) (red) \ldots 1
\( d_1 = |AE| = |AF| = |AH| = |AK| = |AL| = |BG| = |BJ| = |CD| = |CI| = |DJ| = |EF| = |GJ| \) (blue) \ldots 12
\( d_2 = |GN| \) (green) \ldots 1

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} = \frac{d_2}{d_5} \) (24 occurrences)
Construction MM114

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2 , \ D \in k_1 \cap k_2 \)
3. \( k_3 = k(A, |CD|) \)
   \( E \in k_2 \cap k_3 , \ F \in k_2 \cap k_3 \)
4. \( k_4 = k(D, |DE|) \)
   \( G \in k_1 \cap k_4 , \ H \in k_3 \cap k_4 \)
5. \( k_5 = k(H, |AB|) \)
   \( I \in k_1 \cap k_5 , \ J \in k_1 \cap k_5 , \ K \in k_3 \cap k_5 , \ L \in k_3 \cap k_5 , \ M \in k_4 \cap k_5 , \)
   \( N \in k_4 \cap k_5 \)

Distances

\( d_0 = |GN| \) (red) \ldots 1
\( d_1 = |AE| = |AF| = |AH| = |AK| = |AL| = |BG| = |BJ| = |CD| = |CI| = |DJ| = |EF| = |GJ| \) (blue) \ldots 12
\( d_2 = |GM| \) (green) \ldots 1

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} = \frac{d_2}{d_5} \) (24 occurrences)
2.78.3 Construction MM141

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2, D \in k_1 \cap k_2 \)
3. \( k_3 = k(B, |CD|) \)
   \( E \in k_1 \cap k_3, F \in k_1 \cap k_3 \)
4. \( k_4 = k(C, |DE|) \)
   \( G \in k_2 \cap k_4, H \in k_3 \cap k_4 \)
5. \( k_5 = k(H, |AB|) \)
   \( I \in k_2 \cap k_5, J \in k_2 \cap k_5, K \in k_3 \cap k_5, L \in k_3 \cap k_5, M \in k_4 \cap k_5, \)
   \( N \in k_4 \cap k_5 \)

Distances

\( d_0 = |GM| \) (red) ... 1
\( d_1 = |AG| = |AI| = |BE| = |BF| = |BH| = |BK| = |BL| = |CD| = |CJ| = |DJ| = |EF| = |GI| \) (blue) ... 12
\( d_2 = |GN| \) (green) ... 1

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} = \frac{d_2}{d_2} \) (24 occurrences)
2.78.4 Construction MM144

Construction Process

\(A, B\) given initial points
1. \(k_1 = k(A, |AB|)\)
2. \(k_2 = k(B, |AB|)\)
\(C \in k_1 \cap k_2 \), \(D \in k_1 \cap k_2\)
3. \(k_3 = k(B, |CD|)\)
\(E \in k_1 \cap k_3 \), \(F \in k_1 \cap k_3\)
4. \(k_4 = k(D, |DE|)\)
\(G \in k_2 \cap k_4 \), \(H \in k_3 \cap k_4\)
5. \(k_5 = k(H, |AB|)\)
\(I \in k_2 \cap k_5 \), \(J \in k_2 \cap k_5 \), \(K \in k_3 \cap k_5 \), \(L \in k_3 \cap k_5 \), \(M \in k_4 \cap k_5 \), \(N \in k_4 \cap k_5\)

Distances

\(d_0 = |GN| \) (red) \ldots 1
\(d_1 = |AG| = |AI| = |BE| = |BF| = |BH| = |BK| = |BL| = |CD| = |CJ| = |DJ| = |EF| = |GI| \) (blue) \ldots 12
\(d_2 = |GM| \) (green) \ldots 1

Occurrences of the Golden Ratio

\[\phi = \frac{d_0}{d_1} = \frac{d_2}{d_1} \] (24 occurrences)
2.79 Class of Similar Constructions No. 70

Contained Constructions
MM115, MM116, MM145, MM146
2.79.1 Construction MM115

Construction Process

$A, B$ given initial points
1. $k_1 = k(A, |AB|)$
2. $k_2 = k(B, |AB|)$
   $C \in k_1 \cap k_2$, $D \in k_1 \cap k_2$
3. $k_3 = k(A, |CD|)$
   $E \in k_2 \cap k_3$, $F \in k_2 \cap k_3$
4. $k_4 = k(E, |DE|)$
   $G \in k_3 \cap k_4$, $H \in k_3 \cap k_4$, $I \in k_3 \cap k_4$
5. $k_5 = k(G, |AB|)$
   $J \in k_1 \cap k_5$, $K \in k_3 \cap k_5$, $L \in k_3 \cap k_5$, $M \in k_4 \cap k_5$, $N \in k_4 \cap k_5$

Distances

\[
\begin{align*}
    d_0 &= |DM| \text{ (red) } \ldots 1 \\
    d_1 &= |JM| \text{ (blue) } \ldots 1 \\
    d_2 &= |AE| = |AF| = |AH| = |AI| = |AK| = |AL| = |BG| = |CD| = |CJ| = |DJ| = |EF| = |EL| = |KL| \text{ (green) } \ldots 13 \\
    d_3 &= |CM| = |CN| \text{ (cyan) } \ldots 2 \\
    d_4 &= |DN| \text{ (magenta) } \ldots 1 \\
    d_5 &= |JN| \text{ (yellow) } \ldots 1
\end{align*}
\]

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5}$ (30 occurrences)
2.79.2 Construction MM116

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2, D \in k_1 \cap k_2 \)
3. \( k_3 = k(A, |CD|) \)
   \( E \in k_2 \cap k_3, F \in k_2 \cap k_3 \)
4. \( k_4 = k(F, |DE|) \)
   \( G \in k_1 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4 \)
5. \( k_5 = k(G, |AB|) \)
   \( J \in k_1 \cap k_5, K \in k_3 \cap k_5, L \in k_3 \cap k_5, M \in k_4 \cap k_5, N \in k_4 \cap k_5 \)

Distances

\[
\begin{align*}
d_0 &= |CN| \text{ (red)} \ldots 1 \\
d_1 &= |JN| \text{ (blue)} \ldots 1 \\
d_2 &= |AE| = |AF| = |AH| = |AI| = |AK| = |AL| = |BG| = |CD| = |CJ| = |DJ| = |EF| = |FL| = |KL| \text{ (green)} \ldots 13 \\
d_3 &= |DM| = |DN| \text{ (cyan)} \ldots 2 \\
d_4 &= |CM| \text{ (magenta)} \ldots 1 \\
d_5 &= |JM| \text{ (yellow)} \ldots 1
\end{align*}
\]

Occurrences of the Golden Ratio

\[
\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (30 occurrences)}
\]
2.79.3 Construction MM145

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(B, |CD|) \)
   \( E \in k_1 \cap k_3 \), \( F \in k_1 \cap k_3 \)
4. \( k_4 = k(E, |DE|) \)
   \( G \in k_2 \cap k_4 \), \( H \in k_3 \cap k_4 \), \( I \in k_3 \cap k_4 \)
5. \( k_5 = k(G, |AB|) \)
   \( J \in k_2 \cap k_5 \), \( K \in k_3 \cap k_5 \), \( L \in k_3 \cap k_5 \), \( M \in k_4 \cap k_5 \), \( N \in k_4 \cap k_5 \)

Distances

\( d_0 = |DM| \) (red) \ldots 1
\( d_1 = |JM| \) (blue) \ldots 1
\( d_2 = |AG| = |BE| = |BF| = |BH| = |BI| = |BK| = |BL| = |CD| = |CJ| = |DJ| = |EF| = |EK| = |KL| \) (green) \ldots 13
\( d_3 = |CM| = |CN| \) (cyan) \ldots 2
\( d_4 = |DN| \) (magenta) \ldots 1
\( d_5 = |JN| \) (yellow) \ldots 1

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \) (30 occurrences)
2.79.4 Construction MM146

Construction Process

A, B given initial points
1. $k_1 = k(A, |AB|)$
2. $k_2 = k(B, |AB|)$
   $C \in k_1 \cap k_2$, $D \in k_1 \cap k_2$
3. $k_3 = k(B, |CD|)$
   $E \in k_1 \cap k_3$, $F \in k_1 \cap k_3$
4. $k_4 = k(F, |DE|)$
   $G \in k_2 \cap k_4$, $H \in k_3 \cap k_4$, $I \in k_3 \cap k_4$
5. $k_5 = k(G, |AB|)$
   $J \in k_2 \cap k_5$, $K \in k_3 \cap k_5$, $L \in k_3 \cap k_5$, $M \in k_4 \cap k_5$, $N \in k_4 \cap k_5$

Distances

$d_0 = |CN|$ (red) ... 1
$d_1 = |JN|$ (blue) ... 1
$d_2 = |AG| = |BE| = |BF| = |BH| = |BI| = |BK| = |BL| = |CD| = |CJ| = |DJ| = |EF| = |FK| = |KL|$ (green) ... 13
$d_3 = |DM| = |DN|$ (cyan) ... 2
$d_4 = |CM|$ (magenta) ... 1
$d_5 = |JM|$ (yellow) ... 1

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5}$ (30 occurrences)
2.80 Class of Similar Constructions No. 71

Contained Constructions
MM150, MM154, MM262, MM266
2.80.1 Construction MM150

\[
\begin{align*}
A, B & \text{ given initial points} \\
1. & k_1 = k(A, |AB|) \\
2. & k_2 = k(B, |AB|) \\
   & C \in k_1 \cap k_2, D \in k_1 \cap k_2 \\
3. & k_3 = k(C, |CD|) \\
   & E \in k_1 \cap k_3, F \in k_2 \cap k_3 \\
4. & k_4 = k(A, |BE|) \\
   & G \in k_3 \cap k_4 \\
5. & k_5 = k(C, |BE|) \\
   & H \in k_1 \cap k_5, I \in k_2 \cap k_5, J \in k_4 \cap k_5, K \in k_4 \cap k_5
\end{align*}
\]

Distances

\[
\begin{align*}
& d_0 = |BJ| \text{ (red)} \ldots 1 \\
& d_1 = |AI| = |BH| = |CD| = |CE| = |CF| = |CG| = |DE| = |DF| = |EG| \text{ (blue)} \ldots 9 \\
& d_2 = |BK| \text{ (green)} \ldots 1
\end{align*}
\]

Occurrences of the Golden Ratio

\[
\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (18 occurrences)}
\]
2.80.2 Construction MM154

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, |CD|) \)
   \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(B, |BE|) \)
   \( G \in k_3 \cap k_4 \)
5. \( k_5 = k(C, |BE|) \)
   \( H \in k_1 \cap k_5 \), \( I \in k_2 \cap k_5 \), \( J \in k_4 \cap k_5 \), \( K \in k_4 \cap k_5 \)

Distances

\[
\begin{align*}
d_0 &= |AK| \quad \text{(red)} \ldots 1 \\
d_1 &= |AI| = |BH| = |CD| = |CE| = |CF| = |CG| = |DE| = |DF| = |FG| \quad \text{(blue)} \ldots 9 \\
d_2 &= |AJ| \quad \text{(green)} \ldots 1
\end{align*}
\]

Occurrences of the Golden Ratio

\[
\phi = \frac{d_0}{d_1} = \frac{d_2}{d_5} \quad \text{(18 occurrences)}
\]
2.80.3 Construction MM262

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2, D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, |CD|) \)
   \( E \in k_1 \cap k_3, F \in k_2 \cap k_3 \)
4. \( k_4 = k(A, |BE|) \)
   \( G \in k_3 \cap k_4 \)
5. \( k_5 = k(D, |BE|) \)
   \( H \in k_1 \cap k_5, I \in k_2 \cap k_5, J \in k_4 \cap k_5, K \in k_4 \cap k_5 \)

Distances

\( d_0 = |BJ| \) (red) \( \ldots 1 \)
\( d_1 = |AI| = |BH| = |CD| = |CE| = |CF| = |DE| = |DF| = |DG| = |EG| \) (blue) \( \ldots 9 \)
\( d_2 = |BK| \) (green) \( \ldots 1 \)

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} = \frac{d_2}{d_5} \) (18 occurrences)
2.80.4 Construction MM266

**Construction Process**

A, B given initial points

1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   
   \( C \in k_1 \cap k_2, \ D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, |CD|) \)
   
   \( E \in k_1 \cap k_3, \ F \in k_2 \cap k_3 \)
4. \( k_4 = k(B, |BE|) \)
   
   \( G \in k_3 \cap k_4 \)
5. \( k_5 = k(D, |BE|) \)
   
   \( H \in k_1 \cap k_5, \ I \in k_2 \cap k_5, \ J \in k_4 \cap k_5, \ K \in k_4 \cap k_5 \)

**Distances**

\[
\begin{align*}
d_0 &= |AK| \quad \text{(red)} \ldots 1 \\
d_1 &= |AI| = |BH| = |CD| = |CE| = |CF| = |DE| = |DF| = |DG| = |FG| \quad \text{(blue)} \ldots 9 \\
d_2 &= |AJ| \quad \text{(green)} \ldots 1
\end{align*}
\]

**Occurrences of the Golden Ratio**

\[
\phi = \frac{d_0}{d_1} = \frac{d_2}{d_5} \quad (18 \text{ occurrences})
\]
2.81 Class of Similar Constructions No. 72

Contained Constructions
MM151, MM155, MM261, MM265
2.81 Class of Similar Constructions No. 72

2.81.1 Construction MM151

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, |CD|) \)
   
   \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(A, |BE|) \)
   
   \( G \in k_3 \cap k_4 \)
5. \( k_5 = k(D, |BE|) \)
   
   \( H \in k_1 \cap k_5 \), \( I \in k_2 \cap k_5 \), \( J \in k_3 \cap k_5 \), \( K \in k_3 \cap k_5 \), \( L \in k_4 \cap k_5 \), \( M \in k_4 \cap k_5 \)

Distances

\( d_0 = |BL| \) (red) \ldots 1

\( d_1 = |AI| = |BH| = |CD| = |CF| = |CG| = |CJ| = |CK| = |DE| = |DF| = |EG| \) (blue) \ldots 11

\( d_2 = |BM| \) (green) \ldots 1

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} = \frac{d_4}{d_5} \) (22 occurrences)
2.81.2 Construction MM155

Construction Process

\( A, B \) given initial points

1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2, D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, |CD|) \)
   \( E \in k_1 \cap k_3, F \in k_2 \cap k_3 \)
4. \( k_4 = k(B, |BE|) \)
   \( G \in k_3 \cap k_4 \)
5. \( k_5 = k(D, |BE|) \)
   \( H \in k_1 \cap k_5, I \in k_2 \cap k_5, J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5, M \in k_4 \cap k_5 \)

Distances

\[ d_0 = |AM| \] \( \text{red} \) \( \ldots 1 \)
\[ d_1 = |AI| = |BI| = |CD| = |CE| = |CF| = |CG| = |CJ| = |CK| = |DE| = |DF| = |FG| \] \( \text{blue} \) \( \ldots 11 \)
\[ d_2 = |AL| \] \( \text{green} \) \( \ldots 1 \)

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \] (22 occurrences)
Construction MM261

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2, \ D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, |CD|) \)
   \( E \in k_1 \cap k_3, \ F \in k_2 \cap k_3 \)
4. \( k_4 = k(A, |BE|) \)
   \( G \in k_3 \cap k_4 \)
5. \( k_5 = k(C, |BE|) \)
   \( H \in k_1 \cap k_5, \ I \in k_2 \cap k_5, \ J \in k_3 \cap k_5, \ K \in k_3 \cap k_5, \ L \in k_4 \cap k_5, \)
   \( M \in k_4 \cap k_5 \)

Distances

\( d_0 = |BL| \) (red) \( \ldots 1 \)
\( d_1 = |AI| = |BH| = |CD| = |CE| = |CF| = |DE| = |DF| = |DG| = |DJ| = |DK| = |EG| \) (blue) \( \ldots 11 \)
\( d_2 = |BM| \) (green) \( \ldots 1 \)

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \) (22 occurrences)
2.81.4 Construction MM265

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, |CD|) \)
   \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(B, |BE|) \)
   \( G \in k_3 \cap k_4 \)
5. \( k_5 = k(C, |BE|) \)
   \( H \in k_1 \cap k_5 \), \( I \in k_2 \cap k_5 \), \( J \in k_3 \cap k_5 \)
   \( K \in k_3 \cap k_5 \), \( L \in k_4 \cap k_5 \), \( M \in k_4 \cap k_5 \)

Distances

\( d_0 = |AM| \) (red) \ldots 1
\( d_1 = |AI| = |BH| = |CD| = |CE| = |CF| = |DE| = |DF| = |DG| = |DJ| = |DK| = |FG| \) (blue) \ldots 11
\( d_2 = |AL| \) (green) \ldots 1

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} = \frac{d_2}{d_1} \) (22 occurrences)
2.82 Class of Similar Constructions No. 73

Contained Constructions
MM158, MM159, MM273, MM274
**2.82.1 Construction MM158**

**Construction Process**

1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
3. \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
4. \( k_3 = k(C, |CD|) \)
5. \( E \in k_1 \cap k_3 \)
6. \( F \in k_2 \cap k_4 \)
7. \( G \in k_2 \cap k_5 \), \( H \in k_2 \cap k_5 \)

**Distances**

\[
\begin{align*}
d_0 &= |FG| \quad \text{(red)} \ldots 1 \\
d_1 &= |AG| = |AH| \quad \text{(blue)} \ldots 2 \\
d_2 &= |FH| \quad \text{(green)} \ldots 1 
\end{align*}
\]

**Occurrences of the Golden Ratio**

\[
\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \quad \text{(4 occurrences)}
\]
2.82.2 Construction MM159

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2, D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, |CD|) \)
   \( E \in k_1 \cap k_3, F \in k_2 \cap k_3 \)
4. \( k_4 = k(C, |BE|) \)
   \( G \in k_1 \cap k_4 \)
5. \( k_5 = k(F, |BE|) \)
   \( H \in k_1 \cap k_5, I \in k_1 \cap k_5 \)

Distances

\( d_0 = |GH| \) (red) \ldots 1
\( d_1 = |BH| = |BI| \) (blue) \ldots 2
\( d_2 = |GI| \) (green) \ldots 1

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \) (4 occurrences)
2.82.3 Construction MM273

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, |CD|) \)
   \( E \in k_1 \cap k_3 \)
4. \( k_4 = k(D, |BE|) \)
   \( F \in k_2 \cap k_4 \)
5. \( k_5 = k(E, |BE|) \)
   \( G \in k_2 \cap k_5 \), \( H \in k_2 \cap k_5 \)

Distances

\( d_0 = |FH| \) (red) \ldots 1
\( d_1 = |AG| = |AH| \) (blue) \ldots 2
\( d_2 = |FG| \) (green) \ldots 1

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \) (4 occurrences)
Construction Process

1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \[ C \in k_1 \cap k_2, D \in k_1 \cap k_2 \]
3. \( k_3 = k(D, |CD|) \)
   \[ E \in k_1 \cap k_3, F \in k_2 \cap k_3 \]
4. \( k_4 = k(D, |BE|) \)
   \[ G \in k_1 \cap k_4 \]
5. \( k_5 = k(F, |BE|) \)
   \[ H \in k_1 \cap k_5, I \in k_1 \cap k_5 \]

Distances

\[ d_0 = |GI| \quad \text{(red)} \quad 1 \]
\[ d_1 = |BH| = |BI| \quad \text{(blue)} \quad 2 \]
\[ d_2 = |GH| \quad \text{(green)} \quad 1 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} = \frac{d_4}{d_5} \quad \text{(4 occurrences)} \]
2.83 Class of Similar Constructions No. 74

Contained Constructions

MM160, MM161, MM275, MM276
2.83.1 Construction MM160

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, |CD|) \)
   \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(C, |BE|) \)
   \( G \in k_1 \cap k_4 \), \( H \in k_2 \cap k_4 \)
5. \( k_5 = k(G, |AB|) \)
   \( I \in k_4 \cap k_5 \), \( J \in k_4 \cap k_5 \)

Distances

\( d_0 = |HI| \) (red) \... 1
\( d_1 = |AH| = |BG| = |CD| = |CE| = |CF| = |DE| = |DF| \) (blue) \... 7
\( d_2 = |HJ| \) (green) \... 1

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} = \frac{d_4}{d_2} \) (14 occurrences)
2.83.2 Construction MM161

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, |CD|) \)
   
   \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(C, |BE|) \)
   
   \( G \in k_1 \cap k_4 \), \( H \in k_2 \cap k_4 \)
5. \( k_5 = k(H, |AB|) \)
   
   \( I \in k_4 \cap k_5 \), \( J \in k_4 \cap k_5 \)

Distances

\[ d_0 = |GJ| \] (red) \ldots 1
\[ d_1 = |AH| = |BG| = |CD| = |CE| = |CF| = |DE| = |DF| \] (blue) \ldots 7
\[ d_2 = |GI| \] (green) \ldots 1

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \] (14 occurrences)
2.83.3 Construction MM275

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, |CD|) \)
   \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(D, |BE|) \)
   \( G \in k_1 \cap k_4 \), \( H \in k_2 \cap k_4 \)
5. \( k_5 = k(G, |AB|) \)
   \( I \in k_4 \cap k_5 \), \( J \in k_4 \cap k_5 \)

Distances

\[
\begin{align*}
d_0 &= |HI| \quad \text{(red)} \quad \ldots 1 \\
d_1 &= |AH| = |BG| = |CD| = |CE| = |CF| = |DE| = |DF| \quad \text{(blue)} \quad \ldots 7 \\
d_2 &= |HJ| \quad \text{(green)} \quad \ldots 1
\end{align*}
\]

Occurrences of the Golden Ratio

\[
\phi = \frac{d_0}{d_1} = \frac{d_2}{d_2} \quad (14 \text{ occurrences})
\]
2.83.4 Construction MM276

Construction Process

\[ A, B \text{ given initial points} \]
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
3. \( k_3 = k(D, |CD|) \)
4. \( k_4 = k(D, |BE|) \)
5. \( k_5 = k(H, |AB|) \)

Distances

\[ d_0 = |GJ| \text{ (red)} \ldots 1 \]
\[ d_1 = |AH| = |BG| = |CD| = |CE| = |CF| = |DE| = |DF| \text{ (blue)} \ldots 7 \]
\[ d_2 = |GI| \text{ (green)} \ldots 1 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \] (14 occurrences)
2.84 Class of Similar Constructions No. 75

Contained Constructions

MM162, MM163, MM269, MM270
2.84.1 Construction MM162

Construction Process

A, B given initial points
1. $k_1 = k(A, |AB|)$
2. $k_2 = k(B, |AB|)$
   $C \in k_1 \cap k_2$, $D \in k_1 \cap k_2$
3. $k_3 = k(C, |CD|)$
   $E \in k_1 \cap k_3$
4. $k_4 = k(D, |BE|)$
   $F \in k_2 \cap k_4$
5. $k_5 = k(E, |BE|)$
   $G \in k_2 \cap k_5$, $H \in k_2 \cap k_5$

Distances

$d_0 = |FH|$ (red) ... 1
$d_1 = |AG| = |AH|$ (blue) ... 2
$d_2 = |FG|$ (green) ... 1

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2}$ (4 occurrences)
Construction Process

A, B given initial points

1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   
3. \( C \in k_1 \cap k_2, D \in k_1 \cap k_2 \)
4. \( k_3 = k(C, |CD|) \)
   
5. \( E \in k_1 \cap k_3, F \in k_2 \cap k_3 \)

Distances

\( d_0 = |GI| \) (red) \ldots 1

\( d_1 = |BH| = |BI| \) (blue) \ldots 2

\( d_2 = |GH| \) (green) \ldots 1

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \) (4 occurrences)
2.84.3 Construction MM269

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, |CD|) \)
   \( E \in k_1 \cap k_3 \)
4. \( k_4 = k(C, |BE|) \)
   \( F \in k_2 \cap k_4 \)
5. \( k_5 = k(E, |BE|) \)
   \( G \in k_2 \cap k_5 \), \( H \in k_2 \cap k_5 \)

Distances

\( d_0 = |FG| \) (red) \ldots 1
\( d_1 = |AG| = |AH| \) (blue) \ldots 2
\( d_2 = |FH| \) (green) \ldots 1

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \) (4 occurrences)
2.84.4 Construction MM270

Construction Process

\(A, B\) given initial points
1. \(k_1 = k(A, |AB|)\)
2. \(k_2 = k(B, |AB|)\)
   \(C \in k_1 \cap k_2, D \in k_1 \cap k_2\)
3. \(k_3 = k(D, |CD|)\)
   \(E \in k_1 \cap k_3, F \in k_2 \cap k_3\)
4. \(k_4 = k(C, |BE|)\)
   \(G \in k_1 \cap k_4\)
5. \(k_5 = k(F, |BE|)\)
   \(H \in k_1 \cap k_5, I \in k_1 \cap k_5\)

Distances

\(d_0 = |GH|\) (red) . . . 1
\(d_1 = |BH| = |BI|\) (blue) . . . 2
\(d_2 = |GI|\) (green) . . . 1

Occurrences of the Golden Ratio

\(\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2}\) (4 occurrences)
2.85 Class of Similar Constructions No. 76

Contained Constructions
MM164, MM165, MM271, MM272
2.85.1 Construction MM164

Construction Process

A, B given initial points
1. $k_1 = k(A, |AB|)$
2. $k_2 = k(B, |AB|)$
   $C \in k_1 \cap k_2$, $D \in k_1 \cap k_2$
3. $k_3 = k(C, |CD|)$
   $E \in k_1 \cap k_3$, $F \in k_2 \cap k_3$
4. $k_4 = k(D, |BE|)$
   $G \in k_1 \cap k_4$, $H \in k_2 \cap k_4$, $I \in k_3 \cap k_4$, $J \in k_3 \cap k_4$
5. $k_5 = k(G, |AB|)$
   $K \in k_3 \cap k_5$, $L \in k_4 \cap k_5$, $M \in k_4 \cap k_5$

Distances

$d_0 = |HL|$ (red) ... 1
$d_1 = |AH| = |BG| = |CD| = |CE| = |CF| = |CI| = |CJ| = |CK| = |DE| = |DF| = |EK|$ (blue) ... 11
$d_2 = |HM|$ (green) ... 1

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_1} = \frac{d_4}{d_5}$ (22 occurrences)
2.85.2 Construction MM165

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2, D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, |CD|) \)
   \( E \in k_1 \cap k_3, F \in k_2 \cap k_3 \)
4. \( k_4 = k(D, |BE|) \)
   \( G \in k_1 \cap k_4, H \in k_2 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4 \)
5. \( k_5 = k(H, |AB|) \)
   \( K \in k_3 \cap k_5, L \in k_4 \cap k_5, M \in k_4 \cap k_5 \)

Distances
\[ d_0 = |GM| \quad \text{(red)} \ldots 1 \]
\[ d_1 = |AH| = |BG| = |CD| = |CE| = |CF| = |CI| = |CJ| = |CK| = |DE| = |DF| = |FK| \quad \text{(blue)} \ldots 11 \]
\[ d_2 = |GL| \quad \text{(green)} \ldots 1 \]

Occurrences of the Golden Ratio
\[ \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \quad \text{(22 occurrences)} \]
2.85.3 Construction MM271

Construction Process

A, B given initial points
1. \(k_1 = k(A, |AB|)\)
2. \(k_2 = k(B, |AB|)\)
   \(C \in k_1 \cap k_2, D \in k_1 \cap k_2\)
3. \(k_3 = k(D, |CD|)\)
   \(E \in k_1 \cap k_3, F \in k_2 \cap k_3\)
4. \(k_4 = k(C, |BE|)\)
   \(G \in k_1 \cap k_4, H \in k_2 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4\)
5. \(k_5 = k(G, |AB|)\)
   \(K \in k_3 \cap k_5, L \in k_4 \cap k_5, M \in k_4 \cap k_5\)

Distances

\[d_0 = |HL| \text{ (red)} \ldots 1\]
\[d_1 = |AH| = |BG| = |CD| = |CE| = |CF| = |DE| = |DF| = |DI| = |DJ| = |DK| = |EK| \text{ (blue)} \ldots 11\]
\[d_2 = |HM| \text{ (green)} \ldots 1\]

Occurrences of the Golden Ratio

\(\phi = \frac{d_0}{x} = \frac{d_4}{x_2} \text{ (22 occurrences)}\)
2.85.4 Construction MM272

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2, \ D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, |CD|) \)
   \( E \in k_1 \cap k_3, \ F \in k_2 \cap k_3 \)
4. \( k_4 = k(C, |BE|) \)
   \( G \in k_1 \cap k_4, \ H \in k_2 \cap k_4, \ I \in k_3 \cap k_4, \ J \in k_3 \cap k_4 \)
5. \( k_5 = k(H, |AB|) \)
   \( K \in k_3 \cap k_5, \ L \in k_4 \cap k_5, \ M \in k_4 \cap k_5 \)

Distances

\( d_0 = |GM| \) (red) ... 1
\( d_1 = |AH| = |BG| = |CD| = |CE| = |CF| = |DE| = |DF| = |DI| = |DJ| = |DK| = |FK| \) (blue) ... 11
\( d_2 = |GL| \) (green) ... 1

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} = \frac{d_4}{d_2} \) (22 occurrences)
2.86 Class of Similar Constructions No. 77

Contained Constructions

MM167, MM213, MM281, MM327
2.86.1 Construction MM167

Construction Process

$A, B$ given initial points
1. $k_1 = k(A, |AB|)$
2. $k_2 = k(B, |AB|)$
   $C \in k_1 \cap k_2$, $D \in k_1 \cap k_2$
3. $k_3 = k(C, |CD|)$
   $E \in k_1 \cap k_3$
4. $k_4 = k(E, |BE|)$
   $F \in k_2 \cap k_4$, $G \in k_2 \cap k_4$
5. $k_5 = k(F, |CD|)$
   $H \in k_2 \cap k_5$, $I \in k_2 \cap k_5$

Distances

$d_0 = |AI|$ (red) ... 1
$d_1 = |AF| = |AG|$ (blue) ... 2
$d_2 = |AH|$ (green) ... 1

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2}$ (4 occurrences)


2.86.2 Construction MM213

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2, D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, |CD|) \)
   \( E \in k_1 \cap k_3, F \in k_2 \cap k_3 \)
4. \( k_4 = k(F, |BE|) \)
   \( G \in k_1 \cap k_4, H \in k_1 \cap k_4 \)
5. \( k_5 = k(G, |CD|) \)
   \( I \in k_1 \cap k_5, J \in k_1 \cap k_5 \)

Distances

\( d_0 = |BI| \) (red) \ldots 1
\( d_1 = |BG| = |BH| \) (blue) \ldots 2
\( d_2 = |BJ| \) (green) \ldots 1

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} = \frac{d_2}{d_4} \) (4 occurrences)
2.86.3 Construction MM281

Construction Process

\(\text{A, B given initial points}\)
1. \(k_1 = k(A, |AB|)\)
2. \(k_2 = k(B, |AB|)\)
   \(C \in k_1 \cap k_2, D \in k_1 \cap k_2\)
3. \(k_3 = k(D, |CD|)\)
   \(E \in k_1 \cap k_3\)
4. \(k_4 = k(E, |BE|)\)
   \(F \in k_2 \cap k_4, G \in k_2 \cap k_4\)
5. \(k_5 = k(G, |CD|)\)
   \(H \in k_2 \cap k_5, I \in k_2 \cap k_5\)

Distances

\(d_0 = |AI| \text{ (red)} \ldots 1\)
\(d_1 = |AF| = |AG| \text{ (blue)} \ldots 2\)
\(d_2 = |AH| \text{ (green)} \ldots 1\)

Occurrences of the Golden Ratio

\(\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (4 occurrences)}\)
2.86.4 Construction MM327

Construction Process

\[ A, B \text{ given initial points} \]
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2, D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, |CD|) \)
   \( E \in k_1 \cap k_3, F \in k_2 \cap k_3 \)
4. \( k_4 = k(F, |BE|) \)
   \( G \in k_1 \cap k_4, H \in k_1 \cap k_4 \)
5. \( k_5 = k(H, |CD|) \)
   \( I \in k_1 \cap k_5, J \in k_1 \cap k_5 \)

Distances

\[ d_0 = |BI| \text{ (red)} \ldots 1 \]
\[ d_1 = |BG| = |BH| \text{ (blue)} \ldots 2 \]
\[ d_2 = |BJ| \text{ (green)} \ldots 1 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} = \frac{d_4}{d_2} \text{ (4 occurrences)} \]
2.87 Class of Similar Constructions No. 78

Contained Constructions

MM170, MM216, MM278, MM324
2.87.1 Construction MM170

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2, \ D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, |CD|) \)
   \( E \in k_1 \cap k_3 \)
4. \( k_4 = k(E, |BE|) \)
   \( F \in k_2 \cap k_4, \ G \in k_2 \cap k_4 \)
5. \( k_5 = k(G, |CD|) \)
   \( H \in k_2 \cap k_5, \ I \in k_2 \cap k_5 \)

Distances

\( d_0 = |AI| \) (red) \ldots 1
\( d_1 = |AF| = |AG| \) (blue) \ldots 2
\( d_2 = |AH| \) (green) \ldots 1

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \) (4 occurrences)
2.87.2 Construction MM216

**Construction Process**

A, B given initial points
1. $k_1 = k(A, |AB|)$
2. $k_2 = k(B, |AB|)$
   $C \in k_1 \cap k_2, \ D \in k_1 \cap k_2$
3. $k_3 = k(C, |CD|)$
   $E \in k_1 \cap k_3, \ F \in k_2 \cap k_3$
4. $k_4 = k(F, |BE|)$
   $G \in k_1 \cap k_4, \ H \in k_1 \cap k_4$
5. $k_5 = k(H, |CD|)$
   $I \in k_1 \cap k_5, \ J \in k_1 \cap k_5$

**Distances**

$d_0 = |BI|$ (red) ... 1
$d_1 = |BG| = |BH|$ (blue) ... 2
$d_2 = |BJ|$ (green) ... 1

**Occurrences of the Golden Ratio**

$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2}$ (4 occurrences)
2.87.3 Construction MM278

Construction Process

1. \(k_1 = k(A, |AB|)\)
2. \(k_2 = k(B, |AB|)\)
3. \(k_3 = k(D, |CD|)\)
4. \(k_4 = k(E, |BE|)\)
5. \(k_5 = k(F, |CD|)\)

Distances

- \(d_0 = |AI|\) (red) \ldots 1
- \(d_1 = |AF| = |AG|\) (blue) \ldots 2
- \(d_2 = |AH|\) (green) \ldots 1

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \] (4 occurrences)
2.87.4 Construction MM324

Construction Process

\[ A, B \text{ given initial points} \]
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2, D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, |CD|) \)
   \( E \in k_1 \cap k_3, F \in k_2 \cap k_3 \)
4. \( k_4 = k(F, |BE|) \)
   \( G \in k_1 \cap k_4, H \in k_1 \cap k_4 \)
5. \( k_5 = k(G, |CD|) \)
   \( I \in k_1 \cap k_5, J \in k_1 \cap k_5 \)

Distances

\[ d_0 = |BI| \text{ (red)} \ldots 1 \]
\[ d_1 = |BG| = |BH| \text{ (blue)} \ldots 2 \]
\[ d_2 = |BJ| \text{ (green)} \ldots 1 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (4 occurrences)} \]
2.88 Class of Similar Constructions No. 79

Contained Constructions

MM172, MM218, MM284, MM330
2.88.1 Construction MM172

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2, D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, |CD|) \)
   \( E \in k_1 \cap k_3, F \in k_2 \cap k_3 \)
4. \( k_4 = k(E, |BE|) \)
   \( G \in k_2 \cap k_4, H \in k_2 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4 \)
5. \( k_5 = k(C, |AG|) \)
   \( K \in k_1 \cap k_5, L \in k_1 \cap k_5, M \in k_2 \cap k_5, N \in k_2 \cap k_5, O \in k_4 \cap k_5, P \in k_4 \cap k_5 \)

Distances

\[ d_0 = |EN| = |FK| \text{ (red)} \ldots 2 \]
\[ d_1 = |DK| = |DN| = |EL| = |FM| \text{ (blue)} \ldots 4 \]
\[ d_2 = |CD| = |CE| = |CF| = |CI| = |CJ| = |DE| = |DF| \text{ (green)} \ldots 7 \]
\[ d_3 = |AG| = |AH| = |CK| = |CL| = |CM| = |CN| = |CO| = |CP| = |KM| = |LN| \text{ (cyan)} \ldots 10 \]
\[ d_4 = |EM| = |FL| \text{ (magenta)} \ldots 2 \]
\[ d_5 = |DL| = |DM| = |EK| = |FN| \text{ (yellow)} \ldots 4 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_4}{d_5} = \frac{d_3}{d_5} \text{ (108 occurrences)} \]
2.88.2 Construction MM218

\[ A, B \text{ given initial points} \]

1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \[ C \in k_1 \cap k_2, \quad D \in k_1 \cap k_2 \]
3. \( k_3 = k(C, |CD|) \)
   \[ E \in k_1 \cap k_3, \quad F \in k_2 \cap k_3 \]
4. \( k_4 = k(F, |BE|) \)
   \[ G \in k_1 \cap k_4, \quad H \in k_1 \cap k_4, \quad I \in k_3 \cap k_4, \quad J \in k_3 \cap k_4 \]
5. \( k_5 = k(C, |BG|) \)
   \[ K \in k_1 \cap k_5, \quad L \in k_1 \cap k_5, \quad M \in k_2 \cap k_5, \quad N \in k_2 \cap k_5, \quad O \in k_4 \cap k_5, \quad P \in k_4 \cap k_5 \]

**Distances**

\[ d_0 = |EN| = |FK| \text{ (red)} \ldots 2 \]
\[ d_1 = |DK| = |DN| = |EL| = |FM| \text{ (blue)} \ldots 4 \]
\[ d_2 = |CD| = |CE| = |CF| = |CI| = |CJ| = |DE| = |DF| \text{ (green)} \ldots 7 \]
\[ d_3 = |BG| = |BH| = |CK| = |CL| = |CM| = |CN| = |CO| = |CP| = |KM| = |LN| \text{ (cyan)} \ldots 10 \]
\[ d_4 = |EM| = |FL| \text{ (magenta)} \ldots 2 \]
\[ d_5 = |DL| = |DM| = |EK| = |FN| \text{ (yellow)} \ldots 4 \]

**Occurrences of the Golden Ratio**

\[ \phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (108 occurrences)} \]
2.88.3 Construction MM284

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, |CD|) \)
   \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(E, |BE|) \)
   \( G \in k_2 \cap k_4 \), \( H \in k_2 \cap k_4 \), \( I \in k_3 \cap k_4 \), \( J \in k_3 \cap k_4 \)
5. \( k_5 = k(D, |AG|) \)
   \( K \in k_1 \cap k_5 \), \( L \in k_1 \cap k_5 \), \( M \in k_2 \cap k_5 \), \( N \in k_2 \cap k_5 \), \( O \in k_4 \cap k_5 \), \( P \in k_4 \cap k_5 \)

Distances

\( d_0 = |EN| = |FK| \) (red) \( \ldots 2 \)
\( d_1 = |CK| = |CN| = |EL| = |FM| \) (blue) \( \ldots 4 \)
\( d_2 = |CD| = |CE| = |CF| = |DE| = |DF| = |DI| = |DJ| \) (green) \( \ldots 7 \)
\( d_3 = |AG| = |AH| = |DK| = |DL| = |DM| = |DN| = |DO| = |DP| = |KM| = |LN| \) (cyan) \( \ldots 10 \)
\( d_4 = |EM| = |FL| \) (magenta) \( \ldots 2 \)
\( d_5 = |CL| = |CM| = |EK| = |FN| \) (yellow) \( \ldots 4 \)

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_4}{d_5} = \frac{d_5}{d_2} \) (108 occurrences)
2.88.4 Construction MM330

Construction Process

1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, |CD|) \)
   \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(F, |BE|) \)
   \( G \in k_1 \cap k_4 \), \( H \in k_1 \cap k_4 \), \( I \in k_3 \cap k_4 \), \( J \in k_3 \cap k_4 \)
5. \( k_5 = k(D, |BG|) \)
   \( K \in k_1 \cap k_5 \), \( L \in k_1 \cap k_5 \), \( M \in k_2 \cap k_5 \), \( N \in k_2 \cap k_5 \), \( O \in k_4 \cap k_5 \), \( P \in k_4 \cap k_5 \)

Distances

\( d_0 = |EN| = |FK| \) \hspace{1em} red \hspace{1em} 2
\( d_1 = |CK| = |CN| = |EL| = |FM| \) \hspace{1em} blue \hspace{1em} 4
\( d_2 = |CD| = |CE| = |CF| = |DE| = |DF| = |DI| = |DJ| \) \hspace{1em} green \hspace{1em} 7
\( d_3 = |BG| = |BH| = |DK| = |DL| = |DM| = |DN| = |DO| = |DP| = |KM| = |LN| \) \hspace{1em} cyan \hspace{1em} 10
\( d_4 = |EM| = |FL| \) \hspace{1em} magenta \hspace{1em} 2
\( d_5 = |CL| = |CM| = |EK| = |FN| \) \hspace{1em} yellow \hspace{1em} 4

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_2} = \frac{d_4}{d_5} = \frac{d_2}{d_4} = \frac{d_5}{d_2} \) \hspace{1em} (108 occurrences)
2.89 Class of Similar Constructions No. 80

Contained Constructions

MM173, MM219, MM283, MM329
2.89.1 Construction MM173

Construction Process

A, B given initial points
1. $k_1 = k(A, |AB|)$
2. $k_2 = k(B, |AB|)$
   $C \in k_1 \cap k_2 , \ D \in k_1 \cap k_2$
3. $k_3 = k(C, |CD|)$
   $E \in k_1 \cap k_3 , \ F \in k_2 \cap k_3$
4. $k_4 = k(E, |BE|)$
   $G \in k_2 \cap k_4 , \ H \in k_2 \cap k_4 , \ I \in k_3 \cap k_4 , \ J \in k_3 \cap k_4$
5. $k_5 = k(D, |AG|)$
   $K \in k_1 \cap k_5 , \ L \in k_1 \cap k_5 , \ M \in k_2 \cap k_5 , \ N \in k_2 \cap k_5 , \ O \in k_3 \cap k_5 , \ P \in k_3 \cap k_5 , \ Q \in k_4 \cap k_5 , \ R \in k_4 \cap k_5$

Distances

$$d_0 = |EN| = |FK| \ (red) \ldots 2$$
$$d_1 = |CK| = |CN| = |EL| = |FM| \ (blue) \ldots 4$$
$$d_2 = |CD| = |CE| = |CF| = |CI| = |CJ| = |CO| = |CP| = |DE| = |DF| \ (green) \ldots 9$$
$$d_3 = |AG| = |AH| = |DK| = |DL| = |DM| = |DN| = |DO| = |DP| = |DQ| = |DR| = |KM| = |LN| \ (cyan) \ldots 12$$
$$d_4 = |EM| = |FL| \ (magenta) \ldots 2$$
$$d_5 = |CL| = |CM| = |EK| = |FN| \ (yellow) \ldots 4$$

Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \ (132 \ occurrences)$$
2.89.2 Construction MM219

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, |CD|) \)
   
   \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(F, |BE|) \)
   
   \( G \in k_1 \cap k_4 \), \( H \in k_1 \cap k_4 \), \( I \in k_3 \cap k_4 \), \( J \in k_3 \cap k_4 \)
5. \( k_5 = k(D, |BG|) \)
   
