Twin-Width and Contraction Sequences - Set 3

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1 Algorithms based on contraction sequences

The k-Dominating Set problem asks, given a graph G and a positive integer k, if there is a subset $S \subseteq V(G)$ of size at most k such that the closed neighborhood of S in G equals the entire set $V(G)$. In other words, every vertex of G shall either be in S, or be a neighbor of some vertex in S.

Question 1. *Design an algorithm with running time $f(d,k) \cdot n$ that solves k-*Dominating Set *on n-vertex graphs given with a d-sequence, for some function f.*

You should manage to get the dependence of f in k to be single-exponential. (Ask for hints, if you're not sure how to get started.)

The Max Induced Matching problems asks, given a graph G, for a largest subset of edges whose endpoints induce in G a 1-regular graph (i.e., matching). Both Max Induced Matching and Min Coloring are as inapproximable as Max Independent Set: for any $\varepsilon > 0$, approximating these problems on n-vertex graphs within ratio $n^{1-\varepsilon}$ is NP-hard.

Question 2. *For either* Min Coloring *or* Max Induced Matching*, design an $O_d(1)$-approximation algorithm in $2^{O_d(\sqrt{n})}$ time on n-vertex graphs given with a d-sequence.*

As we saw during the lecture for Max Independent Set, this leads to polynomial-time n^{ε}-approximation algorithm, for any $\varepsilon > 0$. For Min Coloring: find the suitable problem generalization. For Max Induced Matching: distinguish different types of edges with respect to a given vertex-partition.

2 Other parameters based on contraction sequences

We go back to some facts we did not prove during the lecture.

Question 3. *Show that (conversely to what we saw during the lecture) classes of bounded component twin-width have bounded boolean-width.*

Question 4. *Show that* total twin-width *is tied to* linear boolean-width*.*