$$\int \frac{dx}{x} = \ln|x| + C = \ln|x| + \ln K = \ln K|x|$$
$$C \in \mathbb{R}, K > 0$$

$$\int \frac{dx}{x^2} = -\frac{1}{x} + C, \int \frac{dx}{1+x^2} = \arctan x + C, \int \frac{2x dx}{x^2+1} = \ln(x^2+1) + C$$

$$\int \frac{1}{3x^2-2x-1} dx = \int \left(\frac{1}{4(x-1)} - \frac{3}{4(3x+1)}\right) dx = \int \frac{1}{4(x-1)} dx - \int \frac{3}{4(3x+1)} dx$$
$$\int \frac{1}{4(x-1)} dx = \frac{1}{4}\int \frac{1}{x-1} dx = \begin{vmatrix} y = x-1 \\ dy = dx \end{vmatrix} = \frac{1}{4}\int \frac{dy}{y} = \frac{1}{4}\ln|y| = \frac{1}{4}\ln|x-1| + C$$
$$\int \frac{3}{4(3x+1)} dx = \frac{1}{4}\int \frac{3dx}{3x+1} = \begin{vmatrix} y = 3x+1 \\ dy = 3dx \end{vmatrix} = \frac{1}{4}\int \frac{dy}{y} = \frac{1}{4}\ln|y| = \frac{1}{4}\ln|3x+1| + C$$
$$\int \frac{1}{3x^2-2x-1} dx = \frac{1}{4}(\ln|x-1| - \ln|3x+1|) + C = \frac{1}{4}\ln\left|\frac{x-1}{3x+1}\right| + C \quad x \neq 1 \cup x \neq -\frac{1}{3}$$

$$\int \frac{x}{x^4-2x^2-1} dx = \begin{vmatrix} y = x^2 \\ dy = 2x\,dx \end{vmatrix} = \frac{1}{2}\int \frac{dy}{y^2-2y-1} = \frac{1}{2}\int \frac{dy}{\left(y-(1+\sqrt{2})\right)\left(y-(1-\sqrt{2})\right)} =$$
$$\frac{1}{2}\int \frac{1}{2\sqrt{2}}\left(\frac{1}{y-(1+\sqrt{2})} - \frac{1}{y-(1-\sqrt{2})}\right) dy = \frac{1}{4\sqrt{2}}\ln\left|\frac{y-(1+\sqrt{2})}{y-(1-\sqrt{2})}\right| + C =$$
$$= \frac{1}{4\sqrt{2}}\ln\left|\frac{x^2-(1+\sqrt{2})}{x^2-(1-\sqrt{2})}\right| + C \qquad x \neq \pm\sqrt{1+\sqrt{2}}$$

$$\int \frac{x+1}{x^2+x+1} dx = \frac{1}{2}\int \frac{2x+1}{x^2+x+1} dx + \frac{1}{2}\int \frac{1}{x^2+x+1} dx$$
$$\frac{1}{2}\int \frac{2x+1}{x^2+x+1} dx = \begin{vmatrix} y = x^2+x+1 \\ dy = (2x+1)dx \end{vmatrix} = \frac{1}{2}\int \frac{dy}{y} = \frac{1}{2}\ln|y| = \frac{1}{2}\ln|x^2+x+1|$$
$$\frac{1}{2}\int \frac{1}{x^2+x+1} dx = \frac{1}{2}\int \frac{1}{x^2+2\frac{x}{2}+\frac{1}{2^2}-\frac{1}{2^2}+1} dx = \frac{1}{2}\int \frac{1}{\left(x+\frac{1}{2}\right)^2+\frac{3}{4}} dx =$$
$$= \frac{1}{2}\int \frac{1}{\frac{3}{4}\left(\left(\frac{x+\frac{1}{2}}{\sqrt{\frac{3}{4}}}\right)^2+1\right)} dx = \begin{vmatrix} z = \frac{x+\frac{1}{2}}{\sqrt{\frac{3}{4}}} = \frac{2x+1}{\sqrt{3}} \\ dz = \frac{2}{\sqrt{3}}dx \end{vmatrix} = \frac{2}{3}\frac{\sqrt{3}}{2}\int \frac{1}{(z^2+1)} dx = \frac{1}{\sqrt{3}}\arctan z$$
$$\int \frac{x+1}{x^2+x+1} dx = \frac{1}{2}\ln|x^2+x+1| + \frac{1}{\sqrt{3}}\arctan\frac{2x+1}{\sqrt{3}} + C$$