   \( K \in k_1 \cap k_5 \), \( L \in k_1 \cap k_5 \), \( M \in k_2 \cap k_5 \), \( N \in k_2 \cap k_5 \), \( O \in k_3 \cap k_5 \)
   \( P \in k_3 \cap k_5 \), \( Q \in k_4 \cap k_5 \), \( R \in k_4 \cap k_5 \)

Distances

\[
\begin{align*}
d_0 &= |EN| = |FK| \quad \text{(red)} \ldots 2 \\
d_1 &= |CK| = |CN| = |EL| = |FM| \quad \text{(blue)} \ldots 4 \\
d_2 &= |CD| = |CE| = |CF| = |CI| = |CJ| = |CO| = |CP| = |DE| = |DF| \\
        \quad \text{(green)} \ldots 9 \\
d_3 &= |BG| = |BH| = |DK| = |DL| = |DM| = |DN| = |DO| = |DP| = \\
        |DQ| = |DR| = |KM| = |LN| \quad \text{(cyan)} \ldots 12 \\
d_4 &= |EM| = |FL| \quad \text{(magenta)} \ldots 2 \\
d_5 &= |CL| = |CM| = |EK| = |FN| \quad \text{(yellow)} \ldots 4
\end{align*}
\]

Occurrences of the Golden Ratio

\[
\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_4}{d_5} = \frac{d_5}{d_2} \quad (132 \text{ occurrences})
\]
2.89.3 Construction MM283

Construction Process

A, B given initial points
1. \(k_1 = k(A, |AB|)\)
2. \(k_2 = k(B, |AB|)\)
   \(C \in k_1 \cap k_2, \ D \in k_1 \cap k_2\)
3. \(k_3 = k(D, |CD|)\)
   \(E \in k_1 \cap k_3, \ F \in k_2 \cap k_3\)
4. \(k_4 = k(E, |BE|)\)
   \(G \in k_2 \cap k_4, \ H \in k_2 \cap k_4, \ I \in k_3 \cap k_4, \ J \in k_3 \cap k_4\)
5. \(k_5 = k(C, |AG|)\)
   \(K \in k_1 \cap k_5, \ L \in k_1 \cap k_5, \ M \in k_2 \cap k_5, \ N \in k_2 \cap k_5, \ O \in k_3 \cap k_5, \ P \in k_3 \cap k_5, \ Q \in k_4 \cap k_5, \ R \in k_4 \cap k_5\)

Distances

\[d_0 = |EN| = |FK| \text{ (red)} \ldots 2\]
\[d_1 = |DK| = |DN| = |EL| = |FM| \text{ (blue)} \ldots 4\]
\[d_2 = |CD| = |CE| = |CF| = |DE| = |DF| = |DI| = |DJ| = |DO| = |DP| \text{ (green)} \ldots 9\]
\[d_3 = |AG| = |AH| = |CK| = |CL| = |CM| = |CN| = |CO| = |CP| = |CQ| = |CR| = |KM| = |LN| \text{ (cyan)} \ldots 12\]
\[d_4 = |EM| = |FL| \text{ (magenta)} \ldots 2\]
\[d_5 = |DL| = |DM| = |EK| = |FN| \text{ (yellow)} \ldots 4\]

Occurrences of the Golden Ratio

\[\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (132 occurrences)}\]
2.89.4 Construction MM329

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2, D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, |CD|) \)
   \( E \in k_1 \cap k_3, F \in k_2 \cap k_3 \)
4. \( k_4 = k(F, |BE|) \)
   \( G \in k_1 \cap k_4, H \in k_1 \cap k_4 \)
5. \( k_5 = k(C, |BG|) \)
   \( K \in k_1 \cap k_5, L \in k_1 \cap k_5 \)
   \( M \in k_2 \cap k_5, N \in k_2 \cap k_5 \)
   \( O \in k_3 \cap k_5 \)
   \( P \in k_3 \cap k_5 \)

Distances

\( d_0 = |EN| = |FK| \) (red) .. 2
\( d_1 = |DK| = |DN| = |EL| = |FM| \) (blue) .. 4
\( d_2 = |CD| = |CE| = |CF| = |DE| = |DF| = |DI| = |DJ| = |DO| = |DP| \) (green) .. 9
\( d_3 = |BG| = |BH| = |CK| = |CL| = |CM| = |CN| = |CO| = |CP| = |CQ| = |CR| = |KM| = |LN| \) (cyan) .. 12
\( d_4 = |EM| = |FL| \) (magenta) .. 2
\( d_5 = |DL| = |DM| = |EK| = |FN| \) (yellow) .. 4

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_4}{d_5} = \frac{d_5}{d_2} \) (132 occurrences)
2.90 Class of Similar Constructions No. 81

Contained Constructions
MM174, MM221, MM285, MM332
2.90.1 Construction MM174

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, |CD|) \)
   \( E \in k_1 \cap k_3 \)
4. \( k_4 = k(E, |BE|) \)
   \( F \in k_2 \cap k_4 \), \( G \in k_2 \cap k_4 \)
5. \( k_5 = k(E, |AF|) \)
   \( H \in k_1 \cap k_5 \), \( I \in k_1 \cap k_5 \), \( J \in k_2 \cap k_5 \), \( K \in k_2 \cap k_5 \), \( L \in k_3 \cap k_5 \)

Distances

\[ d_0 = |CI| = |DH| \quad \text{(red)} \quad 2 \]
\[ d_1 = |AF| = |AG| = |EH| = |EI| = |EJ| = |EK| = |EL| = |EM| \quad \text{(blue)} \]
\[ \ldots 8 \]
\[ d_2 = |CH| = |DI| \quad \text{(green)} \quad 2 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} = \frac{d_2}{d_2} \quad \text{(32 occurrences)} \]
2.90.2 Construction MM221

Construction Process

A, B given initial points
1. $k_1 = k(A, |AB|)$
2. $k_2 = k(B, |AB|)$
   $C \in k_1 \cap k_2$, $D \in k_1 \cap k_2$
3. $k_3 = k(C, |CD|)$
   $E \in k_1 \cap k_3$, $F \in k_2 \cap k_3$
4. $k_4 = k(F, |BE|)$
   $G \in k_1 \cap k_4$, $H \in k_1 \cap k_4$
5. $k_5 = k(F, |BG|)$
   $I \in k_1 \cap k_5$, $J \in k_1 \cap k_5$, $K \in k_2 \cap k_5$, $L \in k_2 \cap k_5$, $M \in k_3 \cap k_5$, $N \in k_3 \cap k_5$

Distances

$d_0 = |CL| = |DK|$ (red) \ldots 2
$d_1 = |BG| = |BH| = |FI| = |FJ| = |FK| = |FL| = |FM| = |FN|$ (blue) \ldots 8
$d_2 = |CK| = |DL|$ (green) \ldots 2

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_1} = \frac{d_2}{d_4}$ (32 occurrences)
2.90.3 Construction MM285

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, |CD|) \)
   \( E \in k_1 \cap k_3 \)
4. \( k_4 = k(E, |BE|) \)
   \( F \in k_2 \cap k_4 \), \( G \in k_2 \cap k_4 \)
5. \( k_5 = k(E, |AF|) \)
   \( H \in k_1 \cap k_5 \), \( I \in k_1 \cap k_5 \), \( J \in k_2 \cap k_5 \), \( K \in k_2 \cap k_5 \), \( L \in k_3 \cap k_5 \), \( M \in k_3 \cap k_5 \)

Distances

\( d_0 = |CI| = |DH| \) (red) \( \ldots 2 \)
\( d_1 = |AF| = |AG| = |EH| = |EI| = |EJ| = |EK| = |EL| = |EM| \) (blue)
\( \ldots 8 \)
\( d_2 = |CH| = |DI| \) (green) \( \ldots 2 \)

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} = \frac{d_2}{d_2} \) (32 occurrences)
2.90.4 Construction MM332

Construction Process

\(A, B\) given initial points

1. \(k_1 = k(A, |AB|)\)
2. \(k_2 = k(B, |AB|)\)
3. \(k_3 = k(D, |CD|)\)
   \(C \in k_1 \cap k_2, D \in k_1 \cap k_2\)
4. \(k_4 = k(F, |BE|)\)
   \(E \in k_1 \cap k_3, F \in k_2 \cap k_3\)
5. \(k_5 = k(F, |BG|)\)
   \(G \in k_1 \cap k_4, H \in k_1 \cap k_4\)
   \(I \in k_1 \cap k_5, J \in k_1 \cap k_5, K \in k_2 \cap k_5, L \in k_2 \cap k_5, M \in k_3 \cap k_5, N \in k_3 \cap k_5\)

Distances

\(d_0 = |CL| = |DK|\) (red) \(\ldots 2\)
\(d_1 = |BG| = |BH| = |FI| = |FK| = |FL| = |FM| = |FN|\) (blue) \(\ldots 8\)
\(d_2 = |CK| = |DL|\) (green) \(\ldots 2\)

Occurrences of the Golden Ratio

\(\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2}\) (32 occurrences)
2.91 Class of Similar Constructions No. 82

Contained Constructions

MM175, MM220, MM286, MM331
2.91.1 Construction MM175

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2, D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, |CD|) \)
   \( E \in k_1 \cap k_3, F \in k_2 \cap k_3 \)
4. \( k_4 = k(E, |BE|) \)
   \( G \in k_2 \cap k_4, H \in k_2 \cap k_4 \)
5. \( k_5 = k(F, |AG|) \)
   \( I \in k_1 \cap k_5, J \in k_1 \cap k_5, K \in k_2 \cap k_5, L \in k_2 \cap k_5, M \in k_3 \cap k_5, N \in k_3 \cap k_5, O \in k_4 \cap k_5, P \in k_4 \cap k_5 \)

Distances

\[ d_0 = |CL| = |DK| \] (red) \ldots 2
\[ d_1 = |AG| = |AH| = |FI| = |FJ| = |FK| = |FL| = |FM| = |FN| = |FO| = |FP| \] (blue) \ldots 10
\[ d_2 = |CK| = |DL| \] (green) \ldots 2

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} = \frac{d_2}{d_2} \] (40 occurrences)
2.91.2 Construction MM220

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, |CD|) \)
   \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(F, |BE|) \)
   \( G \in k_1 \cap k_4 \), \( H \in k_1 \cap k_4 \)
5. \( k_5 = k(E, |BG|) \)
   \( I \in k_1 \cap k_5 \), \( J \in k_1 \cap k_5 \), \( K \in k_2 \cap k_5 \), \( L \in k_2 \cap k_5 \), \( M \in k_3 \cap k_5 \), \( N \in k_3 \cap k_5 \), \( O \in k_4 \cap k_5 \), \( P \in k_4 \cap k_5 \)

Distances

\( d_0 = |CJ| = |DI| \) (red) ... 2
\( d_1 = |BG| = |BH| = |EI| = |EJ| = |EK| = |EL| = |EM| = |EN| = |EO| = |EP| \) (blue) ... 10
\( d_2 = |CI| = |DJ| \) (green) ... 2

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \) (40 occurrences)
Construction MM286

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, |CD|) \)
   \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(E, |BE|) \)
   \( G \in k_2 \cap k_4 \), \( H \in k_2 \cap k_4 \)
5. \( k_5 = k(F, |AG|) \)
   \( I \in k_1 \cap k_5 \), \( J \in k_1 \cap k_5 \), \( K \in k_2 \cap k_5 \), \( L \in k_2 \cap k_5 \), \( M \in k_3 \cap k_5 \), \( N \in k_3 \cap k_5 \), \( O \in k_4 \cap k_5 \), \( P \in k_4 \cap k_5 \)

Distances

\( d_0 = |CL| = |DK| \) (red) \( \ldots 2 \)
\( d_1 = |AG| = |AH| = |FJ| = |FK| = |FL| = |FM| = |FN| = |FO| = |FP| \) (blue) \( \ldots 10 \)
\( d_2 = |CK| = |DL| \) (green) \( \ldots 2 \)

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} = \frac{d_2}{d_2} \) (40 occurrences)
2.91.4 Construction MM331

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2, D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, |CD|) \)
   \( E \in k_1 \cap k_3, F \in k_2 \cap k_3 \)
4. \( k_4 = k(F, |BE|) \)
   \( G \in k_1 \cap k_4, H \in k_1 \cap k_4 \)
5. \( k_5 = k(E, |BG|) \)
   \( I \in k_1 \cap k_5, J \in k_1 \cap k_5, K \in k_2 \cap k_5, L \in k_2 \cap k_5, M \in k_3 \cap k_5, N \in k_3 \cap k_5, O \in k_4 \cap k_5, P \in k_4 \cap k_5 \)

Distances

\[ d_0 = |CJ| = |DI| \] (red) ... 2
\[ d_1 = |BG| = |BH| = |EI| = |EJ| = |EK| = |EL| = |EM| = |EN| = |EO| = |EP| \] (blue) ... 10
\[ d_2 = |CI| = |DJ| \] (green) ... 2

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} = \frac{d_2}{d_2} \] (40 occurrences)
2.92 Class of Similar Constructions No. 83

Contained Constructions
MM176, MM223, MM287, MM334
2.92.1 Construction MM176

Construction Process

$A, B$ given initial points
1. $k_1 = k(A, |AB|)$
2. $k_2 = k(B, |AB|)$
   $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
3. $k_3 = k(C, |CD|)$
   $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
4. $k_4 = k(E, |BE|)$
   $G \in k_2 \cap k_4, H \in k_2 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
5. $k_5 = k(A, |CG|)$
   $K \in k_2 \cap k_5, L \in k_2 \cap k_5$

Distances

$d_0 = |FK| = |FL|$ (red) \ldots 2
$d_1 = |CD| = |CE| = |CF| = |CI| = |CJ| = |DE| = |DF|$ (blue) \ldots 7
$d_2 = |AG| = |AH| = |CL| = |DK|$ (green) \ldots 4
$d_3 = |EK| = |EL|$ (cyan) \ldots 2
$d_4 = |CK| = |DL|$ (magenta) \ldots 2

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_2} = \frac{d_3}{d_4} = \frac{d_2}{d_4}$ (30 occurrences)
2.92.2 Construction MM223

Construction Process

$A, B$ given initial points
1. $k_1 = k(A, |AB|)$
2. $k_2 = k(B, |AB|)$
   $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
3. $k_3 = k(C, |CD|)$
   $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
4. $k_4 = k(F, |BE|)$
   $G \in k_1 \cap k_4, H \in k_1 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
5. $k_5 = k(B, |CG|)$
   $K \in k_1 \cap k_5, L \in k_1 \cap k_5$

Distances

$d_0 = |EK| = |EL|$ (red) ... 2
$d_1 = |CD| = |CE| = |CF| = |CI| = |CJ| = |DE| = |DF|$ (blue) ... 7
$d_2 = |BG| = |BH| = |CL| = |DK|$ (green) ... 4
$d_3 = |FK| = |FL|$ (cyan) ... 2
$d_4 = |CK| = |DL|$ (magenta) ... 2

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_2} = \frac{d_2}{d_3} = \frac{d_3}{d_4}$ (30 occurrences)
2.92.3 Construction MM287

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2, D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, |CD|) \)
   \( E \in k_1 \cap k_3, F \in k_2 \cap k_3 \)
4. \( k_4 = k(E, |BE|) \)
   \( G \in k_2 \cap k_4, H \in k_2 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4 \)
5. \( k_5 = k(A, |CG|) \)
   \( K \in k_2 \cap k_5, L \in k_2 \cap k_5 \)

Distances

\[ d_0 = |FK| = |FL| \text{ (red)} \ldots 2 \]
\[ d_1 = |CD| = |CE| = |CF| = |DE| = |DF| = |DI| = |DJ| \text{ (blue)} \ldots 7 \]
\[ d_2 = |AG| = |AH| = |CL| = |DK| \text{ (green)} \ldots 4 \]
\[ d_3 = |EK| = |EL| \text{ (cyan)} \ldots 2 \]
\[ d_4 = |CK| = |DL| \text{ (magenta)} \ldots 2 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_3}{d_5} = \frac{d_4}{d_4} \text{ (30 occurrences)} \]
2.92.4 Construction MM334

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2, \ D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, |CD|) \)
   \( E \in k_1 \cap k_3, \ F \in k_2 \cap k_3 \)
4. \( k_4 = k(F, |BE|) \)
   \( G \in k_1 \cap k_4, \ H \in k_1 \cap k_4, \ I \in k_3 \cap k_4, \ J \in k_3 \cap k_4 \)
5. \( k_5 = k(B, |CG|) \)
   \( K \in k_1 \cap k_5, \ L \in k_1 \cap k_5 \)

Distances

\( d_0 = |EK| = |EL| \) (red) \ldots 2
\( d_1 = |CD| = |CE| = |CF| = |DE| = |DF| = |DI| = |DJ| \) (blue) \ldots 7
\( d_2 = |BG| = |BH| = |CL| = |DK| \) (green) \ldots 4
\( d_3 = |FK| = |FL| \) (cyan) \ldots 2
\( d_4 = | CK| = | DL| \) (magenta) \ldots 2

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_2} = \frac{d_2}{d_3} = \frac{d_3}{d_4} \) (30 occurrences)
2.93 Class of Similar Constructions No. 84

Contained Constructions

MM177, MM222, MM288, MM333
2.93.1 Construction MM177

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2 , D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, |CD|) \)
   \( E \in k_1 \cap k_3 , F \in k_2 \cap k_3 \)
4. \( k_4 = k(E, |BE|) \)
   \( G \in k_2 \cap k_4 , H \in k_2 \cap k_4 , I \in k_3 \cap k_4 , J \in k_3 \cap k_4 \)
5. \( k_5 = k(B, |CG|) \)
   \( K \in k_1 \cap k_5 , L \in k_1 \cap k_5 \)

Distances

\( d_0 = |EK| = |EL| \) (red) \ldots 2
\( d_1 = |CD| = |CE| = |CF| = |CI| = |CJ| = |DE| = |DF| \) (blue) \ldots 7
\( d_2 = |AG| = |AH| = |CL| = |DK| \) (green) \ldots 4
\( d_3 = |FK| = |FL| \) (cyan) \ldots 2
\( d_4 = |CK| = |DL| \) (magenta) \ldots 2

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_2} = \frac{d_3}{d_4} \) (30 occurrences)
2.93.2 Construction MM222

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, |CD|) \)
   \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(F, |BE|) \)
   \( G \in k_1 \cap k_4 \), \( H \in k_1 \cap k_4 \), \( I \in k_3 \cap k_4 \), \( J \in k_3 \cap k_4 \)
5. \( k_5 = k(A, |CG|) \)
   \( K \in k_2 \cap k_5 \), \( L \in k_2 \cap k_5 \)

Distances

\( d_0 = |FK| = |FL| \quad \text{(red)} \ldots 2 \)
\( d_1 = |CD| = |CE| = |CF| = |CI| = |CJ| = |DE| = |DF| \quad \text{(blue)} \ldots 7 \)
\( d_2 = |BG| = |BH| = |CL| = |DK| \quad \text{(green)} \ldots 4 \)
\( d_3 = |EK| = |EL| \quad \text{(cyan)} \ldots 2 \)
\( d_4 = |CK| = |DL| \quad \text{(magenta)} \ldots 2 \)

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_2} = \frac{d_2}{d_4} = \frac{d_4}{d_4} \) (30 occurrences)
2.93.3 Construction MM288

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, |CD|) \)
   \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(E, |BE|) \)
   \( G \in k_2 \cap k_4 \), \( H \in k_2 \cap k_4 \), \( I \in k_3 \cap k_4 \), \( J \in k_3 \cap k_4 \)
5. \( k_5 = k(B, |CG|) \)
   \( K \in k_1 \cap k_5 \), \( L \in k_1 \cap k_5 \)

Distances

\[
\begin{align*}
d_0 &= |EK| = |EL| \text{ (red) } \ldots 2 \\
d_1 &= |CD| = |CE| = |CF| = |DE| = |DF| = |DI| = |DJ| \text{ (blue) } \ldots 7 \\
d_2 &= |AG| = |AH| = |CL| = |DK| \text{ (green) } \ldots 4 \\
d_3 &= |FK| = |FL| \text{ (cyan) } \ldots 2 \\
d_4 &= |CK| = |DL| \text{ (magenta) } \ldots 2
\end{align*}
\]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_2}{d_4} = \frac{d_2}{d_4} \text{ (30 occurrences) } \]
2.93.4 Construction MM333

**Construction Process**

*Given initial points* $A, B$:

1. $k_1 = k(A, |AB|)$
2. $k_2 = k(B, |AB|)$
3. $k_3 = k(D, |CD|)$
   - $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
4. $k_4 = k(F, |BE|)$
   - $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
5. $k_5 = k(A, |CG|)$
   - $G \in k_1 \cap k_4, H \in k_1 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
   - $K \in k_2 \cap k_5, L \in k_2 \cap k_5$

**Distances**

$\begin{align*}
d_0 &= |FK| = |FL| \quad \text{(red)} \ldots 2 \\
d_1 &= |CD| = |CE| = |CF| = |DE| = |DF| = |DI| = |DJ| \quad \text{(blue)} \ldots 7 \\
d_2 &= |BG| = |BH| = |CL| = |DK| \quad \text{(green)} \ldots 4 \\
d_3 &= |EK| = |EL| \quad \text{(cyan)} \ldots 2 \\
d_4 &= |CK| = |DL| \quad \text{(magenta)} \ldots 2
\end{align*}$

**Occurrences of the Golden Ratio**

$\phi = \frac{d_0}{d_2} = \frac{d_2}{d_3} = \frac{d_3}{d_4} \quad (30 \text{ occurrences})$
2.94 Class of Similar Constructions No. 85

Contained Constructions

MM180, MM227, MM291, MM338
2.94.1 Construction MM180

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, |CD|) \)
   \( E \in k_1 \cap k_3 \)
4. \( k_4 = k(E, |BE|) \)
   \( F \in k_2 \cap k_4 \), \( G \in k_2 \cap k_4 \)
5. \( k_5 = k(E, |CF|) \)
   \( H \in k_1 \cap k_5 \), \( I \in k_1 \cap k_5 \)

Distances

\[ d_0 = |BH| = |BI| \text{ (red)} \ldots 2 \]
\[ d_1 = |AF| = |AG| \text{ (blue)} \ldots 2 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} \text{ (4 occurrences)} \]
2.94.2 Construction MM227

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2 , D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, |CD|) \)
   \( E \in k_1 \cap k_3 , F \in k_2 \cap k_3 \)
4. \( k_4 = k(F, |BE|) \)
   \( G \in k_1 \cap k_4 , H \in k_1 \cap k_4 \)
5. \( k_5 = k(F, |CG|) \)
   \( I \in k_2 \cap k_5 , J \in k_2 \cap k_5 \)

Distances

\[ d_0 = |AI| = |AJ| \quad \text{(red)} \quad \ldots 2 \]
\[ d_1 = |BG| = |BH| \quad \text{(blue)} \quad \ldots 2 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} \quad \text{(4 occurrences)} \]
2.94.3 Construction MM291

Construction Process

1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
3. \( C \in k_1 \cap k_2, D \in k_1 \cap k_2 \)
4. \( k_3 = k(D, |CD|) \)
5. \( E \in k_1 \cap k_3 \)
6. \( k_4 = k(E, |BE|) \)
7. \( F \in k_2 \cap k_4, G \in k_2 \cap k_4 \)
8. \( k_5 = k(E, |CF|) \)
9. \( H \in k_1 \cap k_5, I \in k_1 \cap k_5 \)

Distances

\[
d_0 = |BH| = |BI| \text{ (red)} \quad \ldots 2
\]
\[
d_1 = |AF| = |AG| \text{ (blue)} \quad \ldots 2
\]

Occurrences of the Golden Ratio

\[
\phi = \frac{d_0}{d_1} \text{ (4 occurrences)}
\]
2.94.4 Construction MM338

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2, \ D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, |CD|) \)
   \( E \in k_1 \cap k_3, \ F \in k_2 \cap k_3 \)
4. \( k_4 = k(F, |BE|) \)
   \( G \in k_1 \cap k_4, \ H \in k_1 \cap k_4 \)
5. \( k_5 = k(F, |CG|) \)
   \( I \in k_2 \cap k_5, \ J \in k_2 \cap k_5 \)

Distances

\( d_0 = |AI| = |AJ| \) (red) \( \ldots 2 \)
\( d_1 = |BG| = |BH| \) (blue) \( \ldots 2 \)

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} \) (4 occurrences)
2.95 Class of Similar Constructions No. 86

Contained Constructions

MM181, MM226, MM292, MM337
2.95.1 Construction MM181

Construction Process

\[ A, B \text{ given initial points} \]
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \[ C \in k_1 \cap k_2, \quad D \in k_1 \cap k_2 \]
3. \( k_3 = k(C, |CD|) \)
   \[ E \in k_1 \cap k_3, \quad F \in k_2 \cap k_3 \]
4. \( k_4 = k(E, |BE|) \)
   \[ G \in k_2 \cap k_4, \quad H \in k_2 \cap k_4 \]
5. \( k_5 = k(F, |CG|) \)
   \[ I \in k_2 \cap k_5, \quad J \in k_2 \cap k_5 \]

Distances

\[ d_0 = |AI| = |AJ| \text{ (red)} \ldots 2 \]
\[ d_1 = |AG| = |AH| \text{ (blue)} \ldots 2 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} \text{ (4 occurrences)} \]
2.95.2 Construction MM226

Construction Process

1. $k_1 = k(A, |AB|)$
2. $k_2 = k(B, |AB|)$
3. $k_3 = k(C, |CD|)$
4. $k_4 = k(F, |BE|)$
5. $k_5 = k(E, |CG|)$

Distances

$d_0 = |BI| = |BJ|$ (red) ... 2
$d_1 = |BG| = |BH|$ (blue) ... 2

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_1}$ (4 occurrences)
2.95.3 Construction MM292

Construction Process

1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
3. \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
4. \( k_3 = k(D, |CD|) \)
5. \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
6. \( k_4 = k(E, |BE|) \)
7. \( G \in k_2 \cap k_4 \), \( H \in k_2 \cap k_4 \)
8. \( k_5 = k(F, |CG|) \)
9. \( I \in k_2 \cap k_5 \), \( J \in k_2 \cap k_5 \)

Distances

\[ d_0 = |AI| = |AJ| \text{ (red)} \ldots 2 \]
\[ d_1 = |AG| = |AH| \text{ (blue)} \ldots 2 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} \text{ (4 occurrences)} \]
2.95.4 Construction MM337

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2 , D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, |CD|) \)
   \( E \in k_1 \cap k_3 , F \in k_2 \cap k_3 \)
4. \( k_4 = k(F, |BE|) \)
   \( G \in k_1 \cap k_4 , H \in k_1 \cap k_4 \)
5. \( k_5 = k(E, |CG|) \)
   \( I \in k_1 \cap k_5 , J \in k_1 \cap k_5 \)

Distances

\( d_0 = |BI| = |BJ| \) (red) \( \ldots 2 \)
\( d_1 = |BG| = |BH| \) (blue) \( \ldots 2 \)

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} \) (4 occurrences)
2.96 Class of Similar Constructions No. 87

Contained Constructions
MM184, MM231, MM295, MM342
2.96.1 Construction MM184

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, |CD|) \)
   \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(E, |BE|) \)
   \( G \in k_2 \cap k_4 \), \( H \in k_2 \cap k_4 \), \( I \in k_3 \cap k_4 \), \( J \in k_3 \cap k_4 \)
5. \( k_5 = k(A, |DG|) \)
   \( K \in k_2 \cap k_5 \), \( L \in k_2 \cap k_5 \), \( M \in k_3 \cap k_5 \), \( N \in k_3 \cap k_5 \)

Distances

\( d_0 = |EK| = |EL| \) (red) \ldots 2
\( d_1 = |CL| = |DK| \) (blue) \ldots 2
\( d_2 = |CD| = |CE| = |CF| = |CI| = |CJ| = |CM| = |CN| = |DE| = |DF| \) (green) \ldots 9
\( d_3 = |AG| = |AH| = |CK| = |DL| \) (cyan) \ldots 4
\( d_4 = |FK| = |FL| \) (magenta) \ldots 2

Occurences of the Golden Ratio

\( \phi = \frac{d_0}{d_2} = \frac{d_2}{d_4} = \frac{d_4}{d_4} \) (34 occurrences)
2.96.2 Construction MM231

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2, D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, |CD|) \)
   \( E \in k_1 \cap k_3, F \in k_2 \cap k_3 \)
4. \( k_4 = k(F, |BE|) \)
   \( G \in k_1 \cap k_4, H \in k_1 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4 \)
5. \( k_5 = k(B, |DG|) \)
   \( K \in k_1 \cap k_5, L \in k_1 \cap k_5, M \in k_3 \cap k_5, N \in k_3 \cap k_5 \)

Distances

\( d_0 = |FK| = |FL| \) (red) \ldots 2
\( d_1 = |CL| = |DK| \) (blue) \ldots 2
\( d_2 = |CD| = |CE| = |CF| = |CI| = |CJ| = |CM| = |CN| = |DE| = |DF| \) (green) \ldots 9
\( d_3 = |BG| = |BH| = |CK| = |DL| \) (cyan) \ldots 4
\( d_4 = |EK| = |EL| \) (magenta) \ldots 2

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_2} = \frac{d_2}{d_3} = \frac{d_3}{d_4} \) (34 occurrences)
2.96.3 Construction MM295

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2 , D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, |CD|) \)
   \( E \in k_1 \cap k_3 , F \in k_2 \cap k_3 \)
4. \( k_4 = k(E, |BE|) \)
   \( G \in k_2 \cap k_4 , H \in k_2 \cap k_4 , I \in k_3 \cap k_4 , J \in k_3 \cap k_4 \)
5. \( k_5 = k(A, |DG|) \)
   \( K \in k_2 \cap k_5 , L \in k_2 \cap k_5 , M \in k_3 \cap k_5 , N \in k_3 \cap k_5 \)

Distances

\( d_0 = |EK| = |EL| \) (red) \ldots 2
\( d_1 = |CL| = |DK| \) (blue) \ldots 2
\( d_2 = |CD| = |CE| = |CF| = |DE| = |DF| = |DI| = |DJ| = |DM| = |DN| \) (green) \ldots 9
\( d_3 = |AG| = |AH| = |CK| = |DL| \) (cyan) \ldots 4
\( d_4 = |FK| = |FL| \) (magenta) \ldots 2

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_2} = \frac{d_3}{d_3} = \frac{d_4}{d_4} \) (34 occurrences)
2.96.4 Construction MM342

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2, \ D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, |CD|) \)
   \( E \in k_1 \cap k_3, \ F \in k_2 \cap k_3 \)
4. \( k_4 = k(F, |BE|) \)
   \( G \in k_1 \cap k_4, \ H \in k_1 \cap k_4, \ I \in k_3 \cap k_4, \ J \in k_3 \cap k_4 \)
5. \( k_5 = k(B, |DG|) \)
   \( K \in k_1 \cap k_5, \ L \in k_1 \cap k_5, \ M \in k_3 \cap k_5, \ N \in k_3 \cap k_5 \)

Distances

\( d_0 = |FK| = |FL| \) (red) \ldots 2
\( d_1 = |CL| = |DK| \) (blue) \ldots 2
\( d_2 = |CD| = |CE| = |CF| = |DE| = |DF| = |DI| = |DJ| = |DM| = |DN| \) (green) \ldots 9
\( d_3 = |BG| = |BH| = |CK| = |DL| \) (cyan) \ldots 4
\( d_4 = |EK| = |EL| \) (magenta) \ldots 2

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_2} = \frac{d_3}{d_3} = \frac{d_4}{d_4} \) (34 occurrences)
2.97 Class of Similar Constructions No. 88

Contained Constructions

MM185, MM230, MM296, MM341
2.97.1 Construction MM185

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2, D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, |CD|) \)
   \( E \in k_1 \cap k_3, F \in k_2 \cap k_3 \)
4. \( k_4 = k(E, |BE|) \)
   \( G \in k_2 \cap k_4, H \in k_2 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4 \)
5. \( k_5 = k(B, |DG|) \)
   \( K \in k_1 \cap k_5, L \in k_1 \cap k_5, M \in k_3 \cap k_5, N \in k_3 \cap k_5 \)

Distances

\( d_0 = |FK| = |FL| \) (red) ... 2
\( d_1 = |CL| = |DK| \) (blue) ... 2
\( d_2 = |CD| = |CE| = |CF| = |CI| = |CJ| = |CM| = |CN| = |DE| = |DF| \) (green) ... 9
\( d_3 = |AG| = |AH| = |CK| = |DL| \) (cyan) ... 4
\( d_4 = |EK| = |EL| \) (magenta) ... 2

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_2} = \frac{d_2}{d_3} = \frac{d_3}{d_4} \) (34 occurrences)
2.97.2 Construction MM230

Construction Process

A, B given initial points

1. \(k_1 = k(A, |AB|)\)
2. \(k_2 = k(B, |AB|)\)
   \(C \in k_1 \cap k_2, D \in k_1 \cap k_2\)
3. \(k_3 = k(C, |CD|)\)
   \(E \in k_1 \cap k_3, F \in k_2 \cap k_3\)
4. \(k_4 = k(F, |BE|)\)
   \(G \in k_1 \cap k_4, H \in k_1 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4\)
5. \(k_5 = k(A, |DG|)\)
   \(K \in k_2 \cap k_5, L \in k_2 \cap k_5, M \in k_3 \cap k_5, N \in k_3 \cap k_5\)

Distances

\(d_0 = |EK| = |EL| \) (red) \ldots 2
\(d_1 = |CL| = |DK| \) (blue) \ldots 2
\(d_2 = |CD| = |CE| = |CF| = |CI| = |CJ| = |CM| = |CN| = |DE| = |DF| \) (green) \ldots 9
\(d_3 = |BG| = |BH| = |CK| = |DL| \) (cyan) \ldots 4
\(d_4 = |FK| = |FL| \) (magenta) \ldots 2

Occurrences of the Golden Ratio

\(\phi = \frac{d_0}{d_2} = \frac{d_2}{d_3} = \frac{d_3}{d_4} \) (34 occurrences)
2.97.3 Construction MM296

Construction Process

A, B given initial points
1. $k_1 = k(A, |AB|)$
2. $k_2 = k(B, |AB|)$
   $C \in k_1 \cap k_2$, $D \in k_1 \cap k_2$
3. $k_3 = k(D, |CD|)$
   $E \in k_1 \cap k_3$, $F \in k_2 \cap k_3$
4. $k_4 = k(E, |BE|)$
   $G \in k_2 \cap k_4$, $H \in k_2 \cap k_4$, $I \in k_3 \cap k_4$, $J \in k_3 \cap k_4$
5. $k_5 = k(B, |DG|)$
   $K \in k_1 \cap k_5$, $L \in k_1 \cap k_5$, $M \in k_3 \cap k_5$, $N \in k_3 \cap k_5$

Distances

$d_0 = |FK| = |FL| \ (\text{red}) \ldots 2$
$d_1 = |CL| = |DK| \ (\text{blue}) \ldots 2$
$d_2 = |CD| = |CE| = |CF| = |DE| = |DF| = |DI| = |DJ| = |DM| = |DN| \ (\text{green}) \ldots 9$
$d_3 = |AG| = |AH| = |CK| = |DL| \ (\text{cyan}) \ldots 4$
$d_4 = |EK| = |EL| \ (\text{magenta}) \ldots 2$

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_2} = \frac{d_2}{d_3} = \frac{d_3}{d_4} \ (34 \ \text{occurrences})$
2.97.4 Construction MM341

Construction Process

- \( A, B \) given initial points
- 1. \( k_1 = k(A, |AB|) \)
- 2. \( k_2 = k(B, |AB|) \)
- \( C \in k_1 \cap k_2, D \in k_1 \cap k_2 \)
- 3. \( k_3 = k(D, |CD|) \)
- \( E \in k_1 \cap k_3, F \in k_2 \cap k_3 \)
- 4. \( k_4 = k(F, |BE|) \)
- \( G \in k_1 \cap k_4, H \in k_1 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4 \)
- 5. \( k_5 = k(A, |DG|) \)
- \( K \in k_2 \cap k_5, L \in k_2 \cap k_5, M \in k_3 \cap k_5, N \in k_3 \cap k_5 \)

Distances

- \( d_0 = |EK| = |EL| \) (red) \ldots 2
- \( d_1 = |CL| = |DK| \) (blue) \ldots 2
- \( d_2 = |CD| = |CE| = |CF| = |DE| = |DF| = |DI| = |DJ| = |DM| = |DN| \) (green) \ldots 9
- \( d_3 = |BG| = |BH| = |CK| = |DL| \) (cyan) \ldots 4
- \( d_4 = |FK| = |FL| \) (magenta) \ldots 2

Occurrences of the Golden Ratio

- \( \phi = \frac{d_0}{d_2} = \frac{d_3}{d_4} = \frac{d_2}{d_4} \) (34 occurrences)
2.98 Class of Similar Constructions No. 89

Contained Constructions

MM188, MM235, MM299, MM346
2.98.1 Construction MM188

Construction Process

\(A, B\) given initial points
1. \(k_1 = k(A, |AB|)\)
2. \(k_2 = k(B, |AB|)\)
   \(C \in k_1 \cap k_2\), \(D \in k_1 \cap k_2\)
3. \(k_3 = k(C, |CD|)\)
   \(E \in k_1 \cap k_3\)
4. \(k_4 = k(E, |BE|)\)
   \(F \in k_2 \cap k_4\), \(G \in k_2 \cap k_4\)
5. \(k_5 = k(E, |DF|)\)
   \(H \in k_1 \cap k_5\), \(I \in k_1 \cap k_5\)

Distances

\(d_0 = |AF| = |AG|\) (red) \(\ldots 2\)
\(d_1 = |BH| = |BI|\) (blue) \(\ldots 2\)

Occurrences of the Golden Ratio

\(\phi = \frac{d_0}{d_1}\) (4 occurrences)
2.98.2 Construction MM235

Construction Process

1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
3. \( C \in k_1 \cap k_2 , D \in k_1 \cap k_2 \)
4. \( k_3 = k(C, |CD|) \)
5. \( E \in k_1 \cap k_3 , F \in k_2 \cap k_3 \)
6. \( k_4 = k(F, |BE|) \)
7. \( G \in k_1 \cap k_4 , H \in k_1 \cap k_4 \)
8. \( k_5 = k(F, |DG|) \)
9. \( I \in k_2 \cap k_5 , J \in k_2 \cap k_5 \)

Distances

\( d_0 = |BG| = |BH| \) (red) \( \dot{\ldots} 2 \)
\( d_1 = |AI| = |AJ| \) (blue) \( \dot{\ldots} 2 \)

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} \) (4 occurrences)
2.98 Class of Similar Constructions No. 89

2.98.3 Construction MM299

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2, \quad D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, |CD|) \)
   \( E \in k_1 \cap k_3 \)
4. \( k_4 = k(E, |BE|) \)
   \( F \in k_2 \cap k_4, \quad G \in k_2 \cap k_4 \)
5. \( k_5 = k(E, |DF|) \)
   \( H \in k_1 \cap k_5, \quad I \in k_1 \cap k_5 \)

Distances

\( d_0 = |AF| = |AG| \) (red) \ldots 2
\( d_1 = |BH| = |BI| \) (blue) \ldots 2

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} \) (4 occurrences)
2.98.4 Construction MM346

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2 , D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, |CD|) \)
   \( E \in k_1 \cap k_3 , F \in k_2 \cap k_3 \)
4. \( k_4 = k(F, |BE|) \)
   \( G \in k_1 \cap k_4 , H \in k_1 \cap k_4 \)
5. \( k_5 = k(F, |DG|) \)
   \( I \in k_2 \cap k_5 , J \in k_2 \cap k_5 \)

Distances

\( d_0 = |BG| = |BH| \) (red) \( \ldots 2 \)
\( d_1 = |AI| = |AJ| \) (blue) \( \ldots 2 \)

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} \) (4 occurrences)
2.99 Class of Similar Constructions No. 90

Contained Constructions

MM189, MM234, MM300, MM345
2.99.1 Construction MM189

Construction Process

A, B given initial points
1. $k_1 = k(A, |AB|)$
2. $k_2 = k(B, |AB|)$
   $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
3. $k_3 = k(C, |CD|)$
   $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
4. $k_4 = k(E, |BE|)$
   $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
5. $k_5 = k(F, |DG|)$
   $I \in k_2 \cap k_5, J \in k_2 \cap k_5$

Distances

$d_0 = |AG| = |AH|$ (red) \ldots 2
$d_1 = |AI| = |AJ|$ (blue) \ldots 2

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_1}$ (4 occurrences)
2.99.2 Construction MM234

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
3. \( C \in k_1 \cap k_2, D \in k_1 \cap k_2 \)
4. \( k_3 = k(C, |CD|) \)
5. \( E \in k_1 \cap k_3, F \in k_2 \cap k_3 \)
6. \( k_4 = k(F, |BE|) \)
7. \( G \in k_1 \cap k_4, H \in k_4 \cap k_4 \)
8. \( k_5 = k(E, |DG|) \)
9. \( I \in k_1 \cap k_5, J \in k_1 \cap k_5 \)

Distances

\[ d_0 = |BG| = |BH| \text{ (red)} \ldots 2 \]
\[ d_1 = |BI| = |BJ| \text{ (blue)} \ldots 2 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} \text{ (4 occurrences)} \]
2.99.3 Construction MM300

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2, D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, |CD|) \)
   \( E \in k_1 \cap k_3, F \in k_2 \cap k_3 \)
4. \( k_4 = k(E, |BE|) \)
   \( G \in k_2 \cap k_4, H \in k_2 \cap k_4 \)
5. \( k_5 = k(F, |DG|) \)
   \( I \in k_2 \cap k_5, J \in k_2 \cap k_5 \)

Distances

\( d_0 = |AG| = |AH| \) (red) \ldots 2
\( d_1 = |AI| = |AJ| \) (blue) \ldots 2

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} \) (4 occurrences)
2.99.4 Construction MM345

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, |CD|) \)
   \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(F, |BE|) \)
   \( G \in k_1 \cap k_4 \), \( H \in k_1 \cap k_4 \)
5. \( k_5 = k(E, |DG|) \)
   \( I \in k_1 \cap k_5 \), \( J \in k_1 \cap k_5 \)

Distances

\[ d_0 = |BG| = |BH| \quad \text{(red)} \ldots 2 \]
\[ d_1 = |BI| = |BJ| \quad \text{(blue)} \ldots 2 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} \quad (4 \text{ occurrences}) \]
2.100  Class of Similar Constructions No. 91

Contained Constructions

MM192, MM239, MM303, MM350
\section*{2.100.1 Construction MM192}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{construction_mm192}
\caption{Construction MM192}
\end{figure}

\textbf{Construction Process}

\begin{enumerate}
\item $k_1 = k(A, |AB|)$
\item $k_2 = k(B, |AB|)$
\item $C \in k_1 \cap k_2 \ , \ D \in k_1 \cap k_2$
\item $k_3 = k(C, |CD|)$
\item $E \in k_1 \cap k_3 \ , \ F \in k_2 \cap k_3$
\item $k_4 = k(E, |BE|)$
\item $G \in k_2 \cap k_4 \ , \ H \in k_2 \cap k_4$
\item $k_5 = k(A, |FG|)$
\end{enumerate}

\textbf{Distances}

\begin{align*}
  d_0 &= |CJ| = |DI| \text{ (red)} \ldots 2 \\
  d_1 &= |AG| = |AH| = |FI| = |FJ| \text{ (blue)} \ldots 4 \\
  d_2 &= |CI| = |DJ| \text{ (green)} \ldots 2
\end{align*}

\textbf{Occurrences of the Golden Ratio}

\[ \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \] (16 occurrences)
2.100.2 Construction MM239

