

State Final Examination (Sample Questions)

Spring 2019

1 Automata (3 points)

Let A and B be regular languages over the alphabet $\{0, 1\}$. Decide whether the following languages are regular and explain why.

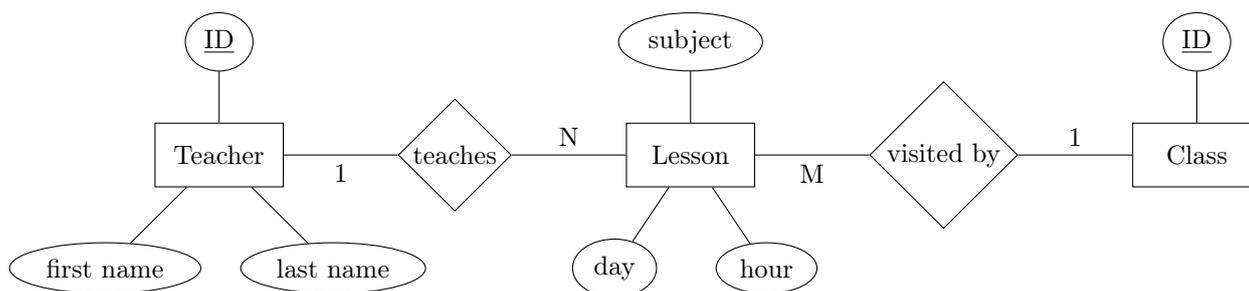
1. $A \cap B$
2. $A^R = \{a^R \mid a \in A\}$, where a^R is the word a reversed
3. $\{a \mid a \in A \wedge |a| \bmod 3 = 0\}$

2 Sorting (3 points)

1. Describe how to sort a sequence x_1, \dots, x_n using a binary search tree.
2. What does follow for the minimum possible complexity of operations on binary search trees? Let us assume that keys stored in the tree can only be compared.
3. Describe an algorithm which sorts n ordered pairs $(x_1, y_1), \dots, (x_n, y_n) \in \{1, \dots, n\}^2$ in time $\mathcal{O}(n)$.

3 Database systems (3 points)

The following E-R diagram depicts a simplified elementary-school timetable:



The diagram is rather informal and not necessarily complete; for instance, you may need to guess how to handle weak entity types, based on your real-world experience.

1. Convert the diagram into a relational schema (use a diagram or SQL data-definition language). Include also primary and foreign keys of the relations.
2. Write a query in SQL which displays all situations (i.e., errors in the timetable) where a teacher is expected to teach two different lessons at the same day and hour. Each output row shall contain the name of the teacher, the day, and the hour when the conflict occurs. Pay attention to display every error only once.
3. Write a query in SQL which generates an alphabetically sorted list of all teachers. Each line shall contain the name of the teacher and the number of different classes he/she sees at his/her lessons. Include also teachers who teach nothing.

4 Programming languages (3 points)

Design a data structure in C++, C# or Java that will represent a mathematical expression and will be used for repeated quick evaluation of the expression for different parameter values. Such a structure can be used, for example, when drawing graphs of functions of one, two or more variables.

The expression may include constants (numbers), variables, unary and binary mathematical operators, and mathematical functions from an extensive library. All calculations are performed in double-precision floating point arithmetic.

The structure should be designed to make the addition of new operators or functions to the library as simple as possible.

1. Describe the most important classes and other elements of your solution, in particular the external and internal interfaces used to evaluate the expression. Draw the objects and their mutual links for the expression $\sin(x + 1)/\sin(x - 1)$.
2. Suggest an approach to handling of errors (division by zero, overflow, etc.) during evaluation. The system shall allow, for instance, drawing the function $1/\sqrt{x}$ over the interval $\langle -1, 1 \rangle$.
3. Describe a modification of the structure which can prevent re-evaluation of common sub-expressions, e.g., in the expression $(1 - \sin(x \cdot y))/(1 + \sin(x \cdot y))$ where the sub-expression $\sin(x \cdot y)$ shall be evaluated only once (for each pair of x and y values).

Do NOT describe the way how the structure is built (i.e., the parsing of an expression or finding common sub-expressions).

5 Processes, threads, scheduling (3 points)

Assume a typical operating system where processes or threads can be in one of states called READY, RUNNING, WAITING/SLEEPING, TERMINATED.

1. Draw a state transition diagram whose nodes are the process states and whose edges denote possible state transitions. For each edge explain when such a transition can take place (describe at least one specific event that triggers the transition).
2. Is the number of processes that can simultaneously exist in the READY state limited in any way? Explain why.
3. Is the number of processes that can simultaneously exist in the RUNNING state limited in any way? Explain why.

