A general remark, valid for all programmes in Mathematics

A part of the study plan of each student of the doctoral study in the framework of the study programmes in Mathematics should be an active participation in a conference. An appropriate preparation for the participation in an international conference is an active participation in the internal conference Day of Doctoral Students of the School of Mathematics (DDS-M) which is organized by the School of Mathematics for this purpose every year, see [http://karlin.mff.cuni.cz/wds-m/](http://karlin.mff.cuni.cz/wds-m/). An active participation in DDS-M is not a general duty of all students, however, it becomes obligatory if an active participation in DDS-M is contained in the individual study plan of the student for the respective academic year. Thus, the obligation of an active participation in DDS-M is mainly given by the decision of the supervisor, resp. by his or her agreement with the respective student.

Study programme P4M1 Algebra, number theory and mathematical logic

Subject-area board

Current composition of the board is at the address [http://mff.cuni.cz/phd/or/p4m1](http://mff.cuni.cz/phd/or/p4m1).

Cooperating institutes

- Institute of Mathematics of the CAS, v.v.i.
  Žitná 25, 115 67 Praha 1

- Institute of Computer Science of the CAS, v.v.i.
  Pod Vodárenskou věží 2, 182 07 Praha 8

Offered topics

The topics can be found in SIS at the address [http://mff.cuni.cz/phd/temata/p4m1](http://mff.cuni.cz/phd/temata/p4m1)
## Provided teaching

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List of requirements for taking the state doctoral exam

Student will choose - in agreement with his or her advisor - one of three topics: "Algebra", "Mathematical logic" or "Number theory." The state doctoral exam is then conducted in the chosen topic according to the following list of requirements:

**Algebra**

I. Basics

Obligatory part.

I.1 Basic algebra

Group theory: finite groups, the Sylow theorems, the structure of finitely generated abelian groups, free groups and their subgroups.

Galois theory: Galois extensions and Galois groups, radical field extensions, unsolvability of polynomial equations by radicals.

Representation theory and algebraic geometry: Representations of finite groups, Maschke’s theorem, characters. The correspondence between affine algebraic sets and ideals in polynomial rings, Hilbert’s Nullstellensatz.

Universal algebra: varieties of algebras, subdirect decompositions, free algebras, the Birkhoff theorem. Lattices, complete lattices, closure operators and Galois correspondences.

II. Advanced topics

After agreement with the advisor, the student will select two different topics from the advanced topics of the specialization "Algebra", "Number theory" or "Mathematical logic". However, at least one of them must be one of the following (II.1–II.10):

II.1. Group theory

Group action on a set. Permutation, solvable and nilpotent groups. Linear groups.

Finite simple groups, simplicity of $A_n$ and $PSL_n(K)$. Basics of the theory of groups extensions, semidirect products of groups. Induced representations of groups and the Frobenius reciprocity, Mackey’s theorem and its consequences.

II.2 Binary systems

Left distributive groupoids (free, monogenerated, the word problem), relation to braid groups. Medial and two–sided distributive groupoids, the equational theory of medial idempotent groupoids. Normal subquasigroups and congruences of loops and quasi–groups, nuclei, the center, nilpotence. Relations to the multiplication group. LCC, CC,
extra, Bol and Moufang loops. Inverse properties, diassociativity. Isotopy, central and medial quasigroups. Toyoda’s theorem.

II.3 Commutative algebra


II.4 Algebraic geometry


II.5 Module theory and homological algebra

Projective and injective modules. Chain conditions on ideals, the Hopkins–Levitzki theorem, Faith’s characterization of noetherian rings. The Morita equivalence. Categories of modules, the tensor product, functors Ext and Tor, long exact sequences. Direct limits, pure embeddings, pure–injective modules and relation to model theory. Derived categories.

II.6 Approximations of modules and infinite combinatorics

Cotorsion pairs, filtrations, the Eklof and Hill lemmas. Deconstructibility for regular and singular cardinals (dependence on the set theory, Shelah’s Singular compactness theorem). The structure of Whitehead and Baer modules.