Construction Process

A, B given initial points
1. $k_1 = k(A, |AB|)$
2. $k_2 = k(B, |AB|)$
   $C \in k_1 \cap k_2$, $D \in k_1 \cap k_2$
3. $k_3 = k(C, |CD|)$
   $E \in k_1 \cap k_3$, $F \in k_2 \cap k_3$
4. $k_4 = k(F, |BE|)$
   $G \in k_1 \cap k_4$, $H \in k_1 \cap k_4$
5. $k_5 = k(B, |EG|)$
   $I \in k_1 \cap k_5$, $J \in k_1 \cap k_5$

Distances

$d_0 = |CJ| = |DI|$ (red) . . . 2
$d_1 = |BG| = |BH| = |EI| = |EJ|$ (blue) . . . 4
$d_2 = |CI| = |DJ|$ (green) . . . 2

Occurrences of the Golden Ratio

$\phi = \frac{d_2}{d_1} = \frac{d_4}{d_2}$ (16 occurrences)
2.100.3 Construction MM303

Construction Process

A, B given initial points
1. \(k_1 = k(A, |AB|)\)
2. \(k_2 = k(B, |AB|)\)
   \(C \in k_1 \cap k_2\), \(D \in k_1 \cap k_2\)
3. \(k_3 = k(D, |CD|)\)
   \(E \in k_1 \cap k_3\), \(F \in k_2 \cap k_3\)
4. \(k_4 = k(E, |BE|)\)
   \(G \in k_2 \cap k_4\), \(H \in k_2 \cap k_4\)
5. \(k_5 = k(A, |FG|)\)
   \(I \in k_2 \cap k_5\), \(J \in k_2 \cap k_5\)

Distances

\[d_0 = |CJ| = |DI| \text{ (red) ... 2}\]
\[d_1 = |AG| = |AH| = |FI| = |FJ| \text{ (blue) ... 4}\]
\[d_2 = |CI| = |DJ| \text{ (green) ... 2}\]

Occurrences of the Golden Ratio

\[\phi = \frac{d_0}{d_1} = \frac{d_4}{d_2} \text{ (16 occurrences)}\]
2.100.4  Construction MM350

Construction Process

$A, B$ given initial points
1. $k_1 = k(A, |AB|)$
2. $k_2 = k(B, |AB|)$
   $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
3. $k_3 = k(D, |CD|)$
   $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
4. $k_4 = k(F, |BE|)$
   $G \in k_1 \cap k_4, H \in k_1 \cap k_4$
5. $k_5 = k(B, |EG|)$
   $I \in k_1 \cap k_5, J \in k_1 \cap k_5$

Distances

$d_0 = |CJ| = |DI|$ (red) \ldots 2
$d_1 = |BG| = |BH| = |EI| = |EJ|$ (blue) \ldots 4
$d_2 = |CI| = |DJ|$ (green) \ldots 2

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_2} = \frac{d_4}{d_2}$ (16 occurrences)
2.101 Class of Similar Constructions No. 92

Contained Constructions

MM193, MM238, MM304, MM349
2.101.1 Construction MM193

Construction Process

A, B given initial points
1. $k_1 = k(A, |AB|)$
2. $k_2 = k(B, |AB|)$
   $C \in k_1 \cap k_2$, $D \in k_1 \cap k_2$
3. $k_3 = k(C, |CD|)$
   $E \in k_1 \cap k_3$, $F \in k_2 \cap k_3$
4. $k_4 = k(E, |BE|)$
   $G \in k_2 \cap k_4$, $H \in k_2 \cap k_4$
5. $k_5 = k(B, |FG|)$
   $I \in k_1 \cap k_5$, $J \in k_1 \cap k_5$

Distances

$d_0 = |CJ| = |DI|$ (red) ... 2
$d_1 = |AG| = |AH| = |EI| = |EJ|$ (blue) ... 4
$d_2 = |CI| = |DJ|$ (green) ... 2

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2}$ (16 occurrences)
2.101.2 Construction MM238

Construction Process

A, B given initial points
1. $k_1 = k(A, |AB|)$
2. $k_2 = k(B, |AB|)$
   - $C \in k_1 \cap k_2$ , $D \in k_1 \cap k_2$
3. $k_3 = k(C, |CD|)$
   - $E \in k_1 \cap k_3$ , $F \in k_2 \cap k_3$
4. $k_4 = k(F, |BE|)$
   - $G \in k_1 \cap k_4$ , $H \in k_1 \cap k_4$
5. $k_5 = k(A, |EG|)$
   - $I \in k_2 \cap k_5$ , $J \in k_2 \cap k_5$

Distances

$d_0 = |CJ| = |DI|$ (red) ... 2
$d_1 = |BG| = |BH| = |FI| = |FJ|$ (blue) ... 4
$d_2 = |CI| = |DJ|$ (green) ... 2

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_1} = \frac{d_4}{d_2}$ (16 occurrences)
2.101.3 Construction MM304

Construction Process

A, B given initial points

1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \[ C \in k_1 \cap k_2, \; D \in k_1 \cap k_2 \]
3. \( k_3 = k(D, |CD|) \)
   \[ E \in k_1 \cap k_3, \; F \in k_2 \cap k_3 \]
4. \( k_4 = k(E, |BE|) \)
   \[ G \in k_2 \cap k_4, \; H \in k_2 \cap k_4 \]
5. \( k_5 = k(B, |FG|) \)
   \[ I \in k_1 \cap k_5, \; J \in k_1 \cap k_5 \]

Distances

\[ d_0 = |CJ| = |DI| \text{ (red)} \ldots 2 \]
\[ d_1 = |AG| = |AH| = |EI| = |EJ| \text{ (blue)} \ldots 4 \]
\[ d_2 = |CI| = |DJ| \text{ (green)} \ldots 2 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (16 occurrences)} \]
2.101.4 Construction MM349

Construction Process

A, B given initial points
1. $k_1 = k(A, |AB|)$
2. $k_2 = k(B, |AB|)$
   $C \in k_1 \cap k_2$, $D \in k_1 \cap k_2$
3. $k_3 = k(D, |CD|)$
   $E \in k_1 \cap k_3$, $F \in k_2 \cap k_3$
4. $k_4 = k(F, |BE|)$
   $G \in k_1 \cap k_4$, $H \in k_1 \cap k_4$
5. $k_5 = k(A, |EG|)$
   $I \in k_2 \cap k_5$, $J \in k_2 \cap k_5$

Distances

$d_0 = |CJ| = |DI|$ (red) ... 2
$d_1 = |BG| = |BH| = |FI| = |FJ|$ (blue) ... 4
$d_2 = |CI| = |DJ|$ (green) ... 2

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_1} = \frac{d_2}{d_2}$ (16 occurrences)
2.102 Class of Similar Constructions No. 93

Contained Constructions

MM194, MM240, MM306, MM352
2.102.1 Construction MM194

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
\( C \in k_1 \cap k_2 , D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, |CD|) \)
\( E \in k_1 \cap k_3 , F \in k_2 \cap k_3 \)
4. \( k_4 = k(E, |BE|) \)
\( G \in k_2 \cap k_4 , H \in k_2 \cap k_4 \)
5. \( k_5 = k(C, |FG|) \)
\( I \in k_1 \cap k_5 , J \in k_1 \cap k_5 , K \in k_2 \cap k_5 , L \in k_2 \cap k_5 \)

Distances
\( d_0 = |AL| = |BI| \) (red) \ldots 2
\( d_1 = |AG| = |AH| \) (blue) \ldots 2
\( d_2 = |AK| = |BJ| \) (green) \ldots 2

Occurrences of the Golden Ratio
\( \phi = \frac{d_0}{d_5} = \frac{d_1}{d_2} \) (8 occurrences)
Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, |CD|) \)
   \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(F, |BE|) \)
   \( G \in k_1 \cap k_4 \), \( H \in k_1 \cap k_4 \)
5. \( k_5 = k(C, |EG|) \)
   \( I \in k_1 \cap k_5 \), \( J \in k_1 \cap k_5 \), \( K \in k_2 \cap k_5 \), \( L \in k_2 \cap k_5 \)

Distances

\( d_0 = |AL| = |BI| \) (red) \ldots 2
\( d_1 = |BG| = |BH| \) (blue) \ldots 2
\( d_2 = |AK| = |BJ| \) (green) \ldots 2

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} = \frac{d_2}{d_2} \) (8 occurrences)
2.102.3 Construction MM306

Construction Process

A, B given initial points
1. \( k_1 = k(A, AB) \)
2. \( k_2 = k(B, AB) \)
   \( C \in k_1 \cap k_2, D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, CD) \)
   \( E \in k_1 \cap k_3, F \in k_2 \cap k_3 \)
4. \( k_4 = k(E, BE) \)
   \( G \in k_2 \cap k_4, H \in k_2 \cap k_4 \)
5. \( k_5 = k(D, FG) \)
   \( I \in k_1 \cap k_5, J \in k_1 \cap k_5, K \in k_2 \cap k_5, L \in k_2 \cap k_5 \)

Distances

\( d_0 = |AL| = |BI| \) (red) \( \ldots 2 \)
\( d_1 = |AG| = |AH| \) (blue) \( \ldots 2 \)
\( d_2 = |AK| = |BJ| \) (green) \( \ldots 2 \)

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \) (8 occurrences)
2.102.4 Construction MM352

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
C \( \in k_1 \cap k_2 \), D \( \in k_1 \cap k_2 \)
3. \( k_3 = k(D, |CD|) \)
E \( \in k_1 \cap k_3 \), F \( \in k_2 \cap k_3 \)
4. \( k_4 = k(F, |BE|) \)
G \( \in k_1 \cap k_4 \), H \( \in k_1 \cap k_4 \)
5. \( k_5 = k(D, |EG|) \)
I \( \in k_1 \cap k_5 \), J \( \in k_1 \cap k_5 \), K \( \in k_2 \cap k_5 \), L \( \in k_2 \cap k_5 \)

Distances

\[ d_0 = |AL| = |BI| \text{ (red)} \ldots 2 \]
\[ d_1 = |BG| = |BH| \text{ (blue)} \ldots 2 \]
\[ d_2 = |AK| = |BJ| \text{ (green)} \ldots 2 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} = \frac{d_4}{d_2} \text{ (8 occurrences)} \]
2.103 Class of Similar Constructions No. 94

Contained Constructions

MM195, MM241, MM305, MM351
2.103.1 Construction MM195

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, |CD|) \)
   \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(E, |BE|) \)
   \( G \in k_2 \cap k_4 \), \( H \in k_2 \cap k_4 \)
5. \( k_5 = k(D, |FG|) \)
   \( I \in k_1 \cap k_5 \), \( J \in k_1 \cap k_5 \), \( K \in k_2 \cap k_5 \), \( L \in k_2 \cap k_5 \)

Distances

\( d_0 = |AL| = |BI| \) (red) ... 2
\( d_1 = |AG| = |AH| \) (blue) ... 2
\( d_2 = |AK| = |BJ| \) (green) ... 2

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \) (8 occurrences)
2.103.2 Construction MM241

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2, D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, |CD|) \)
   \( E \in k_1 \cap k_3, F \in k_2 \cap k_3 \)
4. \( k_4 = k(F, |BE|) \)
   \( G \in k_1 \cap k_4, H \in k_1 \cap k_4 \)
5. \( k_5 = k(D, |EG|) \)
   \( I \in k_1 \cap k_5, J \in k_1 \cap k_5, K \in k_2 \cap k_5, L \in k_2 \cap k_5 \)

Distances

\( d_0 = |AL| = |BI| \) (red) \( \ldots 2 \)
\( d_1 = |BG| = |BH| \) (blue) \( \ldots 2 \)
\( d_2 = |AK| = |BJ| \) (green) \( \ldots 2 \)

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \) (8 occurrences)
2.103.3 Construction MM305

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, |CD|) \)
   \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(E, |BE|) \)
   \( G \in k_2 \cap k_4 \), \( H \in k_2 \cap k_4 \)
5. \( k_5 = k(C, |FG|) \)
   \( I \in k_1 \cap k_5 \), \( J \in k_1 \cap k_5 \), \( K \in k_2 \cap k_5 \), \( L \in k_2 \cap k_5 \)

Distances

\[ d_0 = |AL| = |BI| \text{ (red)} \ldots 2 \]
\[ d_1 = |AG| = |AH| \text{ (blue)} \ldots 2 \]
\[ d_2 = |AK| = |BJ| \text{ (green)} \ldots 2 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)} \]
2.103.4 Construction MM351

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, |CD|) \)
   \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(F, |BE|) \)
   \( G \in k_1 \cap k_4 \), \( H \in k_1 \cap k_4 \)
5. \( k_5 = k(C, |EG|) \)
   \( I \in k_1 \cap k_5 \), \( J \in k_1 \cap k_5 \), \( K \in k_2 \cap k_5 \), \( L \in k_2 \cap k_5 \)

Distances

\( d_0 = |AL| = |BI| \) (red) \( \ldots 2 \)
\( d_1 = |BG| = |BH| \) (blue) \( \ldots 2 \)
\( d_2 = |AK| = |BJ| \) (green) \( \ldots 2 \)

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \) (8 occurrences)
2.104 Class of Similar Constructions No. 95

Contained Constructions

MM198, MM251, MM310, MM363
2.104.1 Construction MM198

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2 \) , \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, |CD|) \)
   \( E \in k_1 \cap k_3 \) , \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(E, |BE|) \)
   \( G \in k_2 \cap k_4 \) , \( H \in k_3 \cap k_4 \) , \( I \in k_3 \cap k_4 \)
5. \( k_5 = k(G, |BH|) \)
   \( J \in k_4 \cap k_5 \)

Distances

\( d_0 = |CD| = |CE| = |CF| = |CH| = |CI| = |DE| = |DF| \) (red) \( \ldots 7 \)
\( d_1 = |IJ| \) (blue) \( \ldots 1 \)

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} \) (7 occurrences)
2.104.2 Construction MM251

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2, D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, |CD|) \)
   \( E \in k_1 \cap k_3, F \in k_2 \cap k_3 \)
4. \( k_4 = k(F, |BE|) \)
   \( G \in k_1 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4 \)
5. \( k_5 = k(G, |AI|) \)
   \( J \in k_4 \cap k_5 \)

Distances

\( d_0 = |CD| = |CE| = |CF| = |CH| = |CI| = |DE| = |DF| \) (red) \( \ldots 7 \)
\( d_1 = |HJ| \) (blue) \( \ldots 1 \)

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} \) (7 occurrences)
2.104.3 Construction MM310

Construction Process

1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, |CD|) \)
   \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(E, |BE|) \)
   \( G \in k_2 \cap k_4 \), \( H \in k_3 \cap k_4 \), \( I \in k_3 \cap k_4 \)
5. \( k_5 = k(G, |BH|) \)
   \( J \in k_4 \cap k_5 \)

Distances

\( d_0 = |CD| = |CE| = |CF| = |DE| = |DF| = |DH| = |DI| \quad \text{(red)} \ldots 7 \)
\( d_1 = |IJ| \quad \text{(blue)} \ldots 1 \)

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} \) (7 occurrences)
2.104.4 Construction MM363

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, |CD|) \)
   \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(F, |BE|) \)
   \( G \in k_1 \cap k_4 \), \( H \in k_3 \cap k_4 \), \( I \in k_3 \cap k_4 \)
5. \( k_5 = k(G, |AI|) \)
   \( J \in k_4 \cap k_5 \)

Distances

\[
d_0 = |CD| = |CE| = |CF| = |DE| = |DF| = |DH| = |DI| \text{ (red)} \ldots 7 \\
d_1 = |HJ| \text{ (blue)} \ldots 1
\]

Occurrences of the Golden Ratio

\[
\phi = \frac{d_0}{d_1} \text{ (7 occurrences)}
\]
2.105 Class of Similar Constructions No. 96

Contained Constructions
MM199, MM252, MM309, MM362
2.105.1 Construction MM199

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2, \ D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, |CD|) \)
   \( E \in k_1 \cap k_3, \ F \in k_2 \cap k_3 \)
4. \( k_4 = k(E, |BE|) \)
   \( G \in k_2 \cap k_4, \ H \in k_3 \cap k_4, \ I \in k_3 \cap k_4 \)
5. \( k_5 = k(G, |BH|) \)
   \( J \in k_3 \cap k_5, \ K \in k_3 \cap k_5, \ L \in k_4 \cap k_5 \)

Distances

\( d_0 = |IL| \) (red) \ldots 1
\( d_1 = |CD| = |CE| = |CF| = |CH| = |CI| = |CJ| = |CK| = |DE| = |DF| \) (blue) \ldots 9

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} \) (9 occurrences)
2.105.2 Construction MM252

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, |CD|) \)
   \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(F, |BE|) \)
   \( G \in k_1 \cap k_4 \), \( H \in k_3 \cap k_4 \), \( I \in k_3 \cap k_4 \)
5. \( k_5 = k(G, |AI|) \)
   \( J \in k_3 \cap k_5 \), \( K \in k_3 \cap k_5 \), \( L \in k_4 \cap k_5 \)

Distances

\( d_0 = |HL| \) (red) \ldots 1
\( d_1 = |CD| = |CE| = |CF| = |CH| = |CI| = |CJ| = |CK| = |DE| = |DF| \) (blue) \ldots 9

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} \) (9 occurrences)
2.105.3 Construction MM309

Construction Process

A, B given initial points
1. \(k_1 = k(A, |AB|)\)
2. \(k_2 = k(B, |AB|)\)
   \(C \in k_1 \cap k_2, D \in k_1 \cap k_2\)
3. \(k_3 = k(D, |CD|)\)
   \(E \in k_1 \cap k_3, F \in k_2 \cap k_3\)
4. \(k_4 = k(E, |BE|)\)
   \(G \in k_2 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4\)
5. \(k_5 = k(G, |BH|)\)
   \(J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5\)

Distances
\[d_0 = |IL| \text{ (red)} \ldots 1\]
\[d_1 = |CD| = |CE| = |CF| = |DE| = |DF| = |DH| = |DI| = |DJ| = |DK| \text{ (blue)} \ldots 9\]

Occurrences of the Golden Ratio
\[\phi = \frac{d_0}{d_1} \text{ (9 occurrences)}\]
2.105.4 Construction MM362

Construction Process

\(A, B\) given initial points

1. \(k_1 = k(A, |AB|)\)
2. \(k_2 = k(B, |AB|)\)
   \(C \in k_1 \cap k_2, D \in k_1 \cap k_2\)
3. \(k_3 = k(D, |CD|)\)
   \(E \in k_1 \cap k_3, F \in k_2 \cap k_3\)
4. \(k_4 = k(F, |BE|)\)
   \(G \in k_1 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4\)
5. \(k_5 = k(G, |AI|)\)
   \(J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5\)

Distances

\(d_0 = |HL|\) (red) \ldots 1
\(d_1 = |CD| = |CE| = |CF| = |DE| = |DF| = |DH| = |DI| = |DJ| = |DK|\) (blue) \ldots 9

Occurrences of the Golden Ratio

\(\phi = \frac{d_0}{d_1}\) (9 occurrences)
2.106 Class of Similar Constructions No. 97

Contained Constructions

MM200, MM253, MM311, MM364
2.106.1 Construction MM200

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, |CD|) \)
   \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(E, |BE|) \)
   \( G \in k_2 \cap k_4 \), \( H \in k_2 \cap k_4 \), \( I \in k_3 \cap k_4 \), \( J \in k_3 \cap k_4 \)
5. \( k_5 = k(J, |BI|) \)
   \( K \in k_3 \cap k_5 \), \( L \in k_3 \cap k_5 \), \( M \in k_4 \cap k_5 \)

Distances

\( d_0 = |HM| \) (red) \ldots 1
\( d_1 = |AM| = |CD| = |CE| = |CF| = |CI| = |CJ| = |CK| = |CL| = |DE| = |DF| \) (blue) \ldots 10
\( d_2 = |GM| \) (green) \ldots 1

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \) (20 occurrences)
2.106.2 Construction MM253

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2, D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, |CD|) \)
   \( E \in k_1 \cap k_3, F \in k_2 \cap k_3 \)
4. \( k_4 = k(F, |BE|) \)
   \( G \in k_1 \cap k_4, H \in k_1 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4 \)
5. \( k_5 = k(I, |AJ|) \)
   \( K \in k_3 \cap k_5, L \in k_3 \cap k_5, M \in k_4 \cap k_5 \)

Distances

\( d_0 = |HM| \) (red) \( \ldots 1 \)
\( d_1 = |BM| = |CD| = |CE| = |CF| = |CI| = |CK| = |CL| = |DE| = |DF| \) (blue) \( \ldots 10 \)
\( d_2 = |GM| \) (green) \( \ldots 1 \)

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} = \frac{d_2}{d_2} \) (20 occurrences)
2.106.3 Construction MM311

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, |CD|) \)
   \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(E, |BE|) \)
   \( G \in k_2 \cap k_4 \), \( H \in k_2 \cap k_4 \), \( I \in k_3 \cap k_4 \), \( J \in k_3 \cap k_4 \)
5. \( k_5 = k(J, |BI|) \)
   \( K \in k_3 \cap k_5 \), \( L \in k_3 \cap k_5 \), \( M \in k_4 \cap k_5 \)

Distances

\( d_0 = |GM| \) (red) \ldots 1
\( d_1 = |AM| = |CD| = |CE| = |CF| = |DE| = |DF| = |DI| = |DJ| = |DK| = |DL| \) (blue) \ldots 10
\( d_2 = |HM| \) (green) \ldots 1

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \) (20 occurrences)
2.106.4 Construction MM364

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
\( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, |CD|) \)
\( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(F, |BE|) \)
\( G \in k_1 \cap k_4 \), \( H \in k_1 \cap k_4 \), \( I \in k_3 \cap k_4 \), \( J \in k_3 \cap k_4 \)
5. \( k_5 = k(I, |AJ|) \)
\( K \in k_3 \cap k_5 \), \( L \in k_3 \cap k_5 \), \( M \in k_4 \cap k_5 \)

Distances

\( d_0 = |GM| \) (red) \( \ldots 1 \)
\( d_1 = |BM| = |CD| = |CE| = |CF| = |DE| = |DF| = |DI| = |DJ| = |DK| = |DL| \) (blue) \( \ldots 10 \)
\( d_2 = |HM| \) (green) \( \ldots 1 \)

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \) (20 occurrences)
2.107 Class of Similar Constructions No. 98

Contained Constructions

MM201, MM254, MM314, MM367
2.107 Class of Similar Constructions No. 98

2.107.1 Construction MM201

Construction Process

\( A, B \) given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2, \ D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, |CD|) \)
   \( E \in k_1 \cap k_3, \ F \in k_2 \cap k_3 \)
4. \( k_4 = k(E, |BE|) \)
   \( G \in k_2 \cap k_4, \ H \in k_3 \cap k_4, \ I \in k_3 \cap k_4 \)
5. \( k_5 = k(B, |GH|) \)
   \( J \in k_3 \cap k_5, \ K \in k_3 \cap k_5, \ L \in k_4 \cap k_5 \)

Distances

\( d_0 = |CD| = |CE| = |CF| = |CH| = |CI| = |CJ| = |CK| = |DE| = \)
\( |DF| \) (red) \ldots 9
\( d_1 = |IL| \) (blue) \ldots 1

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} \) (9 occurrences)
2.107.2 Construction MM254

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, |CD|) \)
   \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(F, |BE|) \)
   \( G \in k_1 \cap k_4 \), \( H \in k_3 \cap k_4 \), \( I \in k_3 \cap k_4 \)
5. \( k_5 = k(A, |GI|) \)
   \( J \in k_3 \cap k_5 \), \( K \in k_3 \cap k_5 \), \( L \in k_4 \cap k_5 \)

Distances

\( d_0 = |CD| = |CE| = |CF| = |CH| = |CI| = |CJ| = |CK| = |DE| = |DF| \textcolor{red}{(red)} \ldots 9 \)
\( d_1 = |HL| \textcolor{blue}{(blue)} \ldots 1 \)

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} \) (9 occurrences)
2.107.3 Construction MM314

Construction Process

\(A, B\) given initial points
1. \(k_1 = k(A, |AB|)\)
2. \(k_2 = k(B, |AB|)\)
   \(C \in k_1 \cap k_2, D \in k_1 \cap k_2\)
3. \(k_3 = k(D, |CD|)\)
   \(E \in k_1 \cap k_3, F \in k_2 \cap k_3\)
4. \(k_4 = k(E, |BE|)\)
   \(G \in k_2 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4\)
5. \(k_5 = k(B, |GH|)\)
   \(J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5\)

Distances

\(d_0 = |CD| = |CE| = |CF| = |DE| = |DF| = |DH| = |DI| = |DJ| = |DK|\) (red) \(\ldots 9\)
\(d_1 = |IL|\) (blue) \(\ldots 1\)

Occurrences of the Golden Ratio

\(\phi = \frac{d_0}{d_1}\) (9 occurrences)
2.107.4 Construction MM367

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2, D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, |CD|) \)
   \( E \in k_1 \cap k_3, F \in k_2 \cap k_3 \)
4. \( k_4 = k(F, |BE|) \)
   \( G \in k_1 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4 \)
5. \( k_5 = k(A, |GI|) \)
   \( J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5 \)

Distances

\( d_0 = |CD| = |CE| = |CF| = |DE| = |DF| = |DH| = |DI| = |DJ| = |DK| \) (red) ... 9
\( d_1 = |HL| \) (blue) ... 1

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} \) (9 occurrences)
2.108 Class of Similar Constructions No. 99

Contained Constructions
MM202, MM255, MM315, MM368
2.108.1 Construction MM202

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \[ C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, |CD|) \)
4. \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
5. \( k_4 = k(E, |BE|) \)
   \[ G \in k_2 \cap k_4 \), \( H \in k_3 \cap k_4 \), \( I \in k_3 \cap k_4 \)
6. \( k_5 = k(I, |GH|) \)
   \[ J \in k_3 \cap k_5 \), \( K \in k_3 \cap k_5 \), \( L \in k_4 \cap k_5 \)

Distances

\[ d_0 = |CD| = |CE| = |CF| = |CH| = |CI| = |CK| = |DE| = |DF| \) (red) ... 9
\[ d_1 = |BL| \) (blue) ... 1

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} \) (9 occurrences)
2.108.2 Construction MM255

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, |CD|) \)
   \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(F, |BE|) \)
   \( G \in k_1 \cap k_4 \), \( H \in k_3 \cap k_4 \), \( I \in k_3 \cap k_4 \)
5. \( k_5 = k(H, |GI|) \)
   \( J \in k_3 \cap k_5 \), \( K \in k_3 \cap k_5 \), \( L \in k_4 \cap k_5 \)

Distances

\( d_0 = |CD| = |CE| = |CF| = |CH| = |CI| = |CK| = |DE| = |DF| \) (red) \( \ldots 9 \)
\( d_1 = |AL| \) (blue) \( \ldots 1 \)

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} \) (9 occurrences)
2.108.3 Construction MM315

Construction Process

\( A, B \) given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2, D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, |CD|) \)
   \( E \in k_1 \cap k_3, F \in k_2 \cap k_3 \)
4. \( k_4 = k(E, |BE|) \)
   \( G \in k_2 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4 \)
5. \( k_5 = k(I, |GH|) \)
   \( J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5 \)

Distances

\[ d_0 = |CD| = |CE| = |CF| = |DE| = |DF| = |DH| = |DI| = |DJ| = |DK| \] (red) ... 9
\[ d_1 = |BL| \] (blue) ... 1

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} \] (9 occurrences)
2.108.4 Construction MM368

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, |CD|) \)
   \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(F, |BE|) \)
   \( G \in k_1 \cap k_4 \), \( H \in k_3 \cap k_4 \), \( I \in k_3 \cap k_4 \)
5. \( k_5 = k(H, |GI|) \)
   \( J \in k_3 \cap k_5 \), \( K \in k_3 \cap k_5 \), \( L \in k_4 \cap k_5 \)

Distances

\[ d_0 = |CD| = |CE| = |CF| = |DE| = |DF| = |DH| = |DI| = |DJ| = |DK| \text{ (red)} \ldots 9 \]
\[ d_1 = |AL| \text{ (blue)} \ldots 1 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} \text{ (9 occurrences)} \]
2.109 Class of Similar Constructions No. 100

Contained Constructions
MM203, MM256, MM312, MM365
2.109.1 Construction MM203

Construction Process

\(A, B\) given initial points

1. \(k_1 = k(A, |AB|)\)
2. \(k_2 = k(B, |AB|)\)
   \(C \in k_1 \cap k_2, D \in k_1 \cap k_2\)
3. \(k_3 = k(C, |CD|)\)
   \(E \in k_1 \cap k_3, F \in k_2 \cap k_3\)
4. \(k_4 = k(E, |BE|)\)
   \(G \in k_2 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4\)
5. \(k_5 = k(B, |GH|)\)
   \(J \in k_4 \cap k_5\)

Distances

\(d_0 = |IJ|\) (red) \(\ldots 1\)
\(d_1 = |CD| = |CE| = |CF| = |CH| = |CI| = |DE| = |DF|\) (blue) \(\ldots 7\)

Occurrences of the Golden Ratio

\(\phi = \frac{d_0}{d_1}\) (7 occurrences)
2.109.2 Construction MM256

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   
   C \( \in k_1 \cap k_2 \), D \( \in k_1 \cap k_2 \)
3. \( k_3 = k(C, |CD|) \)
   
   E \( \in k_1 \cap k_3 \), F \( \in k_2 \cap k_3 \)
4. \( k_4 = k(F, |BE|) \)
   
   G \( \in k_1 \cap k_4 \), H \( \in k_3 \cap k_4 \), I \( \in k_3 \cap k_4 \)
5. \( k_5 = k(A, |GI|) \)
   
   J \( \in k_4 \cap k_5 \)

Distances

\[ d_0 = |HJ| \quad \text{(red)} \quad \ldots 1 \]

\[ d_1 = |CD| = |CE| = |CF| = |CH| = |CI| = |DE| = |DF| \quad \text{(blue)} \quad \ldots 7 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} \quad (7 \text{ occurrences}) \]
2.109.3 Construction MM312

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, |CD|) \)
   \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(E, |BE|) \)
   \( G \in k_2 \cap k_4 \), \( H \in k_3 \cap k_4 \), \( I \in k_3 \cap k_4 \)
5. \( k_5 = k(B, |GH|) \)
   \( J \in k_4 \cap k_5 \)

Distances

\( d_0 = |IJ| \) (red) \( \ldots 1 \)
\( d_1 = |CD| = |CE| = |CF| = |DE| = |DF| = |DH| = |DI| \) (blue) \( \ldots 7 \)

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} \) (7 occurrences)
2.109.4 Construction MM365

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2 , D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, |CD|) \)
   \( E \in k_1 \cap k_3 , F \in k_2 \cap k_3 \)
4. \( k_4 = k(F, |BE|) \)
   \( G \in k_1 \cap k_4 , H \in k_3 \cap k_4 , I \in k_3 \cap k_4 \)
5. \( k_5 = k(A, |GI|) \)
   \( J \in k_4 \cap k_5 \)

Distances

\( d_0 = |HJ| \) (red) \ldots 1
\( d_1 = |CD| = |CE| = |CF| = |DE| = |DF| = |DH| = |DI| \) (blue) \ldots 7

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} \) (7 occurrences)
2.110 Class of Similar Constructions No. 101

Contained Constructions
MM204, MM257, MM313, MM366
2.110.1 Construction MM204

Construction Process

\[ A, B \text{ given initial points} \]

1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
3. \( C \in k_1 \cap k_2, D \in k_1 \cap k_2 \)
4. \( k_3 = k(C, |CD|) \)
5. \( E \in k_1 \cap k_3, F \in k_2 \cap k_3 \)
6. \( k_4 = k(E, |BE|) \)
7. \( G \in k_2 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4 \)
8. \( k_5 = k(I, |GH|) \)
9. \( J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5 \)

Distances

\[ d_0 = |BL| \text{ (red)} \ldots 1 \]
\[ d_1 = |CD| = |CE| = |CF| = |CH| = |CI| = |CJ| = |CK| = |DE| = |DF| \text{ (blue)} \ldots 9 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} \text{ (9 occurrences)} \]
2.110.2 Construction MM257

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, \overline{AB}) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, |CD|) \)
   \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(F, |BE|) \)
   \( G \in k_1 \cap k_4 \), \( H \in k_3 \cap k_4 \), \( I \in k_3 \cap k_4 \)
5. \( k_5 = k(H, |GI|) \)
   \( J \in k_3 \cap k_5 \), \( K \in k_3 \cap k_5 \), \( L \in k_4 \cap k_5 \)

Distances

\( d_0 = |AL| \) (red) \( \ldots 1 \)
\( d_1 = |CD| = |CE| = |CF| = |CH| = |CI| = |CJ| = |CK| = |DE| = |DF| \) (blue) \( \ldots 9 \)

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} \) (9 occurrences)
2.110.3 Construction MM313

Construction Process

A, B given initial points
1. $k_1 = k(A, |AB|)$
2. $k_2 = k(B, |AB|)\quad C \in k_1 \cap k_2 , D \in k_1 \cap k_2$
3. $k_3 = k(D, |CD|)\quad E \in k_1 \cap k_3 , F \in k_2 \cap k_3$
4. $k_4 = k(E, |BE|)\quad G \in k_2 \cap k_4 , H \in k_3 \cap k_4 , I \in k_3 \cap k_4$
5. $k_5 = k(I, |GH|)\quad J \in k_3 \cap k_5 , K \in k_3 \cap k_5 , L \in k_4 \cap k_5$

Distances

\[ d_0 = |BL| \quad \text{(red)} \quad \ldots 1 \]
\[ d_1 = |CD| = |CE| = |CF| = |DE| = |DF| = |DH| = |DI| = |DJ| = |DK| \quad \text{(blue)} \quad \ldots 9 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} \quad \text{(9 occurrences)} \]
2.110.4 Construction MM366

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2, \ D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, |CD|) \)
   \( E \in k_1 \cap k_3, \ F \in k_2 \cap k_3 \)
4. \( k_4 = k(F, |BE|) \)
   \( G \in k_1 \cap k_4, \ H \in k_3 \cap k_4, \ I \in k_3 \cap k_4 \)
5. \( k_5 = k(H, |GI|) \)
   \( J \in k_3 \cap k_5, \ K \in k_3 \cap k_5, \ L \in k_4 \cap k_5 \)

Distances

\( d_0 = |AL| \) (red) \( \ldots 1 \)
\( d_1 = |CD| = |CE| = |CF| = |DE| = |DF| = |DH| = |DI| = |DJ| = |DK| \) (blue) \( \ldots 9 \)

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} \) (9 occurrences)
2.111 Class of Similar Constructions No. 102

Contained Constructions

MM205, MM244, MM317, MM356
2.111.1 Construction MM205

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2, D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, |CD|) \)
   \( E \in k_1 \cap k_3, F \in k_2 \cap k_3 \)
4. \( k_4 = k(E, |BE|) \)
   \( G \in k_2 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4 \)
5. \( k_5 = k(G, |BI|) \)
   \( J \in k_4 \cap k_5 \)

Distances

\[ d_0 = |CD| = |CE| = |CF| = |CH| = |CI| = |DE| = |DF| \text{ (red)} \ldots 7 \]
\[ d_1 = |HJ| \text{ (blue)} \ldots 1 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} \text{ (7 occurrences)} \]
Construction MM244

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, |CD|) \)
   \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(F, |BE|) \)
   \( G \in k_1 \cap k_4 \), \( H \in k_3 \cap k_4 \), \( I \in k_3 \cap k_4 \)
5. \( k_5 = k(G, |AH|) \)
   \( J \in k_4 \cap k_5 \)

Distances

\[ d_0 = |CD| = |CE| = |CF| = |CH| = |CI| = |DE| = |DF| \quad \text{(red)} \quad \ldots 7 \]
\[ d_1 = |IJ| \quad \text{(blue)} \quad \ldots 1 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} \quad \text{(7 occurrences)} \]
2.111.3 Construction MM317

Construction Process

\[ A, B \text{ given initial points} \]
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \[ C \in k_1 \cap k_2, D \in k_1 \cap k_2 \]
3. \( k_3 = k(D, |CD|) \)
   \[ E \in k_1 \cap k_3, F \in k_2 \cap k_3 \]
4. \( k_4 = k(E, |BE|) \)
   \[ G \in k_2 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4 \]
5. \( k_5 = k(G, |BI|) \)
   \[ J \in k_4 \cap k_5 \]

Distances

\[ d_0 = |CD| = |CE| = |CF| = |DE| = |DF| = |DH| = |DI| \text{ (red)} \ldots 7 \]
\[ d_1 = |HJ| \text{ (blue)} \ldots 1 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} \text{ (7 occurrences)} \]
2.111.4 Construction MM356

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2 \, , \, D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, |CD|) \)
   \( E \in k_1 \cap k_3 \, , \, F \in k_2 \cap k_3 \)
4. \( k_4 = k(F, |BE|) \)
   \( G \in k_1 \cap k_4 \, , \, H \in k_3 \cap k_4 \, , \, I \in k_3 \cap k_4 \)
5. \( k_5 = k(G, |AH|) \)
   \( J \in k_4 \cap k_5 \)

Distances

\[ d_0 = |CD| = |CE| = |CF| = |DE| = |DF| = |DH| = |DI| \] (red) \ldots 7
\[ d_1 = |IJ| \] (blue) \ldots 1

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} \] (7 occurrences)
2.112 Class of Similar Constructions No. 103

Contained Constructions

MM206, MM245, MM316, MM355
2.112.1 Construction MM206

Construction Process

A, B given initial points
1. $k_1 = k(A, |AB|)$
2. $k_2 = k(B, |AB|)$
   $C \in k_1 \cap k_2, \ D \in k_1 \cap k_2$
3. $k_3 = k(C, |CD|)$
   $E \in k_1 \cap k_3, \ F \in k_2 \cap k_3$
4. $k_4 = k(E, |BE|)$
   $G \in k_2 \cap k_4, \ H \in k_3 \cap k_4, \ I \in k_3 \cap k_4$
5. $k_5 = k(G, |BI|)$
   $J \in k_3 \cap k_5, \ K \in k_3 \cap k_5, \ L \in k_4 \cap k_5$

Distances

$\ d_0 = |HL| \ (\text{red}) \ldots 1$
$\ d_1 = |CD| = |CE| = |CF| = |CH| = |CI| = |CJ| = |CK| = |DE| = |DF| \ (\text{blue}) \ldots 9$

Occurrences of the Golden Ratio

$\ \phi = \frac{d_0}{d_5} \ (9 \ \text{occurrences})$
Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, |CD|) \)
   \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(F, |BE|) \)
   \( G \in k_1 \cap k_4 \), \( H \in k_3 \cap k_4 \), \( I \in k_3 \cap k_4 \)
5. \( k_5 = k(G, |AH|) \)
   \( J \in k_3 \cap k_5 \), \( K \in k_3 \cap k_5 \), \( L \in k_4 \cap k_5 \)

Distances

\( d_0 = |IL| \) (red) \ldots 1
\( d_1 = |CD| = |CE| = |CF| = |CH| = |CI| = |CJ| = |CK| = |DE| = |DF| \) (blue) \ldots 9

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} \) (9 occurrences)
2.112.3 Construction MM316

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2, \ D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, |CD|) \)
   \( E \in k_1 \cap k_3, \ F \in k_2 \cap k_3 \)
4. \( k_4 = k(E, |BE|) \)
   \( G \in k_2 \cap k_4, \ H \in k_3 \cap k_4, \ I \in k_3 \cap k_4 \)
5. \( k_5 = k(G, |BI|) \)
   \( J \in k_3 \cap k_5, \ K \in k_3 \cap k_5, \ L \in k_4 \cap k_5 \)

Distances

\( d_0 = |HL| \) (red) \ldots 1
\( d_1 = |CD| \) = \( |CE| \) = \( |CF| \) = \( |DF| \) = \( |DH| \) = \( |DI| \) = \( |DJ| \) = \( |DK| \) (blue) \ldots 9

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_4} \) (9 occurrences)
2.112.4 Construction MM355

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2 , D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, |CD|) \)
   \( E \in k_1 \cap k_3 , F \in k_2 \cap k_3 \)
4. \( k_4 = k(F, |BE|) \)
   \( G \in k_1 \cap k_4 , H \in k_3 \cap k_4 , I \in k_3 \cap k_4 \)
5. \( k_5 = k(G, |AH|) \)
   \( J \in k_3 \cap k_5 , K \in k_3 \cap k_5 , L \in k_4 \cap k_5 \)

Distances

\( d_0 = |IL| \) (red) \ldots 1
\( d_1 = |CD| = |CE| = |CF| = |DE| = |DF| = |DH| = |DI| = |DJ| = |DK| \) (blue) \ldots 9

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} \] (9 occurrences)
2.113 Class of Similar Constructions No. 104

Contained Constructions
MM207, MM246, MM318, MM357
2.113.1 Construction MM207

Construction Process

- $A, B$ given initial points
- $k_1 = k(A, |AB|)$
- $k_2 = k(B, |AB|)$
- $C \in k_1 \cap k_2$ , $D \in k_1 \cap k_2$
- $k_3 = k(C, |CD|)$
- $E \in k_1 \cap k_3$ , $F \in k_2 \cap k_3$
- $k_4 = k(E, |BE|)$
- $G \in k_2 \cap k_4$ , $H \in k_2 \cap k_4$ , $I \in k_3 \cap k_4$ , $J \in k_3 \cap k_4$
- $k_5 = k(I, |BJ|)$
- $K \in k_3 \cap k_5$ , $L \in k_3 \cap k_5$ , $M \in k_4 \cap k_5$

Distances

- $d_0 = |HM|$ (red) ... 1
- $d_1 = |AM| = |CD| = |CE| = |CF| = |CI| = |CJ| = |CK| = |CL| = |DE| = |DF|$ (blue) ... 10
- $d_2 = |GM|$ (green) ... 1

Occurrences of the Golden Ratio

- $\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2}$ (20 occurrences)
2.113.2 Construction MM246

Construction Process

A, B given initial points
1. $k_1 = k(A, |AB|)$
2. $k_2 = k(B, |AB|)$
   $C \in k_1 \cap k_2$, $D \in k_1 \cap k_2$
3. $k_3 = k(C, |CD|)$
   $E \in k_1 \cap k_3$, $F \in k_2 \cap k_3$
4. $k_4 = k(F, |BE|)$
   $G \in k_1 \cap k_4$, $H \in k_1 \cap k_4$, $I \in k_3 \cap k_4$, $J \in k_3 \cap k_4$
5. $k_5 = k(J, |AI|)$
   $K \in k_3 \cap k_5$, $L \in k_3 \cap k_5$, $M \in k_4 \cap k_5$

Distances

- $d_0 = |HM|$ (red) ... 1
- $d_1 = |BM| = |CD| = |CE| = |CF| = |CI| = |CJ| = |CK| = |CL| = |DE| = |DF|$ (blue) ... 10
- $d_2 = |GM|$ (green) ... 1

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_1} = \frac{d_2}{d_2}$ (20 occurrences)
2.113.3 Construction MM318

Construction Process

A, B given initial points
1. $k_1 = k(A, |AB|)$
2. $k_2 = k(B, |AB|)$
   $C \in k_1 \cap k_2$, $D \in k_1 \cap k_2$
3. $k_3 = k(D, |CD|)$
   $E \in k_1 \cap k_3$, $F \in k_2 \cap k_3$
4. $k_4 = k(E, |BE|)$
   $G \in k_2 \cap k_4$, $H \in k_2 \cap k_4$, $I \in k_3 \cap k_4$, $J \in k_3 \cap k_4$
5. $k_5 = k(I, |BJ|)$
   $K \in k_3 \cap k_5$, $L \in k_3 \cap k_5$, $M \in k_4 \cap k_5$

