State Final Examination (Sample Questions)

Spring 2017

1 SQL

Consider this database schema: Commodity (CId, Name), Seller (Code, Name, Address), Sale (Id, CId, Code, Price, Year), where Transaction.CId is a foreign key that refers to a Commodity and Transaction.Code is a foreign key that refers to a Seller. Write these SQL queries:

1. Names of Sellers that did not trade wheat in 2016.
2. Codes of Sellers whose aggregate Sales price in 2016 exceeded 10 million.

Describe the SQL 92 transaction isolation levels and list possible ways of implementing each isolation level in a database.

2 Optimization

The following questions concern the optimization problem 3VC, which is a variant of the vertex cover problem. It is defined as follows:

An input instance is a 3-uniform hypergraph $G = (V, E)$, where E is a system of 3-element subsets of V.
A feasible solution for $G = (V, E)$, is a subset $U \subseteq V$ such that $(\forall e \in E)(e \cap U \neq \emptyset)$.
The objective is to minimize the **objective function** $|U|$, i.e., the number of elements of U.

We also consider the following linear program LP:

$$\begin{array}{rrcll} \textbf{minimize} & \sum_{v\in V} x_v & & & \\ \textbf{subject to} & & & & \\ & x_v & \geq & 0 & \text{for all } v \in V \\ & x_u + x_v + x_w & \geq & 1 & \text{for all triples } \{u, v, w\} \in E \end{array}$$

1. Define the meaning of "an algorithm for the problem 3VC is an R-approximation algorithm".
2. What is the relation of the integral solutions of the linear program LP to the feasible solutions and the optimum of an instance of 3VC ? Justify.
3. What is the relation of the optimal (not necessarily integral) solution(s) of LP to the optimal solution(s) of an instance of 3VC ?
4. How can we use the previous observations to obtain an approximation algorithm ?

3 NP-completeness

1. Define: decision problem, problem instance, the NP class, the NP-complete class.
2. Describe 3 NP-complete problems. For one of them, show that it belongs to the NP class.
3. Explain how polynomial time reduction can be used to prove that a problem is NP-complete (assuming you already know some NP-complete problems).
4. What are the practical approaches to solving NP-complete problems ?

4 Convergence of infinite series

Define convergence and absolute convergence of infinite series.

1. Does $1 - 1/2 + 1/4 - 1/8 + 1/16 - \dots$ converge absolutely ?
2. Does $1 - 1/2 + 1/3 - 1/4 + 1/5 - \dots$ converge ?
3. Is it true that if $a_1 + a_2 + \dots$ converges then $a_1^2 + a_2^2 + \dots$ converges too ?

Justify your answers.

5 Column space of a matrix

Define the column space of a matrix $A \in \mathbb{R}^{m \times n}$.

Find a basis of the column space of the matrix

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 1 & 3 & 0 \\ 1 & 1 & 2 \end{pmatrix}.$$

Decide and explain whether the system $Ax = b$ (for the matrix A defined above) is solvable

1. for each $b \in \mathbb{R}^3$,
2. for infinitely many values of $b \in \mathbb{R}^3$.

6 Graph Connectivity

Define the term „trail in a graph“.

Given graph $G = (V, E)$, assume two binary relations on $V \times V$:

- for $x, y \in V$ let $(x, y) \in S$ if and only if both x and y belong to the same component of connectivity of G,
- for $x, y \in V$ let $(x, y) \in T$ if and only if there exists a trail from vertex x to vertex y in G.

For both relations show whether they are reflexive, symetric, antisymetric and transitive. Explain your answers.

Assume graph G on vertex set $\{0, 1\}^3$ (consiting of strings of 0's and 1's of length 3) with edges joining pairs of vertices which differ in exactly one position. For what natural numbers k there is a trail of length k from vertex 000 to vertex 111 ?