$$\int \frac{x^5}{x^2+x-2} dx = \int \left(x^3 - x^2 + 3x - 5 + \frac{1}{3(x-1)} + \frac{32}{3(x+2)}\right) dx =$$
$$= \frac{x^4}{4} - \frac{x^3}{3} + \frac{3x^2}{2} - 5x + \frac{\log|1-x|}{3} + \frac{32}{3}\log|2+x| + C \quad x \neq 1 \cup x \neq -2$$

$$\int \frac{dx}{x^4+1} = \int \frac{dx}{(x^2+\sqrt{2}x+1)(x^2-\sqrt{2}x+1)} = \frac{1}{2\sqrt{2}}\int \left(\frac{\sqrt{2}-x}{x^2-\sqrt{2}x+1} + \frac{\sqrt{2}+x}{x^2+\sqrt{2}x+1}\right)dx$$
$$= \frac{1}{4\sqrt{2}}\left(\ln\frac{x^2+\sqrt{2}x+1}{x^2-\sqrt{2}x+1} + 2\arctan(\sqrt{2}x+1) + 2\arctan(\sqrt{2}x-1)\right) + C$$

$$\int \frac{xdx}{x^8-1} = \begin{vmatrix} x^2 = y \\ 2xdx = dy \end{vmatrix} = \frac{1}{2}\int \frac{dy}{y^4-1} = \frac{1}{2}\int \frac{dy}{(y-1)(y+1)(y^2+1)} = \frac{1}{8}\int \left(\frac{dy}{y-1} - \frac{dy}{y+1} - \frac{2dy}{1+y^2}\right)$$
$$= \frac{1}{8}\log\left|\frac{x^2-1}{x^2+1}\right| - \frac{1}{4}\arctan x^2 \qquad x \neq 1$$

$$\int \frac{x^3}{x^8+3}dx = \begin{vmatrix} x^4 = y \\ 4x^3dx = dy \end{vmatrix} = \frac{1}{4}\int \frac{dy}{y^2+3} = \frac{1}{12}\int \frac{dy}{\left(\frac{y}{\sqrt{3}}\right)^2+1} = \frac{1}{4\sqrt{3}}\arctan\left(\frac{x^4}{\sqrt{3}}\right) + C$$

$$\int \frac{x^3}{(x-1)^{100}}dx = \begin{vmatrix} x-1 = y \\ dx = dy \end{vmatrix} = \int \frac{(y+1)^3}{y^{100}}dy = \int \frac{y^3+3y^2+3y+1}{y^{100}}dy =$$
$$= -\frac{y^{-96}}{96} - \frac{3y^{-97}}{97} - \frac{98y^{-98}}{98} - \frac{y^{-99}}{99} = -\frac{1}{96(x-1)^{96}} - \frac{3}{97(x-1)^{97}} - \frac{3}{98(x-1)^{98}} - \frac{1}{99(x-1)^{99}} + C$$

$$\int \frac{dx}{x^5+x^4-2x^3-2x^2+x+1} = \int \frac{dx}{(x+1)^3(x-1)^2} =$$
$$= \frac{1}{16}\int \left(\frac{2}{(x-1)^2} - \frac{3}{x-1} + \frac{4}{(x+1)^3} + \frac{4}{(x+1)^2} + \frac{3}{x+1}\right)dx =$$
$$= \frac{1}{16}\left(\frac{4-6x-6x^2}{(x-1)(1+x)^2} + 3\log\left|\frac{x+1}{x-1}\right|\right) + C \qquad x \neq \pm 1$$

$$t=\sqrt[n]{\frac{x+a}{x+b}} \qquad x=\frac{a-bt^n}{t^n-1} \qquad dx=\frac{(b-a)nt^{n-1}}{(t^n-1)^2}$$

$$t=\sqrt[n]{\frac{x+a}{b-x}} \qquad x=\frac{bt^n-a}{t^n+1} \qquad dx=\frac{(a+b)nt^{n-1}}{(t^n+1)^2}$$