Distances

$$d_0 = |GM| \quad \text{(red)} \quad \ldots 1$$
$$d_1 = |AM| = |CD| = |CE| = |CF| = |DE| = |DF| = |DI| = |DJ| = |DK| = |DL| \quad \text{blue} \quad \ldots 10$$
$$d_2 = |HM| \quad \text{green} \quad \ldots 1$$

Occurrences of the Golden Ratio

$$\phi = \frac{d_1}{d_0} = \frac{d_2}{d_1} \quad \text{(20 occurrences)}$$
2.113.4 Construction MM357

Construction Process

1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
3. \( k_3 = k(D, |CD|) \)
4. \( k_4 = k(F, |BE|) \)
5. \( k_5 = k(J, |AI|) \)

Distances

- \( d_0 = |GM| \) (red) \ldots 1
- \( d_1 = |BM| = |CD| = |CE| = |CF| = |DE| = |DF| = |DI| = |DJ| = |DK| = |DL| \) (blue) \ldots 10
- \( d_2 = |HM| \) (green) \ldots 1

Occurrences of the Golden Ratio


\[
\phi = \frac{d_0}{d_1} = \frac{d_4}{d_5} \quad (20 \text{ occurrences})
\]
2.114 Class of Similar Constructions No. 105

Contained Constructions
MM208, MM247, MM321, MM360
2.114.1 Construction MM208

Construction Process

A, B given initial points
1. $k_1 = k(A, |AB|)$
2. $k_2 = k(B, |AB|)$
   $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
3. $k_3 = k(C, |CD|)$
   $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
4. $k_4 = k(E, |BE|)$
   $G \in k_2 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
5. $k_5 = k(B, |GI|)$
   $J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5$

Distances

$d_0 = |CD| = |CE| = |CF| = |CH| = |CI| = |CJ| = |CK| = |DE| = |DF|$ (red) ... 9
$d_1 = |HL|$ (blue) ... 1

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_1}$ (9 occurrences)
2.114.2 Construction MM247

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2, \; D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, |CD|) \)
   \( E \in k_1 \cap k_3, \; F \in k_2 \cap k_3 \)
4. \( k_4 = k(F, |BE|) \)
   \( G \in k_1 \cap k_4, \; H \in k_3 \cap k_4, \; I \in k_3 \cap k_4 \)
5. \( k_5 = k(A, |GH|) \)
   \( J \in k_3 \cap k_5, \; K \in k_3 \cap k_5, \; L \in k_4 \cap k_5 \)

Distances

\( d_0 = |CD| = |CE| = |CF| = |CH| = |CI| = |CJ| = |CK| = |DE| = |DF| (\text{red}) \ldots 9 \)
\( d_1 = |IL| (\text{blue}) \ldots 1 \)

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} \) (9 occurrences)
2.114.3 Construction MM321

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2, D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, |CD|) \)
   \( E \in k_1 \cap k_3, F \in k_2 \cap k_3 \)
4. \( k_4 = k(E, |BE|) \)
   \( G \in k_2 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4 \)
5. \( k_5 = k(B, |GI|) \)
   \( J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5 \)

Distances

\[ d_0 = |CD| = |CE| = |CF| = |DE| = |DF| = |DH| = |DI| = |DJ| = |DK| \text{ (red)} \ldots 9 \]
\[ d_1 = |HL| \text{ (blue)} \ldots 1 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} \text{ (9 occurrences)} \]
2.114.4 Construction MM360

Construction Process

$A, B$ given initial points
1. $k_1 = k(A, |AB|)$
2. $k_2 = k(B, |AB|)$
   $C \in k_1 \cap k_2$, $D \in k_1 \cap k_2$
3. $k_3 = k(D, |CD|)$
   $E \in k_1 \cap k_3$, $F \in k_2 \cap k_3$
4. $k_4 = k(F, |BE|)$
   $G \in k_1 \cap k_4$, $H \in k_3 \cap k_4$, $I \in k_3 \cap k_4$
5. $k_5 = k(A, |GH|)$
   $J \in k_3 \cap k_5$, $K \in k_3 \cap k_5$, $L \in k_4 \cap k_5$

Distances

$d_0 = |CD| = |CE| = |CF| = |DE| = |DF| = |DH| = |DI| = |DJ| = |DK|$ (red) ... 9
$d_1 = |IL|$ (blue) ... 1

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_1}$ (9 occurrences)
2.115 Class of Similar Constructions No. 106

Contained Constructions

MM209, MM248, MM322, MM361
2.115.1 Construction MM209

Construction Process

1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2, D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, |CD|) \)
   \( E \in k_1 \cap k_3, F \in k_2 \cap k_3 \)
4. \( k_4 = k(E, |BE|) \)
   \( G \in k_2 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4 \)
5. \( k_5 = k(H, |GI|) \)
   \( J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5 \)

Distances

\( d_0 = |CD| = |CE| = |CF| = |CH| = |CI| = |CK| = |DE| = |DF| \) (red) \( \ldots 9 \)
\( d_1 = |BL| \) (blue) \( \ldots 1 \)

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} \) (9 occurrences)
2.115.2 Construction MM248

Construction Process

A, B given initial points
1. $k_1 = k(A, |AB|)$
2. $k_2 = k(B, |AB|)$
   $C \in k_1 \cap k_2$, $D \in k_1 \cap k_2$
3. $k_3 = k(C, |CD|)$
   $E \in k_1 \cap k_3$, $F \in k_2 \cap k_3$
4. $k_4 = k(F, |BE|)$
   $G \in k_1 \cap k_4$, $H \in k_3 \cap k_4$, $I \in k_3 \cap k_4$
5. $k_5 = k(I, |GH|)$
   $J \in k_3 \cap k_5$, $K \in k_3 \cap k_5$, $L \in k_4 \cap k_5$

Distances

$d_0 = |CD| = |CE| = |CF| = |CH| = |CI| = |CJ| = |CK| = |DE| = |DF|$ (red) ... 9
$d_1 = |AL|$ (blue) ... 1

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_1}$ (9 occurrences)
2.115.3 Construction MM322

Construction Process

A, B given initial points
1. \(k_1 = k(A, |AB|)\)
2. \(k_2 = k(B, |AB|)\)
   \(C \in k_1 \cap k_2\), \(D \in k_1 \cap k_2\)
3. \(k_3 = k(D, |CD|)\)
   \(E \in k_1 \cap k_3\), \(F \in k_2 \cap k_3\)
4. \(k_4 = k(E, |BE|)\)
   \(G \in k_2 \cap k_4\), \(H \in k_3 \cap k_4\), \(I \in k_3 \cap k_4\)
5. \(k_5 = k(H, |GI|)\)
   \(J \in k_3 \cap k_5\), \(K \in k_3 \cap k_5\), \(L \in k_4 \cap k_5\)

Distances

\(d_0 = |CD| = |CE| = |CF| = |DE| = |DF| = |DH| = |DI| = |DJ| = |DK|\) (red) ... 9
\(d_1 = |BL|\) (blue) ... 1

Occurrences of the Golden Ratio

\(\phi = \frac{d_0}{d_1}\) (9 occurrences)
Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2, \ D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, |CD|) \)
   \( E \in k_1 \cap k_3, \ F \in k_2 \cap k_3 \)
4. \( k_4 = k(F, |BE|) \)
   \( G \in k_1 \cap k_4, \ H \in k_3 \cap k_4, \ I \in k_3 \cap k_4 \)
5. \( k_5 = k(I, |GH|) \)
   \( J \in k_3 \cap k_5, \ K \in k_3 \cap k_5, \ L \in k_4 \cap k_5 \)

Distances

\[ d_0 = |CD| = |CE| = |CF| = |DE| = |DF| = |DH| = |DI| = |DJ| = |DK| \] \( (\text{red}) \) \ ... \ 9
\[ d_1 = |AL| \] \( (\text{blue}) \) \ ... \ 1

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} \] \( (9 \text{ occurrences}) \)
2.116 Class of Similar Constructions No. 107

Contained Constructions
MM210, MM249, MM319, MM358
2.116.1 Construction MM210

Construction Process

1. \( k_1 = k(A, |AB|) \)  
2. \( k_2 = k(B, |AB|) \)  
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)  
3. \( k_3 = k(C, |CD|) \)  
   \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)  
4. \( k_4 = k(E, |BE|) \)  
   \( G \in k_2 \cap k_4 \), \( H \in k_3 \cap k_4 \), \( I \in k_3 \cap k_4 \)  
5. \( k_5 = k(B, |GI|) \)  
   \( J \in k_4 \cap k_5 \)

Distances

\( d_0 = |HJ| \) (red) . . . 1  
\( d_1 = |CD| = |CE| = |CF| = |CH| = |CI| = |DE| = |DF| \) (blue) . . . 7

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} \) (7 occurrences)
2.116.2 Construction MM249

Construction Process

A, B given initial points
1. $k_1 = k(A, |AB|)$
2. $k_2 = k(B, |AB|)$
   $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
3. $k_3 = k(C, |CD|)$
   $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
4. $k_4 = k(F, |BE|)$
   $G \in k_1 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
5. $k_5 = k(A, |GH|)$
   $J \in k_4 \cap k_5$

Distances

$d_0 = |IJ| \quad \text{(red)} \ldots 1$

$d_1 = |CD| = |CE| = |CF| = |CH| = |CI| = |DE| = |DF| \quad \text{(blue)} \ldots 7$

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_1} \quad (7 \text{ occurrences})$
2.116.3 Construction MM319

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, |CD|) \)
   \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(E, |BE|) \)
   \( G \in k_2 \cap k_4 \), \( H \in k_3 \cap k_4 \), \( I \in k_3 \cap k_4 \)
5. \( k_5 = k(B, |GI|) \)
   \( J \in k_4 \cap k_5 \)

Distances

\( d_0 = |HJ| \) (red) \ldots 1
\( d_1 = |CD| = |CE| = |CF| = |DE| = |DF| = |DH| = |DI| \) (blue) \ldots 7

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} \) (7 occurrences)
2.116.4 Construction MM358

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, |CD|) \)
   \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(F, |BE|) \)
   \( G \in k_1 \cap k_4 \), \( H \in k_3 \cap k_4 \), \( I \in k_3 \cap k_4 \)
5. \( k_5 = k(A, |GH|) \)
   \( J \in k_4 \cap k_5 \)

Distances

\[ d_0 = |IJ| \text{ (red)} \ldots 1 \]
\[ d_1 = |CD| = |CE| = |CF| = |DE| = |DF| = |DH| = |DI| \text{ (blue)} \ldots 7 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} \text{ (7 occurrences)} \]
2.117 Class of Similar Constructions No. 108

Contained Constructions

MM211, MM250, MM320, MM359
2.117.1 Construction MM211

Construction Process

1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
3. \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
4. \( k_3 = k(C, |CD|) \)
5. \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
6. \( k_4 = k(E, |BE|) \)
7. \( G \in k_2 \cap k_4 \), \( H \in k_3 \cap k_4 \), \( I \in k_3 \cap k_4 \)
8. \( J \in k_3 \cap k_5 \), \( K \in k_3 \cap k_5 \), \( L \in k_4 \cap k_5 \)

Distances

\[ d_0 = |BL| \quad \text{(red)} \quad \ldots 1 \]
\[ d_1 = |CD| = |CE| = |CF| = |CH| = |CI| = |CJ| = |CK| = |DE| = |DF| \quad \text{(blue)} \quad \ldots 9 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} \quad \text{(9 occurrences)} \]
2.117.2 Construction MM250

Construction Process

A, B given initial points
1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, |CD|) \)
   \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(F, |BE|) \)
   \( G \in k_1 \cap k_4 \), \( H \in k_3 \cap k_4 \), \( I \in k_3 \cap k_4 \)
5. \( k_5 = k(I, |GH|) \)
   \( J \in k_3 \cap k_5 \), \( K \in k_3 \cap k_5 \), \( L \in k_4 \cap k_5 \)

Distances

\[ d_0 = |AL| \text{ (red)} \ldots 1 \]
\[ d_1 = |CD| = |CE| = |CF| = |CH| = |CI| = |CJ| = |CK| = |DE| = |DF| \text{ (blue)} \ldots 9 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} \text{ (9 occurrences)} \]
2.117.3 Construction MM320

Construction Process

A, B given initial points

1. \( k_1 = k(A, |AB|) \)
2. \( k_2 = k(B, |AB|) \)
   \( C \in k_1 \cap k_2, \; D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, |CD|) \)
   \( E \in k_1 \cap k_3, \; F \in k_2 \cap k_3 \)
4. \( k_4 = k(E, |BE|) \)
   \( G \in k_2 \cap k_4, \; H \in k_3 \cap k_4, \; I \in k_3 \cap k_4 \)
5. \( k_5 = k(H, |GI|) \)
   \( J \in k_3 \cap k_5, \; K \in k_3 \cap k_5, \; L \in k_4 \cap k_5 \)

Distances

\( d_0 = |BL| \) (red) ... 1
\( d_1 = |CD| = |CE| = |CF| = |DE| = |DF| = |DH| = |DI| = |DJ| = |DK| \) (blue) ... 9

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} \) (9 occurrences)
Construction MM359

Construction Process

$A, B$ given initial points
1. $k_1 = k(A, |AB|)$
2. $k_2 = k(B, |AB|)$
   $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
3. $k_3 = k(D, |CD|)$
   $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
4. $k_4 = k(F, |BE|)$
   $G \in k_1 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
5. $k_5 = k(I, |GH|)$
   $J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5$

Distances

$d_0 = |AL|$ (red) ... 1
$d_1 = |CD| = |CE| = |CF| = |DE| = |DF| = |DH| = |DI| = |DJ| = |DK|$ (blue) ... 9

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_1}$ (9 occurrences)
The constructions presented in this section use only a collapsible compass. The collapsible compass cannot draw a circle with an arbitrary measured radius and only draws a circle given by a center and passing through another given point. The shortest construction of the golden ratio requires 5 steps, and 114 such constructions result from 1048 different processes. The constructions form 21 classes of mutually similar constructions.

The constructions are labeled CCO1 to CCO114.

In this section, we present list of these constructions. These constructions also belong to RCC and MM. Therefore, we also present the RCC and MM labels. CCO96 is identical with the second construction described by K. Hofstetter in [4].

**Remarks**

Similar to MM, some constructions of CCO are analogous to RCC, in which several straight lines are replaced by a circle. Replacing a ruler (straight line) by a compass (circle) is exemplified when comparing constructions RCC35 and CCO99. In RCC35, straight line $p_1$ passes through points $A$ and $B$. The circles $k_2$ and $k_3$ have centers at given points $B$ and $A$ and pass each through the other of the points. These steps could be performed in any order to construct points $D$ and $C$, for which point $D$ will be used to construct the final two circles.

In CCO99, circles with the center at the given points $B$ and $A$ passing through the other points are constructed ($k_1$ and $k_2$). The straight line passing through points $A$ and $B$ is replaced by the circle $k_3$, whose intersections with the circles $k_1$ and $k_2$ (at points $F$ and $E$, respectively) are analogies to points $D$ and $C$ in RCC35, for which point $F$ is used to construct the final two circles.

Similarly, we can compare for example constructions RCC2 and CCO46.

Comparing RCC306 and CCO18 demonstrates the difference caused by replacing the straight line $p_3$ by the circle $k_3$. 
2.119 Constructions Omitted in the Previous Kinds

2.120 Class of Similar Constructions No. 109

Contained Constructions

CCO1, CCO2, CCO3, CCO4, CCO13, CCO14, CCO15, CCO16, CCO17, CCO18, CCO19, CCO20, CCO33, CCO34, CCO35, CCO36, CCO37, CCO38, CCO39, CCO40
2.120.1 Construction CCO1

Construction Process

A, B given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)  
   \( C \in k_1 \cap k_2 , D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, A) \)  
   \( E \in k_1 \cap k_3 , F \in k_2 \cap k_3 \)
4. \( k_4 = k(D, A) \)  
   \( G \in k_1 \cap k_4 , H \in k_2 \cap k_4 \)
5. \( k_5 = k(E, D) \)  
   \( I \in k_4 \cap k_5 , J \in k_4 \cap k_5 \)

Distances

\[
\begin{align*}
    d_0 &= |FJ| \text{ (red)} \ldots 1 \\
    d_1 &= |HJ| \text{ (blue)} \ldots 1 \\
    d_2 &= |AE| = |AG| = |BF| = |BH| = |CD| = |EG| = |FH| \text{ (green)} \ldots 7 \\
    d_3 &= |BI| = |BJ| \text{ (cyan)} \ldots 2 \\
    d_4 &= |FI| \text{ (magenta)} \ldots 1 \\
    d_5 &= |HI| \text{ (yellow)} \ldots 1 \\
\end{align*}
\]

Occurrences of the Golden Ratio

\[
\phi = \frac{d_0}{d_2} = \frac{d_3}{d_4} = \frac{d_4}{d_5} = \frac{d_5}{d_6} \text{ (18 occurrences)}
\]

Identical Constructions

RCC266 MM2
2.120.2 Construction CCO2

Construction Process

A, B given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, A) \)
   \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(D, A) \)
   \( G \in k_1 \cap k_4 \), \( H \in k_2 \cap k_4 \)
5. \( k_5 = k(F, D) \)
   \( I \in k_4 \cap k_5 \), \( J \in k_4 \cap k_5 \)

Distances

\( d_0 = |EI| \) (red) \ldots 1
\( d_1 = |GI| \) (blue) \ldots 1
\( d_2 = |AE| = |AG| = |BF| = |BH| = |CD| = |EG| = |FH| \) (green) \ldots 7
\( d_3 = |AJ| = |AJ| \) (cyan) \ldots 2
\( d_4 = |EJ| \) (magenta) \ldots 1
\( d_5 = |GJ| \) (yellow) \ldots 1

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_2} = \frac{d_2}{d_3} = \frac{d_4}{d_4} = \frac{d_5}{d_5} \) (18 occurrences)

Identical Constructions

RCC267 MM1
2.120.3 Construction CCO3

Construction Process

A, B given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2 , D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, A) \)
   \( E \in k_1 \cap k_3 , F \in k_2 \cap k_3 \)
4. \( k_4 = k(D, A) \)
   \( G \in k_1 \cap k_4 , H \in k_2 \cap k_4 \)
5. \( k_5 = k(G, C) \)
   \( I \in k_3 \cap k_5 , J \in k_3 \cap k_5 \)

Distances

\[ d_0 = |HJ| \text{ (red)} \quad \ldots 1 \]
\[ d_1 = |FJ| \text{ (blue)} \quad \ldots 1 \]
\[ d_2 = |AE| = |AG| = |BF| = |BH| = |CD| = |EG| = |FH| \text{ (green)} \quad \ldots 7 \]
\[ d_3 = |BI| = |BJ| \text{ (cyan)} \quad \ldots 2 \]
\[ d_4 = |HI| \text{ (magenta)} \quad \ldots 1 \]
\[ d_5 = |FI| \text{ (yellow)} \quad \ldots 1 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_1}{d_2} = \frac{d_2}{d_4} = \frac{d_5}{d_5} \text{ (18 occurrences)} \]

Identical Constructions

RCC268 MM4
2.120.4 Construction CCO4

Construction Process

\( A, B \) given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2, D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, A) \)
   \( E \in k_1 \cap k_3, F \in k_2 \cap k_3 \)
4. \( k_4 = k(D, A) \)
   \( G \in k_1 \cap k_4, H \in k_2 \cap k_4 \)
5. \( k_5 = k(H, C) \)
   \( I \in k_3 \cap k_5, J \in k_3 \cap k_5 \)

Distances
\[
\begin{align*}
d_0 &= |GI| \quad \text{(red)} \ldots 1 \\
d_1 &= |EI| \quad \text{(blue)} \ldots 1 \\
d_2 &= |AE| = |AG| = |BF| = |BH| = |CD| = |EG| = |FH| \quad \text{(green)} \ldots 7 \\
d_3 &= |AJ| = |AJ| \quad \text{(cyan)} \ldots 2 \\
d_4 &= |GJ| \quad \text{(magenta)} \ldots 1 \\
d_5 &= |EJ| \quad \text{(yellow)} \ldots 1
\end{align*}
\]

Occurrences of the Golden Ratio
\[
\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \quad (18 \text{ occurrences})
\]

Identical Constructions
RCC269 MM3
2.120.5 Construction CCO13

A, B given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2 , D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, A) \)
   \( E \in k_1 \cap k_3 , F \in k_2 \cap k_3 \)
4. \( k_4 = k(E, B) \)
   \( G \in k_1 \cap k_4 , H \in k_3 \cap k_4 \)
5. \( k_5 = k(D, E) \)
   \( I \in k_4 \cap k_5 , J \in k_4 \cap k_5 \)

Distances
\[
\begin{align*}
  d_0 &= |FJ| \text{ (red)} \ldots 1 \\
  d_1 &= |HJ| \text{ (blue)} \ldots 1 \\
  d_2 &= |AE| = |BF| = |BH| = |CD| = |CG| = |DG| = |FH| \text{ (green)} \ldots 7 \\
  d_3 &= |BI| = |BJ| \text{ (cyan)} \ldots 2 \\
  d_4 &= |FI| \text{ (magenta)} \ldots 1 \\
  d_5 &= |HI| \text{ (yellow)} \ldots 1 \\
\end{align*}
\]

Occurrences of the Golden Ratio
\[
\phi = \frac{d_0}{d_2} = \frac{d_2}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (18 occurrences)}
\]

Identical Constructions
RCC278 MM19
2.120.6 Construction CCO14

Construction Process

A, B given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2 \)
3. \( k_3 = k(C, A) \)
   \( E \in k_1 \cap k_3 \)
4. \( k_4 = k(E, B) \)
   \( G \in k_1 \cap k_4 \)
5. \( k_5 = k(F, D) \)
   \( I \in k_4 \cap k_5 \)

Distances

\[
\begin{align*}
  d_0 &= |DI| \text{ (red)} \ldots 1 \\
  d_1 &= |GI| \text{ (blue)} \ldots 1 \\
  d_2 &= |AE| = |BF| = |BH| = |CD| = |CG| = |DG| = |FH| \text{ (green)} \ldots 7 \\
  d_3 &= |CI| = |CJ| \text{ (cyan)} \ldots 2 \\
  d_4 &= |DJ| \text{ (magenta)} \ldots 1 \\
  d_5 &= |GJ| \text{ (yellow)} \ldots 1 \\
\end{align*}
\]

Occurrences of the Golden Ratio

\[
\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (18 occurrences)}
\]

Identical Constructions
RCC279 MM20
2.120.7 Construction CCO15

Construction Process

A, B given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)

\( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, A) \)

\( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(E, B) \)

\( G \in k_1 \cap k_4 \), \( H \in k_3 \cap k_4 \)
5. \( k_5 = k(G, A) \)

\( I \in k_2 \cap k_5 \), \( J \in k_2 \cap k_5 \)

Distances

\( d_0 = |HJ| \) (red) \ldots 1
\( d_1 = |FJ| \) (blue) \ldots 1
\( d_2 = |AE| = |BF| = |BH| = |CD| = |CG| = |DG| = |FH| \) (green) \ldots 7
\( d_3 = |BI| = |BJ| \) (cyan) \ldots 2
\( d_4 = |HI| \) (magenta) \ldots 1
\( d_5 = |FI| \) (yellow) \ldots 1

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_2} = \frac{d_1}{d_4} = \frac{d_4}{d_5} = \frac{d_4}{d_5} \) (18 occurrences)

Identical Constructions

RCC280 MM21
2.120.8 Construction CCO16

Construction Process

A, B given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2, \ D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, A) \)
   \( E \in k_1 \cap k_3, \ F \in k_2 \cap k_3 \)
4. \( k_4 = k(E, B) \)
   \( G \in k_1 \cap k_4, \ H \in k_3 \cap k_4 \)
5. \( k_5 = k(H, A) \)
   \( I \in k_2 \cap k_5, \ J \in k_2 \cap k_5 \)

Distances

\( d_0 = |GI| \) (red) … 1
\( d_1 = |DI| \) (blue) … 1
\( d_2 = |AE| = |BF| = |BH| = |CD| = |CG| = |DG| = |FH| \) (green) … 7
\( d_3 = |CI| = |CJ| \) (cyan) … 2
\( d_4 = |GJ| \) (magenta) … 1
\( d_5 = |DJ| \) (yellow) … 1

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_4}{d_5} = \frac{d_2}{d_4} = \frac{d_4}{d_5} \) (18 occurrences)

Identical Constructions

RCC281 MM22
2.120.9 Construction CCO17

Construction Process

A, B given initial points
1. $k_1 = k(B, A)$
2. $k_2 = k(A, B)$
   $C \in k_1 \cap k_2 , D \in k_1 \cap k_2$
3. $k_3 = k(C, A)$
   $E \in k_1 \cap k_3 , F \in k_2 \cap k_3$
4. $k_4 = k(E, D)$
5. $k_5 = k(F, A)$
   $G \in k_2 \cap k_5 , H \in k_3 \cap k_5 , I \in k_4 \cap k_5 , J \in k_4 \cap k_5$

Distances

$d_0 = |DI|$ (red) \ldots 1
$d_1 = |GI|$ (blue) \ldots 1
$d_2 = |AE| = |AH| = |BF| = |CD| = |CG| = |DG| = |EH|$ (green) \ldots 7
$d_3 = |CI| = |CJ|$ (cyan) \ldots 2
$d_4 = |DJ|$ (magenta) \ldots 1
$d_5 = |GJ|$ (yellow) \ldots 1

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_2} = \frac{d_3}{d_4} = \frac{d_4}{d_5} = \frac{d_5}{d_5}$ (18 occurrences)

Identical Constructions

RCC282 MM14
2.120.10 Construction CCO18

A, B given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, A) \)
   \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(D, E) \)
5. \( k_5 = k(F, A) \)
   \( G \in k_2 \cap k_5 \), \( H \in k_3 \cap k_5 \), \( I \in k_4 \cap k_5 \), \( J \in k_4 \cap k_5 \)

Distances
\[
\begin{align*}
d_0 &= |EI| \quad \text{(red)} \ldots 1 \\
d_1 &= |HI| \quad \text{(blue)} \ldots 1 \\
d_2 &= |AE| = |AH| = |BF| = |CD| = |CG| = |DG| = |EH| \quad \text{(green)} \ldots 7 \\
d_3 &= |AJ| = |AJ| \quad \text{(cyan)} \ldots 2 \\
d_4 &= |EJ| \quad \text{(magenta)} \ldots 1 \\
d_5 &= |HJ| \quad \text{(yellow)} \ldots 1 
\end{align*}
\]

Occurrences of the Golden Ratio
\[
\phi = \frac{d_0}{d_2} = \frac{d_3}{d_5} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \quad \text{(18 occurrences)}
\]

Identical Constructions
RCC283 MM13
2.120.11 Construction CCO19

Construction Process

A, B given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, A) \)
   \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(F, A) \)
   \( G \in k_2 \cap k_4 \), \( H \in k_3 \cap k_4 \)
5. \( k_5 = k(G, B) \)
   \( I \in k_1 \cap k_5 \), \( J \in k_1 \cap k_5 \)

Distances

\[ d_0 = |HJ| \quad \text{(red)} \quad \ldots \quad 1 \]
\[ d_1 = |EJ| \quad \text{(blue)} \quad \ldots \quad 1 \]
\[ d_2 = |AE| = |AH| = |BF| = |CD| = |CG| = |DG| = |EH| \quad \text{(green)} \quad \ldots \quad 7 \]
\[ d_3 = |AI| = |AJ| \quad \text{(cyan)} \quad \ldots \quad 2 \]
\[ d_4 = |HI| \quad \text{(magenta)} \quad \ldots \quad 1 \]
\[ d_5 = |EI| \quad \text{(yellow)} \quad \ldots \quad 1 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \quad \text{(18 occurrences)} \]

Identical Constructions

RCC284 MM15
2.120.12 Construction CCO20

Construction Process

A, B given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2, D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, A) \)
   \( E \in k_1 \cap k_3, F \in k_2 \cap k_3 \)
4. \( k_4 = k(F, A) \)
   \( G \in k_2 \cap k_4, H \in k_3 \cap k_4 \)
5. \( k_5 = k(H, B) \)
   \( I \in k_1 \cap k_5, J \in k_1 \cap k_5 \)

Distances

\[ d_0 = |GJ| \text{ (red)} \ldots 1 \]
\[ d_1 = |DJ| \text{ (blue)} \ldots 1 \]
\[ d_2 = |AE| = |AH| = |BF| = |CD| = |CG| = |DG| = |EH| \text{ (green)} \ldots 7 \]
\[ d_3 = |CI| = |CJ| \text{ (cyan)} \ldots 2 \]
\[ d_4 = |GI| \text{ (magenta)} \ldots 1 \]
\[ d_5 = |DI| \text{ (yellow)} \ldots 1 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (18 occurrences)} \]

Identical Constructions

RCC285 MM16
2.120.13 Construction CCO33

Construction Process

\(A, B\) given initial points
1. \(k_1 = k(B, A)\)
2. \(k_2 = k(A, B)\)
   \(C \in k_1 \cap k_2, D \in k_1 \cap k_2\)
3. \(k_3 = k(D, A)\)
   \(E \in k_1 \cap k_3, F \in k_2 \cap k_3\)
4. \(k_4 = k(E, B)\)
   \(G \in k_1 \cap k_4, H \in k_3 \cap k_4\)
5. \(k_5 = k(C, E)\)
   \(I \in k_4 \cap k_5, J \in k_4 \cap k_5\)

Distances

\(d_0 = |FJ| \) (red) \ldots 1
\(d_1 = |HJ| \) (blue) \ldots 1
\(d_2 = |AE| = |BF| = |BH| = |CD| = |CG| = |DG| = |FH| \) (green) \ldots 7
\(d_3 = |BI| = |BJ| \) (cyan) \ldots 2
\(d_4 = |FI| \) (magenta) \ldots 1
\(d_5 = |HI| \) (yellow) \ldots 1

Occurrences of the Golden Ratio

\(\phi = \frac{d_0}{d_2} = \frac{d_4}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \) (18 occurrences)

Identical Constructions

RCC322 MM60
2.120.14 Construction CCO34

Construction Process

A, B given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, A) \)
   \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(E, B) \)
   \( G \in k_1 \cap k_4 \), \( H \in k_3 \cap k_4 \)
5. \( k_5 = k(F, C) \)
   \( I \in k_4 \cap k_5 \), \( J \in k_4 \cap k_5 \)

Distances

\[
\begin{align*}
d_0 &= |CJ| \quad \text{(red)} \ldots 1 \\
d_1 &= |GJ| \quad \text{(blue)} \ldots 1 \\
d_2 &= |AE| = |BF| = |BH| = |CD| = |CG| = |DG| = |FH| \quad \text{(green)} \ldots 7 \\
d_3 &= |DI| = |DJ| \quad \text{(cyan)} \ldots 2 \\
d_4 &= |CI| \quad \text{(magenta)} \ldots 1 \\
d_5 &= |GI| \quad \text{(yellow)} \ldots 1
\end{align*}
\]

Occurrences of the Golden Ratio

\[
\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \quad \text{(18 occurrences)}
\]

Identical Constructions

RCC323 MM61
Construction Process

A, B given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, A) \)
   \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(E, B) \)
   \( G \in k_1 \cap k_4 \), \( H \in k_3 \cap k_4 \)
5. \( k_5 = k(G, A) \)
   \( I \in k_2 \cap k_5 \), \( J \in k_2 \cap k_5 \)

Distances

\[
\begin{align*}
  d_0 &= |HI| \text{ (red)} \ldots 1 \\
  d_1 &= |FI| \text{ (blue)} \ldots 1 \\
  d_2 &= |AE| = |BF| = |BH| = |CD| = |CG| = |DG| = |FH| \text{ (green)} \ldots 7 \\
  d_3 &= |BI| = |BJ| \text{ (cyan)} \ldots 2 \\
  d_4 &= |HJ| \text{ (magenta)} \ldots 1 \\
  d_5 &= |FJ| \text{ (yellow)} \ldots 1
\end{align*}
\]

Occurrences of the Golden Ratio

\[\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (18 occurrences)}\]

Identical Constructions

RCC324 MM62
2.120.16 Construction CCO36

Construction Process

A, B given initial points
1. \( k_1 = k(B,A) \)
2. \( k_2 = k(A,B) \)
   \(C \in k_1 \cap k_2, \ D \in k_1 \cap k_2\)
3. \( k_3 = k(D,A) \)
   \(E \in k_1 \cap k_3, \ F \in k_2 \cap k_3\)
4. \( k_4 = k(E,B) \)
   \(G \in k_1 \cap k_4, \ H \in k_3 \cap k_4\)
5. \( k_5 = k(H,A) \)
   \(I \in k_2 \cap k_5, \ J \in k_2 \cap k_5\)

Distances

\[ d_0 = |GI| \ (\text{red}) \ldots 1 \]
\[ d_1 = |CI| \ (\text{blue}) \ldots 1 \]
\[ d_2 = |AE| = |BF| = |BH| = |CD| = |CG| = |DG| = |FH| \ (\text{green}) \ldots 7 \]
\[ d_3 = |DI| = |DJ| \ (\text{cyan}) \ldots 2 \]
\[ d_4 = |GJ| \ (\text{magenta}) \ldots 1 \]
\[ d_5 = |CJ| \ (\text{yellow}) \ldots 1 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_3}{d_4} = \frac{d_3}{d_4} = \frac{d_3}{d_4} \ (18 \text{ occurrences}) \]

Identical Constructions

RCC325 MM63
2.120.17 Construction CCO37

Construction Process

- A, B given initial points
  1. $k_1 = k(B, A)$
  2. $k_2 = k(A, B)$
  - $C \in k_1 \cap k_2$, $D \in k_1 \cap k_2$
  3. $k_3 = k(D, A)$
  - $E \in k_1 \cap k_3$, $F \in k_2 \cap k_3$
  4. $k_4 = k(E, C)$
  5. $k_5 = k(F, A)$
  - $G \in k_2 \cap k_5$, $H \in k_3 \cap k_5$, $I \in k_4 \cap k_5$, $J \in k_4 \cap k_5$

Distances

- $d_0 = |CJ|$ (red) \ldots 1
- $d_1 = |GJ|$ (blue) \ldots 1
- $d_2 = |AE| = |AH| = |BF| = |CD| = |CG| = |DG| = |EH|$ (green) \ldots 7
- $d_3 = |DI| = |DJ|$ (cyan) \ldots 2
- $d_4 = |CI|$ (magenta) \ldots 1
- $d_5 = |GI|$ (yellow) \ldots 1

Occurrences of the Golden Ratio

\[
\phi = \frac{d_0}{d_2} = \frac{d_3}{d_5} = \frac{d_4}{d_4} = \frac{d_5}{d_5} \quad (18 \text{ occurrences})
\]

Identical Constructions

RCC326 MM55
2.120.18 Construction CCO38

Construction Process

A, B given initial points
1. \( k_1 = k(B,A) \)
2. \( k_2 = k(A,B) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(D,A) \)
   \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(C,E) \)
5. \( k_5 = k(F,A) \)
   \( G \in k_2 \cap k_5 \), \( H \in k_3 \cap k_5 \), \( I \in k_4 \cap k_5 \), \( J \in k_4 \cap k_5 \)

Distances

\( d_0 = |EI| \) (red) \ldots 1
\( d_1 = |HI| \) (blue) \ldots 1
\( d_2 = |AE| = |AH| = |BF| = |CD| = |CG| = |DG| = |EH| \) (green) \ldots 7
\( d_3 = |AI| = |AJ| \) (cyan) \ldots 2
\( d_4 = |EJ| \) (magenta) \ldots 1
\( d_5 = |HJ| \) (yellow) \ldots 1

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_2} = \frac{d_3}{d_4} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \) (18 occurrences)

Identical Constructions

RCC327 MM54
2.120.19 Construction CCO39

Construction Process

A, B given initial points
1. $k_1 = k(B, A)$
2. $k_2 = k(A, B)$
   $C \in k_1 \cap k_2$, $D \in k_1 \cap k_2$
3. $k_3 = k(D, A)$
   $E \in k_1 \cap k_3$, $F \in k_2 \cap k_3$
4. $k_4 = k(F, A)$
   $G \in k_2 \cap k_4$, $H \in k_3 \cap k_4$
5. $k_5 = k(G, B)$
   $I \in k_1 \cap k_5$, $J \in k_1 \cap k_5$

Distances

\[
d_0 = |HI| \hspace{1em} \text{(red)} \ldots 1
\]
\[
d_1 = |EI| \hspace{1em} \text{(blue)} \ldots 1
\]
\[
d_2 = |AE| = |AH| = |BF| = |CD| = |CG| = |DG| = |EH| \hspace{1em} \text{(green)} \ldots 7
\]
\[
d_3 = |AI| = |AJ| \hspace{1em} \text{(cyan)} \ldots 2
\]
\[
d_4 = |HJ| \hspace{1em} \text{(magenta)} \ldots 1
\]
\[
d_5 = |EJ| \hspace{1em} \text{(yellow)} \ldots 1
\]

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5}$ (18 occurrences)

Identical Constructions

RCC328 MM56
Construction Process

1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, A) \)
   \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(F, A) \)
   \( G \in k_2 \cap k_4 \), \( H \in k_3 \cap k_4 \)
5. \( k_5 = k(H, B) \)
   \( I \in k_1 \cap k_5 \), \( J \in k_1 \cap k_5 \)

Distances

\[ d_0 = |GJ| \text{ (red)} \ldots 1 \]
\[ d_1 = |CJ| \text{ (blue)} \ldots 1 \]
\[ d_2 = |AE| = |AH| = |BF| = |CD| = |CG| = |DG| = |EH| \text{ (green)} \ldots 7 \]
\[ d_3 = |DI| = |DJ| \text{ (cyan)} \ldots 2 \]
\[ d_4 = |GI| \text{ (magenta)} \ldots 1 \]
\[ d_5 = |CI| \text{ (yellow)} \ldots 1 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_4}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (18 occurrences)} \]

Identical Constructions

RCC329 MM57
2.121 Class of Similar Constructions No. 110

Contained Constructions
CCO5, CCO6, CCO9, CCO10, CCO21, CCO22, CCO27, CCO28, CCO29, CCO30, CCO41, CCO42
Construction CCO5

Construction Process

A, B given initial points
1. \(k_1 = k(B, A)\)
2. \(k_2 = k(A, B)\)
   - \(C \in k_1 \cap k_2\), \(D \in k_1 \cap k_2\)
3. \(k_3 = k(C, A)\)
   - \(E \in k_1 \cap k_3\), \(F \in k_2 \cap k_3\)
4. \(k_4 = k(D, C)\)
   - \(G \in k_1 \cap k_4\), \(H \in k_2 \cap k_4\), \(I \in k_3 \cap k_4\), \(J \in k_3 \cap k_4\)
5. \(k_5 = k(G, A)\)
   - \(K \in k_2 \cap k_5\), \(L \in k_2 \cap k_5\), \(M \in k_3 \cap k_5\), \(N \in k_4 \cap k_5\), \(O \in k_4 \cap k_5\)

Distances

\[d_0 = |LM|\] (red) \ldots 1
\[d_1 = |FL|\] (blue) \ldots 1
\[d_2 = |AE| = |BF| = |BM| = |CD| = |CG| = |CH| = |DG| = |DH| = |DI| = |DJ| = |DN| = |DO| = |FM|\] (green) \ldots 13
\[d_3 = |BK| = |BL|\] (cyan) \ldots 2
\[d_4 = |KM|\] (magenta) \ldots 1
\[d_5 = |FK|\] (yellow) \ldots 1

Occurrences of the Golden Ratio

\[\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \quad (30 \text{ occurrences})\]

Identical Constructions

RCC270 MM12
2.121.2 Construction CCO6

Construction Process

A, B given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, A) \)
   \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(D, C) \)
   \( G \in k_1 \cap k_4 \), \( H \in k_2 \cap k_4 \), \( I \in k_3 \cap k_4 \), \( J \in k_3 \cap k_4 \)
5. \( k_5 = k(H, B) \)
   \( K \in k_1 \cap k_5 \), \( L \in k_1 \cap k_5 \), \( M \in k_3 \cap k_5 \), \( N \in k_4 \cap k_5 \), \( O \in k_4 \cap k_5 \)

Distances

\( d_0 = |LM| \) (red) \ldots 1
\( d_1 = |EL| \) (blue) \ldots 1
\( d_2 = |AE| = |AM| = |BF| = |CD| = |CG| = |CH| = |DG| = |DH| = |DI| = |DJ| = |DN| = |DO| = |EM| \) (green) \ldots 13
\( d_3 = |AK| = |AL| \) (cyan) \ldots 2
\( d_4 = |KM| \) (magenta) \ldots 1
\( d_5 = |EK| \) (yellow) \ldots 1

Occurrences of the Golden Ratio

\( \phi = \frac{d_3}{d_2} = \frac{d_3}{d_3} = \frac{d_4}{d_3} = \frac{d_4}{d_5} \) (30 occurrences)

Identical Constructions

RCC271 MM11
2.121.3 Construction CCO9

Construction Process

A, B given initial points
1. \(k_1 = k(B, A)\)
2. \(k_2 = k(A, B)\)
   \(C \in k_1 \cap k_2, D \in k_1 \cap k_2\)
3. \(k_3 = k(C, A)\)
   \(E \in k_1 \cap k_3, F \in k_2 \cap k_3\)
4. \(k_4 = k(E, A)\)
   \(G \in k_1 \cap k_4, H \in k_2 \cap k_4, I \in k_2 \cap k_4, J \in k_3 \cap k_4\)
5. \(k_5 = k(G, C)\)
   \(K \in k_2 \cap k_5, L \in k_3 \cap k_5, M \in k_3 \cap k_5, N \in k_4 \cap k_5, O \in k_4 \cap k_5\)

Distances

\(d_0 = |KM|\) (red) ... 1
\(d_1 = |FM|\) (blue) ... 1
\(d_2 = |AE| = |AG| = |AJ| = |BF| = |BK| = |CD| = |EG| = |EH| = |EI| = |EJ| = |EN| = |EO| = |FK|\) (green) ... 13
\(d_3 = |BL| = |BM|\) (cyan) ... 2
\(d_4 = |KL|\) (magenta) ... 1
\(d_5 = |FL|\) (yellow) ... 1

Occurrences of the Golden Ratio

\(\phi = \frac{d_0}{d_2} = \frac{d_4}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5}\) (30 occurrences)

Identical Constructions

RCC274 MM23
2.121.4 Construction CCO10

Construction Process

\[ A, B \text{ given initial points} \]
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, A) \)
   \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(E, A) \)
   \( G \in k_1 \cap k_4 \), \( H \in k_2 \cap k_4 \), \( I \in k_3 \cap k_4 \), \( J \in k_3 \cap k_4 \)
5. \( k_5 = k(J, B) \)
   \( K \in k_1 \cap k_5 \), \( L \in k_1 \cap k_5 \), \( M \in k_2 \cap k_5 \), \( N \in k_4 \cap k_5 \), \( O \in k_4 \cap k_5 \)

Distances

\[ d_0 = |LM| (\text{red}) \ldots 1 \]
\[ d_1 = |DL| (\text{blue}) \ldots 1 \]
\[ d_2 = |AE| = |AG| = |AJ| = |BF| = |CD| = |CM| = |DM| = |EG| = |EH| = |EI| = |EJ| = |EN| = |EO| (\text{green}) \ldots 13 \]
\[ d_3 = |CK| = |CL| (\text{cyan}) \ldots 2 \]
\[ d_4 = |KM| (\text{magenta}) \ldots 1 \]
\[ d_5 = |DK| (\text{yellow}) \ldots 1 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (30 occurrences)} \]

