THE PLAN FOR THE DEVELOPMENT OF A RESEARCH AREA (COOPERATIO PROGRAM)

name of the research area:	**Mathematics**
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1/ description of the current state of the research area, reflection of the internal evaluation, SWOT analysis

- Briefly describe the starting point in which the given area finds itself, in terms of personnel (incl. doctoral students and postdocs), material and workspace conditions, internationalisation, internal university cooperation etc.
- Reflection of internal evaluation conclusions and recommendations related to the specific research area and its relevant fields, review of their feasibility (if available for your research area)
- If no relevant recommendations were given by the evaluators, or if you disagree with the evaluation outcomes, please prepare a SWOT analysis of the given research area

State of the research area

The research in Mathematics at Charles University is carried out mostly in the School of Mathematics (SM) of the Faculty of the Mathematics and Physics. SM consists of 6 departments with 23 full professors, 43 associate professors and 37 assistant professors. This is 103 tenure track positions in total, with full and associate professors having permanent positions. The number of tenure track positions has been stable during the last ten years. 7 assistant professors are foreigners and obtained their PhD degrees abroad; most of them were hired in last few years (since 2015, 7 young assistant professors have been hired, among them 5 foreigners). Moreover, there are 7 Czech professors who obtained their PhD degrees abroad.

In addition to the above-mentioned numbers, the School of Mathematics currently employs 32 postdocs who are paid by various projects (e.g., ERC, ERC-CZ, GACR, START). The number of postdoc positions naturally depends on the success in the obtaining the projects, but we are happy to say that the number has been increasing in recent years. Most of the postdocs obtained their PhD degrees abroad.

School of Mathematics guarantees 6 PhD programs in various areas of mathematics; the total number of PhD students is currently 74. This number has been slightly growing in recent years. The number of PhD students from abroad has increased: 2 students in 2017, 4 students in 2018, 7 students in 2019, 5 students in 2020 and 4 students in 2021. Since the academic year 2013/2014, 132 PhD students have defended their PhD theses; among them 66 (50%) defended their theses within 4+1 years. This ratio has slightly improved in the last years: since the academic year 2019/2020, 21 PhD theses have been defended and among them 12 (57%) within 5 years of the start of the study.

The research in Mathematics has also been carried out by a small group at the Department of Logic at the Faculty of Arts of the Charles University. Currently, there are one associate professor, one assistant professor and potentially a few PhD students.

The above-mentioned data indicate that the internationalization of the research area has started some years ago and continues to grow. An extended research stay abroad is a very important criterion in the hiring process of the new academic staff.

The cooperation within the university is mostly based on individual collaboration of researchers, namely within the Faculty of Mathematics and Physis (School of Physics, School of Informatics) and some other faculties of CU (Faculties of Medicine - statistics, Faculty of Social Sciences & CERGE – financial mathematics and econometric, Faculty of Pharmacy in Hradec Králové - mathematics, Faculty of Arts in Prague – discrete and structural mathematics).

The material conditions are sufficient, the library (including on-line accesses) is well-equipped, the computers and other equipment are regularly replaced by modern ones. The largest limitation is the lack of the office spaces; hence we must rent the commercial ones.

Reflection of internal evaluations conclusions

The internal evaluation report (Mathematics Evaluation Report) gave several recommendations which are given in the following table together with our reflection.

Evaluation	Reflection
Overall research strategies should be defined and regularly updated in the various areas to react on new developments and opportunities and to improve their visibility. Also, more efforts should be made to leave the comfort zone of mainstream research and to move towards more risky and cutting-edge topics.	The overall research strategy plan is the part of this plan. We agree that *"more efforts should be made to leave the comfort zone of mainstream research"*. However, we suppose that the current effort was not reflected properly in the Mathematics Evaluation Report, we gave several examples in the documents "*Comments to University Unit Evaluation Report"*.
Positions for leading scientists/professors and postdocs should be filled in an open and transparent international competition.	This process has already started a few years ago. Some relevant data are given above.
Support is necessary for research fellows who won large international grants to succeed with their research and stay at CU. They are the talents CU needs to keep. Priority should be given to their needs for personnel and laboratory/office space.	This support has started as well. Each department which receives an ERC project or ERC-CZ grants obtains additional budget in order to reduce the pedagogical duties of the successful applicant. We rent new office spaces for the holders of larger grants including GACR EXPRO and GACR Junior.
Efforts should be strengthened to increase the attractiveness of PhD positions for foreign students. Salaries (if they exist at all) are small, and even when augmented by scholarships, the PhD positions at CU are not yet fully competitive internationally. The times needed to finish a PhD thesis are much too long on average, partially due to the fact that PhD students have to earn money in parallel.	Making use of internal as well as external resources, the aim is to double the scholarship for each active PhD student. We completely agree that the number of PhD students who defended their theses in the prescribed period is low and should be improved.
We strongly recommend launching a Strategic Advisory Board at the School level, composed of scholars from foreign institutions that would provide help and advice in defining top priorities.	We expect that the coordination committee of the Cooperation program will play the role of the Strategic Advisory Board. Currently, we do not intend to include foreign experts, but this prospect is open.
The efforts to help scientists in raising major international funds, particularly from the European H2020 and follow-up programs, should be continued and if possible intensified.	This is an important and strategic point: there are several ways of supporting the applications, both on the level of university and on the level of faculty. The potential applicants participate in the ERC pipeline: they present their proposals in

