$$1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$
$$1^2 = \frac{1(1+1)(2+1)}{6}$$
$$1 = 1$$
$$1^2 + 2^2 + \cdots + n^2 + (n+1)^2 = \frac{(n+1)(n+2)(2n+3)}{6}$$
$$\frac{n(n+1)(2n+1)}{6} + (n+1)^2 = \frac{(n+1)(n+2)(2n+3)}{6}$$
$$2n^3 + 3n^2 + n + 6n^2 + 12n + 6 = 2n^3 + 9n^2 + 13n + 6$$
$$2n^3 + 13n + 9n^2 + 6 = 2n^3 + 9n^2 + 13n + 6$$

$$1^3 + 2^3 + \cdots + n^3 = (1 + 2 + \cdots + n)^2$$
$$1^3 = 1^2$$
$$1^3 + 2^3 + \cdots + n^3 + (n+1)^3 = (1 + 2 + \cdots + n + n + 1)^2$$
$$(1 + 2 + \cdots + n)^2 + (n+1)^3 = (1 + 2 + \cdots + n + n + 1)^2$$
$$\left(\frac{(1+n)n}{2}\right)^2 + (n+1)^3 = \left(\frac{(1+n+1)(n+1)}{2}\right)^2$$
$$\left(\frac{n+n^2}{2}\right)^2 + n^3 + 3n^2 + 3n + 1 = \left(\frac{n^2+3n+2}{2}\right)^2$$
$$n^2 + n^4 + 2n^3 + 4n^3 + 12n^2 + 12n + 4 = n^4 + 6n^3 + 13n^2 + 12n + 4$$

$$\prod_{i=1}^{n}(1+x_i) \geq 1 + \sum_{i=1}^{n} x_i\,, x_i \geq -2, x_i \textit{ mají stejná znaménka}$$
$$1 + x_1 \geq 1 + x_1$$
$$\prod_{i=1}^{3}(1+x_i) \geq 1 + \sum_{i=1}^{3} x_i$$
$$\prod_{i=1}^{n}(1+x_i)\,(1+x_{i+1}) \geq 1 + \sum_{i=1}^{n} x_i + x_{i+1}$$
$$\prod_{i=1}^{n}(1+x_i)\,(1+x_{i+1}) \geq \prod_{i=1}^{n}(1+x_i) + x_{i+1}$$
$$\prod_{i=1}^{n}(1+x_i)\,(1+x_{i+1}) - \prod_{i=1}^{n}(1+x_i) \geq x_{i+1}$$
$$\prod_{i=1}^{n}(1+x_i)\,x_{i+1} \geq x_{i+1}$$
$$\prod_{i=1}^{n}(1+x_i) \geq 1 \qquad \prod_{i=1}^{n}(1+x_i) \leq 1$$

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$
$$a+b = a+b$$
$$(a+b)^{n+1} = \sum_{k=0}^{n+1} \binom{n+1}{k} a^{n+1-k} b^k$$
$$(a+b)^n(a+b) = \sum_{k=0}^{n+1} \binom{n+1}{k} a^{n+1-k} b^k$$
$$\left(\sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k\right)(a+b) = \sum_{k=0}^{n} \binom{n}{k} a^{n+1-k} b^k + \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^{k+1}$$
$$\sum_{\mathrm{i}=0}^{n} \binom{n}{\mathrm{i}} a^{n+1-i} b^{\mathrm{i}} + \sum_{j=0}^{n} \binom{n}{\mathrm{j}} a^{n-\mathrm{j}} b^{\mathrm{j}+1} = a^{n+1} + \sum_{\mathrm{i}=1}^{n} \binom{n}{\mathrm{i}} a^{n+1-i} b^{\mathrm{i}} + \sum_{j=0}^{n-1} \binom{n}{\mathrm{j}} a^{n-\mathrm{j}} b^{\mathrm{j}+1} + \mathrm{b}^{\mathrm{n}+1}$$
$$a^{n+1} + \sum_{\mathrm{k}=1}^{n} \binom{n}{\mathrm{k}} a^{n+1-\mathrm{k}} b^{\mathrm{k}} + \sum_{k=1}^{n} \binom{n}{\mathrm{k}-1} a^{n-\mathrm{k}+1} b^{\mathrm{k}} + \mathrm{b}^{\mathrm{n}+1} = a^{n+1} + \sum_{k=1}^{n} \binom{n+1}{\mathrm{k}} a^{n-\mathrm{k}+1} b^{\mathrm{k}} + \mathrm{b}^{\mathrm{n}+1}$$
$$\sum_{k=0}^{n} \binom{n+1}{\mathrm{k}} a^{n-\mathrm{k}+1} b^{\mathrm{k}}$$

$$\sum_{k=0}^{n} \binom{n}{k} 1^k 1^{n-k} = (1+1)^n$$
$$\sum_{k=0}^{n} \binom{n}{k} = 2^n$$

Dokážu pro i = 1,2. 2n a n-1

$$\sqrt[n]{x_1 \cdot \ldots \cdot x_n} \leq \frac{1}{n}(x_1 + \cdots + x_n), x_i \geq 0, i = 1,2,\ldots,n$$
$$x_1 \leq x_1$$
$$\sqrt{x_1 x_2} \leq \frac{x_1 + x_2}{2}$$
$$4x_1x_2 \leq x_1^2 + 2x_1x_2 + x_2^2$$
$$0 \leq (x_1 - x_2)^2$$
$$a_1 \geq g_1 \ \cap \ a_2 \geq g_2$$
$$\frac{a_1 + a_2}{2} \geq \frac{g_1 + g_2}{2} \geq \sqrt{g_1 g_2}$$