Identical Constructions

RCC275 MM24
2.121.5 Construction CCO21

Construction Process

A, B given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2 , D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, A) \)
   \( E \in k_1 \cap k_3 , F \in k_2 \cap k_3 \)
4. \( k_4 = k(F, B) \)
   \( G \in k_1 \cap k_4 , H \in k_1 \cap k_4 , I \in k_2 \cap k_4 , J \in k_3 \cap k_4 \)
5. \( k_5 = k(I, C) \)
   \( K \in k_1 \cap k_5 , L \in k_3 \cap k_5 , M \in k_3 \cap k_5 , N \in k_4 \cap k_5 , O \in k_4 \cap k_5 \)

Distances

\[
\begin{align*}
&d_0 = |KL| \text{ (red)} \ldots 1 \\
&d_1 = |EL| \text{ (blue)} \ldots 1 \\
&d_2 = |AE| = |AK| = |BF| = |BI| = |BJ| = |CD| = |EK| = |FG| = \\
&|FH| = |FI| = |FJ| = |FN| = |FO| \text{ (green)} \ldots 13 \\
&d_3 = |AL| = |AM| \text{ (cyan)} \ldots 2 \\
&d_4 = |KM| \text{ (magenta)} \ldots 1 \\
&d_5 = |EM| \text{ (yellow)} \ldots 1
\end{align*}
\]

Occurrences of the Golden Ratio

\[
\phi = \frac{d_0}{d_2} = \frac{d_2}{d_4} = \frac{d_3}{d_4} = \frac{d_4}{d_5} \text{ (30 occurrences)}
\]

Identical Constructions

RCC286 MM17
### 2.121.6 Construction CCO22

**Construction Process**

- **A, B given initial points**
  1. \( k_1 = k(B, A) \)
  2. \( k_2 = k(A, B) \)
     - \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
  3. \( k_3 = k(C, A) \)
     - \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
  4. \( k_4 = k(F, B) \)
     - \( G \in k_1 \cap k_4 \), \( H \in k_1 \cap k_4 \), \( I \in k_2 \cap k_4 \), \( J \in k_3 \cap k_4 \)
  5. \( k_5 = k(J, A) \)
     - \( K \in k_1 \cap k_5 \), \( L \in k_2 \cap k_5 \), \( M \in k_2 \cap k_5 \), \( N \in k_4 \cap k_5 \), \( O \in k_4 \cap k_5 \)

**Distances**

- \( d_0 = |KL| \) (red) \ldots 1
- \( d_1 = |DL| \) (blue) \ldots 1
- \( d_2 = |AE| = |BF| = |BI| = |CD| = |CK| = |DK| = |FG| = |FH| = |FI| = |FJ| = |FN| = |FO| \) (green) \ldots 13
- \( d_3 = |CL| = |CM| \) (cyan) \ldots 2
- \( d_4 = |KM| \) (magenta) \ldots 1
- \( d_5 = |DM| \) (yellow) \ldots 1

**Occurrences of the Golden Ratio**

\[ \phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_4}{d_5} \] (30 occurrences)

**Identical Constructions**

RCC287 MM18
Construction CCO27

Construction Process

A, B given initial points
1. $k_1 = k(B, A)$
2. $k_2 = k(A, B)$
   $C \in k_1 \cap k_2$, $D \in k_1 \cap k_2$
3. $k_3 = k(D, A)$
   $E \in k_1 \cap k_3$, $F \in k_2 \cap k_3$
4. $k_4 = k(C, D)$
   $G \in k_1 \cap k_4$, $H \in k_2 \cap k_4$, $I \in k_3 \cap k_4$, $J \in k_3 \cap k_4$
5. $k_5 = k(G, A)$
   $K \in k_2 \cap k_5$, $L \in k_2 \cap k_5$, $M \in k_3 \cap k_5$, $N \in k_4 \cap k_5$, $O \in k_4 \cap k_5$

Distances

\[ d_0 = |KM| \quad (\text{red}) \ldots 1 \]
\[ d_1 = |FK| \quad (\text{blue}) \ldots 1 \]
\[ d_2 = |AE| = |BF| = |BM| = |CD| = |CG| = |CH| = |CI| = |CJ| = |CN| = |CO| = |DG| = |DH| = |FM| \quad (\text{green}) \ldots 13 \]
\[ d_3 = |BK| = |BL| \quad (\text{cyan}) \ldots 2 \]
\[ d_4 = |LM| \quad (\text{magenta}) \ldots 1 \]
\[ d_5 = |FL| \quad (\text{yellow}) \ldots 1 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_4}{d_4} = \frac{d_5}{d_5} = \frac{d_5}{d_5} \quad (30 \text{ occurrences}) \]

Identical Constructions

RCC316 MM51
Construction Process

A, B given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, A) \)
   \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(C, D) \)
   \( G \in k_1 \cap k_4 \), \( H \in k_2 \cap k_4 \), \( I \in k_3 \cap k_4 \), \( J \in k_4 \cap k_4 \)
5. \( k_5 = k(H, B) \)
   \( K \in k_1 \cap k_5 \), \( L \in k_1 \cap k_5 \), \( M \in k_3 \cap k_5 \), \( N \in k_4 \cap k_5 \), \( O \in k_4 \cap k_5 \)

Distances

\( d_0 = |KM| \) (red) \( \ldots 1 \)
\( d_1 = |EK| \) (blue) \( \ldots 1 \)
\( d_2 = |AE| = |AM| = |BF| = |CD| = |CG| = |CH| = |CI| = |CJ| = |CN| = |CO| = |DG| = |DH| = |EM| \) (green) \( \ldots 13 \)
\( d_3 = |AK| = |AL| \) (cyan) \( \ldots 2 \)
\( d_4 = |LM| \) (magenta) \( \ldots 1 \)
\( d_5 = |EL| \) (yellow) \( \ldots 1 \)

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \] (30 occurrences)

Identical Constructions

RCC317 MM50
2.121.9 Construction CCO29

Construction Process

A, B given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2, \ D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, A) \)
   \( E \in k_1 \cap k_3, \ F \in k_2 \cap k_3 \)
4. \( k_4 = k(E, A) \)
   \( G \in k_1 \cap k_4, \ H \in k_2 \cap k_4, \ I \in k_2 \cap k_4, \ J \in k_3 \cap k_4 \)
5. \( k_5 = k(G, D) \)
   \( K \in k_2 \cap k_5, \ L \in k_3 \cap k_5, \ M \in k_4 \cap k_5, \ N \in k_4 \cap k_5, \ O \in k_4 \cap k_5 \)

Distances

\( d_0 = |KM| \) (red) \ldots 1
\( d_1 = |FM| \) (blue) \ldots 1
\( d_2 = |AE| = |AG| = |AJ| = |BF| = |BK| = |CD| = |EG| = |EH| = |EI| = |EJ| = |EN| = |EO| = |FK| \) (green) \ldots 13
\( d_3 = |BL| = |BM| \) (cyan) \ldots 2
\( d_4 = |KL| \) (magenta) \ldots 1
\( d_5 = |FL| \) (yellow) \ldots 1

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_2} = \frac{d_2}{d_4} = \frac{d_4}{d_5} = \frac{d_2}{d_5} \) (30 occurrences)

Identical Constructions

RCC318 MM64
2.121.10 Construction CCO30

Construction Process

A, B given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2, \ D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, A) \)
   \( E \in k_1 \cap k_3, \ F \in k_2 \cap k_3 \)
4. \( k_4 = k(E, A) \)
   \( G \in k_1 \cap k_4, \ H \in k_2 \cap k_4, \ I \in k_2 \cap k_4, \ J \in k_3 \cap k_4 \)
5. \( k_5 = k(J, B) \)
   \( K \in k_1 \cap k_5, \ L \in k_1 \cap k_5, \ M \in k_2 \cap k_5, \ N \in k_4 \cap k_5, \ O \in k_4 \cap k_5 \)

Distances

\( d_0 = |LM| \) (red) ... 1
\( d_1 = |CL| \) (blue) ... 1
\( d_2 = |AE| = |AG| = |AJ| = |BF| = |CD| = |CM| = |DM| = |EG| = \)
\( |EH| = |EI| = |EJ| = |EN| = |EO| \) (green) ... 13
\( d_3 = |DK| = |DL| \) (cyan) ... 2
\( d_4 = |KM| \) (magenta) ... 1
\( d_5 = |CK| \) (yellow) ... 1

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \) (30 occurrences)

Identical Constructions

RCC319 MM65

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Gergelitsová, Holan: The Golden Ratio... using a Ruler and Compass 754
2.121.11 Construction CCO41

Construction Process

\(A, B\) given initial points
1. \(k_1 = k(B, A)\)
2. \(k_2 = k(A, B)\)
   \(C \in k_1 \cap k_2\), \(D \in k_1 \cap k_2\)
3. \(k_3 = k(D, A)\)
   \(E \in k_1 \cap k_3\), \(F \in k_2 \cap k_3\)
4. \(k_4 = k(F, B)\)
   \(G \in k_1 \cap k_4\), \(H \in k_3 \cap k_4\), \(I \in k_2 \cap k_4\), \(J \in k_3 \cap k_4\)
5. \(k_5 = k(I, D)\)
   \(K \in k_1 \cap k_5\), \(L \in k_3 \cap k_5\), \(M \in k_3 \cap k_5\), \(N \in k_4 \cap k_5\), \(O \in k_4 \cap k_5\)

Distances

\(d_0 = |KL|\) (red) \ldots 1
\(d_1 = |EL|\) (blue) \ldots 1
\(d_2 = |AE| = |AK| = |BF| = |BI| = |BJ| = |CD| = |EK| = |FG| = |FH| = |FI| = |FJ| = |FN| = |FO|\) (green) \ldots 13
\(d_3 = |AL| = |AM|\) (cyan) \ldots 2
\(d_4 = |KM|\) (magenta) \ldots 1
\(d_5 = |EM|\) (yellow) \ldots 1

Occurrences of the Golden Ratio

\(\phi = \frac{d_0}{d_2} = \frac{d_4}{d_3} = \frac{d_2}{d_4} = \frac{d_4}{d_5}\) (30 occurrences)

Identical Constructions

RCC330 MM58
2.121.12 Construction CCO42

Construction Process

A, B given initial points
1. \(k_1 = k(B, A)\)
2. \(k_2 = k(A, B)\)
   \(C \in k_1 \cap k_2\), \(D \in k_1 \cap k_2\)
3. \(k_3 = k(D, A)\)
   \(E \in k_1 \cap k_3\), \(F \in k_2 \cap k_3\)
4. \(k_4 = k(F, B)\)
   \(G \in k_1 \cap k_4\), \(H \in k_1 \cap k_4\), \(I \in k_2 \cap k_4\), \(J \in k_3 \cap k_4\)
5. \(k_5 = k(J, A)\)
   \(K \in k_1 \cap k_5\), \(L \in k_2 \cap k_5\), \(M \in k_2 \cap k_5\), \(N \in k_4 \cap k_5\), \(O \in k_4 \cap k_5\)

Distances

\(d_0 = |KL|\) (red) \(\ldots 1\)
\(d_1 = |CL|\) (blue) \(\ldots 1\)
\(d_2 = |AE| = |BF| = |BJ| = |CD| = |CK| = |DK| = |FG| = |FH| = |FI| = |FJ| = |FN| = |FO|\) (green) \(\ldots 13\)
\(d_3 = |DL| = |DM|\) (cyan) \(\ldots 2\)
\(d_4 = |KM|\) (magenta) \(\ldots 1\)
\(d_5 = |CM|\) (yellow) \(\ldots 1\)

Occurrences of the Golden Ratio

\(\phi = \frac{d_4}{d_3} = \frac{d_3}{d_2} = \frac{d_2}{d_1} = \frac{d_1}{d_0}\) (30 occurrences)

Identical Constructions

RCC331 MM59

Gergelitsóva, Holan: The Golden Ratio... using a Ruler and Compass 756
2.122 Class of Similar Constructions No. 111

Contained Constructions

CCO7, CCO8, CCO11, CCO12, CCO23, CCO24, CCO25, CCO26, CCO31, CCO32, CCO43, CCO44
2.122.1 Construction CCO7

Construction Process

1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, A) \)
   \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(C, D) \)
   \( G \in k_1 \cap k_4 \), \( H \in k_2 \cap k_4 \)
5. \( k_5 = k(G, A) \)
   \( I \in k_2 \cap k_5 \), \( J \in k_2 \cap k_5 \), \( K \in k_3 \cap k_5 \), \( L \in k_4 \cap k_5 \), \( M \in k_4 \cap k_5 \)

Distances

\[ d_0 = |JK| \text{ (red)} \ldots 1 \]
\[ d_1 = |FJ| \text{ (blue)} \ldots 1 \]
\[ d_2 = |AE| = |BF| = |BK| = |CD| = |CG| = |CH| = |CL| = |CM| = |DG| = |DH| = |FK| \text{ (green)} \ldots 11 \]
\[ d_3 = |BI| = |BJ| \text{ (cyan)} \ldots 2 \]
\[ d_4 = |IK| \text{ (magenta)} \ldots 1 \]
\[ d_5 = |FI| \text{ (yellow)} \ldots 1 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \] (26 occurrences)

Identical Constructions

RCC272 MM10
2.122.2 Construction CCO8

Construction Process

1. $k_1 = k(B, A)$
2. $k_2 = k(A, B)$
3. $k_3 = k(C, A)$
4. $k_4 = k(C, D)$
5. $k_5 = k(H, B)$

Distances

- $d_0 = |JK|$ (red) ...
- $d_1 = |EJ|$ (blue) ...
- $d_2 = |AE| = |AK| = |BF| = |CD| = |CG| = |CH| = |CL| = |CM| = |DG| = |DH| = |EK|$ (green) ...
- $d_3 = |AI| = |AJ|$ (cyan) ...
- $d_4 = |IK|$ (magenta) ...
- $d_5 = |EI|$ (yellow) ...

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_1}{d_4} = \frac{d_2}{d_5} = \frac{d_3}{d_5} \]

Identical Constructions

RCC273 MM9
2.122.3 Construction CCO11

Construction Process

A, B given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, A) \)
   \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(A, E) \)
   \( G \in k_1 \cap k_4 \), \( H \in k_3 \cap k_4 \)
5. \( k_5 = k(G, C) \)
   \( I \in k_2 \cap k_5 \), \( J \in k_3 \cap k_5 \), \( K \in k_3 \cap k_5 \), \( L \in k_4 \cap k_5 \), \( M \in k_4 \cap k_5 \)

Distances

\[ d_0 = |IK| \quad (\text{red}) \ldots 1 \]
\[ d_1 = |FK| \quad (\text{blue}) \ldots 1 \]
\[ d_2 = |AE| = |AG| = |AH| = |AL| = |AM| = |BF| = |BI| = |CD| = |EG| = |EH| = |FI| \quad (\text{green}) \ldots 11 \]
\[ d_3 = |BJ| = |BK| \quad (\text{cyan}) \ldots 2 \]
\[ d_4 = |IJ| \quad (\text{magenta}) \ldots 1 \]
\[ d_5 = |FJ| \quad (\text{yellow}) \ldots 1 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_2}{d_1} = \frac{d_3}{d_2} = \frac{d_4}{d_3} = \frac{d_5}{d_4} \quad (26 \text{ occurrences}) \]

Identical Constructions

RCC276 MM5
Construction CCO12

Construction Process

A, B given initial points
1. \(k_1 = k(B, A)\)
2. \(k_2 = k(A, B)\)
   \(C \in k_1 \cap k_2, \ D \in k_1 \cap k_2\)
3. \(k_3 = k(C, A)\)
   \(E \in k_1 \cap k_3, \ F \in k_2 \cap k_3\)
4. \(k_4 = k(A, E)\)
   \(G \in k_1 \cap k_4, \ H \in k_3 \cap k_4\)
5. \(k_5 = k(H, B)\)
   \(I \in k_1 \cap k_5, \ J \in k_1 \cap k_5, \ K \in k_2 \cap k_5, \ L \in k_4 \cap k_5, \ M \in k_4 \cap k_5\)

Distances

\(d_0 = |JK| \) (red) \ldots 1
\(d_1 = |DJ| \) (blue) \ldots 1
\(d_2 = |AE| = |AG| = |AH| = |AL| = |AM| = |BF| = |CD| = |CK| = |DK| = |EG| = |EH| \) (green) \ldots 11
\(d_3 = |CI| = |CJ| \) (cyan) \ldots 2
\(d_4 = |IK| \) (magenta) \ldots 1
\(d_5 = |DI| \) (yellow) \ldots 1

Occurrences of the Golden Ratio

\(\phi = \frac{d_0}{d_2} = \frac{d_4}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \) (26 occurrences)

Identical Constructions

RCC277 MM6
2.122.5 Construction CCO23

Construction Process

A, B given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, A) \)
   \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(B, F) \)
   \( G \in k_2 \cap k_4 \), \( H \in k_3 \cap k_4 \)
5. \( k_5 = k(G, C) \)
   \( I \in k_1 \cap k_5 \), \( J \in k_3 \cap k_5 \), \( K \in k_3 \cap k_5 \), \( L \in k_4 \cap k_5 \), \( M \in k_4 \cap k_5 \)

Distances

\[
\begin{align*}
  d_0 &= |IJ| \text{ (red)} \ldots 1 \\
  d_1 &= |EJ| \text{ (blue)} \ldots 1 \\
  d_2 &= |AE| = |AI| = |BF| = |BG| = |BH| = |BL| = |BM| = |CD| = |EI| = |FG| = |FH| \text{ (green)} \ldots 11 \\
  d_3 &= |AJ| = |AK| \text{ (cyan)} \ldots 2 \\
  d_4 &= |IK| \text{ (magenta)} \ldots 1 \\
  d_5 &= |EK| \text{ (yellow)} \ldots 1
\end{align*}
\]

Occurrences of the Golden Ratio

\[
\phi = \frac{d_2}{d_1} = \frac{d_3}{d_4} = \frac{d_4}{d_5} = \frac{d_5}{d_2} \quad (26 \text{ occurrences})
\]

Identical Constructions

RCC288 MM7
2.122.6 Construction CCO24

Construction Process

A, B given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2 , \ D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, A) \)
   \( E \in k_1 \cap k_3 , \ F \in k_2 \cap k_3 \)
4. \( k_4 = k(B, F) \)
   \( G \in k_2 \cap k_4 , \ H \in k_3 \cap k_4 \)
5. \( k_5 = k(H, A) \)
   \( I \in k_1 \cap k_5 , \ J \in k_2 \cap k_5 , \ K \in k_2 \cap k_5 , \ L \in k_4 \cap k_5 , \ M \in k_4 \cap k_5 \)

Distances

\[
\begin{align*}
&d_0 = |IJ| \quad \text{(red)} \ldots 1 \\
&d_1 = |DJ| \quad \text{(blue)} \ldots 1 \\
&d_2 = |AE| = |BF| = |BG| = |BH| = |BL| = |BM| = |CD| = |CI| = |DI| = |FG| = |FH| \quad \text{(green)} \ldots 11 \\
&d_3 = |CJ| = |CK| \quad \text{(cyan)} \ldots 2 \\
&d_4 = |IK| \quad \text{(magenta)} \ldots 1 \\
&d_5 = |DK| \quad \text{(yellow)} \ldots 1
\end{align*}
\]

Occurrences of the Golden Ratio

\[
\phi = \frac{d_0}{d_2} = \frac{d_4}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \quad (26 \text{ occurrences})
\]

Identical Constructions

RCC289 MM8
Construction Process

1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2 , \ D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, A) \)
   \( E \in k_1 \cap k_3 , \ F \in k_2 \cap k_3 \)
4. \( k_4 = k(D, C) \)
   \( G \in k_1 \cap k_4 , \ H \in k_2 \cap k_4 \)
5. \( k_5 = k(G, A) \)
   \( I \in k_2 \cap k_5 , \ J \in k_2 \cap k_5 , \ K \in k_3 \cap k_5 , \ L \in k_4 \cap k_5 , \ M \in k_4 \cap k_5 \)

Distances

\[
\begin{align*}
    d_0 &= |IK| \text{ (red)} \ldots 1 \\
    d_1 &= |FI| \text{ (blue)} \ldots 1 \\
    d_2 &= |AE| = |BF| = |BK| = |CD| = |CG| = |CH| = |DG| = |DH| = \\
    &\quad |DL| = |DM| = |FK| \text{ (green)} \ldots 11 \\
    d_3 &= |BI| = |BJ| \text{ (cyan)} \ldots 2 \\
    d_4 &= |JK| \text{ (magenta)} \ldots 1 \\
    d_5 &= |FJ| \text{ (yellow)} \ldots 1 \\
\end{align*}
\]

Occurrences of the Golden Ratio

\[
\phi = \frac{d_5}{d_2} = \frac{d_4}{d_3} = \frac{d_3}{d_4} = \frac{d_2}{d_3} \quad \text{(26 occurrences)}
\]

Identical Constructions

RCC314 MM53
2.122.8 Construction CCO26

Construction Process

- A, B given initial points
- \( k_1 = k(B,A) \)
- \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
- \( k_3 = k(D,A) \)
- \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
- \( k_4 = k(D,C) \)
- \( G \in k_1 \cap k_4 \), \( H \in k_2 \cap k_4 \)
- \( k_5 = k(H,B) \)

Distances

- \( d_0 = |IK| \) (red) \ldots 1
- \( d_1 = |EI| \) (blue) \ldots 1
- \( d_2 = |AE| = |AK| = |BF| = |CD| = |CG| = |CH| = |DG| = |DH| = |DL| = |DM| = |EK| \) (green) \ldots 11
- \( d_3 = |AJ| = |AJ| \) (cyan) \ldots 2
- \( d_4 = |JK| \) (magenta) \ldots 1
- \( d_5 = |EJ| \) (yellow) \ldots 1

Occurrences of the Golden Ratio

- \( \phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \) (26 occurrences)

Identical Constructions

RCC315 MM52
2.122.9 Construction CCO31

Construction Process

A, B given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, A) \)
   \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(A, E) \)
   \( G \in k_1 \cap k_4 \), \( H \in k_3 \cap k_4 \)
5. \( k_5 = k(G, D) \)
   \( I \in k_2 \cap k_5 \), \( J \in k_3 \cap k_5 \), \( K \in k_3 \cap k_5 \), \( L \in k_4 \cap k_5 \), \( M \in k_4 \cap k_5 \)

Distances

\( d_0 = |IK| \) (red) ... 1
\( d_1 = |FK| \) (blue) ... 1
\( d_2 = |AE| = |AG| = |AH| = |AL| = |AM| = |BF| = |BI| = |CD| = |EG| = |EH| = |FI| \) (green) ... 11
\( d_3 = |BJ| = |BK| \) (cyan) ... 2
\( d_4 = |IJ| \) (magenta) ... 1
\( d_5 = |FJ| \) (yellow) ... 1

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_1}{d_4} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \] (26 occurrences)

Identical Constructions

RCC320 MM46
2.122.10 Construction CCO32

Construction Process

A, B given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, A) \)
   \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(A, E) \)
   \( G \in k_1 \cap k_4 \), \( H \in k_3 \cap k_4 \)
5. \( k_5 = k(H, B) \)
   \( I \in k_1 \cap k_5 \), \( J \in k_1 \cap k_5 \), \( K \in k_2 \cap k_5 \), \( L \in k_4 \cap k_5 \), \( M \in k_4 \cap k_5 \)

Distances
\[
\begin{align*}
d_0 &= |JK| \text{(red)} \ldots 1 \\
d_1 &= |CJ| \text{(blue)} \ldots 1 \\
d_2 &= |AE| = |AG| = |AH| = |AL| = |AM| = |BF| = |CD| = |CK| = \\
d_3 &= |DK| = |EG| = |EH| \text{(green)} \ldots 11 \\
d_4 &= |DI| = |DJ| \text{(cyan)} \ldots 2 \\
d_5 &= |IK| \text{(magenta)} \ldots 1 \\
d_6 &= |CI| \text{(yellow)} \ldots 1
\end{align*}
\]

Occurrences of the Golden Ratio
\[
\phi = \frac{d_0}{d_2} = \frac{d_2}{d_3} = \frac{d_3}{d_4} = \frac{d_4}{d_5} \ (26 \ occurrences)
\]

Identical Constructions
RCC321 MM47
Construction CCO43

Construction Process

$A, B$ given initial points
1. $k_1 = k(B, A)$
2. $k_2 = k(A, B)$
   $C \in k_1 \cap k_2$, $D \in k_1 \cap k_2$
3. $k_3 = k(D, A)$
   $E \in k_1 \cap k_3$, $F \in k_2 \cap k_3$
4. $k_4 = k(B, F)$
   $G \in k_2 \cap k_4$, $H \in k_3 \cap k_4$
5. $k_5 = k(G, D)$
   $I \in k_1 \cap k_5$, $J \in k_3 \cap k_5$, $K \in k_3 \cap k_5$, $L \in k_4 \cap k_5$, $M \in k_4 \cap k_5$

Distances

\[
\begin{align*}
  d_0 &= |IJ| \text{ (red)} \ldots 1 \\
  d_1 &= |EJ| \text{ (blue)} \ldots 1 \\
  d_2 &= |AE| = |AI| = |BF| = |BG| = |BH| = |BL| = |BM| = |CD| = |EI| = |FG| = |FH| \text{ (green)} \ldots 11 \\
  d_3 &= |AJ| = |AK| \text{ (cyan)} \ldots 2 \\
  d_4 &= |IK| \text{ (magenta)} \ldots 1 \\
  d_5 &= |EK| \text{ (yellow)} \ldots 1
\end{align*}
\]

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (26 occurrences)}$

Identical Constructions

RCC332 MM48
2.122.12 Construction CCO44

Construction Process

\( A, B \) given initial points

1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, A) \)
   \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(B, F) \)
   \( G \in k_2 \cap k_4 \), \( H \in k_3 \cap k_4 \)
5. \( k_5 = k(H, A) \)
   \( I \in k_1 \cap k_5 \), \( J \in k_2 \cap k_5 \), \( K \in k_2 \cap k_5 \), \( L \in k_4 \cap k_5 \), \( M \in k_4 \cap k_5 \)

Distances

\( d_0 = |IJ| \) (red) \ldots 1
\( d_1 = |CJ| \) (blue) \ldots 1
\( d_2 = |AE| = |BF| = |BG| = |BH| = |BL| = |BM| = |CD| = |CI| = |DI| = |FG| = |FH| \) (green) \ldots 11
\( d_3 = |DJ| = |DK| \) (cyan) \ldots 2
\( d_4 = |IK| \) (magenta) \ldots 1
\( d_5 = |CK| \) (yellow) \ldots 1

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_4}{d_3} = \frac{d_5}{d_4} = \frac{d_2}{d_5} \] (26 occurrences)

Identical Constructions

RCC333 MM49
2.123 Class of Similar Constructions No. 112

Contained Constructions

CCO45, CCO64, CCO80, CCO99
2.123.1 Construction CCO45

Construction Process

A, B given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, C) \)
   \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(E, A) \)
   \( G \in k_2 \cap k_4 \), \( H \in k_2 \cap k_4 \), \( I \in k_3 \cap k_4 \), \( J \in k_3 \cap k_4 \)
5. \( k_5 = k(A, E) \)
   \( K \in k_3 \cap k_5 \), \( L \in k_4 \cap k_5 \), \( M \in k_4 \cap k_5 \)

Distances

\( d_0 = |GM| = |HL| \) (red) \ldots 2
\( d_1 = |BL| = |BM| = |CD| = |CE| = |CF| = |DE| = |DF| = |DI| = |DJ| = |DK| = |FK| \) (blue) \ldots 11
\( d_2 = |GL| = |HM| \) (green) \ldots 2

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} = \frac{d_2}{d_5} \) (44 occurrences)

Identical Constructions

RCC348 MM263
2.123.2 Construction CCO64

Construction Process

1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2 \) , \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, C) \)
   \( E \in k_1 \cap k_3 \) , \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(F, B) \)
   \( G \in k_1 \cap k_4 \) , \( H \in k_1 \cap k_4 \) , \( I \in k_3 \cap k_4 \) , \( J \in k_3 \cap k_4 \)
5. \( k_5 = k(B, F) \)
   \( K \in k_3 \cap k_5 \) , \( L \in k_4 \cap k_5 \) , \( M \in k_4 \cap k_5 \)

Distances

\[ d_0 = |GM| = |HL| \quad \text{(red)} \cdots 2 \]
\[ d_1 = |AL| = |AM| = |CD| = |CE| = |CF| = |DE| = |DF| = |DI| = |DJ| = |DK| = |EK| \quad \text{(blue)} \cdots 11 \]
\[ d_2 = |GL| = |HM| \quad \text{(green)} \cdots 2 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_0} = \frac{d_2}{d_5} \quad \text{(44 occurrences)} \]

Identical Constructions

RCC379 MM267
2.123.3 Construction CCO80

Construction Process

A, B given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2, \ D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, D) \)
   \( E \in k_1 \cap k_3, \ F \in k_2 \cap k_3 \)
4. \( k_4 = k(E, A) \)
   \( G \in k_2 \cap k_4, \ H \in k_2 \cap k_4, \ I \in k_3 \cap k_4, \ J \in k_3 \cap k_4 \)
5. \( k_5 = k(A, E) \)
   \( K \in k_3 \cap k_5, \ L \in k_4 \cap k_5, \ M \in k_4 \cap k_5 \)

Distances

\[ d_0 = |GM| = |HL| \quad (\text{red}) \quad \ldots 2 \]
\[ d_1 = |BL| = |BM| = |CD| = |CE| = |CF| = |CI| = |CJ| = |CK| = |DE| = |DF| = |FK| \quad (\text{blue}) \quad \ldots 11 \]
\[ d_2 = |GL| = |HM| \quad (\text{green}) \quad \ldots 2 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} = \frac{d_2}{d_5} \quad (44 \text{ occurrences}) \]

Identical Constructions

RCC403 MM152
2.123.4 Construction CCO99

Construction Process

\(A, B\) given initial points
1. \(k_1 = k(B, A)\)
2. \(k_2 = k(A, B)\)
   \(C \in k_1 \cap k_2\), \(D \in k_1 \cap k_2\)
3. \(k_3 = k(C, D)\)
   \(E \in k_1 \cap k_3\), \(F \in k_2 \cap k_3\)
4. \(k_4 = k(F, B)\)
   \(G \in k_1 \cap k_4\), \(H \in k_1 \cap k_4\), \(I \in k_3 \cap k_4\), \(J \in k_3 \cap k_4\)
5. \(k_5 = k(B, F)\)
   \(K \in k_3 \cap k_5\), \(L \in k_4 \cap k_5\), \(M \in k_4 \cap k_5\)

Distances

\[d_0 = |GM| = |HL| (\text{red}) \ldots 2\]
\[d_1 = |AL| = |AM| = |CD| = |CE| = |CF| = |CI| = |CJ| = |CK| = |DE| = |DF| = |EK| (\text{blue}) \ldots 11\]
\[d_2 = |GL| = |HM| (\text{green}) \ldots 2\]

Occurrences of the Golden Ratio

\[\phi = \frac{d_0}{d_1} = \frac{d_2}{d_5} \text{ (44 occurrences)}\]

Identical Constructions
RCC434 MM156
2.124 Class of Similar Constructions No. 113

Contained Constructions

CCO46, CCO63, CCO81, CCO98
Construction CCO46

Construction Process

1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, C) \)
   \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(E, A) \)
   \( G \in k_2 \cap k_4 \), \( H \in k_2 \cap k_4 \)
5. \( k_5 = k(F, A) \)
   \( I \in k_2 \cap k_5 \), \( J \in k_2 \cap k_5 \)

Distances

\( d_0 = |GJ| = |HI| \) (red) \( \ldots 2 \)
\( d_1 = |BG| = |BH| \) (blue) \( \ldots 2 \)
\( d_2 = |GI| = |HJ| \) (green) \( \ldots 2 \)

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \) (8 occurrences)

Identical Constructions

RCC349 MM258
2.124.2 Construction CCO63

Construction Process

A, B given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2 \) , \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, C) \)
   \( E \in k_1 \cap k_3 \) , \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(E, B) \)
   \( G \in k_1 \cap k_4 \) , \( H \in k_1 \cap k_4 \)
5. \( k_5 = k(F, B) \)
   \( I \in k_1 \cap k_5 \) , \( J \in k_1 \cap k_5 \)

Distances

\[ d_0 = |GJ| = |HI| \text{ (red)} \ldots 2 \]
\[ d_1 = |AI| = |AJ| \text{ (blue)} \ldots 2 \]
\[ d_2 = |GI| = |HJ| \text{ (green)} \ldots 2 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} = \frac{d_2}{d_1} \text{ (8 occurrences)} \]

Identical Constructions

RCC376 MM259
2.124.3 Construction CCO81

- **Construction Process**
  - A, B given initial points
  - \( k_1 = k(B, A) \)
  - \( k_2 = k(A, B) \)
  - \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
  - \( k_3 = k(C, D) \)
  - \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
  - \( k_4 = k(E, A) \)
  - \( G \in k_2 \cap k_4 \), \( H \in k_2 \cap k_4 \)
  - \( k_5 = k(F, A) \)
  - \( I \in k_2 \cap k_5 \), \( J \in k_2 \cap k_5 \)

- **Distances**
  - \( d_0 = |GJ| = |HI| \) (red) \( \ldots 2 \)
  - \( d_1 = |BG| = |BH| \) (blue) \( \ldots 2 \)
  - \( d_2 = |GI| = |HJ| \) (green) \( \ldots 2 \)

- **Occurrences of the Golden Ratio**
  \( \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \) (8 occurrences)

- **Identical Constructions**
  RCC404 MM147
2.124.4 Construction CCO98

Construction Process

A, B given initial points
1. \(k_1 = k(B,A)\)
2. \(k_2 = k(A,B)\)
   \(C \in k_1 \cap k_2\), \(D \in k_1 \cap k_2\)
3. \(k_3 = k(C, D)\)
   \(E \in k_1 \cap k_3\), \(F \in k_2 \cap k_3\)
4. \(k_4 = k(E, B)\)
   \(G \in k_1 \cap k_4\), \(H \in k_1 \cap k_4\)
5. \(k_5 = k(F, B)\)
   \(I \in k_1 \cap k_5\), \(J \in k_1 \cap k_5\)

Distances

\(d_0 = |GJ| = |HI|\) (red) \ldots 2
\(d_1 = |AI| = |AJ|\) (blue) \ldots 2
\(d_2 = |GI| = |HJ|\) (green) \ldots 2

Occurrences of the Golden Ratio

\(\phi = \frac{d_0}{x} = \frac{d_1}{x}\) (8 occurrences)

Identical Constructions

RCC431 MM148
2.125 Class of Similar Constructions No. 114

Contained Constructions
CCO47, CCO65, CCO89, CCO107
2.125.1 Construction CCO47

Construction Process

A, B given initial points
1. $k_1 = k(B, A)$
2. $k_2 = k(A, B)$
   $C \in k_1 \cap k_2$, $D \in k_1 \cap k_2$
3. $k_3 = k(D, C)$
   $E \in k_1 \cap k_3$, $F \in k_2 \cap k_3$
4. $k_4 = k(E, A)$
   $G \in k_2 \cap k_4$, $H \in k_2 \cap k_4$, $I \in k_3 \cap k_4$, $J \in k_3 \cap k_4$
5. $k_5 = k(G, A)$
   $K \in k_1 \cap k_5$, $L \in k_2 \cap k_5$, $M \in k_2 \cap k_5$, $N \in k_3 \cap k_5$, $O \in k_3 \cap k_5$

Distances

$d_0 = |EL|$ (red) \ldots 1
$d_1 = |DK| = |DL| = |FM| = |KL|$ (blue) \ldots 4
$d_2 = |CD| = |CE| = |CF| = |DE| = |DF| = |DI| = |DJ| = |DN| = |DO| = |LM|$ (green) \ldots 10
$d_3 = |BG| = |BH| = |CL| = |DM| = |EK|$ (cyan) \ldots 5
$d_4 = |EM|$ (magenta) \ldots 1
$d_5 = |CK| = |CM| = |FL| = |KM|$ (yellow) \ldots 4

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5}$ (60 occurrences)

Identical Constructions

RCC351 MM323
2.125.2 Construction CCO65

Construction Process

1. $k_1 = k(B, A)$  
   $C \in k_1 \cap k_2 , D \in k_1 \cap k_2$
2. $k_2 = k(A, B)$  
   $E \in k_1 \cap k_3 , F \in k_2 \cap k_3$
3. $k_3 = k(D, C)$  
   $G \in k_1 \cap k_4 , H \in k_1 \cap k_4 , I \in k_3 \cap k_4 , J \in k_3 \cap k_4$
4. $k_4 = k(F, B)$  
   $K \in k_1 \cap k_5 , L \in k_1 \cap k_5 , M \in k_2 \cap k_5 , N \in k_3 \cap k_5 , O \in k_3 \cap k_5$

Distances

- $d_0 = |FL|$ (red) \ldots 1
- $d_1 = |DL| = |DM| = |EK| = |LM|$ (blue) \ldots 4
- $d_2 = |CD| = |CE| = |CF| = |DE| = |DF| = |DI| = |DJ| = |DN| = |DO|$ (green) \ldots 10
- $d_3 = |AG| = |AH| = |CL| = |DK| = |FM|$ (cyan) \ldots 5
- $d_4 = |FK|$ (magenta) \ldots 1
- $d_5 = |CK| = |CM| = |EL| = |KM|$ (yellow) \ldots 4

Occurrences of the Golden Ratio

\[ \phi = \frac{d_4}{d_3} = \frac{d_5}{d_4} = \frac{d_5}{d_2} = \frac{d_6}{d_5} \] (60 occurrences)

Identical Constructions

RCC381 MM277
2.125.3 Construction CCO89

Construction Process

\[ A, B \text{ given initial points} \]

1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \[ C \in k_1 \cap k_2, \quad D \in k_1 \cap k_2 \]
3. \( k_3 = k(C, D) \)
   \[ E \in k_1 \cap k_3, \quad F \in k_2 \cap k_3 \]
4. \( k_4 = k(E, A) \)
   \[ G \in k_2 \cap k_4, \quad H \in k_2 \cap k_4, \quad I \in k_3 \cap k_4, \quad J \in k_3 \cap k_4 \]
5. \( k_5 = k(H, A) \)
   \[ K \in k_1 \cap k_5, \quad L \in k_2 \cap k_5, \quad M \in k_2 \cap k_5, \quad N \in k_3 \cap k_5, \quad O \in k_3 \cap k_5 \]

Distances

\[ d_0 = |EL| \text{ (red)} \ldots 1 \]
\[ d_1 = |CK| = |CL| = |FM| = |KL| \text{ (blue)} \ldots 4 \]
\[ d_2 = |CD| = |CE| = |CF| = |CI| = |CJ| = |CN| = |CO| = |DE| = |DF| = |LM| \text{ (green)} \ldots 10 \]
\[ d_3 = |BG| = |BH| = |CM| = |DL| = |EK| \text{ (cyan)} \ldots 5 \]
\[ d_4 = |EM| \text{ (magenta)} \ldots 1 \]
\[ d_5 = |DK| = |DM| = |FL| = |KM| \text{ (yellow)} \ldots 4 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (60 occurrences)} \]

Identical Constructions

RCC417 MM215

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Gergelitsová, Holan: The Golden Ratio... using a Ruler and Compass 783
2.125.4 Construction CCO107

Construction Process

A, B given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2, D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, D) \)
   \( E \in k_1 \cap k_3, F \in k_2 \cap k_3 \)
4. \( k_4 = k(F, B) \)
   \( G \in k_1 \cap k_4, H \in k_1 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4 \)
5. \( k_5 = k(H, B) \)
   \( K \in k_1 \cap k_5, L \in k_1 \cap k_5, M \in k_2 \cap k_5, N \in k_3 \cap k_5, O \in k_3 \cap k_5 \)

Distances

\[ d_0 = |FL| \text{ (red)} \ldots 1 \]
\[ d_1 = |CL| = |CM| = |EK| = |LM| \text{ (blue)} \ldots 4 \]
\[ d_2 = |CD| = |CE| = |CF| = |CI| = |CJ| = |CN| = |CO| = |DE| = |DF| = |KL| \text{ (green)} \ldots 10 \]
\[ d_3 = |AG| = |AH| = |CK| = |DL| = |FM| \text{ (cyan)} \ldots 5 \]
\[ d_4 = |FK| \text{ (magenta)} \ldots 1 \]
\[ d_5 = |DK| = |DM| = |EL| = |KM| \text{ (yellow)} \ldots 4 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_4}{d_5} = \frac{d_4}{d_3} = \frac{d_5}{d_2} \] (60 occurrences)

Identical Constructions

RCC447 MM169
2.126 Class of Similar Constructions No. 115

Contained Constructions

CCO48, CCO66, CCO92, CCO110
2.126.1 Construction CCO48

Construction Process

1. \( k_1 = k(B,A) \)
2. \( k_2 = k(A,B) \)
3. \( C \in k_1 \cap k_2 \) , \( D \in k_1 \cap k_2 \)
4. \( k_3 = k(D,C) \)
5. \( k_4 = k(E,A) \)
6. \( k_5 = k(G,C) \)

Distances

\( d_0 = |EJ| \) (red) \ldots 1
\( d_1 = |FJ| \) (blue) \ldots 1
\( d_2 = |BG| = |BH| \) (green) \ldots 2
\( d_3 = |AI| \) (cyan) \ldots 1
\( d_4 = |CJ| \) (magenta) \ldots 1

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} = \frac{d_2}{d_3} = \frac{d_3}{d_4} \] (4 occurrences)

Identical Constructions

RCC353 MM339
2.126.2 Construction CCO66

Construction Process

$A$, $B$ given initial points
1. $k_1 = k(B, A)$
2. $k_2 = k(A, B)$
   $C \in k_1 \cap k_2$, $D \in k_1 \cap k_2$
3. $k_3 = k(D, C)$
   $E \in k_1 \cap k_3$, $F \in k_2 \cap k_3$
4. $k_4 = k(F, B)$
   $G \in k_1 \cap k_4$, $H \in k_1 \cap k_4$
5. $k_5 = k(G, C)$
   $I \in k_2 \cap k_5$, $J \in k_3 \cap k_5$

Distances

\[
\begin{align*}
  d_0 & = |FJ| \quad \text{(red)} \quad \ldots 1 \\
  d_1 & = |EJ| \quad \text{(blue)} \quad \ldots 1 \\
  d_2 & = |AG| = |AH| \quad \text{(green)} \quad \ldots 2 \\
  d_3 & = |BI| \quad \text{(cyan)} \quad \ldots 1 \\
  d_4 & = |CJ| \quad \text{(magenta)} \quad \ldots 1
\end{align*}
\]

Occurrences of the Golden Ratio

\[
\phi = \frac{d_0}{d_1} = \frac{d_2}{d_3} = \frac{d_3}{d_4} \quad (4 \text{ occurrences})
\]

Identical Constructions

RCC383 MM293
2.126.3 Construction CCO92

Construction Process

$A, B$ given initial points

1. $k_1 = k(B, A)$
2. $k_2 = k(A, B)$
   $C \in k_1 \cap k_2$, $D \in k_1 \cap k_2$
3. $k_3 = k(C, D)$
   $E \in k_1 \cap k_3$, $F \in k_2 \cap k_3$
4. $k_4 = k(E, A)$
   $G \in k_2 \cap k_4$, $H \in k_2 \cap k_4$
5. $k_5 = k(H, D)$
   $I \in k_1 \cap k_5$, $J \in k_3 \cap k_5$

Distances

$d_0 = |EJ|$ (red) ... 1
$d_1 = |FJ|$ (blue) ... 1
$d_2 = |BG| = |BH|$ (green) ... 2
$d_3 = |AI|$ (cyan) ... 1
$d_4 = |DJ|$ (magenta) ... 1