Nechť je $a<b \Rightarrow x\in(a,b) \Rightarrow b>x$

$$\int\frac{dx}{\sqrt{(x-a)(b-x)}}=\int\frac{dx}{(b-x)\sqrt{\frac{x-a}{b-x}}}=\left|\begin{array}{c} t=\sqrt{\frac{x-a}{b-x}} \\ x=\frac{bt^2+a}{t^2+1} \\ dx=\frac{(b-a)2t}{(t^2+1)^2}\end{array}\right|=$$

$$\int\frac{\frac{2t(b-a)}{(t^2+1)^2}dt}{\left(b-\frac{bt^2+a}{t^2+1}\right)t}=\int\frac{\frac{2(b-a)}{(t^2+1)^2}dt}{\frac{bt^2+b-bt^2-a}{t^2+1}}=\int\frac{2dt}{t^2+1}=2\arctan t+C=2\arctan\sqrt{\frac{x-a}{b-x}}+C$$

$$\int\frac{\sqrt{x+1}-\sqrt{x-1}}{\sqrt{x+1}+\sqrt{x-1}}dx=\int\frac{\sqrt{\frac{x+1}{x-1}}-1}{\sqrt{\frac{x+1}{x-1}}+1}dx=\left|\begin{array}{c} \sqrt{\frac{x+1}{x-1}}=t \\ x=\frac{1+t^2}{t^2-1} \\ dx=\frac{-2\,2t}{(t^2-1)^2}\end{array}\right|=\int\frac{t-1}{t+1}\frac{-2\,2t}{(t^2-1)^2}dt=$$

$$=\int\frac{-4t}{(t+1)^3(t-1)}dt=\int\left(-\frac{1}{2\,(-1\ +\ t)}-\frac{2}{(1\ +\ t)^3}+\frac{1}{(1\ +\ t)^2}+\frac{1}{2\,(1\ +\ t)}\right)dt=$$

$$=-\frac{\ln(t-1)}{2}+\frac{1}{(1+t)^2}-\frac{1}{t+1}+\frac{\ln(t+1)}{2}+C=\frac{1}{(1+t)^2}-\frac{1}{t+1}-\frac{1}{2}\ln\frac{t-1}{t+1}+C=$$

$$=\frac{x-1}{\left(\sqrt{x-1}+\sqrt{x+1}\right)^2}-\frac{\sqrt{x-1}}{\sqrt{x+1}+\sqrt{x-1}}-\frac{1}{2}\ln\left|\frac{\sqrt{x+1}-\sqrt{x-1}}{\sqrt{x+1}+\sqrt{x-1}}\right|+C \qquad x\geq 1$$

$$\int \sqrt[4]{\frac{x-4}{x+4}}\,dx = \left| \begin{matrix} & t = \sqrt[4]{\frac{x-4}{x+4}} & \\ x = \frac{-4-4t^4}{t^4-1} & & dx = \frac{(8)4t^3}{(t^4-1)^2} \end{matrix} \right| = \int \frac{32t^4 dt}{(t^4-1)^2}$$

$$\frac{t^4}{(t^4-\ 1)^2} = \frac{1}{16\,(-1\ +\ t)^2} + \frac{1}{16\,(-1\ +\ t)} + \frac{1}{16\,(1\ +\ t)^2} - \frac{1}{16\,(1\ +\ t)} + \frac{1}{4\,(1\ +\ t^2)^2} - \frac{1}{4\,(1\ +\ t^2)}$$

$$\int \frac{32t^4}{(t^4-1)^2}dt = \left| \begin{matrix} u' = \frac{4t^3}{(t^4-1)^2} & u = -\frac{1}{t^4-1} \\ v = t & v' = 1 \end{matrix} \right| = -\frac{8t}{t^4-1} + \int \frac{8dt}{t^4-1} =$$

$$= -\frac{8\sqrt[4]{\frac{x-4}{x+4}}}{\frac{x-4}{x+4}-1} - 4\arctan\sqrt[4]{\frac{x-4}{x+4}} + 2\log\left(\frac{1-\sqrt[4]{\frac{x-4}{x+4}}}{1+\sqrt[4]{\frac{x-4}{x+4}}}\right) + C$$