II.7 Representations of finite dimensional algebras

Finite dimensional algebras as factors of path algebras. The Krull–Schmidt theorem. Finite, tame and wild representation types. Hereditary algebras and Gabriel’s characterisation of the finite representation type. Tilting modules and tilted algebras. Almost split maps, the AR–sequences, the AR–quiver of a finite dimensional algebra.

II.8 Universal algebra and lattice theory


II.9 Universal algebraic methods in the CSP

Relational and algebraic clones, homomorphisms of clones and primitive positive interpretation of relational structures. Complexity of the CSP and clones of polymorphisms, Taylor clones, Maltsev CSP’s and finite width problems, Schaefer’s theorem classifying CSP’s on a two–element set.

II.10 Combinatorics on words

Dickson’s lemma. F–semigroups (the minimal generating set, codes, the stability condition, rank of semigroup). The Chomsky hierarchy (formal grammars and the corresponding automata, Kleene’s theorem, pumping lemmas, Parikh’s theorem). Equations in free monoids (the Compactness theorem, the Graph lemma, properties of the defect, equivalence and test sets). The Post correspondence problem.
Recommended literature


Mathematical logic

I. Common background

Obligatory part.

I.1 Basic logic

Propositional and predicate logic. First-order structures, Tarski’s definition of satisfiability. Provability, the completeness and the compactness theorems. Set theory as a first-order theory. Godel’s theorems on the incompleteness and on the unprovability of consistency. Turing machines: universal machine, algorithmically undecidable problems, the halting problem. Quantifier elimination in the ordered field of real numbers.
II. Advanced topics

After agreement with the advisor, the student will select two different topics from the advanced topics of the specialization "Algebra", "Number theory" or "Mathematical logic". However, at least one of them must be one of the following (II.1–II.6):

II.1. General model theory


II.2 Applied model theory

Real closed ordered fields and their reducts and expansions, theorems of Tarski and of Wilkie. O-minimal structures and their basic geometric and topological properties. Stable and omega-stable theories, uncountable categoricity, Morley’s theorem. Minimal and strongly minimal structures, general closer operator, geometries and dimension in strongly minimal structures. Omega-stable groups, the Cherlin-Zilber hypothesis. Hrushovski’s amalgamation method.

II.3. Set theory


II.4. Computability theory


II.5 Proof theory and formal arithmetic


II.6 Logic and complexity

Time and space complexity of algorithms, main complexity classes. Boolean circuits and main known circuit lower bounds. Natural proofs of Razborov and Rudich. Finite model theory, descriptive complexity. Definability in finite structures, Fagin’s
MATHEMATICS


Recommended literature


**Number Theory**

*I. Basics*

Obligatory part.

Basic number theory: Density of primes, Legendre and Jacobi symbol, quadratic reciprocity, continued fractions, quadratic number fields, Rabin–Miller algorithm, RSA.


*II. Advanced topics*

After agreement with the advisor, the student will select two different topics from the advanced topics of the specialization ”Algebra”, ”Number theory” or ”Mathematical logic”. However, at least one of them must be one of the following (II.1–II.6):

*II.1 Cryptology and factorization*

Pseudo-random number generators, symmetric and stream ciphers, hash functions, provable security, cryptographic protocols, zero-knowledge proofs. Number field sieve and its constituent algorithms (finding the root, choice of polynomial, etc.). Further factorization algorithms (p-1, p+1, rho, use of elliptic curves) and their importance for number field sieve. Primality tests and proofs (quadratic Frobenius, N-1 test, ECPP, polynomial-time algorithms).

*II.2 Advanced cryptoanalysis methods*

Theory of boolean functions, S-boxes, their cryptographic properties, linear and differential cryptoanalysis, LLL–algorithm and its cryptoanalytic applications.

*II.3 Self-correcting codes*

Classical theory of cyclic codes. Self-dual codes and invariant theory. Convolution codes. Turbo codes. Decoding algorithms, esp. Viterbi’s and various algorithms
MATHEMATICS


II.4 Elliptic curves


II.5 Algebraic number theory II

Quadratic forms: class group of binary forms, genus theory, primes represented by a binary form, universal forms, Hasse–Minkowski theorem, Hilbert symbol. Adeles and ideles, locally compact groups. Global and local class field theory.

