Problem 1 (25 points)
Evaluate
\[ \int_M xy \, dx \, dy , \]
where \( M = \{(x,y) \in \mathbb{R}^2; 0 < x < 2, 0 < y, y < x, y < 2 - x \} \).

Problem 2 (25 points)
Function \( f \) is given by
\[ f(x) = \ln \left| \frac{x - 1}{x - 2} \right| . \]

(i) Find the domain of definition \( D(f) \) of the function \( f \).
(ii) Find the intervals of continuity of \( f \).
(iii) Find the limits of \( f \) in the boundary point(s) of \( D(f) \) as well as in the improper point(s).
(iv) Find the intervals of monotonicity of \( f \). Find the local minima and maxima of \( f \) if they exist. Does \( f \) attain its largest and smallest value in \( D(f) \)?
(v) Find the intervals of convexity and concavity of \( f \).
(vi) Find the asymptotes of \( f \).
(vii) Using the solutions (i)-(vi), sketch the plot of \( f \).

Problem 3 (25 points)
Determine, whether function
\[ f(x, y, z) = e^{xyz} \]
attains its maximum and minimum on set
\[ M = \{(x, y, z) \in \mathbb{R}^3; x^2 + 2y^2 + 3z^2 = 30 \} \].
If maximum or minimum exists then find it.

Problem 4 (25 points)
Find the determinant of the real matrix
\[
A = \begin{pmatrix}
ad + 2 & 1 & 0 & 2 \\
2 & 2 & 2 & 5 \\
d + 3 & b & 0 & 3 \\
1 & 2 & 2 & 4 \\
\end{pmatrix}
\]
depending on parameters \( a, b \). For which \( a, b \) is \( A \) invertible?