6 Optimization (3 body)

Consider the following primal linear program P:

$$\min 2x_1 + x_2 + x_3 \tag{1}$$

$$\text{s.t. } 3x_1 + x_2 \geq 1 \tag{2}$$

$$x_1 + 2x_2 + x_3 \geq 4 \tag{3}$$

$$x_1, x_2, x_3 \geq 0 \tag{4}$$

1. Determine the dual linear program.
2. Find the optimal solution of the LP D.
3. State precisely the strong duality theorem.
4. Exploiting the knowledge of the optimal solution of the dual LP and the strong duality theorem, determine the primal optimal solution, if it exists.

7 Language modeling (specialization question – 3 points)

Explain the following notions:

1. Noisy channel method

2. Language model in statistical machine learning
3. Smoothing

8 Basic Formalisms for Description of a Natural Language (specialization question – 3 points)

Describe basic characteristics of following formalisms for the description of natural language syntax:

1. Lexical Functional Grammar
2. Tree-adjoining Grammar
3. Categorical Grammar

9 Morphological, Syntactic and Semantic Analysis of Natural Languages (specialization question – 3 points)

One of the basic concepts of the theory of Functional Generative Description is valency.

1. What is the most important difference between an inner participant and a free modifier?
2. List at least three out of five types of inner participants used in the theory of Functional Generative Description.
3. Define a valency frame.

10 Functions (3 points)

For the function $f(x) = 3x\sqrt[3]{(x+4)^2}$, calculate derivative and find limits at ∞ and $-\infty$. Sketch the graph of the function f (convexity and concavity is not required).

11 Riemann integral (3 points)

Define upper and lower Riemann sum. Using them, define Riemann integral. Determine the value of $\int_1^4 \lfloor x \rfloor dx$. (Where $\lfloor x \rfloor$ is the floor function, its value is the largest integer less than or equal to x .)

12 Metric spaces (3 points)

Define a continuous mapping between metric spaces. Let $id : \mathbb{R} \rightarrow \mathbb{R}$ be an identity mapping (that is $f(x) = x$), d_1 is the usual metric on \mathbb{R} (that is $d_1(x, y) = |x - y|$) and d_{disc} is a discrete metric (that is $d_{disc}(x, y) = 1$ whenever $x \neq y$ and $d_{disc}(x, y) = 0$ for $x = y$). Is id a continuous mapping from (\mathbb{R}, d_1) to (\mathbb{R}, d_{disc}) ? Is id a continuous mapping from (\mathbb{R}, d_{disc}) to (\mathbb{R}, d_1) ? Justify.

13 Linear mappings (3 points)

Consider the linear mapping $f : \mathbb{R}^n \rightarrow \mathbb{R}^n$ given by $f(x) = Ax$, where $A \in \mathbb{R}^{n \times n}$ is a square matrix. Prove the following:

1. If mapping f is surjective (onto), then matrix A is nonsingular.
2. If matrix A is nonsingular, then mapping f is surjective.

14 Positive definite matrices (3 points)

Define the term *positive definite matrix*.

Let

$$A = \begin{pmatrix} 4 & -2 & 0 \\ -2 & 2 & 3 \\ 0 & 3 & 10 \end{pmatrix}.$$

Decide whether A is positive definite. If it is the case, find the Cholesky decomposition of A .

15 Algebraic fields (3 points)

Let T be a field. Decide whether the following statements are true. Explain your answers!

1. $\forall a, b, c \in T : ab = ac \Rightarrow b = c$.

2. Matrix $\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$ is nonsingular if and only if the characteristic of T is 2.

3. If T is a finite field, then the characteristic of T is a finite number.

16 Inclusion exclusion principle (3 points)

Determine the size of the union of four sets A_1, \dots, A_4 provided that the intersection of k (pro $k \geq 1$) of them is $7 - k$, when the set A_1 takes part in the intersection. In other cases the size of an intersection of k sets is $10 - k$.

17 Graphs (3 points)

Using the fact that every planar graph contains a vertex of small degree, show that the vertex set of each planar graph with at least three vertices can be partitioned into three sets V_1, V_2, V_3 such that each set V_i induces a forest.

18 Logic (3 points)

Let T be the theory (of dense linear orders with the smallest and the largest elements) in language $L = \langle \leq \rangle$ with equality consisting of the following axioms:

$$\begin{aligned} x &\leq x \\ (x \leq y \wedge y \leq x) &\rightarrow x = y \\ (x \leq y \wedge y \leq z) &\rightarrow x \leq z \\ x \leq y \vee y \leq x & \\ (\exists x) (\exists y) (\forall z) (x \leq z \wedge z \leq y \wedge x \neq y) & \\ x < y &\rightarrow (\exists z) (x < z \wedge z < y) \end{aligned}$$

where $x < y$ is a shortcut for $x \leq y \wedge x \neq y$.

1. Give a definition of a *complete theory* (in predicate logic).
2. Applying skolemization to the last two axioms find an open conservative extension T' of the theory T (possibly in an extended language).
3. Are theories T and T' complete? Explain why (not).