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_1} = \frac{d_2}{d_3} = \frac{d_3}{d_4}$ (4 occurrences)

Identical Constructions

RCC422 MM229
2.126.4 Construction CCO110

Construction Process

A, B given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2, D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, D) \)
   \( E \in k_1 \cap k_3, F \in k_2 \cap k_3 \)
4. \( k_4 = k(F, B) \)
   \( G \in k_1 \cap k_4, H \in k_1 \cap k_4 \)
5. \( k_5 = k(H, D) \)
   \( I \in k_2 \cap k_5, J \in k_3 \cap k_5 \)

Distances

\( d_0 = |FJ| \) (red) \ldots 1
\( d_1 = |EJ| \) (blue) \ldots 1
\( d_2 = |AG| = |AH| \) (green) \ldots 2
\( d_3 = |BI| \) (cyan) \ldots 1
\( d_4 = |DJ| \) (magenta) \ldots 1

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} = \frac{d_2}{d_3} = \frac{d_3}{d_4} \) (4 occurrences)

Identical Constructions

RCC452 MM183
2.127  Class of Similar Constructions No. 116

Contained Constructions

CCO49, CCO67, CCO93, CCO111
2.127.1 Construction CCO49

Construction Process

A, B given initial points
1. $k_1 = k(B, A)$
2. $k_2 = k(A, B)$
   $C \in k_1 \cap k_2$, $D \in k_1 \cap k_2$
3. $k_3 = k(D, C)$
   $E \in k_1 \cap k_3$, $F \in k_2 \cap k_3$
4. $k_4 = k(E, A)$
   $G \in k_2 \cap k_4$, $H \in k_2 \cap k_4$, $I \in k_3 \cap k_4$, $J \in k_3 \cap k_4$
5. $k_5 = k(C, G)$
   $K \in k_1 \cap k_5$, $L \in k_1 \cap k_5$, $M \in k_2 \cap k_5$, $N \in k_3 \cap k_5$, $O \in k_3 \cap k_5$, $P \in k_4 \cap k_5$

Distances

$d_0 = |CD| = |CE| = |CF| = |DE| = |DF| = |DI| = |DJ| = |DN| = |DO| = |HM|$ (red) ... 10
$d_1 = |AL| = |BG| = |BH|$ (blue) ... 3
$d_2 = |AP|$ (green) ... 1
$d_3 = |AK| = |BM|$ (cyan) ... 2
$d_4 = |GM| = |KL|$ (magenta) ... 2
$d_5 = |LP|$ (yellow) ... 1
$d_6 = |KP|$ (grass) ... 1

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_4}{d_6} = \frac{d_5}{d_4}$ (20 occurrences)

Identical Constructions

RCC354 MM335
2.127.2 Construction CCO67

Construction Process

A, B given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, C) \)
   \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(F, B) \)
   \( G \in k_1 \cap k_4 \), \( H \in k_1 \cap k_4 \), \( I \in k_3 \cap k_4 \), \( J \in k_3 \cap k_4 \)
5. \( k_5 = k(C, G) \)
   \( K \in k_1 \cap k_5 \), \( L \in k_2 \cap k_5 \), \( M \in k_2 \cap k_5 \), \( N \in k_3 \cap k_5 \), \( O \in k_3 \cap k_5 \), \( P \in k_4 \cap k_5 \)

Distances

\[ d_0 = |CD| = |CE| = |CF| = |DE| = |DF| = |DI| = |DJ| = |DN| = |DO| = |HK| \] (red) ... 10
\[ d_1 = |AG| = |AH| = |BL| \] (blue) ... 3
\[ d_2 = |BP| \] (green) ... 1
\[ d_3 = |AK| = |BM| \] (cyan) ... 2
\[ d_4 = |GK| = |LM| \] (magenta) ... 2
\[ d_5 = |LP| \] (yellow) ... 1
\[ d_6 = |MP| \] (grass) ... 1

Occurrences of the Golden Ratio

\[ \phi = \frac{d_5}{d_2} = \frac{d_4}{d_3} = \frac{d_6}{d_5} = \frac{d_5}{d_6} \] (20 occurrences)

Identical Constructions

RCC384 MM289
2.127.3 Construction CCO93

Construction Process

\( A, B \) given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, D) \)
   \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(E, A) \)
   \( G \in k_2 \cap k_4 \), \( H \in k_2 \cap k_4 \), \( I \in k_3 \cap k_4 \), \( J \in k_3 \cap k_4 \)
5. \( k_5 = k(D, H) \)
   \( K \in k_1 \cap k_5 \), \( L \in k_1 \cap k_5 \), \( M \in k_2 \cap k_5 \), \( N \in k_3 \cap k_5 \), \( O \in k_3 \cap k_5 \), \( P \in k_4 \cap k_5 \)

Distances

\( d_0 = |CD| = |CE| = |CF| = |CI| = |CN| = |CO| = |DE| = |DF| = |GM| \) (red) \( \ldots 10 \)
\( d_1 = |AL| = |BG| = |BH| \) (blue) \( \ldots 3 \)
\( d_2 = |AP| \) (green) \( \ldots 1 \)
\( d_3 = |AK| = |BM| \) (cyan) \( \ldots 2 \)
\( d_4 = |HM| = |KL| \) (magenta) \( \ldots 2 \)
\( d_5 = |LP| \) (yellow) \( \ldots 1 \)
\( d_6 = |KP| \) (grass) \( \ldots 1 \)

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_4}{d_5} = \frac{d_5}{d_6} \) (20 occurrences)

Identical Constructions

RCC423 MM225
2.127.4 Construction CCO111

Construction Process

A, B given initial points
1. $k_1 = k(B, A)$
2. $k_2 = k(A, B)$
   $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
3. $k_3 = k(C, D)$
   $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
4. $k_4 = k(F, B)$
   $G \in k_1 \cap k_4, H \in k_1 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
5. $k_5 = k(D, H)$
   $K \in k_1 \cap k_5, L \in k_2 \cap k_5, M \in k_2 \cap k_5, N \in k_3 \cap k_5, O \in k_3 \cap k_5, P \in k_4 \cap k_5$

Distances

$d_0 = |CD| = |CE| = |CF| = |CI| = |CJ| = |CN| = |CO| = |DE| = |DF| = |GK| (red) \ldots 10$
$d_1 = |AG| = |AH| = |BL| (blue) \ldots 3$
$d_2 = |BP| (green) \ldots 1$
$d_3 = |AK| = |BM| (cyan) \ldots 2$
$d_4 = |HK| = |LM| (magenta) \ldots 2$
$d_5 = |LP| (yellow) \ldots 1$
$d_6 = |MP| (grass) \ldots 1$

Occurrences of the Golden Ratio

$\phi = \frac{d_5}{d_2} = \frac{d_4}{d_2} = \frac{d_4}{d_2} = \frac{d_4}{d_2}$ (20 occurrences)

Identical Constructions

RCC453 MM179
2.128 Class of Similar Constructions No. 117

Contained Constructions

CCO50, CCO68, CCO90, CCO108
2.128.1 Construction CCO50

Construction Process

\[ A, B \text{ given initial points} \]
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, C) \)
   \( E \in k_1 \cap k_3 \)
4. \( k_4 = k(E, A) \)
   \( F \in k_2 \cap k_4 \), \( G \in k_2 \cap k_4 \)
5. \( k_5 = k(F, D) \)
   \( H \in k_1 \cap k_5 \)

Distances

\[ d_0 = |AH| \text{ (red)} \ldots 1 \]
\[ d_1 = |BF| = |BG| \text{ (blue)} \ldots 2 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} \text{ (2 occurrences)} \]

Identical Constructions

RCC356 MM347
2.128.2 Construction CCO68

Construction Process

A, B given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2, \; D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, C) \)
   \( E \in k_2 \cap k_3 \)
4. \( k_4 = k(E, B) \)
   \( F \in k_1 \cap k_4, \; G \in k_1 \cap k_4 \)
5. \( k_5 = k(F, D) \)
   \( H \in k_2 \cap k_5 \)

Distances

\[ d_0 = |BH| \; \text{(red)} \ldots 1 \]
\[ d_1 = |AF| = |AG| \; \text{(blue)} \ldots 2 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} \; \text{(2 occurrences)} \]

Identical Constructions

RCC386 MM301
2.128.3 Construction CCO90

Construction Process

A, B given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, D) \)
   \( E \in k_1 \cap k_3 \)
4. \( k_4 = k(E, A) \)
   \( F \in k_2 \cap k_4 \), \( G \in k_2 \cap k_4 \)
5. \( k_5 = k(G, C) \)
   \( H \in k_1 \cap k_5 \)

Distances

\( d_0 = |AH| \) (red) \ldots 1
\( d_1 = |BF| = |BG| \) (blue) \ldots 2

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} \) (2 occurrences)

Identical Constructions

RCC419 MM237
2.128.4 Construction CCO108

Construction Process

A, B given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, D) \)
   \( E \in k_2 \cap k_3 \)
4. \( k_4 = k(E, B) \)
   \( F \in k_1 \cap k_4 \), \( G \in k_1 \cap k_4 \)
5. \( k_5 = k(G, C) \)
   \( H \in k_2 \cap k_5 \)

Distances

\( d_0 = |BH| \) (red) \ldots 1
\( d_1 = |AF| = |AG| \) (blue) \ldots 2

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} \) (2 occurrences)

Identical Constructions

RCC449 MM191
2.129 Contained Constructions

CCO51, CCO69, CCO91, CCO109
2.129.1 Construction CCO51

Construction Process

A, B given initial points
1. \(k_1 = k(B, A)\)
2. \(k_2 = k(A, B)\)
   \(C \in k_1 \cap k_2\), \(D \in k_1 \cap k_2\)
3. \(k_3 = k(D, C)\)
   \(E \in k_1 \cap k_3\), \(F \in k_2 \cap k_3\)
4. \(k_4 = k(E, A)\)
   \(G \in k_2 \cap k_4\), \(H \in k_2 \cap k_4\), \(I \in k_3 \cap k_4\), \(J \in k_3 \cap k_4\)
5. \(k_5 = k(D, G)\)
   \(K \in k_1 \cap k_5\), \(L \in k_1 \cap k_5\), \(M \in k_2 \cap k_5\), \(N \in k_4 \cap k_5\)

Distances

\(d_0 = |KN|\) (red) \(\ldots 1\)
\(d_1 = |AN|\) (blue) \(\ldots 1\)
\(d_2 = |LN|\) (green) \(\ldots 1\)
\(d_3 = |AL| = |BM|\) (cyan) \(\ldots 2\)
\(d_4 = |CD| = |CE| = |CF| = |DE| = |DF| = |DI| = |DJ| = |HM|\) (magenta) \(\ldots 3\)
\(d_5 = |GM| = |KL|\) (yellow) \(\ldots 2\)
\(d_6 = |AK| = |BG| = |BH|\) (grass) \(\ldots 3\)

Occurrences of the Golden Ratio

\[\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} = \frac{d_2}{d_3} = \frac{d_3}{d_4} = \frac{d_4}{d_5} = \frac{d_5}{d_6}\] (18 occurrences)

Identical Constructions

RCC357 MM344
2.129.2 Construction CCO69

Construction Process

\(A, B\) given initial points
1. \(k_1 = k(B, A)\)
2. \(k_2 = k(A, B)\)
   \(C \in k_1 \cap k_2\), \(D \in k_1 \cap k_2\)
3. \(k_3 = k(D, C)\)
   \(E \in k_1 \cap k_3\), \(F \in k_2 \cap k_3\)
4. \(k_4 = k(F, B)\)
   \(G \in k_1 \cap k_4\), \(H \in k_1 \cap k_4\), \(I \in k_3 \cap k_4\), \(J \in k_3 \cap k_4\)
5. \(k_5 = k(D, G)\)
   \(K \in k_1 \cap k_5\), \(L \in k_2 \cap k_5\), \(M \in k_2 \cap k_5\), \(N \in k_4 \cap k_5\)

Distances

\(d_0 = |MN| \) (red) \ldots 1
\(d_1 = |BN| \) (blue) \ldots 1
\(d_2 = |LN| \) (green) \ldots 1
\(d_3 = |AK| = |BL| \) (cyan) \ldots 2
\(d_4 = |CD| = |CE| = |CF| = |DE| = |DF| = |DI| = |DJ| = |HK| \) (magenta) \ldots 8
\(d_5 = |GK| = |LM| \) (yellow) \ldots 2
\(d_6 = |AG| = |AH| = |BM| \) (grass) \ldots 3

Occurrences of the Golden Ratio

\(\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} = \frac{d_2}{d_3} = \frac{d_3}{d_4} = \frac{d_4}{d_5} = \frac{d_5}{d_6} \) (18 occurrences)

Identical Constructions

RCC387 MM298
2.129.3 Construction CCO91

Construction Process

A, B given initial points
1. $k_1 = k(B,A)$
2. $k_2 = k(A,B)$
   $C \in k_1 \cap k_2, \ D \in k_1 \cap k_2$
3. $k_3 = k(C,D)$
   $E \in k_1 \cap k_3, \ F \in k_2 \cap k_3$
4. $k_4 = k(E,A)$
   $G \in k_2 \cap k_4, \ H \in k_2 \cap k_4, \ I \in k_3 \cap k_4, \ J \in k_3 \cap k_4$
5. $k_5 = k(C,H)$
   $K \in k_1 \cap k_5, \ L \in k_1 \cap k_5, \ M \in k_2 \cap k_5, \ N \in k_4 \cap k_5$

Distances

\[ d_0 = |KN| \text{ (red)} \ldots 1 \]
\[ d_1 = |AN| \text{ (blue)} \ldots 1 \]
\[ d_2 = |LN| \text{ (green)} \ldots 1 \]
\[ d_3 = |AL| = |BM| \text{ (cyan)} \ldots 2 \]
\[ d_4 = |CD| = |CE| = |CF| = |CI| = |DE| = |DF| = |GM| \text{ (magenta)} \ldots 8 \]
\[ d_5 = |HM| = |KL| \text{ (yellow)} \ldots 2 \]
\[ d_6 = |AK| = |BG| = |BH| \text{ (grass)} \ldots 3 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_3} = \frac{d_1}{d_4} = \frac{d_2}{d_5} = \frac{d_5}{d_6} \text{ (18 occurrences)} \]

Identical Constructions

RCC420 MM232
2.129.4 Construction CCO109

Construction Process

A, B given initial points
1. \( k_1 = k(B,A) \)
2. \( k_2 = k(A,B) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(C,D) \)
   \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(F,B) \)
   \( G \in k_1 \cap k_4 \), \( H \in k_1 \cap k_4 \)
   \( I \in k_3 \cap k_4 \), \( J \in k_3 \cap k_4 \)
5. \( k_5 = k(C,H) \)
   \( K \in k_1 \cap k_5 \), \( L \in k_2 \cap k_5 \)
   \( M \in k_2 \cap k_5 \), \( N \in k_4 \cap k_5 \)

Distances

\[
\begin{align*}
d_0 &= |MN| \text{ (red)} \ldots 1 \\
d_1 &= |BN| \text{ (blue)} \ldots 1 \\
d_2 &= |LN| \text{ (green)} \ldots 1 \\
d_3 &= |AK| = |BL| \text{ (cyan)} \ldots 2 \\
d_4 &= |CD| = |CE| = |CF| = |CI| = |CJ| = |DE| = |DF| = |GK| \text{ (magenta)} \ldots 8 \\
d_5 &= |HK| = |LM| \text{ (yellow)} \ldots 2 \\
d_6 &= |AG| = |AH| = |BM| \text{ (grass)} \ldots 3
\end{align*}
\]

Occurrences of the Golden Ratio

\[
\phi = \frac{d_0}{d_3} = \frac{d_1}{d_4} = \frac{d_2}{d_5} = \frac{d_4}{d_6} \quad (18 \text{ occurrences})
\]

Identical Constructions

RCC450 MM186
2.130 Class of Similar Constructions No. 119

Contained Constructions

CC052, CCO71, CCO94, CCO113
2.130.1 Construction CCO52

Construction Process

- A, B given initial points
- \( k_1 = k(B, A) \)
- \( k_2 = k(A, B) \)
- \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
- \( k_3 = k(D, C) \)
- \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
- \( k_4 = k(E, A) \)
- \( G \in k_2 \cap k_4 \), \( H \in k_2 \cap k_4 \), \( I \in k_3 \cap k_4 \), \( J \in k_3 \cap k_4 \)
- \( k_5 = k(G, E) \)
- \( K \in k_1 \cap k_5 \), \( L \in k_2 \cap k_5 \), \( M \in k_3 \cap k_5 \), \( N \in k_4 \cap k_5 \), \( O \in k_4 \cap k_5 \)

Distances

- \( d_0 = |EM| = |KO| \) (red) \( \ldots 2 \)
- \( d_1 = |AO| \) (blue) \( \ldots 1 \)
- \( d_2 = |CL| \) (green) \( \ldots 1 \)
- \( d_3 = |CD| = |CE| = |CF| = |DE| = |DF| = |DI| = |DJ| = |DM| \) (cyan) \( \ldots 8 \)
- \( d_4 = |AK| = |BG| = |BH| = |FL| \) (magenta) \( \ldots 4 \)
- \( d_5 = |AN| = |LM| \) (yellow) \( \ldots 2 \)
- \( d_6 = |DL| \) (grass) \( \ldots 1 \)
- \( d_7 = |FM| = |KN| \) (sea) \( \ldots 2 \)

Occurrences of the Golden Ratio

- \( \phi = \frac{d_5}{d_4} = \frac{d_5}{d_3} = \frac{d_5}{d_2} = \frac{d_5}{d_1} = \frac{d_5}{d_0} \) (36 occurrences)

Identical Constructions

RCC358 MM325
2.130.2 Construction CCO71

Construction Process

A, B given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2, \ D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, C) \)
   \( E \in k_1 \cap k_3, \ F \in k_2 \cap k_3 \)
4. \( k_4 = k(F, B) \)
   \( G \in k_1 \cap k_4, \ H \in k_1 \cap k_4, \ I \in k_3 \cap k_4, \ J \in k_3 \cap k_4 \)
5. \( k_5 = k(G, F) \)
   \( K \in k_1 \cap k_5, \ L \in k_2 \cap k_5, \ M \in k_3 \cap k_5, \ N \in k_4 \cap k_5, \ O \in k_4 \cap k_5 \)

Distances

\( d_0 = \vert FM \vert = \vert LN \vert \) (red) \( \ldots 2 \)
\( d_1 = \vert BN \vert \) (blue) \( \ldots 1 \)
\( d_2 = \vert CK \vert \) (green) \( \ldots 1 \)
\( d_3 = \vert CD \vert = \vert CE \vert = \vert CF \vert = \vert DF \vert = \vert DI \vert = \vert DJ \vert = \vert DM \vert \) (cyan) \( \ldots 8 \)
\( d_4 = \vert AG \vert = \vert AH \vert = \vert BL \vert = \vert EK \vert \) (magenta) \( \ldots 4 \)
\( d_5 = \vert BO \vert = \vert KM \vert \) (yellow) \( \ldots 2 \)
\( d_6 = \vert DK \vert \) (grass) \( \ldots 1 \)
\( d_7 = \vert EM \vert = \vert LO \vert \) (sea) \( \ldots 2 \)

Occurrences of the Golden Ratio

\( \phi = \frac{d_4}{d_5} = \frac{d_1}{d_2} = \frac{d_2}{d_3} = \frac{d_4}{d_5} = \frac{d_6}{d_7} \) (36 occurrences)

Identical Constructions
RCC390 MM279
### 2.130.3 Construction CCO94

**Construction Process**

\[ A, B \text{ given initial points} \]

1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
3. \( k_3 = k(C, D) \)
4. \( k_4 = k(E, A) \)
5. \( k_5 = k(H, E) \)

Distances

\[
\begin{align*}
\phi &= \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} = \frac{d_4}{d_6} = \frac{d_6}{d_7} \quad \text{(36 occurrences)}
\end{align*}
\]

**Occurrences of the Golden Ratio**

\[ RCC424 \text{ MM217} \]

Gergelitsova, Holan: The Golden Ratio . . . using a Ruler and Compass 808
2.130.4 Construction CCO113

Construction Process

A, B given initial points

1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, D) \)
   \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(F, B) \)
   \( G \in k_1 \cap k_4 \), \( H \in k_1 \cap k_4 \), \( I \in k_3 \cap k_4 \), \( J \in k_3 \cap k_4 \)
5. \( k_5 = k(H, F) \)
   \( K \in k_1 \cap k_5 \), \( L \in k_2 \cap k_5 \), \( M \in k_3 \cap k_5 \), \( N \in k_4 \cap k_5 \), \( O \in k_4 \cap k_5 \)

Distances

\( d_0 = |FM| = |LN| \) (red) \ldots 2
\( d_1 = |BN| \) (blue) \ldots 1
\( d_2 = |DK| \) (green) \ldots 1
\( d_3 = |CD| = |CE| = |CF| = |CI| = |CM| = |DE| = |DF| \) (cyan)
\ldots 8
\( d_4 = |AG| = |AH| = |BL| = |EK| \) (magenta) \ldots 4
\( d_5 = |BO| = |KM| \) (yellow) \ldots 2
\( d_6 = |CK| \) (grass) \ldots 1
\( d_7 = |EM| = |LO| \) (sea) \ldots 2

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} = \frac{d_4}{d_6} = \frac{d_6}{d_7} \) (36 occurrences)

Identical Constructions

RCC456 MM171
2.131 Class of Similar Constructions No. 120

Contained Constructions
CCO53, CCO70, CCO95, CCO112
2.131.1 Construction CCO53

Construction Process

1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, C) \)
   
   \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(E, A) \)
   
   \( G \in k_2 \cap k_4 \), \( H \in k_2 \cap k_4 \)
5. \( k_5 = k(G, F) \)
   
   \( I \in k_2 \cap k_5 \), \( J \in k_3 \cap k_5 \)

Distances

\[ d_0 = |FI| = |GH| \text{ (red)} \ldots 2 \]
\[ d_1 = |IJ| \text{ (blue)} \ldots 1 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} \text{ (2 occurrences)} \]

Identical Constructions

RCC360 MM353

Gergelitsová, Holan: The Golden Ratio... using a Ruler and Compass 811
Construction Process

1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, C) \)
   \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(F, B) \)
   \( G \in k_1 \cap k_4 \), \( H \in k_1 \cap k_4 \)
5. \( k_5 = k(G, E) \)
   \( I \in k_1 \cap k_5 \), \( J \in k_3 \cap k_5 \)

Distances

\[ d_0 = |EI| = |GH| \text{ (red)} \ldots 2 \]
\[ d_1 = |IJ| \text{ (blue)} \ldots 1 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} \text{ (2 occurrences)} \]

Identical Constructions

RCC389 MM307
2.131.3 Construction CCO95

Construction Process

1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
3. \( k_3 = k(C, D) \)
4. \( k_4 = k(E, A) \)
5. \( k_5 = k(H, F) \)

Distances

\[ d_0 = |FI| = |GH| \text{ (red)} \ldots 2 \]
\[ d_1 = |IJ| \text{ (blue)} \ldots 1 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} \text{ (2 occurrences)} \]

Identical Constructions

RCC426 MM243
2.131.4 Construction CCO112

Construction Process

A, B given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2 , D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, D) \)
   \( E \in k_1 \cap k_3 , F \in k_2 \cap k_3 \)
4. \( k_4 = k(F, B) \)
   \( G \in k_1 \cap k_4 , H \in k_1 \cap k_4 \)
5. \( k_5 = k(H, E) \)
   \( I \in k_1 \cap k_5 , J \in k_3 \cap k_5 \)

Distances

\( d_0 = |EI| = |GH| \) (red) ... 2
\( d_1 = |IJ| \) (blue) ... 1

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} \) (2 occurrences)

Identical Constructions

RCC455 MM197
2.132 Class of Similar Constructions No. 121

Contained Constructions
CCO54, CCO72, CCO82, CCO100
2.132.1 Construction CCO54

Construction Process

A, B given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, C) \)
   \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(E, A) \)
   \( G \in k_2 \cap k_4 \), \( H \in k_2 \cap k_4 \), \( I \in k_3 \cap k_4 \), \( J \in k_3 \cap k_4 \)
5. \( k_5 = k(H, A) \)
   \( K \in k_1 \cap k_5 \), \( L \in k_2 \cap k_5 \), \( M \in k_2 \cap k_5 \)

Distances

\( d_0 = |EL| \) (red) \( \ldots 1 \)
\( d_1 = |CK| = |CL| = |FM| = |KL| \) (blue) \( \ldots 4 \)
\( d_2 = |CD| = |CE| = |CF| = |DE| = |DF| = |DI| = |DJ| = |LM| \) (green) \( \ldots 8 \)
\( d_3 = |BG| = |BH| = |CM| = |DL| = |EK| \) (cyan) \( \ldots 5 \)
\( d_4 = |EM| \) (magenta) \( \ldots 1 \)
\( d_5 = |DK| = |DM| = |FL| = |KM| \) (yellow) \( \ldots 4 \)

Occurrences of the Golden Ratio

\( \phi = \frac{d_5}{d_2} = \frac{d_4}{d_3} = \frac{d_3}{d_4} = \frac{d_1}{d_2} \) (56 occurrences)

Identical Constructions

RCC362 MM326
2.132.2 Construction CCO72

Construction Process

A, B given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, C) \)
   \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(F, B) \)
   \( G \in k_1 \cap k_4 \), \( H \in k_1 \cap k_4 \), \( I \in k_3 \cap k_4 \), \( J \in k_3 \cap k_4 \)
5. \( k_5 = k(H, B) \)
   \( K \in k_1 \cap k_5 \), \( L \in k_1 \cap k_5 \), \( M \in k_2 \cap k_5 \)

Distances

\[ d_0 = |FL| \text{ (red)} \ldots 1 \]
\[ d_1 = |CL| = |CM| = |EK| = |LM| \text{ (blue)} \ldots 4 \]
\[ d_2 = |CD| = |CE| = |CF| = |DE| = |DF| = |DI| = |DJ| = |KL| \text{ (green)} \ldots 8 \]
\[ d_3 = |AG| = |AH| = |CK| = |DL| = |FM| \text{ (cyan)} \ldots 5 \]
\[ d_4 = |FK| \text{ (magenta)} \ldots 1 \]
\[ d_5 = |DK| = |DM| = |EL| = |KM| \text{ (yellow)} \ldots 4 \]

Occurrences of the Golden Ratio
\[ \phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (56 occurrences)} \]

Identical Constructions
RCC392 MM280
2.132.3 Construction CCO82

Construction Process

A, B given initial points
1. $k_1 = k(B, A)$
2. $k_2 = k(A, B)$
   $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
3. $k_3 = k(C, D)$
   $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
4. $k_4 = k(E, A)$
   $G \in k_2 \cap k_4, H \in k_2 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
5. $k_5 = k(G, A)$
   $K \in k_1 \cap k_5, L \in k_2 \cap k_5, M \in k_2 \cap k_5$

Distances

\[
\begin{align*}
d_0 &= |EL| \text{ (red)} \ldots 1 \\
d_1 &= |DK| = |DL| = |FM| = |KL| \text{ (blue)} \ldots 4 \\
d_2 &= |CD| = |CE| = |CF| = |CI| = |CJ| = |DE| = |DF| = |LM| \text{ (green)} \ldots 8 \\
d_3 &= |BG| = |BH| = |CL| = |DM| = |EK| \text{ (cyan)} \ldots 5 \\
d_4 &= |EM| \text{ (magenta)} \ldots 1 \\
d_5 &= |CK| = |CM| = |FL| = |KM| \text{ (yellow)} \ldots 4
\end{align*}
\]

Occurrences of the Golden Ratio

\[
\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (56 occurrences)}
\]

Identical Constructions

RCC406 MM212
2.132.4 Construction CCO100

Construction Process

A, B given initial points

1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2, \ D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, D) \)
   \( E \in k_1 \cap k_3, \ F \in k_2 \cap k_3 \)
4. \( k_4 = k(F, B) \)
   \( G \in k_1 \cap k_4, \ H \in k_1 \cap k_4, \ I \in k_3 \cap k_4, \ J \in k_3 \cap k_4 \)
5. \( k_5 = k(G, B) \)
   \( K \in k_1 \cap k_5, \ L \in k_1 \cap k_5, \ M \in k_2 \cap k_5 \)

Distances

- \( d_0 = |FL| \) (red) \( \ldots 1 \)
- \( d_1 = |DL| = |DM| = |EK| = |LM| \) (blue) \( \ldots 4 \)
- \( d_2 = |CD| = |CE| = |CF| = |CI| = |DE| = |DF| = |KL| \) (green) \( \ldots 8 \)
- \( d_3 = |AG| = |AH| = |CL| = |DK| = |FM| \) (cyan) \( \ldots 5 \)
- \( d_4 = |FK| \) (magenta) \( \ldots 1 \)
- \( d_5 = |CK| = |CM| = |EL| = |KM| \) (yellow) \( \ldots 4 \)

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \] (56 occurrences)

Identical Constructions

RCC436 MM166
2.133 Class of Similar Constructions No. 122

Contained Constructions
CCO55, CCO73, CCO85, CCO103
2.133.1 Construction CCO55

Construction Process

\[A, B \text{ given initial points}
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2 \) , \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, C) \)
   \( E \in k_1 \cap k_3 \) , \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(E, A) \)
   \( G \in k_2 \cap k_4 \) , \( H \in k_2 \cap k_4 \)
5. \( k_5 = k(H, C) \)
   \( I \in k_1 \cap k_5 \) , \( J \in k_3 \cap k_5 \)

Distances

\[d_0 = |FJ| \text{ (red)} \ldots 1\]
\[d_1 = |CJ| \text{ (blue)} \ldots 1\]
\[d_2 = |EJ| \text{ (green)} \ldots 1\]
\[d_3 = |AI| \text{ (cyan)} \ldots 1\]
\[d_4 = |BG| = |BH| \text{ (magenta)} \ldots 2\]

Occurrences of the Golden Ratio

\[\phi = \frac{d_0}{d_2} = \frac{d_2}{d_4} = \frac{d_3}{d_4} \text{ (4 occurrences)}\]

Identical Constructions

RCC364 MM348
2.133.2 Construction CCO73

Construction Process

- **A, B** given initial points
- 1. \( k_1 = k(B, A) \)
- 2. \( k_2 = k(A, B) \)
  - \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
- 3. \( k_3 = k(D, C) \)
  - \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
- 4. \( k_4 = k(F, B) \)
  - \( G \in k_1 \cap k_4 \), \( H \in k_1 \cap k_4 \)
- 5. \( k_5 = k(H, C) \)
  - \( I \in k_2 \cap k_5 \), \( J \in k_3 \cap k_5 \)

Distances

- \( d_0 = |EJ| \) (red) \ldots 1
- \( d_1 = |CJ| \) (blue) \ldots 1
- \( d_2 = |FJ| \) (green) \ldots 1
- \( d_3 = |BI| \) (cyan) \ldots 1
- \( d_4 = |AG| = |AH| \) (magenta) \ldots 2

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_2}{d_4} = \frac{d_4}{d_5} \] (4 occurrences)

Identical Constructions

RCC394 MM302
2.133.3 Construction CCO85

Construction Process

A, B given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2, D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, D) \)
   \( E \in k_1 \cap k_3, F \in k_2 \cap k_3 \)
4. \( k_4 = k(E, A) \)
   \( G \in k_2 \cap k_4, H \in k_2 \cap k_4 \)
5. \( k_5 = k(G, D) \)
   \( I \in k_1 \cap k_5, J \in k_3 \cap k_5 \)

Distances
\[
\begin{align*}
  d_0 &= |FJ| \, \text{(red)} \ldots 1 \\
  d_1 &= |DJ| \, \text{(blue)} \ldots 1 \\
  d_2 &= |EJ| \, \text{(green)} \ldots 1 \\
  d_3 &= |AI| \, \text{(cyan)} \ldots 1 \\
  d_4 &= |BG| = |BH| \, \text{(magenta)} \ldots 2
\end{align*}
\]

Occurrences of the Golden Ratio
\[
\phi = \frac{d_0}{d_2} = \frac{d_2}{d_4} = \frac{d_0}{d_4} \, (4 \text{ occurrences})
\]

Identical Constructions
RCC411 MM236
2.133.4 Construction CCO103

Construction Process

A, B given initial points
1. \(k_1 = k(B, A)\)
2. \(k_2 = k(A, B)\)
   \(C \in k_1 \cap k_2\), \(D \in k_1 \cap k_2\)
3. \(k_3 = k(C, D)\)
   \(E \in k_1 \cap k_3\), \(F \in k_2 \cap k_3\)
4. \(k_4 = k(F, B)\)
   \(G \in k_1 \cap k_4\), \(H \in k_1 \cap k_4\)
5. \(k_5 = k(G, D)\)
   \(I \in k_2 \cap k_5\), \(J \in k_3 \cap k_5\)

Distances

\(d_0 = |EJ|\) (red) \ldots 1
\(d_1 = |DJ|\) (blue) \ldots 1
\(d_2 = |FJ|\) (green) \ldots 1
\(d_3 = |BI|\) (cyan) \ldots 1
\(d_4 = |AG| = |AH|\) (magenta) \ldots 2

Occurrences of the Golden Ratio

\(\phi = \frac{d_2}{d_4} = \frac{d_4}{d_5} = \frac{d_5}{d_4}\) (4 occurrences)

Identical Constructions

RCC441 MM190
2.134 Class of Similar Constructions No. 123

Contained Constructions

CCO56, CCO74, CCO86, CCO104
2.134.1 Construction CCO56

Construction Process

A, B given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, C) \)
   
   \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(E, A) \)
   
   \( G \in k_2 \cap k_4 \), \( H \in k_2 \cap k_4 \), \( I \in k_3 \cap k_4 \), \( J \in k_3 \cap k_4 \)
5. \( k_5 = k(C, H) \)
   
   \( K \in k_1 \cap k_5 \), \( L \in k_1 \cap k_5 \), \( M \in k_2 \cap k_5 \), \( N \in k_3 \cap k_5 \), \( O \in k_3 \cap k_5 \), \( P \in k_4 \cap k_5 \)

Distances

\( d_0 = |KP| \) (red) ... 1
\( d_1 = |AP| \) (blue) ... 1
\( d_2 = |LP| \) (green) ... 1
\( d_3 = |AL| = |BM| \) (cyan) ... 2
\( d_4 = |CD| = |CE| = |CF| = |DE| = |DF| = |DI| = |DJ| = |DN| = |DO| = |GM| \) (magenta) ... 10
\( d_5 = |HM| = |KL| \) (yellow) ... 2
\( d_6 = |AK| = |BG| = |BH| \) (grass) ... 3

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} = \frac{d_2}{d_3} = \frac{d_3}{d_4} \) (20 occurrences)

Identical Constructions

RCC365 MM343
2.134.2 Construction CCO74

Construction Process

A, B given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2, \ D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, C) \)
   \( E \in k_1 \cap k_3, \ F \in k_2 \cap k_3 \)
4. \( k_4 = k(F, B) \)
   \( G \in k_1 \cap k_4, \ H \in k_1 \cap k_4, \ I \in k_3 \cap k_4, \ J \in k_3 \cap k_4 \)
5. \( k_5 = k(C, H) \)
   \( K \in k_1 \cap k_5, \ L \in k_2 \cap k_5, \ M \in k_2 \cap k_5, \ N \in k_3 \cap k_5, \ O \in k_3 \cap k_5, \ P \in k_4 \cap k_5 \)

Distances

\( d_0 = |MP| \) (red) \ldots 1
\( d_1 = |BP| \) (blue) \ldots 1
\( d_2 = |LP| \) (green) \ldots 1
\( d_3 = |AK| = |BL| \) (cyan) \ldots 2
\( d_4 = |CD| = |CE| = |CF| = |DE| = |DF| = |DI| = |DJ| = |DN| = |DO| = |GK| \) (magenta) \ldots 10
\( d_5 = |HK| = |LM| \) (yellow) \ldots 2
\( d_6 = |AG| = |AH| = |BM| \) (grass) \ldots 3

Occurrences of the Golden Ratio

\( \phi = \frac{d_4}{d_1} = \frac{d_5}{d_3} = \frac{d_6}{d_5} = \frac{d_1}{d_3} \) (20 occurrences)

Identical Constructions

RCC395 MM297
Construction Process

A, B given initial points

1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, D) \)
   \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(E, A) \)
   \( G \in k_2 \cap k_4 \), \( H \in k_2 \cap k_4 \), \( I \in k_3 \cap k_4 \), \( J \in k_3 \cap k_4 \)
5. \( k_5 = k(D, G) \)
   \( K \in k_1 \cap k_5 \), \( L \in k_1 \cap k_5 \), \( M \in k_2 \cap k_5 \), \( N \in k_3 \cap k_5 \), \( O \in k_3 \cap k_5 \), \( P \in k_4 \cap k_5 \)

Distances

\( d_0 = |KP| \) (red) . . . 1
\( d_1 = |AP| \) (blue) . . . 1
\( d_2 = |LP| \) (green) . . . 1
\( d_3 = |AL| = |BM| \) (cyan) . . . 2
\( d_4 = |CD| = |CE| = |CF| = |CI| = |CJ| = |CN| = |CO| = |DE| = |DF| = |HM| \) (magenta) . . . 10
\( d_5 = |GM| = |KL| \) (yellow) . . . 2
\( d_6 = |AK| = |BG| = |BH| \) (grass) . . . 3

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_3} = \frac{d_1}{d_4} = \frac{d_2}{d_5} = \frac{d_3}{d_6} \) (20 occurrences)

Identical Constructions

RCC412 MM233
2.134.4 Construction CCO104

Construction Process

A, B given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2, \ D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, D) \)
   \( E \in k_1 \cap k_3, \ F \in k_2 \cap k_3 \)
4. \( k_4 = k(F, B) \)
   \( G \in k_1 \cap k_4 , \ H \in k_1 \cap k_4 , \ I \in k_3 \cap k_4 , \ J \in k_3 \cap k_4 \)
5. \( k_5 = k(D, G) \)
   \( K \in k_1 \cap k_5, \ L \in k_2 \cap k_5, \ M \in k_2 \cap k_5, \ N \in k_3 \cap k_5, \ O \in k_3 \cap k_5, \ P \in k_4 \cap k_5 \)

Distances

\[
\begin{align*}
  d_0 &= |MP| \text{ (red)} \ldots 1 \\
  d_1 &= |BP| \text{ (blue)} \ldots 1 \\
  d_2 &= |LP| \text{ (green)} \ldots 1 \\
  d_3 &= |AK| = |BL| \text{ (cyan)} \ldots 2 \\
  d_4 &= |CD| = |CE| = |CF| = |CI| = |CJ| = |CN| = |CO| = |DE| = |DF| = |HK| \text{ (magenta)} \ldots 10 \\
  d_5 &= |GK| = |LM| \text{ (yellow)} \ldots 2 \\
  d_6 &= |AG| = |AH| = |BM| \text{ (grass)} \ldots 3
\end{align*}
\]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_6}{d_5} = \frac{d_5}{d_4} = \frac{d_4}{d_3} = \frac{d_3}{d_2} \] (20 occurrences)

Identical Constructions

RCC442 MM187
2.135 Class of Similar Constructions No. 124

Contained Constructions
CCO57, CCO75, CCO83, CCO101
2.135 Class of Similar Constructions No. 124

2.135.1 Construction CCO57

Construction Process

A, B given initial points
1. $k_1 = k(B, A)$
2. $k_2 = k(A, B)$
   $C \in k_1 \cap k_2$, $D \in k_1 \cap k_2$
3. $k_3 = k(D, C)$
   $E \in k_1 \cap k_3$
4. $k_4 = k(E, A)$
   $F \in k_2 \cap k_4$, $G \in k_2 \cap k_4$
5. $k_5 = k(G, D)$
   $H \in k_1 \cap k_5$

Distances

$d_0 = |BF| = |BG|$ (red) \ldots 2
$d_1 = |AH|$ (blue) \ldots 1

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_1}$ (2 occurrences)

Identical Constructions

RCC367 MM340
2.135.2 Construction CCO75

Construction Process

1. $k_1 = k(B, A)$
2. $k_2 = k(A, B)$
   $C \in k_1 \cap k_2$, $D \in k_1 \cap k_2$
3. $k_3 = k(D, C)$
   $E \in k_2 \cap k_3$
4. $k_4 = k(E, B)$
   $F \in k_1 \cap k_4$, $G \in k_1 \cap k_4$
5. $k_5 = k(G, D)$
   $H \in k_2 \cap k_5$

Distances

$d_0 = |AF| = |AG|$ (red) \ldots 2
$d_1 = |BH|$ (blue) \ldots 1

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_1}$ (2 occurrences)

Identical Constructions

RCC397 MM294
2.135.3 Construction CCO83

Construction Process

A, B given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, D) \)
   \( E \in k_1 \cap k_3 \)
4. \( k_4 = k(E, A) \)
   \( F \in k_2 \cap k_4 \), \( G \in k_2 \cap k_4 \)
5. \( k_5 = k(F, C) \)
   \( H \in k_1 \cap k_5 \)

Distances

\[ d_0 = |BF| = |BG| \text{ (red)} \ldots 2 \]
\[ d_1 = |AH| \text{ (blue)} \ldots 1 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} \] (2 occurrences)

Identical Constructions

RCC408 MM228
2.135.4 Construction CCO101

Construction Process

1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2, \ D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, D) \)
   \( E \in k_2 \cap k_3 \)
4. \( k_4 = k(E, B) \)
   \( F \in k_1 \cap k_4, \ G \in k_1 \cap k_4 \)
5. \( k_5 = k(F, C) \)
   \( H \in k_2 \cap k_5 \)

Distances

\[ d_0 = |AF| = |AG| \text{ (red)} \ldots 2 \]
\[ d_1 = |BH| \text{ (blue)} \ldots 1 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} \text{ (2 occurrences)} \]

Identical Constructions

RCC438 MM182
2.136 Class of Similar Constructions No. 125

Contained Constructions

CCO58, CCO76, CCO84, CCO102
2.136.1 Construction CCO58

Construction Process

A, B given initial points
1. $k_1 = k(B, A)$
2. $k_2 = k(A, B)$
   \[ C \in k_1 \cap k_2, \quad D \in k_1 \cap k_2 \]
3. $k_3 = k(D, C)$
   \[ E \in k_1 \cap k_3, \quad F \in k_2 \cap k_3 \]
4. $k_4 = k(E, A)$
   \[ G \in k_2 \cap k_4, \quad H \in k_2 \cap k_4, \quad I \in k_3 \cap k_4, \quad J \in k_3 \cap k_4 \]
5. $k_5 = k(D, H)$
   \[ K \in k_1 \cap k_5, \quad L \in k_1 \cap k_5, \quad M \in k_2 \cap k_5, \quad N \in k_4 \cap k_5 \]

Distances

\[ d_0 = |CD| = |CE| = |CF| = |DE| = |DF| = |DI| = |DJ| = |GM| \quad \text{(red)} \]
\[ \ldots 8 \]
\[ d_1 = |AL| = |BG| = |BH| \quad \text{(blue)} \quad \ldots 3 \]
\[ d_2 = |AN| \quad \text{(green)} \quad \ldots 1 \]
\[ d_3 = |AK| = |BM| \quad \text{(cyan)} \quad \ldots 2 \]
\[ d_4 = |HM| = |KL| \quad \text{(magenta)} \quad \ldots 2 \]
\[ d_5 = |LN| \quad \text{(yellow)} \quad \ldots 1 \]
\[ d_6 = |KN| \quad \text{(grass)} \quad \ldots 1 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_4}{d_5} = \frac{d_6}{d_7} \quad (18 \text{ occurrences}) \]

