Each problem is awarded a maximum of 25 points. Justify your answers!

1. We want to cover the whole grid of size $2 \times 10$ squares using ten rectangles of size $1 \times 2$ squares. Determine how many different ways we can do it. Rectangles can be rotated anyway.

Find a general solution for the grid of sizes $2 \times N$ squares using $N$ rectangles of size $1 \times 2$ squares.

2. A directed graph is specified by the number of vertices and list of edges. You have to check whether this graph contains a cycle (i.e. a path that runs along the edges in the direction of their orientation and returns to its starting vertex). Describe an algorithm for solving this task and justify its correctness. Determine its asymptotic time complexity.

3. Design a deterministic finite automaton over the alphabet \{0, 1\} that accepts words containing exactly two letters 1 and having odd length. For example, it accepts the words 011, 1001, 0010100, but it rejects the words 100, 1001, 10101. Write the transition function of the automaton as a table and draw the automaton in the form of a transition diagram. Try to design the simplest possible automaton, i.e. an automaton with the smallest number of states.

4. The following program is given (the Pascal form and the C form are equivalent):

```pascal
program AA;
var a, b, k: integer;
begin
  b := 0;
  k := 3;
  while b <= 1000 do
  begin
    a := 0;
    while a <= b do a := a+k;
    b := a;
    k := 8-k;
  end;
  writeln(b)
end.
```

```c
main() /* AA */
{ int a, b, k;
  b = 0;
  k = 3;
  while (b <= 1000)
  { a = 0;
    while (a <= b) a += k;
    b = a;
    k = 8-k;
  }
  printf("%d", b);
}
```

a) Determine how many times the outer while-cycle will be executed.
b) Determine the final value of the variable $b$. 