	front of the coordination committee of the Cooperation program supplemented by experts in the relevant research areas. In order to reduce the administration of the applications and the administration of successful projects, we hired a grant manager.

2/ main research directions and objectives

- What are the main research directions you will focus on?
- Where do you plan to get with the research in the next five years?
- Do you have a benchmark institution (a faculty or a university institute) the quality of which you would like to approach over the next years?

Main research directions

According to the classification of the Charles University, the research area Mathematics consists of the following fields: Mathematical Analysis, Mathematical and Computer Modelling, Probability and Mathematical Statistics, Structural Mathematics. However, the boundaries among these fields are not strict and moreover, some fields intersect with other research fields such as physics, computing, economics, medicine.

Mathematical Analysis

The research in Analysis at our faculty can be divided into *Geometric Analysis, Function Spaces, Functional and Real Analysis* and *Partial Differential Equations* with natural cooperation between these fields.

In the field of *Geometric Analysis,* we plan to extend our study of mappings that can serve as deformations in Nonlinear Elasticity. Special emphasis will be given to injectivity of these maps and to the approximation and extension of homeomorphisms in the Sobolev class. We plan to study this Ball-Evans approximation problem in the plane for models that involve second derivative and to start the study of extension properties of homeomorphisms in dimension n=3. Special attention will be also given to the study of interaction of solids and fluids.

In the field of *Function Spaces,* we plan to study fractional Sobolev spaces, Banach spaces generated by quasinorm, interpolation characterization of Gauss embedding using Calderon operators, regularity of solution of Ornstein-Uhlenbeck PDE and many other open problems. In the field of *Harmonic Analysis,* we plan to continue the study of singular operators with special emphasis on singular Brascamp-Lieb inequalities, where integrable functions are replaced by kernels with nonintegrable singularities. Further topics include multilinear singular operators and operators with rough kernels.

In the field of *Functional and Real Analysis,* we plan to focus on problems connecting various structures on Banach spaces. This includes both remaining problems in well-established areas (like smooth approximations in Banach spaces or differentiability properties of functions with weak convexity type properties) and rather new areas which either provide a new and a deeper view on known properties and result (like the vector-valued Choquet theory or the quantitative approach to Banach spaces including the study of measures of weak non-compactness in classical spaces) or reveal unexpected connections to other areas of mathematics (like the investigation of the algebraic structures on Banach spaces, including C*-algebras and Jordan structures, from the point of view of Banach space theory).

In the field of *Partial Differential Equations,* we plan to study the equations arising in continuum thermodynamics. We want to focus on the qualitative properties as well as on the existence theory for underlying problems. There are two main directions of the researcher, namely, the equations describing the fluid (incompressible/compressible/mixtures/viscoelastic fluid) and the equations describing the solid mechanics (elastic/plastic/viscoelastic/linearized elasticity/nonlinear elasticity) and possibly also the interaction between the solid and fluid phase. In particular, we want to collaborate with and use the results obtained by the *Function spaces* group (this should lead to proper regularity estimates) and by the *Geometric Analysis* group (to study the equations arising in Nonlinear elasticity and to study the properties of solutions).

Mathematical and Computer Modelling

The group of *Mathematical modelling* plan to investigate the problems connected to many real word applications. To name few of them, we focus on modelling in planetary physics, flows of blood in aorta, flows of blood near aneurysm, viscoelastic fluids (polymers), viscoelastic solids, flows of mixtures, modelling of porous media, modelling of radiative heating, modelling of hydrogen cells, flows of liquid helium (Helium cryogenics). The mentioned problem will be studied from many perspectives. First, the proper physically well sounded model must be developed, second it must be numerically tested and consequently designed to fit the available data and last, the model (typically the partial differential equations - PDEs) will be analysed. All these steps cannot be done without the support of other groups and therefore we will strongly benefit from the cooperation with them.