Identical Constructions

RCC368 MM336
2.136.2 Construction CCO76

Construction Process

\(A, B\) given initial points

1. \(k_1 = k(B, A)\)
2. \(k_2 = k(A, B)\)
   \(C \in k_1 \cap k_2, D \in k_1 \cap k_2\)
3. \(k_3 = k(D, C)\)
   \(E \in k_1 \cap k_3, F \in k_2 \cap k_3\)
4. \(k_4 = k(F, B)\)
   \(G \in k_1 \cap k_4, H \in k_1 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4\)
5. \(k_5 = k(D, H)\)
   \(K \in k_1 \cap k_5, L \in k_2 \cap k_5, M \in k_2 \cap k_5, N \in k_4 \cap k_5\)

Distances

\(d_0 = |CD| = |CE| = |CF| = |DE| = |DF| = |DI| = |DJ| = |GK| \) (red)
\( \ldots 8 \)
\(d_1 = |AG| = |AH| = |BL| \) (blue) \( \ldots 3 \)
\(d_2 = |BN| \) (green) \( \ldots 1 \)
\(d_3 = |AK| = |BM| \) (cyan) \( \ldots 2 \)
\(d_4 = |HK| = |LM| \) (magenta) \( \ldots 2 \)
\(d_5 = |LN| \) (yellow) \( \ldots 1 \)
\(d_6 = |MN| \) (grass) \( \ldots 1 \)

Occurrences of the Golden Ratio

\(\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} = \) (18 occurrences)

Identical Constructions

RCC398 MM290
2.136.3 Construction CCO84

Construction Process

\(A, B\) given initial points
1. \(k_1 = k(B,A)\)
2. \(k_2 = k(A,B)\)
   \(C \in k_1 \cap k_2, \ D \in k_1 \cap k_2\)
3. \(k_3 = k(C,D)\)
   \(E \in k_1 \cap k_3, \ F \in k_2 \cap k_3\)
4. \(k_4 = k(E,A)\)
   \(G \in k_2 \cap k_4, \ H \in k_2 \cap k_4, \ I \in k_3 \cap k_4, \ J \in k_3 \cap k_4\)
5. \(k_5 = k(C,G)\)
   \(K \in k_1 \cap k_5, \ L \in k_1 \cap k_5, \ M \in k_2 \cap k_5, \ N \in k_4 \cap k_5\)

Distances

\(d_0 = |CD| = |CE| = |CF| = |CI| = |CJ| = |DE| = |DF| = |HM| \ (\text{red})\)
\(\ldots 8\)
\(d_1 = |AL| = |BG| = |BH| \ (\text{blue}) \ldots 3\)
\(d_2 = |AN| \ (\text{green}) \ldots 1\)
\(d_3 = |AK| = |BM| \ (\text{cyan}) \ldots 2\)
\(d_4 = |GM| = |KL| \ (\text{magenta}) \ldots 2\)
\(d_5 = |LN| \ (\text{yellow}) \ldots 1\)
\(d_6 = |KN| \ (\text{grass}) \ldots 1\)

Occurrences of the Golden Ratio

\(\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_4}{d_5} = \frac{d_6}{d_7} \ (18 \text{ occurrences})\)

Identical Constructions

RCC409 MM224
2.136.4 Construction CCO102

Construction Process

A, B given initial points
1. $k_1 = k(B, A)$
2. $k_2 = k(A, B)$
   $C \in k_1 \cap k_2$, $D \in k_1 \cap k_2$
3. $k_3 = k(C, D)$
   $E \in k_1 \cap k_3$, $F \in k_2 \cap k_3$
4. $k_4 = k(F, B)$
   $G \in k_1 \cap k_4$, $H \in k_1 \cap k_4$, $I \in k_3 \cap k_4$, $J \in k_3 \cap k_4$
5. $k_5 = k(C, G)$
   $K \in k_1 \cap k_5$, $L \in k_2 \cap k_5$, $M \in k_2 \cap k_5$, $N \in k_4 \cap k_5$

Distances

$d_0 = |CD| = |CE| = |CF| = |CI| = |DJ| = |DE| = |DF| = |HK|$ (red)

... 8
$d_1 = |AG| = |AH| = |BL|$ (blue) ... 3
$d_2 = |BN|$ (green) ... 1
$d_3 = |AK| = |BM|$ (cyan) ... 2
$d_4 = |GK| = |LM|$ (magenta) ... 2
$d_5 = |LN|$ (yellow) ... 1
$d_6 = |MN|$ (grass) ... 1

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_4}{d_6}$ (18 occurrences)

Identical Constructions

RCC439 MM178
2.137 Class of Similar Constructions No. 126

Contained Constructions

CCO59, CCO78, CCO87, CCO106
2.137 Class of Similar Constructions No. 126

2.137.1 Construction CCO59

Construction Process

A, B given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2 , \ D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, C) \)
   \( E \in k_1 \cap k_3 , \ F \in k_2 \cap k_3 \)
4. \( k_4 = k(E, A) \)
   \( G \in k_2 \cap k_4 , \ H \in k_2 \cap k_4 , \ I \in k_3 \cap k_4 , \ J \in k_3 \cap k_4 \)
5. \( k_5 = k(H, E) \)
   \( K \in k_1 \cap k_5 , \ L \in k_2 \cap k_5 , \ M \in k_3 \cap k_5 , \ N \in k_4 \cap k_5 , \ O \in k_4 \cap k_5 \)

Distances

\( d_0 = |FM| = |KO| \) (red) \ldots 2
\( d_1 = |AO| = |LM| \) (blue) \ldots 2
\( d_2 = |DL| \) (green) \ldots 1
\( d_3 = |CD| = |CE| = |CF| = |DE| = |DF| = |DI| = |DJ| = |DM| \)
  (cyan) \ldots 8
\( d_4 = |AK| = |BG| = |BH| = |FL| \) (magenta) \ldots 4
\( d_5 = |AN| \) (yellow) \ldots 1
\( d_6 = |CL| \) (grass) \ldots 1
\( d_7 = |EM| = |KN| \) (sea) \ldots 2

Occurrences of the Golden Ratio

\( \phi = \frac{d_6}{d_2} = \frac{d_4}{d_2} = \frac{d_5}{d_2} = \frac{d_3}{d_2} = \frac{d_7}{d_2} = \frac{d_6}{d_2} \) (36 occurrences)

Identical Constructions

RCC369 MM328
2.137.2 Construction CCO78

Construction Process

A, B given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, C) \)
   \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(F, B) \)
   \( G \in k_1 \cap k_4 \), \( H \in k_1 \cap k_4 \), \( I \in k_3 \cap k_4 \), \( J \in k_3 \cap k_4 \)
5. \( k_5 = k(H, F) \)
   \( K \in k_1 \cap k_5 \), \( L \in k_2 \cap k_5 \), \( M \in k_3 \cap k_5 \), \( N \in k_4 \cap k_5 \), \( O \in k_4 \cap k_5 \)

Distances

\[
\begin{align*}
d_0 &= |EM| = |LN| \quad \text{(red)} \ldots 2 \\
d_1 &= |BN| = |KM| \quad \text{(blue)} \ldots 2 \\
d_2 &= |DK| \quad \text{(green)} \ldots 1 \\
d_3 &= |CD| = |CE| = |CF| = |DE| = |DF| = |DI| = |DJ| = |DM| \\
& \quad \text{(cyan)} \ldots 8 \\
d_4 &= |AG| = |AH| = |BL| = |EK| \quad \text{(magenta)} \ldots 4 \\
d_5 &= |BO| \quad \text{(yellow)} \ldots 1 \\
d_6 &= |CK| \quad \text{(grass)} \ldots 1 \\
d_7 &= |FM| = |LO| \quad \text{(sea)} \ldots 2
\end{align*}
\]

Occurrences of the Golden Ratio

\( \phi = \frac{d_6}{d_7} = \frac{d_4}{d_5} = \frac{d_2}{d_3} = \frac{d_1}{d_2} = \frac{d_0}{d_1} \) (36 occurrences)

Identical Constructions

RCC401 MM282
2.137.3 Construction CCO87

Construction Process

A, B given initial points
1. $k_1 = k(B, A)$
2. $k_2 = k(A, B)$
   $C \in k_1 \cap k_2$, $D \in k_1 \cap k_2$
3. $k_3 = k(C, D)$
   $E \in k_1 \cap k_3$, $F \in k_2 \cap k_3$
4. $k_4 = k(E, A)$
   $G \in k_2 \cap k_4$, $H \in k_2 \cap k_4$, $I \in k_3 \cap k_4$, $J \in k_3 \cap k_4$
5. $k_5 = k(G, E)$
   $K \in k_1 \cap k_5$, $L \in k_2 \cap k_5$, $M \in k_3 \cap k_5$, $N \in k_4 \cap k_5$, $O \in k_4 \cap k_5$

Distances

- $d_0 = |FM| = |KO|$ (red) ... 2
- $d_1 = |AO| = |LM|$ (blue) ... 2
- $d_2 = |CL|$ (green) ... 1
- $d_3 = |CD| = |CE| = |CF| = |CI| = |CJ| = |CM| = |DE| = |DF|$ (cyan)
  ... 8
- $d_4 = |AK| = |BG| = |BH| = |FL|$ (magenta) ... 4
- $d_5 = |AN|$ (yellow) ... 1
- $d_6 = |DL|$ (grass) ... 1
- $d_7 = |EM| = |KN|$ (sea) ... 2

Occurrences of the Golden Ratio

- $\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} = \frac{d_4}{d_6} = \frac{d_5}{d_7}$ (36 occurrences)

Identical Constructions

RCC413 MM214
2.137.4 Construction CCO106

Construction Process

A, B given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, D) \)
   \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(F, B) \)
   \( G \in k_1 \cap k_4 \), \( H \in k_3 \cap k_4 \), \( I \in k_3 \cap k_4 \), \( J \in k_3 \cap k_4 \)
5. \( k_5 = k(G, F) \)
   \( K \in k_1 \cap k_5 \), \( L \in k_2 \cap k_5 \), \( M \in k_3 \cap k_5 \), \( N \in k_4 \cap k_5 \), \( O \in k_4 \cap k_5 \)

Distances

\( d_0 = |EM| = |LN| \) (red) \( \ldots 2 \)
\( d_1 = |BN| = |KM| \) (blue) \( \ldots 2 \)
\( d_2 = |CK| \) (green) \( \ldots 1 \)
\( d_3 = |CD| = |CE| = |CF| = |CI| = |CJ| = |CM| = |DE| = |DF| \) (cyan)
   \( \ldots 8 \)
\( d_4 = |AG| = |AH| = |BL| = |EK| \) (magenta) \( \ldots 4 \)
\( d_5 = |BO| \) (yellow) \( \ldots 1 \)
\( d_6 = |DK| \) (grass) \( \ldots 1 \)
\( d_7 = |FM| = |LO| \) (sea) \( \ldots 2 \)

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_3}{d_7} = \frac{d_7}{d_5} = \frac{d_5}{d_6} = \frac{d_6}{d_4} \) (36 occurrences)

Identical Constructions

RCC445 MM168

Gergelitsová, Holan: The Golden Ratio... using a Ruler and Compass 844
2.138 Class of Similar Constructions No. 127

Contained Constructions
CCO60, CCO77, CCO88, CCO105
2.138.1 Construction CCO60

Construction Process

A, B given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, C) \)
   \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(E, A) \)
   \( G \in k_2 \cap k_4 \), \( H \in k_2 \cap k_4 \)
5. \( k_5 = k(H, F) \)
   \( I \in k_2 \cap k_5 \), \( J \in k_3 \cap k_5 \)

Distances

\[ d_0 = |IJ| \text{ (red)} \ldots 1 \]
\[ d_1 = |FI| = |GH| \text{ (blue)} \ldots 2 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} \text{ (2 occurrences)} \]

Identical Constructions

RCC371 MM354
2.138.2 Construction CCO77

Construction Process

\[ \begin{align*}
A, B \text{ given initial points} \\
1. & \quad k_1 = k(B, A) \\
2. & \quad k_2 = k(A, B) \\
   & \quad C \in k_1 \cap k_2, \quad D \in k_1 \cap k_2 \\
3. & \quad k_3 = k(D, C) \\
   & \quad E \in k_1 \cap k_3, \quad F \in k_2 \cap k_3 \\
4. & \quad k_4 = k(F, B) \\
   & \quad G \in k_1 \cap k_4, \quad H \in k_1 \cap k_4 \\
5. & \quad k_5 = k(H, E) \\
   & \quad I \in k_1 \cap k_5, \quad J \in k_3 \cap k_5
\end{align*} \]

Distances

\[ \begin{align*}
d_0 &= |IJ| \quad \text{(red)} \ldots 1 \\
d_1 &= |EI| = |GH| \quad \text{(blue)} \ldots 2
\end{align*} \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_1} \quad \text{(2 occurrences)} \]

Identical Constructions

RCC400 MM308
2.138.3 Construction CCO88

Construction Process

A, B given initial points
1. $k_1 = k(B, A)$
2. $k_2 = k(A, B)$
   $C \in k_1 \cap k_2$, $D \in k_1 \cap k_2$
3. $k_3 = k(C, D)$
   $E \in k_1 \cap k_3$, $F \in k_2 \cap k_3$
4. $k_4 = k(E, A)$
   $G \in k_2 \cap k_4$, $H \in k_2 \cap k_4$
5. $k_5 = k(G, F)$
   $I \in k_2 \cap k_5$, $J \in k_3 \cap k_5$

Distances

$d_0 = |IJ|$ (red) \ldots 1
$d_1 = |FI| = |GH|$ (blue) \ldots 2

Occurrences of the Golden Ratio

$\phi = \frac{d_0}{d_1}$ (2 occurrences)

Identical Constructions

RCC415 MM242
2.138.4 Construction CCO105

Construction Process

1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2 , \ D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, D) \)
   \( E \in k_1 \cap k_3 , \ F \in k_2 \cap k_3 \)
4. \( k_4 = k(F, B) \)
   \( G \in k_1 \cap k_4 , \ H \in k_1 \cap k_4 \)
5. \( k_5 = k(G, E) \)
   \( I \in k_1 \cap k_5 , \ J \in k_3 \cap k_5 \)

Distances

\[
d_0 = |IJ| \, (\text{red}) \ldots 1
\]
\[
d_1 = |EI| = |GH| \, (\text{blue}) \ldots 2
\]

Occurrences of the Golden Ratio

\[
\phi = \frac{d_0}{d_1} \, (2 \text{ occurrences})
\]

Identical Constructions

RCC444 MM196
2.139 Class of Similar Constructions No. 128

Contained Constructions
CCO61, CCO96
2.139.1 Construction CCO61

Construction Process

\[ A, B \text{ given initial points} \]

1. \[ k_1 = k(B, A) \]
2. \[ k_2 = k(A, B) \]
   \[ C \in k_1 \cap k_2, \ D \in k_1 \cap k_2 \]
3. \[ k_3 = k(D, C) \]
   \[ E \in k_1 \cap k_3, \ F \in k_2 \cap k_3 \]
4. \[ k_4 = k(A, E) \]
   \[ G \in k_3 \cap k_4 \]
5. \[ k_5 = k(B, F) \]
   \[ H \in k_3 \cap k_5, \ I \in k_4 \cap k_5, \ J \in k_4 \cap k_5 \]

Distances

\[ d_0 = |GI| = |HI| \text{ (red) } \ldots 2 \]
\[ d_1 = |CJ| = |DI| \text{ (blue) } \ldots 2 \]
\[ d_2 = |EI| = |EJ| = |FI| = |FJ| \text{ (green) } \ldots 4 \]
\[ d_3 = |CD| = |CE| = |CF| = |DE| = |DF| = |DG| = |DH| = |EH| = |FG| \text{ (cyan) } \ldots 9 \]
\[ d_4 = |GJ| = |HJ| \text{ (magenta) } \ldots 2 \]
\[ d_5 = |CI| = |DJ| \text{ (yellow) } \ldots 2 \]

Occurrences of the Golden Ratio

\[ \phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_1}{d_4} = \frac{d_4}{d_5} \text{ (52 occurrences)} \]

Identical Constructions

RCC372 MM260
2.139.2 Construction CCO96

Construction Process

A, B given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2 \) , \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, D) \)
   \( E \in k_1 \cap k_3 \) , \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(A, E) \)
   \( G \in k_3 \cap k_4 \)
5. \( k_5 = k(B, F) \)
   \( H \in k_3 \cap k_5 \) , \( I \in k_4 \cap k_5 \) , \( J \in k_4 \cap k_5 \)

Distances

\[
\begin{align*}
    d_0 &= |GJ| = |HJ| \quad \text{(red) \ldots 2} \\
    d_1 &= |CJ| = |DI| \quad \text{(blue) \ldots 2} \\
    d_2 &= |EI| = |EJ| = |FI| = |FJ| \quad \text{(green) \ldots 4} \\
    d_3 &= |CD| = |CE| = |CF| = |CG| = |CH| = |DE| = |DF| = |EH| = \\
    &\quad |FG| \quad \text{(cyan) \ldots 9} \\
    d_4 &= |GI| = |HI| \quad \text{(magenta) \ldots 2} \\
    d_5 &= |CI| = |DJ| \quad \text{(yellow) \ldots 2}
\end{align*}
\]

Occurrences of the Golden Ratio

\[
\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \quad \text{(52 occurrences)}
\]

Identical Constructions

RCC427 MM149
2.140 Class of Similar Constructions No. 129

Contained Constructions

CCO62, CCO79, CCO97, CCO114
2.140.1 Construction CCO62

Construction Process

- Given initial points A, B
- \( k_1 = k(B, A) \)
- \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
- \( k_3 = k(D, C) \)
- \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
- \( k_4 = k(A, E) \)
- \( G \in k_3 \cap k_4 \)
- \( k_5 = k(G, A) \)
- \( H \in k_2 \cap k_5 \), \( I \in k_2 \cap k_5 \), \( J \in k_3 \cap k_5 \), \( K \in k_3 \cap k_5 \), \( L \in k_4 \cap k_5 \)
- \( M \in k_4 \cap k_5 \)

Distances

- \( d_0 = |HM| = |IL| \) (red) . . . 2
- \( d_1 = |BM| = |CD| = |CE| = |CF| = |DE| = |DF| = |DG| = |DJ| = |DK| = |FG| \) (blue) . . . 10
- \( d_2 = |HL| = |IM| \) (green) . . . 2

Occurrences of the Golden Ratio

- \( \phi = \frac{d_0}{d_1} = \frac{d_2}{d_5} \) (40 occurrences)

Identical Constructions

RCC375 MM264
Construction Process

1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(D, C) \)
   
   \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(B, F) \)
   
   \( G \in k_3 \cap k_4 \)
5. \( k_5 = k(G, B) \)
   
   \( H \in k_1 \cap k_5 \), \( I \in k_1 \cap k_5 \), \( J \in k_3 \cap k_5 \), \( K \in k_3 \cap k_5 \), \( L \in k_4 \cap k_5 \), \( M \in k_4 \cap k_5 \)

Distances

\[
\begin{align*}
d_0 &= |HM| = |IL| \quad \text{(red)} \ldots 2 \\
d_1 &= |AL| = |CD| = |CE| = |CF| = |DE| = |DF| = |DG| = |DJ| = |DK| = |EG| \quad \text{(blue)} \ldots 10 \\
d_2 &= |HL| = |IM| \quad \text{(green)} \ldots 2
\end{align*}
\]

Occurrences of the Golden Ratio

\[
\phi = \frac{d_0}{d_1} = \frac{d_2}{d_1} \quad \text{(40 occurrences)}
\]

Identical Constructions

RCC402 MM268
2.140.3 Construction CCO97

Construction Process

A, B given initial points
1. \( k_1 = k(B, A) \)
2. \( k_2 = k(A, B) \)
   \( C \in k_1 \cap k_2 \), \( D \in k_1 \cap k_2 \)
3. \( k_3 = k(C, D) \)
   \( E \in k_1 \cap k_3 \), \( F \in k_2 \cap k_3 \)
4. \( k_4 = k(A, E) \)
   \( G \in k_3 \cap k_4 \)
5. \( k_5 = k(G, A) \)
   \( H \in k_2 \cap k_5 \), \( I \in k_2 \cap k_5 \), \( J \in k_3 \cap k_5 \), \( K \in k_3 \cap k_5 \), \( L \in k_4 \cap k_5 \), \( M \in k_4 \cap k_5 \)

Distances

\( d_0 = |HM| = |IL| \) (red) \( \ldots 2 \)
\( d_1 = |BM| = |CD| = |CE| = |CF| = |CG| = |CJ| = |CK| = |DE| = |DF| = |FG| \) (blue) \( \ldots 10 \)
\( d_2 = |HL| = |IM| \) (green) \( \ldots 2 \)

Occurrences of the Golden Ratio

\( \phi = \frac{d_0}{d_1} = \frac{d_0}{d_2} \) (40 occurrences)

Identical Constructions

RCC430 MM153
Construction Process

A, B given initial points
1. $k_1 = k(B, A)$
2. $k_2 = k(A, B)$
   \[ C \in k_1 \cap k_2 \text{, } D \in k_1 \cap k_2 \]
3. $k_3 = k(C, D)$
   \[ E \in k_1 \cap k_3 \text{, } F \in k_2 \cap k_3 \]
4. $k_4 = k(B, F)$
   \[ G \in k_3 \cap k_4 \]
5. $k_5 = k(G, B)$
   \[ H \in k_1 \cap k_5 \text{, } I \in k_1 \cap k_5 \text{, } J \in k_3 \cap k_5 \text{, } K \in k_3 \cap k_5 \text{, } L \in k_4 \cap k_5 \text{, } M \in k_4 \cap k_5 \]

Distances
\[
\begin{align*}
d_0 &= |HM| = |IL| \text{ (red)} \ldots 2 \\
d_1 &= |AL| = |CD| = |CE| = |CF| = |CG| = |CJ| = |CK| = |DE| = |DF| = |EG| \text{ (blue)} \ldots 10 \\
d_2 &= |HL| = |IM| \text{ (green)} \ldots 2
\end{align*}
\]

Occurrences of the Golden Ratio
\[
\phi = \frac{d_0}{d_1} = \frac{d_2}{d_5} \text{ (40 occurrences)}
\]

Identical Constructions
RCC457 MM157
3 Proofs

3.1 The Golden Ratio – Really?

Only lists of lengths and golden ratios of these lengths in the constructions are provided in this book. Whether these ratios are really golden ratios is not immediately known. These statements need to be shown. Additionally, we do not support the stated congruence of listed lengths. Many are easy to show, others are not, necessitating this chapter. This chapter addresses proofs.

We are not going to prove all listed statements. This opportunity we leave to the reader. Additionally, many stated golden ratios (constructed using different sequences of steps) appear in multiple constructions and thus must be proven once. However, plenty of statements remain to be proven.

The goal of this chapter is to provide the reader with the main ideas and hints of how to construct the necessary proofs.

All constructions are based on few lines – straight lines and circles. Therefore, discovering some basic constructions and patterns in which the golden ratio can be found is possible. The term pattern indicates a particular couple or triple of segments (or distances of constructed points) in a specific geometric relation (for instance, the chords of one circle or of congruent circles intercepted by central, or inscribed, angles of particular values). We describe the patterns required for our proofs, indicate them by names and prove that the ratios of the lengths characterized with a given relation are the golden ratio. These patterns occur in the majority of the presented constructions. The other ratios must be proven individually by using known geometry and trigonometric identities.

In the following text, we present the patterns mentioned above and the main ideas as to how the desired ratios can be found (calculated).

3.2 The Properties of the Golden Ratio

As we indicated in the preface, the geometric definition of the ”golden section” is that it divides a length into a larger and smaller part for which the length of the larger part forms the identical ratio with the length of the smaller part as the whole length does with the larger part. This rule implies the value of the ratio of the two lengths.

This number is usually denoted \( \phi \) and

\[
\phi = \frac{\sqrt{5} + 1}{2}
\]

which implies

\[
\phi - 1 = \frac{1}{\phi} = \frac{\sqrt{5} - 1}{2}.
\]

We will prove that the listed ratios for several constructions are equal to these numbers.

To avoid misunderstandings and confusion, we will not label any angle \( \varphi \) or \( \phi \) in this chapter.
3.3 Symbols Used

We denote the line segment with endpoints $A$ and $B$ by $AB$, and we denote the length of this segment and the distance of points $A$, $B$ as $|AB|$. Not to consider various variants of the positions of points on the circle we use oriented angles. The value of the oriented angle $\angle ABC$ is the value of the angle by which we have to rotate the ray $BA$ counterclockwise to the position of ray $BC$. We denote the value of angle $\angle ABC$ as $|\angle ABC|$. We consider arcs as oriented, too.

In this text, we denote circle $k$ with the center $S$ passing through point $A$ by $k(S, A)$, circle $m$ with the center $S$ and radius $|PQ|$ by $m(S, |PQ|)$. Because in the proofs we do not need to distinguish the tool the circle was constructed with ("collapsible" compass from the "normal" compass), we will denote circle $k(A, |AB|) = k(A, B)$ by $k(A, B)$.

In the figures that illustrate the proofs we use the identical captions for points and lines as in the original construction figures in the list. Therefore, the similar placed points and lines in the different figures in this text may not have identical captions. When different figures are linked, this fact is stressed in the text. The illustrative figures may contain lines and points which are not a part of the discussed construction, but which are necessary for performing the proof. The initial points $A$ and $B$ of each construction are drawn as empty circles.

3.4 The Basic Length and Constructed Lengths

All constructions are based on segment $AB$ (which is not drawn in the constructions). This segment can be considered the unit length, $|AB| = 1$. We do not require this premise in our proofs; however, if the reader wishes to construct their own proofs, then the unity length may be helpful.

The first steps in constructions must construct any of the circles $k_1(A, B)$ and $k_2(B, A)$ or the straight line $p(A, B)$. These circles, with the radius $r = |AB|$, intersect, and their common chord is the side $KL$ of two equilateral inscribed triangles; $|KL| = \sqrt{3} \cdot |AB|$. The vertices $M_1$ and $M_2$ of these equilateral triangles are then drawn.
3.5 Pattern A – Concentric Circles of Radii \( r \), \( 2r \)

Triangles are the endpoints of the diameters \( BM_1 \) and \( AM_2 \) of circles \( k_1 \) and \( k_2 \), i.e., the segments of the length \( 2 \cdot |AB| \), (or \( 2r \) in the case of circles with radius \( r \)). The points \( M_1 \) and \( M_2 \) are the points of intersection of the circles \( k_1 \) or \( k_2 \) with any of the circles \( k_3(K,L) \) or \( k_4(L,K) \) or line \( p(A,B) \) (see Figure 1).

The triangles \( \triangle KAB \) and \( \triangle LAB \) are equilateral. Any of the circles \( (K,A) \) and \( (L,A) \) (there is the circle \( (K,A) \) drawn in Figure 2) intersect the circles \( k_1 \) and \( k_2 \) (except for the points \( B \) and \( A \)) at the endpoints of its diameter. Thus, we can determine the length \( 2r \) as the length of the side of an equilateral triangle with medians in the sides of the triangle \( ABK \) (in Figure 2 triangle \( P_1P_2L \)).

In case the first two steps in the construction are to draw the circles \( k_1(A,B) \) and \( k_2(B,A) \), we can construct a perpendicular bisector of \( AB \) in the third step. This process does not create a new point (of intersection) in the construction.

After constructing any triple from the circles and straight lines mentioned above, we continue the construction by adding more lines and circles. The new intersection points form new lengths. Because all of the constructions in this book consist of at most five lines (straight lines and circles), recurring patterns arise frequently.

3.5 Pattern A – Common Chord of Concentric Circles of Radii \( r \), \( 2r \)

Well-known golden ratio we can find in following construction: A straight line intersects two concentric circles of radii \( r \), \( 2r \). The distance of the line from the common center of the circles is half the radius of the inner circle. The circles cut segments on the line, lengths of which provide the golden ratio. In Figure 3 we note intersection points \( K, L \) and \( Q \) that are the endpoints of such segments.
Proof. Because (by Pythagorean Theorem)

\[ |KL| = r\sqrt{3}, \quad |SL| = r\frac{\sqrt{3}}{2}, \quad |SQ| = r\frac{\sqrt{15}}{2}, \]

we have

\[ \frac{|KQ|}{|KL|} = \frac{\sqrt{15} + \sqrt{3}}{2\sqrt{3}} = \frac{\sqrt{5} + 1}{2}, \]

and from the definition of the golden section, it follows that

\[ \frac{|KL|}{|QL|} = \frac{2\sqrt{3}}{\sqrt{15} - \sqrt{3}} = \frac{2}{\sqrt{5} - 1} = \frac{\sqrt{5} + 1}{2}. \]

This construction is obtained by starting from two given points, A and B, using the circles mentioned above. After constructing the length 2r, we draw a concentric circle of this radius to any of circles \( k_1 \) or \( k_2 \) and intersect both the concentric circles with line \( KL \) (Figure 4), which is the common chord of circles \( k_1 \) and \( k_2 \).
This pattern can be found in multiple constructions, e.g., in the constructions of classes 16, 18, 39, 50, 58, 59, 71 (e.g., in the constructions RCC27, RCC30, RCC44, MM33, MM125, MM96 and MM150).

Remark 1. This pattern can also be interpreted as a ratio of the lengths of the "longer" diagonals of a kite and a rhombus as can be seen in Figure 5.

For a rhombus $PQRS$, in which $a = |PQ|$ and a kite $PQRT$, in which $|RT| = 2a$, $|PR| = a$ is the common "shorter" diagonal of these quadrilaterals. Then, the
ratio of lengths of the "longer" diagonal of the kite to the "longer" diagonal of the rhombus is the golden ratio.

If we find the rhombus and the kite mentioned above in a particular construction, we can prove an occurrence of the golden ratio in this construction. In construction RCC25 (the class of similar constructions 15), we prove that $\triangle FEG$ and $\triangle FEH$ are equilateral triangles and that $|ED| = 2|EG| = 2|EH|$ (see Figure 6).

**Proof.** Points $H, E$ and $G$ lie on one circle with center $F$ and $GH$ is the perpendicular bisector of segment $EF$; therefore, $\triangle FEG$ and $\triangle FEH$ are equilateral triangles.

Angle $\angle FCA, |\angle FCA| = \alpha$ is a central angle and $\angle FDA$ and $\angle FEA$ are inscribed angles intercepting the arc $FA$ in circle $k_4(C, A)$. Therefore, $|\angle ADE| = |\angle FDA| = |\angle FEA| = \frac{\alpha}{2}$ and right triangles $\triangle AED, \triangle EJD$ (and also $\triangle JAE$ and many others) are similar. Therefore, $|ED| : |EJ| = |AD| : |AE| = 4 : 1$, hence $|ED| = 4|EJ| = 2|EG| = 2|EH|$. Therefore,

$$\frac{|DG|}{|GH|} = \frac{|GH|}{|DH|} = \phi.$$ 

![Figure 6](image-url)

**Figure 6**

*Remark 2.* The identical pattern can be found in numerous clever constructions of the golden ratio. However, these constructions use other tools, not only the straightedge and compass. [8] shows the construction displayed in Figure 7.
We start with two orthogonal straight lines in the drawing ("vertical" and "horizontal") and a strip of paper with the center marked. We attach the strip of paper to one of the lines (in Figure 7, the line is the "vertical" one) so the "bottom" endpoint coincides with the intersection point of the drawn lines. We mark the position of the center of the strip on the drawing (point $S_1$) and the position of the endpoint of the strip on the "horizontal" line – point $A$. We slide the strip. The "upper" endpoint moves along the "vertical" line to position $S_1$; the "bottom" endpoint moves along the "horizontal" line. We mark the position of the center of the strip again (point $S_2$) and the position of the endpoint of the strip on the "horizontal" line – point $M$. We slide the strip again. The "upper" endpoint slides to point $S_2$; the "bottom" slides along the "horizontal" line. We mark the position of the "bottom" endpoint on the "horizontal" line – point $B$. Point $M$ divides the segment $AB$ in the golden section.

**3.6 Pattern B – Chords on One Circle**

(construction using circles of radii $r$, $2r$)
3.6 Pattern B – Chords on One Circle

Draw a circle $k_1(A, B)$ and intersect it with circle $k_3(C, A)$, $B$ being the midpoint of the $AC$. Denote the points of intersection $G$ and $H$. Consider the equilateral triangle $BKL$ inscribed in the circle $k_1$. Then, the ratios of the lengths of segments $HB$, $HK$ and $HL$ (see Figure 8) provide the golden ratio

$$\frac{|HB|}{|HK|} = \frac{|HL|}{|HB|} = \phi.$$

**Proof.** See Figure 9. Denote $|\angle BAH| = \alpha$. Because of

$$|\angle BAH| = \alpha, \; |\angle HAK| = 120^\circ - \alpha, \; |\angle HAL| = 240^\circ - \alpha,$$

it follows that

$$\frac{|HB|}{|HK|} = \frac{\sin \frac{\alpha}{2}}{\sin \frac{120^\circ - \alpha}{2}}, \quad \frac{|HL|}{|HB|} = \frac{\sin \frac{240^\circ - \alpha}{2}}{\sin \frac{\alpha}{2}}.$$

![Figure 9](image_url)

Because of $\cos \alpha = \frac{|AH|}{2|AC|} = \frac{1}{4}$, it follows that $\sin \frac{\alpha}{2} = \sqrt{\frac{3}{8}}$, $\cos \frac{\alpha}{2} = \sqrt{\frac{5}{8}}$, hence

$$\frac{|HB|}{|HK|} = \frac{\sin \frac{\alpha}{2}}{\sin (60^\circ - \frac{\alpha}{2})} = \frac{\sqrt{\frac{3}{8}}}{\frac{\sqrt{3}}{2} \sqrt{\frac{5}{8}} - \frac{1}{2} \sqrt{\frac{3}{8}}} = \frac{1}{\frac{\sqrt{3} - 1}{2}} = \frac{\sqrt{5} + 1}{2},$$

$$\frac{|HL|}{|HB|} = \frac{\sin (120^\circ - \frac{\alpha}{2})}{\sin \frac{\alpha}{2}} = \frac{\sqrt{3} \sqrt{\frac{5}{8}} + \frac{1}{2} \sqrt{\frac{3}{8}}}{\sqrt{\frac{3}{8}}} = \frac{\sqrt{5} + 1}{2}.$$

This pattern is frequent in the constructions, e.g., in constructions of class 3 (e.g., RCC2) and in constructions of class 20 (e.g., RCC64).
3.7 One Important Angle and Patterns A, B

The perpendicular bisector $AB$ intersects circle $k_1$ at points $E, F$. Consider an isosceles triangle $ABQ$ with base $AB$ and legs of length $b = 2|AB|$. Then, $\triangle ABQ$, $\triangle AHC$ in Figure 10 are congruent triangles. Therefore, their angles are congruent and point $H$ lies on segment $AQ$. Therefore, line $AH$ intersects the perpendicular bisector of $AB$ at point $Q$, for which the points $A, B, F, E$ and $Q$ create pattern A. Denote angle $|\angle EAQ|$ in this pattern by $\psi$.

\[ \psi \]

Figure 10

Circle $k_2(B, A)$, intersects circle $k_1(A, B)$ at the identical intersection points $E, F$ as above because $\triangle ABE$ and $\triangle ABF$ are equilateral triangles (see Figure 11).

Angle $\angle EAQ = \angle EAH$ (and its value $\psi = |\angle EAH|$) and angles whose values we are able to derive from value $\psi$, are frequently used in our proofs.

If we describe the geometric relations in pattern B using angle $\psi$, then values of the central angles corresponding to intercepted chords $HK, HB$ and $HL$ are equal to $|\angle HAK| = 60^\circ - \psi$, $|\angle BAH| = 60^\circ + \psi$ and $|\angle HAL| = 180^\circ - \psi$, respectively (see Figure 12). This proposition proven for pattern B can also be formulated as follows:

Chord lengths $d_1, d_2$ and $d_3$ are subtended on a circle by corresponding central angles of values $180^\circ - \psi$, $60^\circ + \psi$ and $60^\circ - \psi$ respectively; these lengths provide ratios

\[ \frac{d_1}{d_2} = \frac{d_2}{d_3} = \frac{\sqrt{5} + 1}{2} = \phi. \]

In the examined constructions, the chords mentioned above are the chords not only in one circle, but frequently in congruent circles.
If we rotate segment $FQ$ in rotation about point $A$ by angle $-\psi$ (see Figure 13), then point $F$ rotates to point $G$, point $Q$ rotates to point $Q'$ on circle $k_3(C,A)$, $|AQ| = |AQ'|$, $\angle BAQ' = 60^\circ$, and point $E$ rotates to point $E'$, for which $|GE'| = |EF|$.  

**Remark.** $\triangle AQ'C$ is an equilateral triangle, and point $E$ is the midpoint of segment $AQ'$.  

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3.7 One Important Angle and Patterns A, B

The ratios obtained in similar ways can be found in many constructions and configurations of circles and straight lines. These constructions are different at first sight. Figure 14 explains the pattern, in which segment $PG$ (divided by point $H$ on circle $k_1$ in the golden section) is rotated in rotation about point $A$ by angle $-60^\circ$ to position $CD$. Point $H$ must rotate to point $K$ on circle $k_1$, which divides segment $CD$ in the identical ratio as point $H$ divides segment $PG$. Therefore,

$$\frac{|PG|}{|HG|} = \frac{|HG|}{|PH|} = \frac{|CD|}{|KD|} = \frac{|KD|}{|CK|} = \phi.$$

This pattern we denote as pattern $A_2$. 

Figure 13
Figure 14
The circles mentioned above and the pairs of points that correspond to pairs $PG$ and $CD$ in Figure 14 can be found in many constructions, but the lines are not part of these constructions – e.g., straight line (segment) $PG$. Therefore, point $H$ which divides the segment $PG$ in the golden section is not constructed. However, some necessary length (in Figure 14 length $|GH|$ or $|HP|$) can be found as the distance of another pair of points in the construction. In Figure 14 we have pairs of points (segments) $KD$ and $EC$ and – denoting the next intersection point of circles $k_1$ and $k_2$ by $F$ and the next intersection point of circles $k_5$ and $k_2$ by $J$ – we have $|EF| = |FC| = |AJ| = |GH| = \sqrt{3}r$.

This implies that segments in the following pattern form the golden ratio.

### 3.8 Pattern C – Chords on a Circle of Radius 2$r$

#### a Doubled Golden Ratio

![Diagram of Figure 15](image_url)

Figure 15 displays construction RCC1 (class 2), which (in various modifications) appears in other constructions, e.g., in construction RCC31 (class 19), in which two patterns can be found (pattern $B$ and $C$) and in construction RCC3 (class 4), in which circle $k_5$ is replaced by straight line $AE$ (and point $J$ is the intersection point of circle $k_4$ with line $AE$).

**Proposition.**

$$\frac{|GJ|}{|BI|} = \frac{|BI|}{|GI|} = \frac{\sqrt{5} + 1}{2}.$$

**Proof.** This statement is a corollary of the reasoning above. This statement follows from the congruence of segments $|BI| = |FE| = |KG|$ (point $K$ is not drawn in the construction; we have added the point for the proof) and $|GI| = |HJ| = |KJ|$. The first congruence results from the symmetry about line $p_3$, and the next congruence results from symmetry about line $AJ$. (A is the center of arc $HG$; thus, $AJ$ is the angle bisector of $\angle HJG$).

Therefore, $|BI| = \frac{1}{2}|JI| = |FE| = |KG| = \sqrt{3}r$, where $r = |AB|$. 

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Points J, K and G in Figure 15 correspond to points P, H and G in the pattern in Figure 14, which we have denoted by A2.

**Golden Ratio Squared (a Doubled Ratio)**

Only two chords are constructed in circle \( k_4 \) in Figure 15 and their lengths provide the golden ratio with a particular third length. The ratio of the lengths between these two chords is equal to the ratio of the length of the entire segment divided by the golden section to the length of its smaller part; thus, this ratio is the square of \( \phi \).

\[
\frac{|GJ|}{|GI|} = \frac{\sqrt{5} + 1}{\sqrt{5} - 1} = \left(\frac{\sqrt{5} + 1}{2}\right)^2 = \phi^2 = \frac{\sqrt{5} + 3}{2}.
\]

**Description of Pattern C Using Central Angles**

**Proposition.**

Chords GI and GJ are intercepted on circle \( k_4 \) in Figure 15 by central angles \( |\angle GCI| = 2\psi \) and \( |\angle JCG| = 120^\circ - 2\psi \), respectively.

![Figure 16](image)

**Proof.** The triangles \( \triangle AJC \) and \( \triangle AIC \) are equilateral; thus \( |\angle IAJ| = 120^\circ \), see Figure 16.

Point F is the midpoint of AI, thus \( |\angle GAI| = |\angle GAF| = \psi \). Angle \( \angle GAI \) is an inscribed angle corresponding to the intercepted chord GI and to the central angle \( \angle GCI \), therefore \( |\angle GCI| = 2\psi \).

(Point F does not lie on segment GC.)
3.9 Pattern D – Chords on Circle of Radius $3r$

Another Doubled Golden Ratio

The lengths of two chords on a circle of the radius $3r$ that are constructed in Figure 17 give a square of the golden ratio

$$\frac{|HE|}{|EG|} = \phi^2 = \frac{\sqrt{5} + 3}{2}.$$

![Figure 17](image)

**Proof.** We illustrate the proof in Figure 18. Denote angles in the isosceles triangles $\triangle HCD$ and $\triangle BDE$ by $\angle HDC = \delta$ and $\angle BDE = \gamma$, respectively. Then,

$$\cos \frac{\delta}{2} = \frac{3}{4}, \quad \sin \frac{\gamma}{2} = \frac{1}{6},$$

from which

$$\cos \frac{\delta}{2} = \sqrt{\frac{7}{8}}, \quad \sin \frac{\delta}{2} = \sqrt{\frac{1}{8}}, \quad \cos \frac{\gamma}{2} = \sqrt{\frac{35}{6}}.$$

Because

$$\frac{|HE|}{|EG|} = \frac{\sin \frac{\delta + \gamma}{2}}{\sin \frac{\delta - \gamma}{2}} = \frac{\sin \frac{\delta}{2} \cos \frac{\gamma}{2} + \cos \frac{\delta}{2} \sin \frac{\gamma}{2}}{\sin \frac{\delta}{2} \cos \frac{\gamma}{2} - \cos \frac{\delta}{2} \sin \frac{\gamma}{2}},$$

we have

$$\frac{|HE|}{|EG|} = \frac{\sqrt{35} + \sqrt{7}}{\sqrt{35} - \sqrt{7}} = \frac{\sqrt{5} + 1}{\sqrt{5} - 1} = \frac{\sqrt{5} + 3}{2}.$$
Missing Length Providing the Golden Ratio

Similar to pattern C, we must find a third length that forms the golden ratio with the lengths of these chords. We will prove that this missing length is a length of segment $EJ$ in Figure 19. Point $J$ is the intersection point of segment $HE$ with circle $k_1(B, A)$. Thus, this pattern is similar to pattern C.

The symmetry about line $AB$ implies congruence $|EG| = |FH|$. Because $\angle HBF$ and $\angle HEF$ are inscribed angles that intercept the identical arc and chord $HF$ on circle $(D, B)$, they are equal. Because the central angle $\angle JBF$ corresponds to the inscribed angle $\angle JEF$, it follows that $|\angle JBF| = 2|\angle JEF| = 2|\angle HEF|$. Thus, straight line $HB$ is the bisector of $\angle JBF$.

Because $B$ is the midpoint of arc $FE$, line $HB$ is the bisector of $\angle FHE$, see Figure 19.

