

Examples of good and bad texts for the Final English Exam

Bad text

Calculus Textbook

This text is mostly equations and doesn't contain terms and ideas that can be discussed.

When $x = a$, the function is of the form $\frac{0}{0}$, and is indeterminate.

Let $x = a + h$. Then

$$\frac{x^n - a^n}{x - a} = \frac{(a + h)^n - a^n}{(a + h) - a}$$

Expanding $(a + h)^n$ by the binomial theorem:

$$\begin{aligned} \frac{x^n - a^n}{x - a} &= \frac{\left\{ a^n + na^{n-1}h + \frac{n(n-1)}{2!} a^{n-2}h^2 + \dots \right\} - a^n}{h} \\ &= \left\{ na^{n-1} + \frac{n(n-1)}{2!} a^{n-2}h + \dots \right\} \end{aligned}$$

But since $x = a + h$, when $x \rightarrow a$, $h \rightarrow 0$.

Therefore

$$\begin{aligned} \lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} &= \lim_{h \rightarrow 0} \left\{ na^{n-1} + \frac{n(n-1)}{2!} a^{n-2} h + \dots \right\} \\ &= na^{n-1} \end{aligned}$$

since all the other terms have a power of h as a factor, and therefore vanish when $h \rightarrow 0$.

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Example 2.6.3

Find the limit of $\frac{x-3}{\sqrt{x-2} - \sqrt{4-x}}$ as $x \rightarrow 3$.

Both the numerator and the denominator are zero when $x = 3$, so the function takes the form $\frac{0}{0}$ and is indeterminate.

Multiplying the numerator and denominator by $\sqrt{x-2} + \sqrt{4-x}$ gives:

Good texts

Thermodynamics and Phase Diagrams of Materials

On pages 11 – 12 there are ideas that can be discussed. The teacher might ask:

“What is tangent construction?”

“What is the Gibbs – Duhem Equation?”

“What is activity?”

“What are ideal Raoultian solutions?”

1.4.3 Tangent Construction

An important construction is illustrated in Fig. 1-4. If a tangent is drawn to the curve of g^1 at a certain composition ($X_{Cu}=0.6$ in Fig. 1-4), then the intercepts of this tangent on the axes at $X_{Au}=1$ and $X_{Cu}=1$ are equal to g_{Au} and g_{Cu} respectively at this composition.

To prove this, we first consider that the Gibbs energy of the solution at constant T and P is a function of n_{Au} and n_{Cu} . Hence:

$$\begin{aligned} dG_{T,P}^1 &= \left(\frac{\partial G^1}{\partial n_{Au}} \right) dn_{Au} + \left(\frac{\partial G^1}{\partial n_{Cu}} \right) dn_{Cu} \\ &= g_{Au} dn_{Au} + g_{Cu} dn_{Cu} \end{aligned} \quad (1-25)$$

Eq. (1-25) can be integrated as follows:

$$\begin{aligned} \int_0^{G^1} dG^1 &= \int_0^{n_{Au}} g_{Au} dn_{Au} + \int_0^{n_{Cu}} g_{Cu} dn_{Cu} \\ G^1 &= g_{Au} n_{Au} + g_{Cu} n_{Cu} \end{aligned} \quad (1-26)$$

where the integration is performed at constant composition so that the intensive properties g_{Au} and g_{Cu} are constant. This integration can be thought of as describing a process in which a pre-mixed solution of constant composition is added to the system, which initially contains no material.

Dividing Eqs. (1-26) and (1-25) by $(n_{Au} + n_{Cu})$ we obtain expressions for the molar Gibbs energy and its derivative:

$$g^1 = X_{Au} g_{Au} + X_{Cu} g_{Cu} \quad (1-27)$$

and

$$dg^1 = g_{Au} dX_{Au} + g_{Cu} dX_{Cu} \quad (1-28)$$

Since $dX_{Au} = -dX_{Cu}$, it can be seen that Eqs. (1-27) and (1-28) are equivalent to the tangent construction shown in Fig. 1-4.

These equations may also be rearranged to give the following useful expression for a binary system:

$$g_i = g + (1 - X_i) dg/dX_i \quad (1-29)$$

1.4.4 Gibbs–Duhem Equation

Differentiation of Eq. (1-27) yields:

$$\begin{aligned} dg^1 &= (X_{Au} dg_{Au} + X_{Cu} dg_{Cu}) \\ &\quad + (g_{Au} dX_{Au} + g_{Cu} dX_{Cu}) \end{aligned} \quad (1-30)$$

Comparison with Eq. (1-28) then gives the *Gibbs–Duhem equation* at constant T and P :

$$X_{Au} dg_{Au} + X_{Cu} dg_{Cu} = 0 \quad (1-31)$$

1.4.5 Relative Partial Properties

The difference between the partial Gibbs energy g_i of a component in solution and the partial Gibbs energy g_i^0 of the same component in a *standard state* is called the *relative partial Gibbs energy* (or *relative chemical potential*), Δg_i . It is most usual to choose as standard state the pure component in the same phase at the same temperature. The activity a_i of the component relative to the chosen standard state is then defined in terms of Δg_i by the following equation, as illustrated in Fig. 1-4.

$$\Delta g_i = g_i - g_i^0 = \mu_i - \mu_i^0 = RT \ln a_i \quad (1-32)$$

Note that g_i and μ_i are equivalent symbols, as are g_i^0 and μ_i^0 , see Eq. (1-23).

