Twin-Width and Contraction Sequences - Set 4

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CSSDM 2024, July 5th

1 First-order transductions

Question 1. *Show that full twin-models can transduce ordered twin-models, and vice versa.*

A *segment graph* is the intersection graph of segments in the plane.

Question 2. *Show that there is some function $f : \mathbb{N} \to \mathbb{N}$ such that the class of all segment graphs of maximum degree d has twin-width at most $f(d)$.*

2 Oriented twin-width

In the definition of the partition sequence of a graph G, instead of putting an undirected red *edge* between two parts P, P', we now put a red *arc* from P to P' if there are $u, v \in P, w \in P'$ such that $uw \in E(G)$ and $vw \notin E(G)$, and/or a red *arc* from P' to P if there are $u, v \in P', w \in P$ such that $uw \in E(G)$ and $vw \notin E(G)$. (Note that it is possible to have red arcs in both directions between P and P'). We now have a *red digraph* instead of a *red graph*.

We then define the *oriented twin-width* as the minimum, taken among every partition sequence, of the maximum, taken among every red digraph of the sequence, of the maximum *outdegree*.

Question 3. *Revisit the proof that tww $\equiv$ mn to show that otww $\equiv$ tww.*

A *minor* of G is any graph obtained from G after a series of vertex deletions, edge deletions, and edge contractions. We also recall that a pair of *false twins* are two distinct non-adjacent vertices having the same neighborhood.

Lemma 1 ([1])**.** *Every K_t-minor free graph with at least two vertices admits two distinct vertices of degree at most $2^{O(t \log t)}$ that are either adjacent or false twins.*

Question 4. *Deduce that the class of all K_t-minor free graphs has bounded twin-width.*

References

[1] S. Norine, P. D. Seymour, R. Thomas, and P. Wollan. Proper minor-closed families are small. *J. Comb. Theory, Ser. B*, 96(5):754–757, 2006.