$$= (x+4)\sqrt[4]{\frac{x-4}{x+4}} - 4\arctan\sqrt[4]{\frac{x-4}{x+4}} + 2\log\left(\frac{\sqrt[4]{x+4}-\sqrt[4]{x-4}}{\sqrt[4]{x+4}+\sqrt[4]{x-4}}\right) + C$$

$$\int \frac{1-\sqrt{x+1}}{1+\sqrt[3]{x+1}}dx = \left| \begin{matrix} x+1 = y \\ dx = dy \end{matrix} \right| = \int \frac{1-\sqrt{y}}{1+\sqrt[3]{y}}dx = \left| \begin{matrix} \sqrt[6]{y} = z \\ y = z^6 \\ dy = 6z^5 \end{matrix} \right| = \int \frac{1-z^3}{1+z^2}6z^5 dz =$$

$$= 6\int \left(1 - z - z^2 + z^3 + z^4 - z^6 + \frac{z}{1+z^2} - \frac{1}{1+z^2}\right)dz =$$

$$= 6\left(1z - \frac{z^2}{2} - \frac{z^3}{3} + \frac{z^4}{4} + \frac{z^5}{5} - \frac{z^7}{7} + \frac{\log(1+z^2)}{2} - \arctan z + C\right) =$$

$$= 6\left(\sqrt[6]{x+1} - \frac{\sqrt[3]{x+1}}{2} - \frac{\sqrt{x+1}}{3} + \frac{\sqrt[3]{x+1}^2}{4} + \frac{\sqrt[6]{x+1}^5}{5} - \frac{\sqrt[6]{x+1}^7}{7} + \frac{\log\left(1+\sqrt[3]{x+1}\right)}{2} - \arctan\sqrt[6]{x+1}\right)$$

$$\int \frac{x}{\sqrt[4]{x^3(1-x)}}dx = \int \frac{x}{\sqrt[4]{\frac{x^4(1-x)}{x}}}dx = \int \sqrt[4]{\frac{x}{1-x}}dx = \left| \begin{matrix} t = \sqrt[4]{\frac{x}{1-x}} \\ x = \frac{t^4}{t^4+1} \\ dx = \frac{4t^3}{(t^4+1)^2} \end{matrix} \right| = \int \frac{4t^4}{(t^4+1)^2}dx$$

$$\int \frac{4t^4}{(t^4+1)^2}dx = \left| \begin{matrix} u' = \frac{4t^3}{(t^4+1)^2} & u = -\frac{1}{t^4+1} \\ v = t & v' = 1 \end{matrix} \right| = -\frac{t}{t^4+1} + \int \frac{dt}{t^4+1} =$$

$$-\frac{t}{t^4+1} + \frac{1}{4\sqrt{2}}\left(\ln\frac{t^2+\sqrt{2}t+1}{t^2-\sqrt{2}t+1} + 2\arctan\left(\sqrt{2}t+1\right) + 2\arctan\left(\sqrt{2}t-1\right)\right)$$

$$\int \frac{x}{(1+x)(1-x^3)\sqrt{\frac{1-x}{1+x}}}dx = \left| \begin{array}{c} \sqrt{\frac{1-x}{1+x}} = t \\ x = \frac{-t^2+1}{t^2+1} \\ dx = \frac{(-2)2tdt}{(t^2+1)^2} \end{array} \right| = \int \frac{\frac{-t^2+1}{t^2+1}\frac{(-2)2t}{(t^2+1)^2}}{\left(1+\frac{-t^2+1}{t^2+1}\right)\left(1-\left(\frac{-t^2+1}{t^2+1}\right)^3\right)t} dt =$$

$$= \int \frac{\frac{-t^2+1}{t^2+1}\frac{(-2)2}{(t^2+1)^2}}{\left(\frac{t^2+1-t^2+1}{t^2+1}\right)\left(\frac{t^6+3t^4+3t^2+1}{(t^2+1)^3} - \frac{1-3t^2+3t^4-t^6}{(t^2+1)^3}\right)} dt =$$

$$= -4\int \frac{-t^2+1}{\left(\frac{2}{t^2+1}\right)(2t^6+6t^2)}dt = \int \frac{(t^2+1)(t^2-1)}{(t^6+3t^2)}dt = \int \frac{t^4-1}{t^2(t^4+3)}dt = \int \left(\frac{4t^2}{3(t^4+3)} - \frac{1}{3t^2}\right)dt$$