Thus, line $BH$ is the perpendicular bisector of $FJ$. Therefore, $|FH| = |JH|$, and point $J$ divides segment $EH$ in the golden section.
In Figures 20 and 21, we can see other segments of identical length (equal to $|EJ|$). Hints to the proofs are also displayed in these Figures.

In Figure 20, we find construction RCC86 from class 28. Because the right triangles $\triangle ADH$, $\triangle HDM$, $\triangle AHM$ and $\triangle IBM$ are similar, we have

$$\frac{|AI|}{2r} = \sin \frac{180^\circ - \delta}{2} = \sin \left(90^\circ - \frac{\delta}{2}\right) = \cos \frac{\delta}{2}.$$ 

Therefore,

$$\frac{|HE|}{|AI|} = \frac{3 \sin \frac{\delta + \gamma}{2}}{2 \cos \frac{\delta}{2}} = \frac{\sqrt{3} + \sqrt{7}}{2 \sqrt{8}} = \frac{\sqrt{5} + 1}{2} = \phi.$$

Figure 21 displays construction RCC81 from class 23. Denote $|\angle IDC| = \delta$ (the labels for the points follow the labels in the original construction). Then,

$$|\angle DCI| = 180^\circ - 2\delta \quad \text{and} \quad |\angle IEB| = \frac{1}{2} |\angle IDB| = \frac{\delta}{2} \quad \text{(central and inscribed angle)}.$$ 

Because $|\angle DCI| = 180^\circ - 2\delta$, we have

$$|\angle CIJ| = |\angle IJC| = \frac{3\delta}{2},$$

from which $|\angle JCI| = 180^\circ - 3\delta$.

Therefore,

$$\frac{|IF|}{3r} = \frac{\sin \frac{\delta + \gamma}{2}}{\sin (90^\circ - 3\delta)} = \frac{\sin \frac{\delta + \gamma}{2}}{\cos \frac{\delta}{2}},$$

which results in

$$\frac{|IF|}{|IJ|} = \frac{\sqrt{5} + 1}{2}.$$
3.9 Pattern D – Chords on Circle of Radius $3r$

Figure 20

Figure 21
3.10 More Patterns and Approaches – Circles

We must often find angles in the circles (central and inscribed angles) and triangles (isosceles, equilateral and scalene) in our proofs. Figure 22 illustrates the process to find the values of angles, which can be derived from the value $|\angle EAH| = \psi$ (patterns $A, B$) in constructions:

- $\angle EAH$ is a central angle in circle $k_1(A, B)$ corresponding to inscribed angles $\angle EFH$ and $\angle EGH$; therefore, $|\angle EFH| = |\angle EGH| = \frac{\psi}{2}$.

- $\angle ACG$ is a central angle corresponding to any angle $\angle AMG$, where $M$ is any point on arc $GA$ on circle $k_3(C, A)$; thus, $\angle ACG$ is corresponding to inscribed angle $\angle AKG$.

- Because $|\angle GAF| = \psi$, we have $|\angle GAC| = 60^\circ + \psi$ and in isosceles triangle $ACG$ we have $|\angle ACG| = 60^\circ - 2\psi$. Therefore, $|\angle AKG| = 30^\circ - \psi$.

- Because equal chords subtend angles that are equal, we can write $|\angle HGA| = |\angle AHG| = |\angle AKG| = |\angle AMG| = 30^\circ - \psi$.

- $\angle HGA = \angle HJG$, which is an inscribed angle corresponding to central angle $\angle HAJ$; therefore $|\angle HAJ| = 60^\circ - 2\psi$. $|\angle EAJ| = 60^\circ - \psi$ then can be written.

- In the isosceles triangle $HAE$, we find $|\angle AHE| = |\angle HEA| = 90^\circ - \frac{\psi}{2}$ (not drawn in Figure 22).

In a similar manner and using the properties of the angles in the triangles, we derive the values of angles required for performing the proofs.

Figure 22

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I Rotation of Circles

Each pair of congruent circles forms a preimage and an image in infinitely many rotations. Each circle centered in the center of such a rotation (and intersecting the given circles) intersects them in congruent chords that are mapped (the first to the second chord) in this rotation.

Example: Triangle $FHJ$ in Figure 23 is equilateral.

Proof. Circle $k_5(F,H)$ intersects circles $k_1$ and $k_2$. A rotation about center $F$ by $60^\circ$ rotates $k_2$ to $k_1$. The "first" of the intersection points of $k_5$ with $k_1$ (when moving counterclockwise) we denote by $H$; the "first" of the intersection points of $k_5$ with $k_2$ we denote by $J$. Thus, $J$ rotates to $H$ in the identical rotation. Therefore, $|\angle JFH| = 60^\circ$.

Figure 23

Point $E$ "seems to lie" on side $HJ$ of triangle $FJH$ in Figure 23. We will prove this statement.

Proof. Triangle $FCE$ is an equilateral triangle. From the rotation mentioned above, we have $|\angle EFH| = |\angle CFJ|$ and $|\angle EAH| = |\angle CBJ|$.

$|\angle EAH| = \psi$, thus $|\angle FAH| = \psi + 120^\circ$ (central angle in $k_1(A,B)$), thus $|\angle HEF| = 120^\circ - \frac{\psi}{2}$ (complementary inscribed angle in $k_1(A,B)$).

$|\angle CBJ| = \psi$, thus $|\angle FBJ| = \psi + 120^\circ$ (central angle in $k_2(B,A)$), thus $|\angle FEJ| = 60^\circ + \frac{\psi}{2}$ (inscribed angle in $k_2(B,A)$).

Therefore, $|\angle HEF| + |\angle FEJ| = 180^\circ$, and point $E$ lies on segment $HJ$.

The relations explained above imply that circles $e(E,H)$, $c(C,J)$ and $f(F,G)$ in Figure 24 are congruent, rotated about center $B$ by $120^\circ$ and the values of the central angles in circle $e(E,H)$ are $|\angle HEL| = |\angle KEI| = 60^\circ$.

Points $G$ and $H$ do not map to each other in any of these rotations.
3.10 More Patterns and Approaches – Circles

Figure 24

Each of the circles \( e = (E, H) \) and \( f = (F, G) \) intersects circle \( k_4 \) at one more point, see circle \( e(E, H) \) in Figure 25.

Figure 25
Because \( \triangle ACQ \) in Figure 25 is an equilateral triangle, circle \( k_1 \) is mapped to circle \( q(Q,E) \) in the reflection about the straight line \( CE \) that is the common tangent of these circles, in the reflection about point \( E \) and in the rotation about center \( C \) by \(-60^\circ\). This mapping implies the following corollaries:

- Point \( G \) maps to point \( N \) in rotation about center \( C \) by \(-60^\circ\); thus \( |AQ| = |GN| = 2r \), and \( \triangle GCN \) is an equilateral triangle.

- Because the value of the angle between chord \( HE \) and tangent \( t = CE \) to the circle is equal to half the central angle \( \angle EAH \), \( \frac{\angle EAH}{2} = \frac{\psi}{2} \). Thus \( |\angle NEH| = \psi \).

- \( NK \) is the diameter of circle \( e(E,H) \) (central symmetry of \( k_1 \) and \( q \)).

- The values of the central angles in circle \( e \) are \( |\angle NEH| = \psi \), \( |\angle HEL| = 60^\circ \), \( |\angle LEK| = 120^\circ - \psi \), \( |\angle KEI| = 60^\circ \), \( |\angle IEN| = 120^\circ \).

- Circle \( s(G,C) \) of radius \( 2r \) passes through point \( N \).

- Line \( GA \) (not drawn in Figure 25) is the common diameter of circles \( k_1 \) and \( s \) and intersects the circles at the point of tangency.

- Circles \( k_2(B,A) \), \( k_5(F,H) \), \( e(E,H) \) and \( s(G,C) \) meet at point \( I \).

- The intersection point \( M \) of circles \( k_5(F,H) \) and \( s(G,C) \) lies on circle \( k_4 \). This statement results from the rotation of circle \( k_2 \) about center \( F \) by \(-60^\circ\), which is not drawn in Figure 25. \( |\angle MFI| = 120^\circ \).

Figure 26 (without explanation) illustrates the relations in constructions containing line \( AB \), circles \( k_1(A,B) \), \( k_2(B,A) \) and \( k_4(C,A) \) and some additional lines and circles. Circles \( f(C,F) \), \( f_H(F,H) \), \( g_C(G,C) \) and \( g_E(G,E) \) are also constructed.
Figures 25 and 26 do not display all of the straight lines and circles that can be found in any construction. These Figures only illustrate the rules and relations that can assist us in finding angles in circles that we will use to calculate ratios of chords lengths or find the values of angles and lengths of sides in triangles.

II Examples – Chords on One Circle

Example 1.
To prove the ratio of lengths, we must (except for the known patterns) examine the ratios of lengths of chords on congruent circles.
In construction RCC28 from class 17 (see Figure 27), we get

\[
\frac{|DH|}{|CE|} = \frac{|CE|}{|DG|} = \frac{|CG|}{|FG|} = \frac{|FG|}{|CH|} = \phi.
\]

Figure 27

Because of \(|CE| = |DE|\) and \(|EH| = |GD|\), we find that two ratios

\[
\frac{|DH|}{|CE|} = \frac{|CE|}{|DG|} = \phi
\]

satisfy the relations in pattern A.

From the symmetry about line \(p_5\), \(GF\) is a chord of a circle of radius \(|JF| = |AC| = 2|AB|\). From the values of the angles in pattern A, we obtain \(\angle D J G = \psi\) and \(\angle E A H = \psi\); therefore, \(\angle H A C = 60^\circ - \psi\), \(\angle F J G = 60^\circ + \psi\) and \(\angle C A G = 180^\circ - \psi\). Chords \(CH, FG\) and \(CG\), are in pattern B in circles of radius \(2|AB|\). Therefore,

\[
\frac{|CG|}{|FG|} = \frac{|FG|}{|CH|} = \phi.
\]

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Example 2.
In construction MM200 from class 97, we find pattern C (see Figure 28). However, this statement must be proven. Point $M$ is the intersection point of circle $k_3(C, D)$ that we have already examined with circle $k_5(J, |IB|)$ previously unknown.

Figure 28
To prove that the configuration of circles, lines and points satisfies pattern C, we can prove (except for other possible proofs) any of following statements. Others follow from relations in pattern C.

- $C$ is the midpoint of $BM$ ($C$ lies on $BM$)
- $AM$ is perpendicular to $AB$
- $|AM| = |CD|$
- $|\angle MEG| = 2\psi$
- $|\angle MEH| = 120^\circ - 2\psi$.

The first statement can be seen in Figures 28 and 29; $EC$ is the bisector of $\angle IJE$ and $|\angle BCE| = 90^\circ$.

To show another proof, we prove the statement $|\angle MEG| = 2\psi$.
We will examine the central angles for arcs (chords) subtended on circle $k_4(E, B)$. Their values can be seen in Figure 29.
We have already proven 
\[ |\angle CEB| = |\angle BED| = 30^\circ, \quad |\angle GEB| = |\angle BEH| = 60^\circ - 2\psi, \quad \text{hence} \]
\[ |\angle CEG| = |\angle HED| = 2\psi - 30^\circ. \]

Denote \( |\angle IEC| = \omega \). Points \( I \) and \( J \) are intersection points of circles \( k_3(C, E) \) and \( k_4(E, B) \); thus, the congruent triangles \( \triangle ECI \) and \( \triangle ECJ \) are isosceles with bases \( EI, EJ \), \( |EI| = |EB| = 2|AB| \) and \( |EC| = \sqrt{3}|AB| \). Therefore, we can calculate the value of \( \omega \). However, this calculation is not necessary.

We must find \( |\angle MEG| \). Because 
\[ |IB| = |MJ|, \quad \text{thus,} \quad |\angle IEB| = |\angle MEB| \quad (M \text{ is the intersection point of } k_4 \text{ and } k_5). \]

Therefore,
\[ |\angle IEM| = |\angle BEJ|, \quad |\angle MEC| = |\angle CEB| \quad (EC \text{ is the angle bisector of } \angle IEB). \]

Thus, we have
\[ |\angle MEG| = |\angle MEC| + |\angle CEG| = |\angle CEB| + |\angle CEG| = 30^\circ + (2\psi - 30^\circ) = 2\psi. \]

Similarly, we can examine the central angles for chords in circle \( k_4(A, B) \) (or its analogy in any symmetrical construction) in constructions of classes 95–108. These constructions imply each other, as seen in the following example.
Example 3.
In the following proof, we use only reflection about a line and known patterns. Figure 30 displays construction MM211 from class 108. Similar to Example 2, we use the reflection about bisector $EC$ of angle $\angle HEI$.
Point $L$ is the intersection point of circles $k_5(H, |GI|)$ and $k_4(E, B)$. We have

$$\frac{|BL|}{|CD|} = \phi.$$ 

We reflect circles $k_2$ and $k_5$ about line $CE$ (that connects the centers of circles $k_3(C,D)$ and $k_4(E,B)$ meeting at points $I$ and $H$, as seen in Figure 30). $\triangle EBM$ is an equilateral triangle, $|EB| = 2r = 2|AB|$. C is the midpoint of MB, circle $k_5(H, |GI|)$ is reflected to $k'_5(I, G)$. Therefore, $|LB| = |MG|$, and we obtain pattern C.

III Useful Trigonometric Entities and Values
Except for the proof in pattern D in which we used sine and cosine of sum identities, we have not previously used numerous calculations. The distances or lengths of segments in our proofs are usually not required. Instead, we need their ratios. Therefore, we often examine angles in constructions and then use trigonometric identities for these angles. Because we calculate such ratios frequently in the following examples, we calculate several useful trigonometric values in advance. Except for these values, we often use trigonometric functions with double and half angles, the sum and difference formulae and product-to-sum and sum-to-product identities.
Because of \( \cos (60^\circ + \psi) = \sin (30^\circ - \psi) = \frac{1}{4} \)
with trigonometric identities, we have
\[
\begin{align*}
\sin (60^\circ + \psi) &= \cos (30^\circ - \psi) = \frac{\sqrt{15}}{4}, \\
\cos \left(30^\circ + \frac{\psi}{2}\right) &= \sin \left(60^\circ - \frac{\psi}{2}\right) = \frac{\sqrt{5}}{8}, \\
\sin \left(30^\circ + \frac{\psi}{2}\right) &= \cos \left(60^\circ - \frac{\psi}{2}\right) = \frac{\sqrt{3}}{8}, \\
\tan \left(30^\circ + \frac{\psi}{2}\right) &= \frac{\sqrt{3}}{5}.
\end{align*}
\]

The values of the trigonometric functions of \( \psi \) and \( \frac{\psi}{2} \) can be calculated using known values of trigonometric functions of angle \( \nu = 60^\circ + \psi \), which results in \( \psi = \nu - 60^\circ \).
\[
\begin{align*}
\sin \psi &= \sin (\nu - 60^\circ) = \frac{\sqrt{3}}{8} \left(\sqrt{5} - 1\right), \\
\cos \psi &= \cos (\nu - 60^\circ) = \frac{1}{8} \left(3\sqrt{5} + 1\right), \\
\cos \frac{\psi}{2} &= \frac{\sqrt{9 + 3\sqrt{5}}}{4} = \frac{\sqrt{6}}{8} \left(\sqrt{5} + 1\right), \\
\sin \frac{\psi}{2} &= \frac{\sqrt{7 - 3\sqrt{5}}}{4} = \frac{\sqrt{2}}{8} \left(3 - \sqrt{5}\right)
\end{align*}
\]
and finally
\[
\begin{align*}
\tan \frac{\psi}{2} &= \frac{\sqrt{5} - 2}{\sqrt{3}}, \\
\cot \frac{\psi}{2} &= \sqrt{3} \left(\sqrt{5} + 2\right).
\end{align*}
\]

### IV Examples – Chords in Circles of Different Radii

Numerous golden ratios in our constructions are ratios of lengths of chords in circles of different radii. Several have been examined in the denoted patterns. Others remain to be calculated. We will use properties and relations in triangles, notably relations between angles and legs in isosceles triangles.

**Example 4.**

The distances of points in construction RCC9 from class 8 form several golden ratios. A few can be explained using known patterns; to prove others, we must calculate the ratios of lengths of chords in circles of different radii.

In construction RCC9, we obtain
\[
\frac{|IL|}{|EJ|} = \frac{|EJ|}{|AI|} = \frac{|AI|}{|FJ|} = \frac{|FJ|}{|IK|} = \frac{|AL|}{|CE|} = \frac{|CE|}{|AK|} = \phi.
\]

We will first show the known patterns (see Figure 31).

The equality
\[
\frac{|AL|}{|CE|} = \frac{|CE|}{|AK|} = \phi
\]
follows from pattern C in circle \( k_4(C, A) \) because \(|GL| = |GK| = 2r \) and \(|AH| = |AG| = r \) and because \( k_4 \) passes through the center \( A \) of circle \( k_3(A, B) \).
Because of $|AI| = |BH|$, the equality

$$\frac{|EJ|}{|AI|} = \frac{|AI|}{|FJ|} = \phi$$

follows from pattern B in circle $k_3(A, B)$. Segment $GJ$ is the diameter of circle $k_3$; thus, $\angle FAJ = 60^\circ - \psi$. 
3.10 More Patterns and Approaches – Circles

Proofs for the next two equalities require several calculations.

Let us prove \( \frac{|IL|}{|EJ|} = \phi \).

In Figure 32, we have drawn the circle \( c(E, H) \) that intersects circle \( k_3(G, C) \), and (following the rotation of circles and corollaries in Figures 25 and 26) these circles have the common chord \( IL \). Furthermore, \( \angle LEI = 120^\circ \). Therefore,

\[
|IL| = \sqrt{3} |EI| = \sqrt{3} |EH|.
\]

Because of \( \angle EAJ = 180^\circ - \psi \) and \( \angle EAH = 120^\circ + \psi \), we obtain

\[
\frac{|IL|}{|EJ|} = \frac{\sqrt{3} |EI|}{|EJ|} = \frac{\sqrt{3} |EH|}{|EJ|} = \frac{\sqrt{3} \sin \frac{120^\circ + \psi}{2}}{\sin \frac{180^\circ - \psi}{2}} \quad \text{and}
\]

\[
\frac{|IL|}{|EJ|} = \frac{\sqrt{3} \sin (60^\circ + \frac{\psi}{2})}{\sin (90^\circ - \frac{\psi}{2})} = \frac{\sqrt{3} \sin (60^\circ + \frac{\psi}{2})}{\cos \frac{\psi}{2}} = \frac{\sqrt{3} \sin 60^\circ \cos \frac{\psi}{2} + \cos 60^\circ \sin \frac{\psi}{2}}{\cos \frac{\psi}{2}}.
\]

After manipulation and substitution of known values, we obtain

\[
\frac{|IL|}{|EJ|} = 1^2 \left( 3 + \sqrt{3} \tan \frac{\psi}{2} \right) = \frac{5 + 1}{2} = \phi.
\]

In Figure 32, we have drawn the intersection point \( M \) of segment \( IL \) with circle \( k_2 \) (not drawn in construction RCC9 because segment \( IL \) is not constructed; we have added \( IL \) for the proof).

Because \( IEL \) is an isosceles triangle, we have \( \angle EIL = 30^\circ \).

The properties of the central and inscribed angles in circles \( k_2 \) and \( k_3 \) give

\[
\angle IEF = \frac{1}{2} \angle IBF \quad \text{and} \quad \frac{1}{2} \angle FAJ = 30^\circ.
\]

Therefore, \( JE \) and \( IL \) are parallel. Because \( JI \) and \( AB \) are parallel as well (an easy proof for the reader), we obtain \( |JE| = |IM| \). Thus, point \( M \) divides segment \( IL \) in the golden ratio.

Further, let us prove

\[
\frac{|FJ|}{|IK|} = \phi.
\]

Figure 33
We have already shown (see Figures 24 and 25) \(|\angle IFK| = 120^\circ\) and \(|FK| = |FH|).

Similar to the previous proof

\[ |IK| = \sqrt{3} |FK| = \sqrt{3} |FH|, \]

where segments \(FJ\) and \(FH\) are chords in circle \(k_3\) (see zoomed Figure 33).

Because \(\angle FAJ = 60^\circ - \psi\) and \(\angle FAH = \psi\),

\[ \frac{|FJ|}{|IK|} = \frac{|FJ|}{\sqrt{3} |FH|} = \frac{\sin \frac{60^\circ - \psi}{2}}{\sqrt{3} \sin \frac{\psi}{2}} = \frac{\sin (30^\circ - \frac{\psi}{2})}{\sqrt{3} \sin \frac{\psi}{2}}. \]

After trigonometric manipulation (see 3.10–III) we obtain

\[ \frac{|FJ|}{|IK|} = \frac{1}{2\sqrt{3}} \left( \cot \frac{\psi}{2} - \sqrt{3} \right) = \frac{\sqrt{5} + 1}{2} = \phi. \]

**Example 5.**

Individual constructions often differ only in a single line (a straight line or a circle). In this case, the proofs may be similar. This example shows construction RCC15, from class 14. By examining the ratios, we can find them based on previous calculations.

![Figure 34](image_url)

In construction RCC15 (see Figure 34), we have

\[ \frac{|AJ|}{|AI|} = \frac{|IL|}{|AJ|} = \frac{|AL|}{|CE|} = \frac{|JL|}{|KG|} = \phi. \]

The equality

\[ \frac{|AJ|}{|AI|} = \phi \]
is given by pattern \( B \) in circle \( k_2(B, A) \).

The equality
\[
\frac{|JL|}{|GK|} = \phi
\]
is given by pattern \( B \) in circle \( k_3(F, G) \). Because \( |IJ| = |GK| \) (intercepted chords on rotated circles) and because triangles \( \triangle FEC \) and \( \triangle FLC \) are equilateral, \( |\angle EFL| = 120^\circ \).

Because \( |\angle EFC| = 60^\circ \) and \( |\angle JFC| = |\angle EFI| = |\angle GFE| = \frac{1}{2}|\angle GAE| = \frac{\psi}{2} \), we have \( |\angle IFJ| = 60^\circ - 2\psi = 60^\circ - \psi \).

Therefore, \( |\angle JFL| = 120^\circ - (60^\circ - \psi) = 60^\circ + \psi \).

Figure 35
Constructions RCC15 and RCC9 differ in one circle and in reflection about line \( AB \). Compare Figures 32 (circles \( k_5(G, C) \) and dashed \( e(E, H) \)) and 35 (dashed circle \( h(H, I) \) and circle \( k_5(F, G) \)). Thus, the equality
\[
\frac{|IL|}{|AJ|} = \phi
\]
was already proven in Example 4.

The equality
\[
\frac{|AL|}{|CE|} = \phi
\]
is given by pattern \( C \) in circle \( k_4(C, A) \). We draw segment \( AA' \) with the midpoint \( F \). Then, \( |AA'| = 2r \) and \( |FA'| = |LA'| = r \) (see circle \( m(A', F) \) that completes pattern \( C \) in Figure 35).
Example 6.
We prove construction RCC8 from class 7. In RCC8 (in Figure 36), we have
\[
\frac{|DL|}{|BG|} = \frac{|BG|}{|DK|} = \frac{|CK|}{|CE|} = \frac{|IE|}{|EI|} = \frac{|IK|}{|IL|} = \phi.
\]

Figure 36
We first show the equalities given by known patterns.

The equality
\[
\frac{|DL|}{|BG|} = \frac{|BG|}{|DK|} = \phi
\]
is given by pattern B in circle \( k_3(A,B) \) because circle \( k_5(G,A) \) is rotated to
circle \( k_2(B,A) \) about center \( A \) by angle \( \angle GAB \); \( \angle GAB = 60^\circ + \psi \).

The equality
\[
\frac{|CK|}{|CE|} = \frac{|CK|}{|KL|} = \frac{|CE|}{|CL|} = \phi
\]
is given by patterns A2 and C.

Proposition. Circle \( k_5(G,A) \) intersects segment \( AB \) at its midpoint.
Proof. Let \( S \) be the midpoint of segment \( AB \). Therefore, \( |AH| : |AC| = 1 : 2 = |AS| : |AG| \) and \( \angle CAH = \angle GAB = \angle GAS \).
Thus, triangles \( \triangle ACH \) and \( \triangle AGS \) are similar; both are isosceles triangles, and
point \( S \) lies on circle \( k_5(G,A) \). Therefore, \( S = I \).

Now, we can prove the equality
\[
\frac{|IK|}{|EI|} = \phi.
\]
Figure 37

The central and inscribed angles in circles \( k_5 \) and \( k_3 \) in Figure 37 provide

\[
|\angle LKB| = |\angle LDB| = \frac{1}{2}|\angle LAB| = \frac{1}{2}|\angle LAI| = \frac{1}{2}|\angle LKI|;
\]

thus \( |\angle CKB| = \frac{1}{2}|\angle CKI| \). Therefore, line \( KB \) is a bisector of \( \angle CKI \). The bisector of the internal angle in the triangle intersects the opposite side at the point that divides that side into segments with lengths of the identical ratio as the lengths of the sides on the legs of the angle. Thus, in triangle \( CKI \) we have

\[
\frac{|IK|}{|CK|} = \frac{|IB|}{|CB|} = \frac{1}{2}.
\]

Then, \( |IK| = \frac{1}{2}|CK| \) and \( |EI| = \frac{1}{2}|EF| = \frac{1}{2}|CE| \), and thus

\[
\frac{|IK|}{|EI|} = \frac{|CK|}{|CE|} = \phi.
\]

The equality

\[
\frac{|EI|}{|IL|} = \phi
\]

can be similarly proven.

We have \( |LC| = 2|IL| \) because point \( L \) lies on circle \( k_3(A, B) \), which is the locus of point \( M \) for which \( |CM| : |IM| = 2 \) (Apollonius circle). Therefore,

\[
\frac{|EI|}{|IL|} = \frac{|EC|}{|CL|} = \phi.
\]
3.11 More Patterns and Approaches – Circles and Lines

If a straight line is a part of a construction, then the segments and pairs of points whose distance we examine may not be chords of any "known" circle. However, because such lines are given by points \(A\) and \(B\) and the points of intersection of previously constructed straight lines and circles, the examined lengths are the lengths of the sides of triangles for which any length and internal angle values are known. Therefore, we use the law of sines.

Our proofs follow only one of many possible ways of how to prove the given statements. We leave other ways to the reader.

Example 7.
We use the law of sines in the proof of construction RCC22 from class 9.

![Figure 38](image)

In construction RCC22 (in Figure 38), we have

\[
\frac{|AJ|}{|BG|} = \frac{|AI|}{|BI|} = \frac{|BI|}{|AB|} = \frac{|FK|}{|EI|} = \frac{|EI|}{|IK|} = \frac{|GI|}{|DG|} = \phi.
\]

The equality

\[
\frac{|AJ|}{|BG|} = \phi
\]

follows from pattern \(B\) in circles \(k_3(A, B)\) and \(k_3(B, A)\). As we have shown, circle \(k_3(A, B)\) is rotated to circle \(k_2(B, A)\) about point \(F\) by angle 60° and \(|\angle CEF| = 60°\). Therefore, \(|\angle CBJ| = \psi\).

From the following group of three equalities,

\[
\frac{|AI|}{|BI|} = \frac{|BI|}{|AB|} = \frac{|AB|}{|CT|} = \phi,
\]

we must prove only one because of the properties of the golden ratio. Because \(|AB| = |BC|\), \(|AI| = |AB| + |BI|\) and \(|BI| = |BC| + |CI| = |AB| + |CI|\), these ratios are ratios of lengths of the entire segment to the length of the longer part.
and the ratio of lengths of the longer and shorter part of the divided segment. We have selected to examine the ratio $|BI| : |AB|$.

The equality

$$\frac{|BI|}{|AB|} = \frac{|BJ|}{|BJ|} = \phi$$

can be proven using angle values and the lengths of the sides of triangle $BIJ$ in Figure 39.

**Figure 39**

$\triangle FAH$ is an isosceles triangle with vertex angle $\psi$ at vertex $A$.

Thus $|\angle AHF| = 90^\circ - \frac{\psi}{2}$.

In triangle $AIH$, we have $|\angle IAH| = 60^\circ + \psi$ and $|\angle AHI| = |\angle AHF| = 90^\circ - \frac{\psi}{2}$, thus $|\angle HIA| = 30^\circ - \frac{\psi}{2}$.

In triangle $BJI$, we have $|\angle IBJ| = \psi$ and because $|\angle JIB| = |\angle HIA| = 30^\circ - \frac{\psi}{2}$, we have $|\angle BJI| = 150^\circ - \frac{\psi}{2}$.

Because $|BJ| = |AB|$, the law of sines in triangle $BJI$ can be used to find the ratio of the given lengths:

$$\frac{|BI|}{|BJ|} = \frac{\sin \left(150^\circ - \frac{\psi}{2}\right)}{\sin \left(30^\circ - \frac{\psi}{2}\right)}.$$  

Using trigonometric identities, we obtain

$$\frac{\cos \frac{\psi}{2} + \sqrt{3} \sin \frac{\psi}{2}}{\cos \frac{\psi}{2} - \sqrt{3} \sin \frac{\psi}{2}} = \frac{1 + \sqrt{3} \tan \frac{\psi}{2}}{1 - \sqrt{3} \tan \frac{\psi}{2}} = \frac{\sqrt{5} + 1}{2} = \phi.$$  

This calculation is not necessary. Using known formulae, we obtain

$$\frac{|BI|}{|BJ|} = \frac{\sin \left(150^\circ - \frac{\psi}{2}\right)}{\sin \left(30^\circ - \frac{\psi}{2}\right)} = \frac{\sin \left(30^\circ + \frac{\psi}{2}\right)}{\sin \left(30^\circ - \frac{\psi}{2}\right)}.$$
which expresses the geometric principle of pattern B.
We could prove this equality using the law of sines in triangle BIE.

Because |FI| = |EI|, we must prove only one of the following pair of equalities
\[
\frac{|FK|}{|EI|} = \frac{|EI|}{|IK|} = \phi.
\]

Let us prove \[\frac{|FK|}{|EI|} = \frac{|FK|}{|FI|} = \phi\] using the ratios of lengths of the sides in triangles \(\triangle AIF\) and \(\triangle AKF\) in Figure 40.

We must find the values of the internal angles in the triangles.
We have already found the following angles in triangle \(\triangle AIF\):
\[|\angle IAF| = 60^\circ, \quad |\angle FIA| = 30^\circ - \frac{\psi}{2}\]
and \[|\angle AFI| = 90^\circ + \frac{\psi}{2}\].

For internal angles in triangle \(\triangle AKF\), we have:
\[|\angle AFK| = |\angle AFI| = 90^\circ + \frac{\psi}{2}\].

From the proof of pattern B, we obtain \[|\angle HCA| = 60^\circ - 2\psi\]. This angle is a central angle in circle \(k_4(C,A)\); thus, for the corresponding inscribed angle \(\angle HKA\), we have \[|\angle HKA| = 30^\circ - \psi\]. Thus \[|\angle KAF| = 60^\circ + \frac{\psi}{2}\].

From the law of sines in triangle \(\triangle AIF\), we obtain
\[
\frac{|FI|}{|FA|} = \frac{\sin 60^\circ}{\sin (30^\circ - \frac{\psi}{2})}.
\]

From the law of sines in triangle \(\triangle AKF\), we obtain
\[
\frac{|FK|}{|FA|} = \frac{\sin \left(60^\circ + \frac{\psi}{2}\right)}{\sin (30^\circ - \psi)}.
\]

Therefore,
\[
\frac{|FK|}{|FI|} = \frac{\sin \left(60^\circ + \frac{\psi}{2}\right)}{\sin (30^\circ - \psi)} \cdot \frac{\sin (30^\circ - \frac{\psi}{2})}{\sin 60^\circ}.
\]
After substitution of known values, we obtain
\[
\frac{4}{\sqrt{3}} \cdot 2 \cos \left( 30^\circ - \frac{\psi}{2} \right) \cdot \sin \left( 30^\circ - \frac{\psi}{2} \right) = \frac{4}{\sqrt{3}} \sin (60^\circ - \psi),
\]
and using trigonometric identities and known values (see 3.10–III), we have

\[
\frac{|FK|}{|FI|} = \frac{\sqrt{5} + 1}{2}.
\]

The last equality in this construction,

\[
\frac{|GI|}{|DG|} = \phi,
\]

can be similarly proven. One length in the ratio is the length of the chord in circle \( k_3(A, B) \), the other is the length of the side of triangle \( AGI \) (see Figure 41). Internal angles of the triangle can be easily found.

![Figure 41](image)

We have: \( \angle GAI = 60^\circ + \psi, \angle AIG = 30^\circ - \frac{\psi}{2} \) and \( \angle IGA = 90^\circ - \frac{\psi}{2} \).

From the isosceles triangle \( DAG \) and the law of sines in triangle \( GAI \) we obtain

\[
\frac{|DG|}{2|GA|} = \sin \left( 60^\circ - \frac{\psi}{2} \right),
\]

\[
\frac{|GI|}{|GA|} = \sin \left( 60^\circ + \psi \right) \cdot \sin \left( 30^\circ - \frac{\psi}{2} \right).
\]

Therefore,

\[
\frac{|GI|}{|DG|} = \frac{\sin (60^\circ + \psi)}{\sin \left( 30^\circ - \frac{\psi}{2} \right)} \cdot \frac{1}{2 \sin \left( 60^\circ - \frac{\psi}{2} \right)} = \frac{\sqrt{15}}{4} \cdot \frac{1}{2} \cdot \sin \left( 30^\circ - \frac{\psi}{2} \right) \cdot \sin \left( 60^\circ - \frac{\psi}{2} \right).
\]
and using product-to-sum entity we obtain

\[
\frac{|GI|}{|GD|} = \frac{\sqrt{15}}{4} \cdot \cos 30^\circ - \cos (90^\circ - \psi) = \frac{\sqrt{15}}{4} \cdot \frac{1}{2} \sqrt{3} - \sin \psi.
\]

After some manipulation we have

\[
\frac{|GI|}{|GD|} = \frac{2}{\sqrt{5} - 1} = \frac{\sqrt{5} + 1}{2} = \phi.
\]

This proof for construction RCC22 is also a proof for constructions from class 41 (e.g., RCC382).

**Example 8. Golden Section of Segment AB**

In all examined constructions, the golden ratio was formed by pairs of lengths of segments in various positions. However, none of these ratios was the golden section of the *given segment AB* (or of pair of given points A and B). We can find the golden section in class of similar constructions 12. We show construction RCC13 (see Figure 42).

We will prove the equalities for all golden ratios in this construction as we did in previous examples.

\[\text{Figure 42}\]

In construction RCC13 (in Figure 42), we have

\[
\frac{|BG|}{|AJ|} = \frac{|DG|}{|GI|} = \frac{|IK|}{|EI|} = \frac{|CI|}{|AB|} = \frac{|BI|}{|AI|} = \phi.
\]

The equality

\[
\frac{|BG|}{|AJ|} = \phi.
\]
follows from pattern B in congruent circles \( k_2(B, A) \) and \( k_3(A, B) \) (Figure 43). To prove this equality, we must find the central angles that subtend chords \( BG \) and \( AJ \).

The chord \( GB \) in circle \( k_3(A, B) \) is intercepted by the central angle \( \angle GAB \); 
\( |\angle GAB| = 60^\circ + \psi \).

The central angle \( \angle GAE \) in circle \( k_3(A, B) \) corresponds to the inscribed angle \( \angle GFE \). Because \( \angle GFE = \angle JFE \) (which is the inscribed angle in circle \( k_2(B, A) \) corresponding to central angle \( \angle JBE \)), we have \( |\angle JBE| = \psi \) and 
\( |\angle ABJ| = 60^\circ - \psi \).

Remark. Because circle \( k_2(B, A) \) rotates to circle \( k_3(A, B) \) when rotated about point \( E \) by angle \( 60^\circ \) and from the congruence of arcs \( GE \) and \( JE \) that we have just proven, the triangle \( GEJ \) in Figure 43 is therefore an equilateral triangle.
The equality
\[ \frac{|DG|}{|GI|} = \phi \]
is proven using the law of sines in triangle \( \triangle DIG \) (see Figure 44).

Because \( |\angle IFE| = \frac{\psi}{2} \) (as we have already shown) and line \( FE \) is perpendicular
to line \( IB \), we have \( |\angle BIF| = |\angle DIG| = 90^\circ - \frac{\psi}{2} \).

In the isosceles triangle \( \triangle DAG \), we have \( |\angle GDA| = 30^\circ + \frac{\psi}{2} \).

From the law of sines in triangle \( \triangle DIG \), we have
\[ \frac{|DG|}{|GI|} = \frac{\sin \left( 90^\circ - \frac{\psi}{2} \right)}{\sin \left( 30^\circ + \frac{\psi}{2} \right)}, \]

From which
\[ \frac{|DG|}{|GI|} = 2 \cdot \frac{\cos \frac{\psi}{2}}{\cos \frac{\psi}{2} + \sqrt{3} \sin \frac{\psi}{2}} = 2 \cdot \frac{1}{1 + \sqrt{3} \tan \frac{\psi}{2}}; \]
after substitution of known values, we obtain
\[ \frac{|DG|}{|GI|} = \frac{2}{\sqrt{5} - 1} = \frac{\sqrt{5} + 1}{2} = \phi. \]

Because \( |EI| = |FI| \) and because of the definition of the golden section, only
one from following equalities must be proven:
\[ \frac{|IK|}{|EI|} = \frac{|EI|}{|FK|} = \phi. \]

The equality
\[ \frac{|IK|}{|EI|} = \frac{|IK|}{|FF|} = \phi \]
can be proven using the ratios of lengths in triangles \( \triangle AIK \) and \( \triangle AIF \) (see Figure 45).

![Figure 45](image_url)
We must find the internal angles in triangles $\triangle AIK$ and $\triangle AIF$.

Because line $AK$ passes through the midpoint $A$ of arc $HG$, $AK$ is the angle bisector of $\angle HGK$.

Because $|\angle KAH| = |\angle KGH|$ and lines $HG$ and $FE$ are parallel, we have $|\angle HKG| = |\angle HGF| = \frac{\psi}{2}$. Thus, $|\angle KAF| = \frac{\psi}{2}$.

Because $|\angle IAF| = 60^\circ$ then $|\angle IAK| = 60^\circ + \frac{\psi}{2}$.

$|\angle FIA| = |\angle KIA| = 90^\circ + \frac{\psi}{2}$, thus $|\angle AFI| = 30^\circ - \frac{\psi}{2}$ and $|\angle AKI| = 30^\circ - \psi$.

From the law of sines in triangle $AIK$, we obtain

\[
\frac{|KI|}{|AI|} = \frac{\sin(60^\circ + \frac{\psi}{2})}{\sin(30^\circ - \psi)}.
\]

From the law of sines in triangle $AIF$, we obtain

\[
\frac{|FI|}{|AI|} = \frac{\sin 60^\circ}{\sin \left(30^\circ - \frac{\psi}{2}\right)}.
\]

Thus

\[
\frac{|KI|}{|FI|} = \frac{\sin(60^\circ + \frac{\psi}{2})}{\sin(30^\circ - \psi)} \cdot \frac{\sin(30^\circ - \frac{\psi}{2})}{\sin 60^\circ} = \frac{\cos \left(30^\circ - \frac{\psi}{2}\right)}{\frac{\sqrt{3}}{2}}.
\]

and after substitution of known values and simple manipulations, we have

\[
\frac{|KI|}{|FI|} = \frac{4}{\sqrt{3}} \cdot \sin(60^\circ - \psi) = \frac{1}{2} \left(\sqrt{5} + 1\right) = \phi.
\]

**The Golden Section of $AB$**

Following equalities are formulae of golden ratios of segment $AB$ and its parts and corollaries. Again, we must prove only one of the equalities

\[
\frac{|CI|}{|AB|} = \frac{|AB|}{|BI|} = \frac{|BI|}{|AI|} = \phi.
\]

The law of sines in triangle $AIF$ allows us to show

\[
\frac{|AB|}{|AI|} = \frac{|AF|}{|AI|} = \phi^2.
\]

We have decided to prove the formula

\[
\frac{|BI|}{|AI|} = \phi
\]

using the ratios of lengths of sides in triangles $\triangle AIF$ and $\triangle BIF$ in Figure 46.
We will prove that line $FG$ meets the side $AB$ of an equilateral triangle $ABF$ at point $I$ that divides the segment $AB$ in the golden section. For the internal angles in the triangles $\triangle AIF$ and $\triangle BIF$ we have $|\angle IAF| = |\angle FBI| = 60^\circ$, $|\angle FIA| = 90^\circ + \frac{\psi}{2}$, $|\angle AFI| = 30^\circ - \frac{\psi}{2}$, $|\angle BIF| = 90^\circ - \frac{\psi}{2}$, $|\angle IFB| = 30^\circ + \frac{\psi}{2}$.

Thus from the law of sines in triangle $AIF$, we can write
\[
\frac{|FI|}{|AI|} = \frac{\sin 60^\circ}{\sin \left(30^\circ - \frac{\psi}{2}\right)},
\]
and from the law of sines in triangle $BIF$, we can write
\[
\frac{|BI|}{|FI|} = \frac{\sin \left(30^\circ + \frac{\psi}{2}\right)}{\sin 60^\circ}.
\]

Therefore,
\[
\frac{|BI|}{|AI|} = \frac{\sin \left(30^\circ + \frac{\psi}{2}\right)}{\sin \left(30^\circ - \frac{\psi}{2}\right)} = \frac{\sqrt{5} + 1}{2} = \phi.
\]

However, it is not necessary to calculate and prove the last equality $\frac{\sin \left(30^\circ + \frac{\psi}{2}\right)}{\sin \left(30^\circ - \frac{\psi}{2}\right)} = \frac{\sqrt{5} + 1}{2} = \phi$. This equality is the ratio of the lengths of chords intercepted in the circle by the central angles $60^\circ + \psi$ and $60^\circ - \psi$ (shown in pattern B).

The last two equalities
\[
\frac{|AB|}{|BI|} = \frac{|CI|}{|AB|} = \phi
\]
are corollaries of the proof above, equalities $|CI| = |CB| + |BI| = |AB| + |BI|$ and from the definition of the golden section.

Construction RCC13 was described by and the golden section of segment $AB$ was proved by Kurt Hofstetter in [6].
3.12 Finally...

We have already mentioned that the proofs we present in this part of the book provide hints of how to prove the other constructions in this book. Let us stress that other possibilities of how the necessary proofs could be performed are available, and our approach is only one of many.

In our method, we purposely avoided calculating explicit lengths, using algebraic equations and employing analytic geometry. Moreover, the proofs shown can be similarly performed using properties in other triangles. The reader can certainly use their own ideas and inspiration.
4 Conclusion

We have described four different types of constructions, and for each type of construction, we have presented a list of all of the shortest constructions of the golden ratio (715 in total).

To simplify the search, we added an image table of all 129 classes of similarity; the clickable link to this table can be found in the headline of every page. We also described a method to prove why ratio of those and these distances should be the golden ratio.

We (arrogantly) imagine that you, the reader, could find similar enjoyment in browsing and investigating of individual constructions as we have found in searching them. Whether you are teacher, student or somebody who found this book accidentally.

If you would like to provide comments or report errors, please write to Sarka@gbn.cz or to Tomas.Holan@mff.cuni.cz. Alternatively, please visit the webpage dedicated to this book at http://ksvi.mff.cuni.cz/~holan/GoldenRatio. Thank you!

P.S.: Try to estimate how many different occurrences of the golden ratio is in all presented constructions; the answer can be found on the next page after a list of references.
References


Answer: $32 + 8308 + 9080 + 2624 = 20044$