II.6 Analytic number theory


Recommended literature

Study programme P4M2 Geometry, topology, and global analysis

Subject-area board

Current composition of the board is at the address [http://mff.cuni.cz/phd/or/](http://mff.cuni.cz/phd/or/)

Cooperating institutes

- Institute of Mathematics of the CAS, v.v.i.  
  Žitná 25, 115 67 Praha 1  

Offered topics

The topics can be found in SIS at the address [http://mff.cuni.cz/phd/temata/](http://mff.cuni.cz/phd/temata/)

Provided teaching

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List of requirements for taking the state doctoral exam

I. Common background

At least three of the following subjects:

I.1. General Topology


I.2. Set theory


I.3. Category theory

Categories and functors, examples. Natural transformations and equivalences, examples. Limits, colimits, completeness, their forms in concrete categories. Adjunction, reflectivity and coreflectivity. Closed and Cartesian closed categories. Small categories, Mac Lane representation.

I.4. Selected topics from algebra

Tensor algebra, in particular multilinear algebra. Selected topics from ring and module theory (extensions, resolvents, gradings, filtrations). Basics of homological algebra (homology of complexes, cohomology of groups and other algebraic systems).

I.5. Riemannian manifolds


I.6. Analysis on manifolds

I.7. Lie groups and algebras
Classification of simple Lie groups and their finite dimensional representations. Decomposition of tensor product into irreducible components. Klimyk formula. Characters of representations and character formulas (Weyl, Freudenthal, etc.).

I.8. Algebraic topology
Homology and cohomology groups (either simplicial or singular) and their computation. Borsuk theorems, theorems concerning invariance of domains and dimensions, fundamental theorem of algebra. Euler theorem. Map degree. Lefschetz fixed point theorem. De Rham cohomology. Basics of homotopy theory.

II. Advanced topics
One of the following:

II.1. General topology

II.2. Set theory

II.3. Category theory


II.5. Parabolic structures on manifolds
Graded Lie algebras, their real forms. Principal fibre bundles, connections, covariant derivatives and their curvatures. Homogeneous differential operators. Cartan and parabolic geometries, Cartan connection and its curvature. Conformal, projective, quaternionic geometry and further examples of parabolic geometries.

II.6. Integral geometry and complex analysis
II.7. Invariant differential operators


II.8. Algebraic topology


Recommended literature


# Study programme P4M3 Mathematical analysis

## Subject-area board

Current composition of the board is at the address [http://mff.cuni.cz/phd/or/p4m3](http://mff.cuni.cz/phd/or/p4m3).

## Cooperating institutes

- Institute of Mathematics of the CAS, v.v.i.
  Žitná 25, 115 67 Praha 1

## Homepage of the study programme

[http://karlin.mff.cuni.cz/studium/phd/4m3/](http://karlin.mff.cuni.cz/studium/phd/4m3/).

## Offered topics

The topics can be found in SIS at the address [http://mff.cuni.cz/phd/temata/p4m3](http://mff.cuni.cz/phd/temata/p4m3).

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<tr>
<td>NMMO461</td>
<td>Seminar in Continuum Mechanics</td>
<td>0/2 C</td>
<td>0/2 C</td>
</tr>
<tr>
<td>NMMA452</td>
<td>Seminar on Partial Differential Equations</td>
<td>0/2 C</td>
<td>0/2 C</td>
</tr>
<tr>
<td>NMMA458</td>
<td>Seminar on Topology</td>
<td>0/2 C</td>
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</tr>
</tbody>
</table>