Důkaz pro $n-1$ zde chybí. Můžete si zkusit rozmyslet. Jedná se o trik kdy k nerovnosti o $n-1$ členech přidám člen který má hodnotu právě aritmetického průměru

$$(2n)! < 2^{2n}(n!)^2$$
$$2! < 2^2 1^2$$
$$(2n+2)! < 2^{2n+2}\big((n+1)!\big)^2$$
$$(2n+2)(2n+1)(2n)! < 2^{2n}2^2(n+1)^2(n!)^2$$
$$(2n+2)(2n+1)(2n)! < (2n)!\,4(n+1)^2$$
$$(2n+2)(2n+1) < 4(n+1)(n+1)$$
$$4n^2 + 6n + 2 < 4n^2 + 8n + 4$$
$$0 < 2n + 2$$

$$n! \leq \left(\frac{n+1}{2}\right)^n$$
$$1! \leq 1^1$$
$$(n+1)! \leq \left(\frac{n+2}{2}\right)^{n+1}$$
$$(n+1)n! \leq \frac{n+2}{2}\frac{(n+2)^n}{2^n}$$
$$(n+1)\left(\frac{n+1}{2}\right)^n \leq \frac{n+2}{2}\frac{(n+2)^n}{2^n}$$
$$(n+1)\frac{(n+1)^n}{2^n} \leq \frac{n+2}{2}\frac{(n+2)^n}{2^n}$$
$$2(n+1)^{n+1} \leq (n+2)^{n+1}$$
$$2 \leq \frac{(n+2)^{n+1}}{(n+1)^{n+1}}$$
$$2 \leq \left(\frac{n+2}{n+1}\right)^{n+1}$$
$$2 \leq \left(1+\frac{1}{n+1}\right)^{n+1}$$
$$2 \leq \sum_{i=0}^{n+1}\binom{n+1}{i}\left(\frac{1}{n+1}\right)^i$$
$$2 \leq 1+\frac{n+1}{n+1}+\sum_{i=2}^{n+1}\binom{n+1}{i}\left(\frac{1}{n+1}\right)^i$$
$$2 \leq 2+\sum_{i=2}^{n+1}\binom{n+1}{i}\left(\frac{1}{n+1}\right)^i$$

$$\left|\sin\left(\sum_{k=1}^{n} x_k\right)\right| \leq \sum_{k=1}^{n} \sin x_k,\ x_k \in [0,\pi],\ k = 1,2\ldots,n$$

$$\left|\sin\left(\sum_{k=1}^{n} x_k\right)\right| \leq \sum_{k=1}^{n} \sin x_k\ , \qquad x_k \in [0,\pi], \qquad k = 1,2\ldots,n$$
$$|\sin x| \leq \sin x$$
$$\left|\sin\left(\sum_{k=1}^{n} x_k + x_{k+1}\right)\right| \leq \sum_{k=1}^{n} \sin x_k + \sin x_{k+1}$$
$$\left|\sin\left(\sum_{k=1}^{n} x_k\right)\cos x_{k+1} + \cos\left(\sum_{k=1}^{n} x_k\right)\sin x_{k+1}\right| \leq \sum_{k=1}^{n} \sin x_k + \sin x_{k+1}$$
$$\left|\sin\left(\sum_{k=1}^{n-1} x_k\right)\cos(x_k + x_{k+1}) + \cos\left(\sum_{k=1}^{n-1} x_k\right)\sin(x_k + x_{k+1})\right|$$
$$\leq \left|\sin\left(\sum_{k=1}^{n-1} x_k\right)\cos x_{k+1} + \cos\left(\sum_{k=1}^{n-1} x_k\right)\sin x_{k+1}\right| + \sin x_{k+1}$$

$$\prod_{i=1}^{n}\frac{2i-1}{2i}<\frac{1}{\sqrt{2n+1}}$$
$$\frac{1}{2}<\frac{1}{\sqrt{3}}$$
$$\prod_{i=1}^{n}\frac{2i-1}{2i}\cdot\frac{2n+1}{2n+2}<\frac{1}{\sqrt{2n+3}}$$
$$\frac{1}{\sqrt{2n+1}}\frac{2n+1}{2n+2}<\frac{1}{\sqrt{2n+3}}$$
$$\frac{2n+1}{2n+2}<\frac{\sqrt{2n+1}}{\sqrt{2n+3}}$$
$$(2n+1)(2n+3)<(2n+2)^2$$
$$4n^2+8n+3<4n^2+8n+4$$

$$n^{n+1}>(n+1)^n$$
$$nn^n>(n+1)^n$$
$$n>\frac{(n+1)^n}{n^n}$$
$$n>\left(\frac{n+1}{n}\right)^n$$
$$n>\left(1+\frac{1}{n}\right)^n$$
$$3^4>4^3$$
$$(n+1)^{n+2}>(n+2)^{n+1}$$
$$(n+1)(n+1)^{n+1}>(n+2)^{n+1}$$
$$n+1>\frac{(n+2)^{n+1}}{(n+1)^{n+1}}$$
$$n+1>\left(1+\frac{1}{n+1}\right)^{n+1}$$