Efficient numerical solution of the mathematical models represented by PDEs is a complex problem consisting of several mutually connected parts: discretization technique, solvers for arising algebraic systems, error analysis and implementation aspects.

In the area of *discretization of PDEs* we intend to focus on modern higher-order methods which can be easily combined with algebraic solvers suitable for High performance computing (HPC). These techniques require typically higher number of degrees of freedom in comparison with traditional approaches, but this disadvantage is compensated by the possible use of more efficient algebraic solvers working on multiprocessors computers. We intend to deal not only with the simple linear models but to develop techniques for real-life (nonlinear) applications.

The efficiency of numerical methods strongly depends on *error estimates*, which allow to adaptively control the accuracy of the numerical approximations. The attention will be paid not only to rigorous numerical analysis but also on their applicability to realistic applications, namely from the point of view of robustness and the computational costs of the methods.

In the area of *matrix computations*, we focus on the analysis and development of efficient and reliable algorithms for the numerical solution of challenging matrix problems with large scale and often ill-conditioned matrices, including computational aspects. In particular, we focus on the analysis and development of iterative solvers and preconditioners for solving matrix problems arising from the discretization of PDE's, constrained optimization problems, or image processing. Concerning computational aspects, we are going to deal with the influence of various sources of inexactness in computations (rounding errors, mixed-precision, intentional inexactness) on the robustness and performance of algorithms and guarantees on the quality of the computed results.

High performance computing (HPC) represents a fast developed indivisible part of scientific areas and industrial applications. Issues related to the use of high-performance computer architectures form a part of our research. In the push toward exascale-level computational science, it will be necessary to redesign algorithms to exploit the massive parallelism of the hardware while considering numerical behaviour of the algorithms.

Probability and Mathematical Statistics

In the field of *statistics*, we will concentrate mainly on the analysis of multivariate and high-dimensional functional data, and on data in spaces without a linear structure such as directional data or data in more general manifolds. Joining the state-of-the-art research in different fields of mathematics, e.g., stochastics, mathematical analysis, geometry, and functional analysis, will be used for designing principally novel statistical procedures. Attention will be paid to developing both parametric and nonparametric methods based on characteristic functions, generalized ranks, and data depth, allowing, among others, to measure associations and dependencies for complex data, and to establish a new paradigm in modeling changes and disorder recognition under uncertainty. Identification of structural breaks, both abrupt and smooth, in functional data and in a series of random functions, will help to tie specific health, legal, or economic changes to the time epochs in which they occurred, and the environments or industries that were affected. Unsupervised classification of complex data structures and closely related machine learning tools will be also an integral part of our research. We will concentrate especially on model-based clustering.

In the field of *stochastic analysis*, research will be focused on the theory of stochastic infinite-dimensional systems, especially stochastic partial differential equations and stochastic differential systems on lattices. In particular, the equations of stochastic fluid dynamics will be studied. Special attention will be paid to non-Markovian solutions, in which case the driving noise is not white in time (such as fractional noise or Volterra processes) and a new approach to stochastic integration has to be developed. To this end, the theory of rough paths will be intensively studied. Some more applied problems for such systems will also be solved, such as control and parameter identification.

In the field of s*tochastic geometry*, new models for random spatial structures will be investigated, among them Gibbs particle processes with random interaction, point processes with special marking, and random marked tessellations. Particular attention will be paid to the asymptotic properties of the models. In statistical inference of spatial structures, challenging problems are Bayesian MCMC-based inference for inhomogeneous spatial structures, nonparametric tests of independence between point processes and covariates, and connections between spatial statistics and functional data analysis.

In the field of *econometrics and optimization*, we will study the theoretical properties of various econometric and optimization models and apply them in forecasting and optimal decision making under risk and uncertainty. Among these models, special attention will be paid to stochastic programming and robust optimization problems including their dynamic versions. Moreover, new problems with endogenous randomness will be analyzed and the sensitivity analysis will be performed. Finally, new algorithms for input selection and problem solving motivated by machine learning techniques will be constructed. Besides the theoretical analysis of econometric and optimization models, we plan to apply these models in various fields, mainly finance, logistics, waste management, production problems, etc.