List of requirements for taking the state doctoral exam

For the purpose of the state doctoral exam please visit the webpage of the subject-area board [http://karlin.mff.cuni.cz/studium/phd/4m3](http://karlin.mff.cuni.cz/studium/phd/4m3) where two lists of topics, denoted List A and List B, can be found.
List A
1. Distribution theory
2. Advanced spectral theory
3. Complex analysis
4. Introduction to abstract harmonic analysis
5. Introduction to the approximation theory
6. Classical harmonic analysis
7. Hausdorff measure and change of variables in an integral
8. Spaces of functions of bounded variation and approximation by smooth functions
9. Qualitative theory of ordinary differential equations
10. Classical potential theory
11. Introduction to the hyperbolic conservation laws
12. Introduction to optimization theory
13. Sturm-Liouville theory of second-order linear equations
14. Integral equations and the eigenvalue problem
15. Laplace transform

List B
1. Introduction to interpolation theory
2. Topological degree
3. Integral representation on compacts
4. Theory of C*-algebras
5. Descriptive set theory
6. Function spaces
7. Singular integrals
8. Littlewood-Paley theory
9. Riesz and Bessel potentials
10. Hardy spaces
11. Finite distortion mappings
12. Isoperimetric inequality
13. Differentiability of convex functions
14. Introduction to the homogenization theory
15. Introduction to the theory of stochastic parabolic equations
16. Existence theory for the Navier-Stokes-Fourier system
17. Attractor: structure and dimensional estimates
18. Volterra integral equations
19. Regularity of Navier-Stokes equations

Topics on both lists are of unified extent corresponding approximately to 70-100 pages in a monograph. The supervisor of each student who intends to take the exam will choose one topic from the list A and one topic from the list B. These two topics
will be complemented by a third one (of the same extent) according to the supervisor’s choice. This third topic might be chosen from one of the lists, or it can be a new topic which (so far) has not been included to the lists. The third topic should be close to the topic of the student’s dissertation or his/her field of research.

The supervisor will then apply to the subject-area board for approval of the list of three chosen topics before applying for fixing of the exam date. The subject-area board will assess the appropriateness of the suggested list and decide by acclamation whether or not it gives its approval. When the application was approved, the three topics are considered to be established. The exam then consists of three parts each of those corresponding to one of the topics. If the third topic had not been included to one of the lists before the exam, could be included to one of those on the decision of the subject-area board. More details concerning the lists can be found at http://karlin.mff.cuni.cz/studium/phd/p4m3/phdzkouska.php.

Recommended literature


Study programme P4M6 Computational mathematics

Subject-area board

Current composition of the board is at the address http://mff.cuni.cz/phd/or/p4m6.

Cooperating institutes

- Institute of Computer Science of the CAS, v.v.i.
  Pod Vodárenskou věží 2, 182 07 Praha 8
  http://www.ustavinformatiky.cz/

- Institute of Mathematics of the CAS, v.v.i.
  Žitná 25, 115 67 Praha 1
  http://www.math.cas.cz/

- Institute of Thermomechanics of the CAS, v.v.i.
  Dolejškova 1402/5, 182 00 Praha 8
  http://www.it.cas.cz/

- Institute of Information Theory and Automation of the CAS, v.v.i.
  Pod Vodárenskou věží 4/1143, 182 08 Praha 8
  http://www.utia.cas.cz/

Offered topics

The topics can be found in SIS at the address http://mff.cuni.cz/phd/temata/p4m6.

Provided teaching

<table>
<thead>
<tr>
<th>Code</th>
<th>Subject</th>
<th>Winter</th>
<th>Summer</th>
</tr>
</thead>
<tbody>
<tr>
<td>NMNO461</td>
<td>Seminar in Continuum Mechanics</td>
<td>0/2 C</td>
<td>0/2 C</td>
</tr>
<tr>
<td>NMNO533</td>
<td>Nonlinear Differential Equations and Inequalities 1</td>
<td>3/1 C+Ex</td>
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<tr>
<td>NMNO535</td>
<td>Mathematical Methods in Mechanics of Solids</td>
<td>2/0 Ex</td>
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<tr>
<td>NMNO536</td>
<td>Mathematical Methods in Mechanics of Compressible Fluids</td>
<td>—</td>
<td>2/0 Ex</td>
</tr>
<tr>
<td>NMNO537</td>
<td>Saddle Point Problems and Their Solution</td>
<td>—</td>
<td>2/2 C+Ex</td>
</tr>
<tr>
<td>NMNO538</td>
<td>Mathematical Methods in Mechanics of Non-Newtonian Fluids</td>
<td>2/0 Ex</td>
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<tr>
<td>NMNV451</td>
<td>Seminar in Numerical Mathematics</td>
<td>0/2 C</td>
<td>0/2 C</td>
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<tr>
<td>NMNV461</td>
<td>Techniques for a posteriori error estimation</td>
<td>2/0 Ex</td>
<td>—</td>
</tr>
</tbody>
</table>
List of requirements for taking the state doctoral exam

