Computer Science - continuing to the MSc study programme
Computer Science Entrance Examination - 2016 - Version A

Each problem is awarded a maximum of 25 points.
Justify your answers!

1. Determine how many times you write an even digit, if you write the first 2000 positive integers.

2. Transform the following logical formula into disjunctive normal form:
   \[ ( (~ x \land y) \lor (x \land ~ z)) \land ( ~ x \lor y) \]
The symbols x, y, z denote logical variables, the symbol & stands for conjunction, \lor denotes disjunction, and ~ x denotes negation of the variable x.
   a) Find any solution.
   b) Find a solution in the form of a disjunction with at most two clauses.

3. Design a deterministic finite automaton over the alphabet \{0, 1\} that accepts words containing exactly two letters 0 and having two identical trailing letters. For example, it accepts the words 00, 01011, 111100, but it rejects the words 11, 00111, 10101. Write the transition function of the automaton as a table and draw the automaton in the form of a transition diagram. Try to design the simplest possible automaton, that is, one with the smallest number of states.

4. The following program is given (the Pascal form and the C form are equivalent):

   ```
   program A;
   var N, X: integer;
   begin
      read(N);
      X := 0;
      while N > 1 do
      begin
         X := X * 2 + N mod 2;
         N := N div 2
      end;
      write(X)
   end.
   main() /* A */
   {
      int n, x;
      scanf( "%d", &n);
      x = 0;
      while (n > 1)
      begin
         { x = x * 2 + n % 2;
           n /= 2;
      end;
      printf( "%d", x);
   }
   ```

   a) Determine the smallest input value \( N \) for which the result of the computation is \( X = 77 \).
   b) Determine three smallest input values \( N \) for which the result of the computation is a two-digit number.
   c) Determine all input values \( N \) for which the result of the computation equals to the input value, that is, \( X = N \).