In the field of *insurance*, we will concentrate on stochastic forecasting of future losses, mainly of nonfinancial nature, based on already reported, occurred but not reported, and yet not occurred subevents. Attention will be put to regime switching in empirical economics and inference for econometric marked processes. Financial mathematics tasks will include modeling, measuring and managing risks, valuation of derivatives as well as pricing problems. Special attention will be paid to financial econometric models.

Finally, we will continue in our broad multidisciplinary cooperation with researchers from biomedical, technical, economic, financial, and social sciences, fight against a ceaseless misinterpretation of data analyses and pursue the line of honest interpretation of the data.

Structural Mathematics

In the field of *representation theory*, we will cover a range of topics of algebraic and geometric nature. We will use tools of homological algebra in research of derived commutative algebra and derived algebraic geometry, and modern representation theory of rings and algebras and its connections to non-commutative geometry. At the same time, we will actively collaborate with experts both from the Czech Republic and abroad to move frontiers of the homological and homotopical methods themselves, using combinatorial and approximation-theoretic methods.

The main directions in *logic* and *general algebra* concern interactions with, and applications to other areas of mathematics and computer science. We will study proof complexity by methods of mathematical logic; we will study computational and descriptive complexity (of constraint problems, in particular) using universal algebra, combinatorics, and model theory; and, motivated by questions in theoretical computer science and cryptography, we will continue developing the structure theory of general algebras (finite, oligomorphic), non-associative algebras (loops, quasigroups, quandles), associative algebras (semigroups), and other algebraic structures (clones, minions) using techniques from universal algebra, category theory, group theory, and combinatorics.

Relatedly, in foundations of mathematics, specifically *set theory*, we will continue the study of compactness principles which can hold at small cardinals. These principles extend the usual compactness of the first-order logic to uncountable cardinals and have deep connections to model theory and descriptive set theory. Specifically, we will focus on compactness of (generalized) trees, and compactness related to closed unbounded sets (stationary reflection).

In *number theory and cryptography*, we will study the arithmetic of number fields using algebraic and analytic techniques. In particular, we will focus on topics related to the distribution of class numbers and to quadratic forms and lattices. We will also consider their cryptographic applications, as they are the top candidates for post-quantum cryptography. We will continue to work on cryptography based on finite fields in its various aspects.

In the broad field of geometry, we will focus mainly on the development of the following areas: *differential geometry* (properties of invariant differential operators on manifolds with a given symmetry structure), *contact geometry* (geography and botany of Legendrian submanifolds and their Lagrangian fillings), *symplectic geometry* (properties of Liouville and Weinstein domains and cobordisms by using the Floer theory of Lagrangian cobordisms, and the questions of mirror symmetry that appear in these settings), geometry defined and prescribed by *Cartan connection*, leading to a close relationship with the structural properties and representation theory of Lie groups and Lie algebras, *integral geometry* (in particular, the new development dealing with sets defined using DC functions), *hypercomplex analysis* (developing function theory for various systems of rotationally invariant differential equations).

In the related field of *mathematical physics*, we will concentrate on the interplay between geometry, algebra and physics. Higher and generalized geometric and algebraic structures appear naturally in quantum field theory and string theory where they play a prominent role in our understanding of the fundamental mathematical aspects of these theories their underlying (generalized and higher) symmetries. We will focus on homology and homotopy methods and tools relevant to modern mathematical physics.

Benchmark institutions

As the benchmark universities can serve the universities used in the Bibliometric Support for Evaluation of Research at Charles University 2014-2018, i.e., University of Copenhagen, University of Heidelberg, KU Leuven, University of Milan, University of Vienna, University of Warsaw.

Five years vision

We would like to increase the number of prestigious international projects such as ERC grants (currently, we have one ERC-starting grant and in the call from spring 2021, we have candidates in the second round). We would like to attract promising master and PhD students, and to hire excellent postdocs and researcher from all over the world.

The Mathematics Evaluation Report, which compares the research in mathematics at Charles University with the benchmark universities mentioned above, contains the sentence: "*The number of papers of high quality is comparable to that of these universities (with the exception of Vienna with a significantly higher number).*" Furthermore, according to the Shanghai Ranking, Mathematics at Charles University belongs among the best ranked subjects (rank 101-150 together with Physics, Metallurgical Engineering and Economics) http://www.shanghairanking.com/institution/charles-university-in-prague. Our goal is to improve this level of evaluations.