1. **Mathematical and functional analysis**
   

2. **Numerical methods**
   

3. **Elective subjects related to the topic of the PhD thesis**

Recommended literature


Study programme P4M8 General questions of mathematics and computer science

Subject-area board

Current composition of the board is at the address [http://mff.cuni.cz/phd/or/](http://mff.cuni.cz/phd/or/)

Cooperating institutes

- Institute of Mathematics of the CAS, v.v.i.
  Žitná 25, 115 67 Praha 1

Offered topics

The topics can be found in SIS at the address [http://mff.cuni.cz/phd/temata/](http://mff.cuni.cz/phd/temata/)

Provided teaching

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<tr>
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<th>Summer</th>
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<tbody>
<tr>
<td>NMUM603</td>
<td>Mathematics in the ancient times I</td>
<td>2/0 Ex</td>
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<tr>
<td>NMUM604</td>
<td>Mathematics in the ancient times II</td>
<td>—</td>
<td>2/0 Ex</td>
</tr>
<tr>
<td>NMUM602</td>
<td>Didactics of Mathematics for Ph.D. Students</td>
<td>—</td>
<td>2/2 C+Ex</td>
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<tr>
<td>NMINV054</td>
<td>ICT in Mathematics Teaching I</td>
<td>0/2 C</td>
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<tr>
<td>NMINV055</td>
<td>ICT in Mathematics Teaching II</td>
<td>—</td>
<td>0/2 C</td>
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<tr>
<td>NMIN258</td>
<td>Beginners’ course in Mathematica</td>
<td>0/2 C</td>
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<tr>
<td>NMIN263</td>
<td>Advanced course in Mathematica</td>
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<td>0/2 C</td>
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<tr>
<td>NMUM610</td>
<td>Applications of Mathematics for Teachers</td>
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<td>0/2 coloquium</td>
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<tr>
<td>NMINV056</td>
<td>Greek Mathematical Texts I</td>
<td>0/2 C</td>
<td>—</td>
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<tr>
<td>NMINV057</td>
<td>Greek Mathematical Texts II</td>
<td>—</td>
<td>0/2 C</td>
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<tr>
<td>NMIN201</td>
<td>Advanced probability for mathematicians</td>
<td>—</td>
<td>2/0 Ex</td>
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<tr>
<td>NATH102</td>
<td>Philosophical problems of Informatics</td>
<td>0/1 C</td>
<td>0/1 C</td>
</tr>
<tr>
<td>NPOZ102</td>
<td>Philosophical Problems of Physics</td>
<td>0/1 C</td>
<td>—</td>
</tr>
<tr>
<td>NPOZ202</td>
<td>Geometry for Computer Graphics</td>
<td>—</td>
<td>2/0 Ex</td>
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<tr>
<td>NPOZ201</td>
<td>Geometric Modelling</td>
<td>2/2 C+Ex</td>
<td>—</td>
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<tr>
<td>NMINA316</td>
<td>Seminar in combinatorics and graph theory</td>
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</table>

Programme description

The programme is intended mainly for graduates of education-oriented study programmes involving mathematics or computer science for high-school teachers, and for university teachers of mathematics, computer science, and didactics of these subjects.

A necessary condition for admission into this programme is an excellent knowledge of high-school mathematics, as well as basic university courses of mathematics.