$$= \frac{x^2}{x^4+1} = \frac{x}{2\sqrt{2}\left(x^2-\sqrt{2}x+1\right)} - \frac{x}{2\sqrt{2}\left(x^2+\sqrt{2}x+1\right)}$$

$$\int \frac{4t^2}{3(t^4+3)}dt = \frac{1}{3\sqrt{2}\sqrt[4]{3}}\left(2\arctan\left(1+\frac{\sqrt{2}t}{\sqrt[4]{3}}\right) - 2\arctan\left(1+\frac{\sqrt{2}t}{\sqrt[4]{3}}\right) + \log\left|\frac{3-\sqrt{2}\sqrt[4]{3}t+\sqrt{3}t^2}{3+\sqrt{2}\sqrt[4]{3}t+\sqrt{3}t^2}\right|\right)$$

$$\int \frac{dx}{x\sqrt{x^2+x+1}} = \left| \begin{array}{c} \sqrt{x^2+x+1} = t-x \\ x^2+x+1 = t^2-2xt+x^2 \\ x(1+2t) = t^2-1 \\ x = \frac{t^2-1}{1+2t} \quad dx = \frac{2t^2+2t+2}{(1+2t)^2}dt \\ \sqrt{x^2+x+1} = t - \frac{t^2-1}{1+2t} = \frac{t^2+t+1}{2t+1} \end{array} \right| = \int \frac{\frac{2t^2+2t+2}{(1+2t)^2}}{\frac{t^2-1}{1+2t}\left(\frac{t^2+t+1}{2t+1}\right)}dt =$$

$$= \int \frac{2}{t^2-1}dt = \ln\left|\frac{t-1}{t+1}\right| + C = \ln\left|\frac{\sqrt{x^2+x+1}+x-1}{\sqrt{x^2+x+1}+x+1}\right| + C$$

$$\int \frac{xdx}{\sqrt{x^2+x+1}} = \left| \begin{array}{c} \sqrt{x^2+x+1} = t-x \\ x^2+x+1 = t^2-2xt+x^2 \\ x(1+2t) = t^2-1 \\ x = \frac{t^2-1}{1+2t} \quad dx = \frac{2t^2+2t+2}{(1+2t)^2}dt \\ \sqrt{x^2+x+1} = t - \frac{t^2-1}{1+2t} = \frac{t^2+t+1}{2t+1} \end{array} \right| = \int \frac{\frac{t^2-1}{1+2t}\frac{2t^2+2t+2}{(1+2t)^2}}{\left(\frac{t^2+t+1}{2t+1}\right)} =$$

$$= \int \frac{2t^2-1}{(1+2t)^2}dt = \int \left(\frac{1}{2} - \frac{1}{2t+1} - \frac{1}{2(1+2t)^2}\right)dt = \sqrt{1+x+x^2} + \frac{1}{2}\ln\left|-1-2x+2\sqrt{1+x+x^2}\right|$$

$$\int \frac{x^2+1}{x\sqrt{x^4+1}}dx = \left|\begin{matrix} x^2 = y \\ x\,dx = dy\end{matrix}\right| = \int \frac{y+1}{y\sqrt{y^2+1}}dy = \left|\begin{matrix} \sqrt{y^2+1} = t - y \\ y^2+1 = t^2 - 2yt + y^2 \\ y2t = t^2 - 1 \\ y = \dfrac{t^2-1}{2t} \quad dx = \dfrac{t^2+1}{t^2}dt \\ \sqrt{y^2+1} = t - \dfrac{t^2-1}{2t} = \dfrac{t^2+1}{2t}\end{matrix}\right| =$$

$$= \int \frac{\frac{t^2-1}{2t}+1}{\frac{t^2-1}{2t}\frac{t^2+1}{2t}}\frac{t^2+1}{t^2}dt = \int \frac{t^2-1+2t}{2t(t^2-1)}dt = \int \left(\frac{1}{2t} + \frac{1}{2(t-1)} - \frac{1}{2(t+1)}\right)dt$$