From Fig. 1-4, it can be seen that:

$$\begin{aligned} \Delta g_m &= X_{Au} \Delta g_{Au} + X_{Cu} \Delta g_{Cu} \\ &= RT (X_{Au} \ln a_{Au} + X_{Cu} \ln a_{Cu}) \end{aligned} \quad (1-33)$$

The Gibbs energy of mixing can be divided into enthalpy and entropy terms, as can the relative partial Gibbs energies:

$$\Delta g_m = \Delta h_m - T \Delta s_m \quad (1-34)$$

$$\Delta g_i = \Delta h_i - T \Delta s_i \quad (1-35)$$

Hence, the enthalpy and entropy of mixing may be expressed as:

$$\Delta h_m = X_{Au} \Delta h_{Au} + X_{Cu} \Delta h_{Cu} \quad (1-36)$$

$$\Delta s_m = X_{Au} \Delta s_{Au} + X_{Cu} \Delta s_{Cu} \quad (1-37)$$

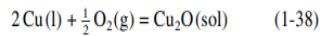
and tangent constructions similar to that of Fig. 1-4 can be used to relate the relative partial enthalpies and entropies Δh_i and Δs_i to the integral molar enthalpy of mixing Δh_m and integral molar entropy of mixing Δs_m respectively.

1.4.6 Activity

The activity of a component in a solution was defined by Eq. (1-32).

Since a_i varies monotonically with g_i it follows that when two or more phases are in equilibrium the activity of any component is the same in all phases, provided that the activity in every phase is expressed with respect to the same standard state.

The use of activities in calculations of chemical equilibrium conditions is illustrated by the following example. A liquid solution of Au and Cu at 1400 K with $X_{\text{Cu}}=0.6$ is exposed to an atmosphere in which the oxygen partial pressure is $p_{\text{O}_2}=10^{-4}$ bar. Will Cu_2O be formed? The reaction is:



where the Cu(l) is in solution. If the reaction proceeds with the formation of dn moles of Cu_2O , then $2dn$ moles of Cu are consumed, and the Gibbs energy of the Au-Cu solution changes by

$$-2(dG^l/dn_{\text{Cu}})dn$$

The total Gibbs energy then varies as:

$$\begin{aligned} dG/dn &= g_{\text{Cu}_2\text{O}} - \frac{1}{2}g_{\text{O}_2} - 2(dG^l/dn_{\text{Cu}}) \\ &= g_{\text{Cu}_2\text{O}} - \frac{1}{2}g_{\text{O}_2} - 2g_{\text{Cu}} \\ &= (g_{\text{Cu}_2\text{O}}^0 - \frac{1}{2}g_{\text{O}_2}^0 - 2g_{\text{Cu}}^0) \\ &\quad - \frac{1}{2}RT \ln p_{\text{O}_2} - 2RT \ln a_{\text{Cu}} \\ &= \Delta G^0 + RT \ln (p_{\text{O}_2}^{-1/2} a_{\text{Cu}}^2) \\ &= \Delta G \end{aligned} \quad (1-39)$$

For the reaction, Eq. (1-38), at 1400 K, $\Delta G^0 = -68.35$ kJ (Barin et al., 1977). The activity of Cu in the liquid alloy at $X_{\text{Cu}}=0.6$ is $a_{\text{Cu}}=0.43$ (Hultgren et al., 1973). Substitution into Eq. (1-39) with $p_{\text{O}_2}=10^{-4}$ bar gives:

$$dG/dn = \Delta G = -50.84 \text{ kJ}$$

Hence under these conditions the reaction entails a decrease in the total Gibbs energy and so the copper will be oxidized.

1.4.7 Ideal Raoultian Solutions

An ideal solution or Raoultian solution is usually defined as one in which the activity of a component is equal to its mole fraction:

$$a_i^{\text{ideal}} = X_i \quad (1-40)$$

(With a judicious choice of standard state, this definition can also encompass ideal Henrian solutions, as discussed in Sec. 1.5.11.)

However, this Raoultian definition of ideality is generally only useful for simple substitutional solutions. There are more useful definitions for other types of solutions such as interstitial solutions, ionic solutions, solutions of defects, polymer solutions, etc. That is, the most convenient definition of ideality depends upon the solution model. This subject will be discussed in Sec. 1.10. In the present section, Eq. (1-40) for an ideal substitutional solution will be developed with the Au-Cu solution as example.

In the ideal substitutional solution model it is assumed that Au and Cu atoms are nearly alike, with nearly identical radii and electronic structures. This being the case, there will be no change in bonding energy or volume upon mixing, so that the enthalpy of mixing is zero:

$$\Delta h_m^{\text{ideal}} = 0 \quad (1-41)$$