The programme has three branches:
1. Elementary mathematics
2. History of mathematics/computer science
3. Teaching of mathematics/computer science

The branch *Elementary mathematics* offers a number of ways for increasing the level of mathematical culture of high-school teachers, and provides better qualification for teaching in general, especially for working with gifted students. Elementary mathematics encompasses classical mathematical disciplines related to high-school mathematics and education-oriented study programmes at universities, and provides a suitable extension of these subjects. Its goals include keeping the historical continuity of mathematics, and strengthening the respect towards traditional mathematical values.

In the branch *History of mathematics/computer science*, attention is paid mainly to the 19th and 20th century, and Czech mathematics/computer science, including various biographical and bibliographic aspects. History of mathematics is closely related to the problems of teaching, because the development of mathematics has been influenced by textbooks and teachers.

The branch *Teaching of mathematics/computer science* is primarily intended for those who already have a teaching experience. A dissertation thesis might include educational texts, collections of problems etc., including methodical guidelines and analysis of difficult parts; resulting materials should be tested in class.

**List of requirements for taking the state doctoral exam**

The nature of the state doctoral exam stems from the fact that the goal of this programme is to educate a mathematician/computer scientist with a general broad insight, who is not being purposely prepared for the scientific work in a certain narrow area, but his/her erudition will allow him/her to create high-quality educational texts, who is aware of modern teaching methods, possesses good orientation in the scholarly literature related to his/her field of interest, and regularly publishes his/her own findings.

The doctoral exam consists of three parts, mathematics/computer science, history of mathematics/computer science, and teaching of mathematics/computer science. Since the topics studied in this programme are quite diverse and cover various branches of mathematics/computer science, it is impossible to have a unified list of requirements for all students. Only a general framework is provided, and the requirements will be made precise by each student’s supervisor and the doctoral exam committee.

**Requirements**

1. *Mathematics/computer science*

   It is assumed that the student has mastered all material required for the master’s state exam in education-oriented study programmes at Charles University, Faculty of Mathematics and Physics. Student must be able to show a good understanding of the connections between high-school and university topics, possess orientation in basic textbook literature, and be able to prepare and teach basic courses in mathematics/computer science.

   Additional requirements will be specified by student’s supervisor and the doctoral exam committee (at least several chapters of a scholarly text, whose content is not part of standard university courses). These requirements must go significantly beyond the level specified in the previous paragraph.
2. History of mathematics/computer science

It is assumed that the student understands the essence of historical topics, and possesses orientation in the development of various disciplines. Deeper knowledge of history is required for topics related to student’s research.

Student’s supervisor and the doctoral exam committee will specify at least 200 pages of scholarly literature.

3. Teaching of mathematics/computer science

Assumed is a good orientation in methodics, didactics, and problem solving methods in mathematics/computer science.

Student’s supervisor and the doctoral exam committee will specify at least 100 pages of scholarly literature.

4. Broadening horizons

It is expected that the student shows interest in his/her field of study, follows scholarly journals and literature dealing with mathematics/computer science and their teaching, knows how to cite relevant works and search for bibliographic data, is familiar with databases of scientific works, digital repositories, etc.

Recommended literature


**Study programme P4M9 Probability and statistics, econometrics and financial mathematics**

**Subject-area board**

Current composition of the board is at the address [http://mff.cuni.cz/phd/or/](http://mff.cuni.cz/phd/or/)

**Cooperating institutes**

- Institute of Information Theory and Automation of the CAS, v.v.i.
  Pod Vodárenskou věží 4/1143, 182 08 Praha 8

**Offered topics**

The topics can be found in SIS at the address [http://mff.cuni.cz/phd/temata/](http://mff.cuni.cz/phd/temata/)
## Provided teaching