$$\int \frac{1dx}{(x^2+2)\sqrt{2x^2-2x+5}} = \left|\begin{matrix} \sqrt{2x^2-2x+5} = t - \sqrt{2}x \\ 2x^2-2x+5 = t^2 - 2\sqrt{2}xt + 2x^2 \\ -2x\left(1-\sqrt{2}t\right) = t^2 - 5 \\ x = \dfrac{t^2-5}{2\left(\sqrt{2}t-1\right)} \quad dx = \dfrac{5\sqrt{2}-2t+\sqrt{2}t^2}{\left(\sqrt{2}-2t\right)^2}dt \\ \sqrt{2x^2-2x+5} = t - \sqrt{2}\dfrac{t^2-5}{2\left(\sqrt{2}t-1\right)} = \dfrac{-10+2\sqrt{2}t-2t^2}{2\left(\sqrt{2}t-1\right)} \\ x^2+2 = \left(\dfrac{t^2-5}{2\left(\sqrt{2}t-1\right)}\right)^2 + 2 = \dfrac{33-16\sqrt{2}t+6t^2+t^4}{2(\sqrt{2}-2t)^2}\end{matrix}\right| =$$

$$= \int \frac{\frac{5\sqrt{2}-2t+\sqrt{2}t^2}{\left(\sqrt{2}-2t\right)^2}dt}{\frac{33-16\sqrt{2}t+6t^2+t^4}{2(\sqrt{2}-2t)^2}\frac{-10+2\sqrt{2}t-2t^2}{2\left(\sqrt{2}t-1\right)}} = \int \frac{\frac{1}{-\sqrt{2}}dt}{\frac{33-16\sqrt{2}t+6t^2+t^4}{2}\frac{1}{2\left(\sqrt{2}t-1\right)}} =$$

$$\int \frac{x^3}{(1+x)\sqrt{1+2x-x^2}}dx = \int \frac{x^3}{(1+x)\sqrt{\left(1+\sqrt{2}-x\right)\left(x-1+\sqrt{2}\right)}}dx =$$

$$= \left|\begin{matrix} t = \sqrt{\dfrac{\left(1+\sqrt{2}-x\right)}{\left(x-1+\sqrt{2}\right)}} \quad x = \dfrac{1+\sqrt{2}+t^2\left(1-\sqrt{2}\right)}{1+t^2} \\ dx = -\dfrac{4\sqrt{2}t}{(1+t^2)^2}\end{matrix}\right| = \int \frac{2\left(1+\sqrt{2}-\left(-1+\sqrt{2}\right)t^2\right)^3}{(1+t^2)^3\left(-2-\sqrt{2}+\left(-2+\sqrt{2}\right)t^2\right)}dt =$$

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$$\int \frac{dx}{(x^2+1)\sqrt{x^2-1}} = \frac{\log\left|\frac{\sqrt{2}+1+x^2-x\sqrt{x^2-1}}{\sqrt{2}-1-x^2+x\sqrt{x^2-1}}\right|}{2\sqrt{2}} + C$$

$$\int \frac{\sqrt{x^2+x+1}}{x^2+2x+1}dx = -\frac{\sqrt{x^2+x+1}}{x+1} - \log\left|2\sqrt{x^2+x+1}-2x-1\right| + \frac{1}{2}\log\left|\frac{\sqrt{x^2+x+1}-x}{2+x-\sqrt{x^2+x+1}}\right| + C$$

$$\int \frac{dx}{(x+1)^5\sqrt{x^2+2x}} = \frac{1}{8}\left(\frac{\sqrt{x^2+2x}(3x^2+6x+5)}{(1+x)^4} + 3\arctan\sqrt{x^2+2x}\right) + C$$

$$\int x^2\sin^2 x\,dx = \frac{x^3}{6} - \frac{x\cos 2x}{4} + \frac{\sin 2x}{8} - \frac{x^2\sin 2x}{4} + C$$

$$\int \frac{x}{\cos^2 x}dx = \log|\cos x| + x\tan x + C$$

$$\int 2\arctan\frac{x}{2}dx = 2x\arctan\frac{x}{2} - 2\log\left(\left(\frac{x}{2}\right)^2 + 1\right) + C$$

$$\int \arcsin x\,dx = x\arcsin x + \sqrt{1-x^2} + C$$

$$\int \frac{\arccos x}{x^2}dx = \log\left|\frac{1+\sqrt{1-x^2}}{x}\right| - \frac{\arccos x}{x} + C$$