3/ set of activities and measures for development of the research area

- Briefly describe the planned activities and measures (incl. their frame schedule and monitored indicators), especially in the following areas:
  - Doctoral studies
  - Prevention of inbreeding
  - Postdocs
  - Publication in international environment
  - Internationalisation of the research area, including the acquisition of international students and researchers
- What forms of cooperation within Charles University, but also at the national and international level do you plan to use and set up within your research area?

Doctoral studies

There are two main goals which are mutually connected: increase the financial support of the PhD students and increase the ratio of students who defended their thesis in the standard period of study. In order to attract the international students, we have unified the study conditions in the Czech and English doctoral study programmes. The hiring of the PhD students will be internationally opened, it is necessary to indicate promising candidates during the entrance examination. Further, a regular evaluation of the fulfilling of the individual study plans has to be required. The available financial resources cannot be used for non-perspective students. Moreover, doctoral students should be involved and financially supported by the research projects. PhD students should stay some time abroad. Only the number of the defended PhD students within the prescribed period will be considered in the internal evaluation of departments. We also support the doctoral studies under double supervision (cotoutelle) based on agreements between Charles University and recognized European universities (e.g., 4EU + European University Alliance).

Prevention of inbreeding

Since couple of the last years, all new academic and research positions are filled in open and transparent international competitions. These positions are advertised in international, frequently visited platforms like mathjobs.org. A scientific experience abroad (typically postdoc positions) is a very important criterion of the hiring process and it is often viewed as a necessary condition. Another important criterion in the hiring process will be the potential to open new research directions.

Postdocs positions

These positions (excluding the tenure track positions) are usually related to the large research projects as ERC, ERC-CZ, GACR EXPRO, START, etc. They will be also filled in open and transparent international competitions but the main criterion for the hiring should be the relevance of the candidates' specializations and the possible contribution to the project. On the other hand, the postdoc positions covered by projects such as OPVVV program allow to consider candidates which can bring new research directions. Sometimes, the excellent candidates stay at the University and continue in the tenure track positions. However, the selection should be carefully made since unrelated areas of research can lead to an isolation of the candidates and a insufficient communication with faculty researchers.

Publication in international environment

We consistently prioritize quality of the output over quantity. We always prefer to publish our results in internationally recognized journals. It is necessary to avoid publications in predatory journals or in journals issued by publisher with suspected policy, e.g., https://beallslist.net/. Similarly, as in PRVOUK and PROGRES projects, we plan to annually select the best publications with excellent results and to award the corresponding authors. We plan to present the results at conferences of good reputation and also to organize conferences, workshops and summer/winter schools, attracting recognized experts in the relevant scientific fields.

Internationalisation of the research area, including the acquisition of international students and researchers

This activity is described in previous sections concerning the doctoral students, postdoc positions and hiring process. We note that most of the relevant strategies started to be applied several years ago.

Collaborations

As a good example of the national level of collaboration serves the Necas Center for Mathematical Modelling (NCMM) which has been establish on the agreement of the Faculty of Mathematics and Physics and the Mathematical Institute of the Academy of Science. It is open to any researcher in the area of applied mathematics. The activities of NCMM are various, e.g., regular organization of EMS summer schools, publication of the Necas Lecture Series in Springer Nature

Switzerland AG publisher, annual awards of the best master students in the area of physical models, mathematical analysis of partial differential equations and computational mathematics.

Moreover, our goal is to be active in the cooperation within the 4EU+ European University Alliance in the research as well as education areas. We would establish a common research study programmes at the master level in order to attract promising students and increase the connections among the universities involved in the Alliance.

Collaboration on the level of doctoral students is supported by the Doctoral School for Education in Mathematical Methods and HPC. Doctoral School has been established on the agreement of four partners (Faculty of Mathematics and Physics, Mathematical Institute of the Academy of Sciences, IT4Innovations, and Faculty of Electrical Engineering and Computer Science VSB TU Ostrava). The intention and mission of the Doctoral School is primarily to create a network linking individual departments implementing doctoral programs in the field of mathematical methods and tools in HPC, and to strengthen mutual awareness, cooperation, internationalization, and collaboration with the non-academic sphere.

Furthermore, there are various national and international cooperation based on individual level. These activities are usually supported by the projects (ERC, ERC-CZ, GACR), we will continue in the preparations and applications of new projects. We also intend to increase the bilateral cooperation supported by GACR (projects Lead Agency). However, it is necessary to seek also other possibilities of financing of these types of cooperation.