<table>
<thead>
<tr>
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<th>Summer</th>
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<tbody>
<tr>
<td>NMSA600</td>
<td>Colloquium of the Department of Probability and Mathematical Statistics</td>
<td>0/1 C</td>
<td>0/1 C</td>
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<tr>
<td>NMSA601</td>
<td>Specialized seminar in probability and mathematical statistics</td>
<td>0/2 C</td>
<td>0/2 C</td>
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<tr>
<td>NMEK613</td>
<td>Stochastic Modelling in Economics and Finance</td>
<td>0/2 C</td>
<td>0/2 C</td>
</tr>
<tr>
<td>NMTP613</td>
<td>Seminar on Probability for Ph.D. Students I</td>
<td>0/2 C</td>
<td>—</td>
</tr>
<tr>
<td>NMTP614</td>
<td>Seminar on Probability for Ph.D. Students II</td>
<td>—</td>
<td>0/2 C</td>
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<tr>
<td>NMST611</td>
<td>Advanced Statistical Seminar</td>
<td>0/1 C</td>
<td>0/1 C</td>
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<td>NMTP611</td>
<td>Seminar on Stochastic Evolution Equations</td>
<td>0/2 C</td>
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<tr>
<td>NMAG671</td>
<td>Seminar on Stochastic Geometry</td>
<td>0/1 C</td>
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<tr>
<td>NMEK615</td>
<td>Stochastic Programming and Approximation</td>
<td>0/2 C</td>
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<tr>
<td>NMSA602</td>
<td>Advanced topics of the field</td>
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<tr>
<td>NMSA603</td>
<td>Advanced topics of the field</td>
<td>—</td>
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<tr>
<td>NMST603</td>
<td>Modern methods of mathematical statistics</td>
<td>2/0 Ex</td>
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<tr>
<td>NMEK608</td>
<td>Optimization and variational analysis</td>
<td>2/0 Ex</td>
<td>2/0 Ex</td>
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<tr>
<td>NMTP616</td>
<td>Some topics on insurance and financial mathematics</td>
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<tr>
<td>NMTP617</td>
<td>Selected topics in spatial modeling</td>
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<td>2/0 Ex</td>
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<tr>
<td>NMAM612</td>
<td>Advanced Topics on Risk Theory</td>
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<td>2/0 Ex</td>
</tr>
<tr>
<td>NMST604</td>
<td>Advanced Course in Time Series</td>
<td>2/0 Ex</td>
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<tr>
<td>NMST632</td>
<td>Simulation Methods</td>
<td>—</td>
<td>2/2 C+Ex</td>
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<tr>
<td>NMAM613</td>
<td>Advanced Topics on Financial Mathematics</td>
<td>—</td>
<td>2/0 Ex</td>
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<tr>
<td>NMTP605</td>
<td>Advanced Theory of Stochastic Differential Equations</td>
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<tr>
<td>NMTP432</td>
<td>Stochastic Analysis</td>
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<td>4/2 C+Ex</td>
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<tr>
<td>NMEK607</td>
<td>Chapters on modern optimization and equilibria</td>
<td>2/0 Ex</td>
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<tr>
<td>NMEK608</td>
<td>Chapters on modern optimization and equilibria</td>
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<td>2/0 Ex</td>
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<tr>
<td>NMAM611</td>
<td>Advanced Topics on Non-life Actuarial Mathematics</td>
<td>2/0 Ex</td>
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<tr>
<td>NMAM612</td>
<td>Mathematical methods in the solvency management and in the financial reporting of insurance companies</td>
<td>—</td>
<td>2/0 Ex</td>
</tr>
</tbody>
</table>
List of requirements for taking the state doctoral exam

The exam consists of three parts, the first part is taken from topics I. or II. below. The second part is chosen from I., II., III. or IV., but this choice cannot coincide with the choice in the first part. The third part is related to the topic of dissertation.

I. Probability and stochastic processes.


II. Mathematical statistics.

Estimation theory and hypotheses testing, loss and risk functions, multivariate analysis, regression, sampling theory, robust and non-parametric methods, bayesian and sequential analysis, spatial statistics, computational statistics, simulation methods, survival analysis.

III. Econometrics and operational research.


IV. Financial and insurance mathematics.


Recommended literature

I. Probability and stochastic processes


II. Mathematical statistics


III. Econometrics and operational research


IV. Financial and insurance mathematics