$$\int \ln\left(\sqrt{x+1}-\sqrt{x-1}\right)dx = x\ln\left(\sqrt{x+1}-\sqrt{x-1}\right) + \frac{\sqrt{x^2-1}}{2} + C$$

$$\int \frac{\ln\arctan x}{1+x^2}dx = \arctan x\,(\ln|\arctan x| - 1) + C$$

$$\int \frac{\tan\frac{1}{x}}{x^2}dx = \ln|\cos x| + C$$

$$\int \frac{dx}{\cos^3 x} = \frac{\sin x}{2\cos^2 x} + \frac{1}{4}\ln\left|\frac{1+\sin x}{1-\sin x}\right| + C$$

$$\int \frac{x^2}{(8x^3+27)^{\frac{2}{3}}}dx = \frac{\sqrt[3]{8x^3+27}}{8} + C$$

$$\int \frac{\sin x + \cos x}{\sqrt[3]{\sin x - \cos x}}dx = \frac{3}{2}\sqrt[3]{(\sin x - \cos x)^2} + C$$

$$\int \frac{dx}{\sqrt{e^x - 1}} = 2\arctan\sqrt{e^x - 1} + C$$

$$\int \frac{dx}{2+\sin x} = \frac{2}{\sqrt{3}}\arctan\left(\frac{1+2\tan\frac{x}{2}}{\sqrt{3}}\right) + C_{((2k-1)\pi,(2k+1)\pi)}$$

$$C_{((2k-1)\pi,(2k+1)\pi)} = \frac{2\pi}{\sqrt{3}}k$$

$$\int \frac{dx}{(2+\cos x)\sin x} = \int \frac{\sin x\, dx}{(2+\cos x)(1-\cos^2 x)} = \frac{\ln|2+\cos x|}{3} - \frac{\ln|\cos x + 1|}{2} + \frac{\ln|1-\cos x|}{6} + C$$

$$\int \frac{\sin x \cos x}{\sin x + \cos x}dx = \left|\begin{matrix} \tan\frac{x}{2} = t \\ dx = \frac{2dt}{t^2+1}\end{matrix}\right| = \int\left(\frac{2(1+t)}{(1+t^2)^2} - \frac{1}{1+t^2} + \frac{1}{t^2-2t-1}\right)dt =$$

$$\frac{t-1}{t^2+1} + \frac{\sqrt{2}}{4}\ln\left|\frac{\sqrt{2}-t+1}{\sqrt{2}+t-1}\right| + C$$

$$\int \frac{\cos x}{2\sin x - 3\cos x + 6}dx = -\frac{3x}{13} + \frac{36}{13\sqrt{23}}\arctan\left(\frac{2}{\sqrt{23}} + \frac{9\tan\frac{x}{2}}{\sqrt{23}}\right) + \frac{2}{13}\ln|6 - 3\cos x + 2\sin x| + C$$

$$C_{((2k-1)\pi,(2k+1)\pi)} = \frac{36\pi}{13\sqrt{23}}$$

$$\int \frac{dx}{\sin x} = \int \frac{\sin x\, dx}{1-\cos^2 x} = |\quad| = \int \frac{dt}{1-t^2} = \frac{1}{2}\ln\left|\frac{1-\cos x}{1+\cos x}\right| + C$$

$$\int \frac{\sin^2 x}{1+\sin^2 x}dx = \arctan(\tan x) - \frac{\arctan\left(\sqrt{2}\tan x\right)}{\sqrt{2}} + C_{\left(\frac{(2k-1)}{2}\pi,\frac{(2k+1)}{2}\pi\right)}$$

$$C_{\left(\frac{(2k-1)}{2}\pi,\frac{(2k+1)}{2}\pi\right)}=\left(1-\frac{1}{\sqrt{2}}\right)\pi k$$

$$\int \tan^5 x\,dx=\left|\begin{matrix} t=\tan x \\ dx=\dfrac{dt}{1+t^2}\end{matrix}\right|=\int \frac{t^5}{1+t^2}dt=\frac{\tan^4 x}{4}-\frac{\tan^2 x}{2}-\ln|\cos x|+C$$

$$\int \frac{1-\tan x}{1+\tan x}dx=\log|\cos x+\sin x|+C$$

$$\int \frac{1+\tan^2 x}{2+\tan x}dx=\ln|2+\tan x|+C$$