Entrance examination, MFF UK in Prague
Study programme for Bachelor of Computer Science
2017, version A

For each of the following ten problems five possible answers a, b, c, d, e are offered. For each problem your task is to mark for each answer whether it is true or false, or whether the statement holds or does not hold. The duration of the test is 75 minutes.

**Awarding points.** For each problem you may get 0 up to 10 points. You get +2 points for each correctly marked answer, you get −2 points for each incorrectly marked answer, 0 points for an unmarked answer. If this leads to a negative score for the whole problem, you will be awarded 0 points for it.

**How to mark answers and how to make a correction.** The answer you choose should be marked by filling the corresponding circle. If you have already marked an answer and wish to make a correction, you can cancel your choice by making a large cross over the filled circle, and then correct it by filling the other circle. It is not possible to choose an answer again where the circle has been already crossed out. Answers marked in any other way will be regarded as non-marked. Notice in the following example that the answers in the last two columns are the same, as they differ only by the corrections made to them.

**Example.** As an example we show the scoring for four markings in response to the problem “The sum of 1 + 1 is “:

<table>
<thead>
<tr>
<th>(a) 2</th>
<th>(b) 3</th>
<th>(c) less than 12</th>
<th>(d) a positive number</th>
<th>(e) 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>(+)2</td>
<td>(−2)</td>
<td>(+2)</td>
<td>(−2)</td>
<td>(+2)</td>
</tr>
</tbody>
</table>

Points: 10 2 6 6

---

1A correctly marked answer is one where the right answer is Yes and you only mark Yes or the right answer is No and you only mark No. An incorrect answer is one where the right answer is Yes and you only mark No or the right answer is No and you only mark Yes. All other possibilities are regarded as being unanswered.
For each of the following problems decide which assertions hold and which do not (Yes = Holds, No = Does not hold).

1. Consider the function of real variable given by \( f(x) = (\sin x)^2 \cos x \). Decide which of the following statements about the function \( f \) are valid:
   (a) \( f \) is even.
   (b) \( f \) is odd.
   (c) \( f \) is periodic.
   (d) \( f \) is increasing.
   (e) \( f \) is injective.

2. Let \( x = \frac{1 + \sqrt{5}}{2} \). Decide which of the following statements are valid for this value of \( x \):
   (a) \( x^2 > x \)
   (b) \( x^2 < 2 \)
   (c) \( x^2 < 3 \)
   (d) \( x^3 - 2x > 1 \)
   (e) \( x^3 - 2x \) is an integer.

3. Consider the equation \( 2|x^2 - 2x| + 3x - 3 = 0 \) for variable \( x \in \mathbb{R} \). Decide which of the following statements are valid:
   (a) The equation has more than two solutions.
   (b) At least one solution to the equation is a negative number.
   (c) All solutions to the equation are integers.
   (d) The sum of all solutions to the equation is a positive number.
   (e) The difference between the largest and the smallest solution is equal to 4.

4. Find four numbers so that the first three form three consecutive terms of an arithmetic progression with common difference \( d = -3 \) and the last three form three consecutive terms of a geometric progression with common ratio \( r = 1/2 \).
   Decide which of the following statements are valid:
   (a) The problem has more than one solution.
   (b) There is a solution where all four numbers are integers.
   (c) There is a solution for which the sum of all four numbers is equal to 19.5.
   (d) There is a solution for which the ratio of the first and the fourth number is equal to the second number.
   (e) For each solution, the product of the first and the fourth number is less than the product of the second and the third number.

5. Determine how many different parallelepipeds exist that have as their edge lengths three distinct positive integers less than or equal to 10. Parallelepipeds that differ only by a rotation are not considered to be different. Decide which of the following statements are valid:
   (a) The number of solutions is less than 100.
   (b) The number of solutions is less than 200.
   (c) The number of solutions is less than 500.
   (d) The number of solutions is less than 800.
   (e) The number of solutions is divisible by fifteen.
6. Solve the following system of equations for real variables \( x, y \) in terms of the real parameter \( p \):

\[
\begin{align*}
  x - y &= 2 \\
p x + y &= 4
\end{align*}
\]

Decide which of the following statements are valid:

(a) The system has exactly one solution for every \( p \neq -1 \).
(b) For \( p = 2 \) the system has no solution.
(c) For every \( p > 1 \), each solution \( (x, y) \) satisfies \( x \geq 0 \) and \( y \leq 0 \).
(d) The system has exactly one solution for every \( p \in \mathbb{R} \).
(e) For \( p = 0 \) there is a solution satisfying \( x \geq 0 \) and \( y \geq 0 \).

7. Let \( M \) be the set of all solutions to the equation

\[
\sin |x| - \cos^2 x + 1 = 0
\]

in real numbers.

Decide which of the following statements are valid:

(a) If \( x \in M \) then \( -x \in M \).
(b) If \( x \in M \) then \( (x + \pi) \in M \).
(c) If \( x \in M \) then \( (x + 2\pi) \in M \).
(d) \( M \) is finite.
(e) If \( x \in M \) then \( 5x \in M \).

8. The picture shows a city plan. A mail van has to pass through each street exactly once (in any direction) and then return to its starting point. The van is not allowed to make a U-turn. At points \( A \) and \( C \) it can continue on any street that has not yet been used.

\[
\begin{array}{c}
A \\
B \\
C
\end{array}
\]

Decide which of the following statements are valid:

(a) If the van starts at \( A \), it has at least 20 possible route variants.
(b) If the van starts at \( A \), it has at most 20 possible route variants.
(c) If the van starts at \( B \), it has at least 20 possible route variants.
(d) If the van starts at \( B \), it has at most 20 possible route variants.
(e) If the van starts at \( B \), it has at most 12 possible route variants.
9. The cube $ABCDEFGH$ has edges of length 4.

Decide which of the following statements are valid:
(a) The distance between vertices $A$ and $G$ is $4\sqrt{3}$.
(b) The distance between $A$ and the center of the segment $GH$ is 6.
(c) The distance between $B$ and the center of the segment $AH$ is $4\sqrt{2}$.
(d) The distance between the center of $AC$ and the center of $CG$ is $3\sqrt{2}$.
(e) The distance between the center of $BG$ and the center of $AF$ is $2\sqrt{2}$.

10. The parallelogram $ABCD$ has unit area. The midpoints of segments $AD$ and $BC$ are marked $M$ and $N$, respectively. The lines $CM$ and $DN$ intersect at $O$, while their intersections with the line $AB$ are denoted by $Q$ and $P$, respectively. (See the figure.)

Decide which of the following statements are valid:
(a) The area of the pentagon $ABNOM$ is $3\frac{4}{5}$.
(b) The area of the pentagon $ABNOM$ is $5\frac{8}{9}$.
(c) The area of the triangle $OPQ$ is $\frac{9}{5}$.
(d) The area of the triangle $OPQ$ is $\frac{5}{3}$.
(e) The area of the hexagon $QPNCDM$ is $1 + \frac{\sqrt{2}}{2}$.
Entrance Test 2017 A - Correct Answers

1. a, c.
2. a, c, e.
3. b.
4. c, d, e.
5. b, c, d, e.
6. a, e.
7. a, e.
8. a, d, e.
9. a, b, e.
10. b, c.