For each of the following ten problems five possible answers a, b, c, d, e are offered. For each problem your task is to mark for each answer whether it is true or false, or whether the statement holds or does not hold. The duration of the test is **75 minutes**.

**Awarding points.** 10 points are assigned to each problem. You gain 10 points for a given problem if and only if you mark each of the five offered answers correctly. For a given problem, if one of the answers is marked incorrectly you will be awarded 0 points, even if some of the other answers for the problem have been marked correctly. For each problem where no answer is marked incorrectly you obtain 2 points for each correctly marked answer. If you mark all five answers correctly, you gain the maximum score of 10 points.

**How to mark answers and how to make a correction.** The answer you choose should be marked by filling the corresponding circle. If you have already marked an answer and wish to make a correction, you can cancel your choice by making a large cross over the filled circle, and then correct it by filling the other circle. It is not possible to choose an answer again where the circle has been already crossed out. Answers marked in any other way will be regarded as non-marked. Notice in the following example that the answers in the last two columns are the same, as they differ only by the corrections made to them.

**Example.** As an example we show the scoring for four markings for the problem “The sum of 1 + 1 is ”:

<table>
<thead>
<tr>
<th></th>
<th>Answers</th>
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<tbody>
<tr>
<td></td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>(a)</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(b)</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(c)</td>
<td>Less than 12</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(d)</td>
<td>Positive</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(e)</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
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</tbody>
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**Points:**

| 10 | 0 | 6 | 6 |
For each of the following problems decide which assertions hold and which do not (Yes = Holds, No = Does not hold). Notation for intervals: \((a, b) = \{x \in \mathbb{R} : a < x < b\}\) and \([a, b] = \{x \in \mathbb{R} : a \leq x \leq b\}\).

1. In the sequence of integers beginning 7, 10, 13, 16, 19, 22, \ldots each term is 3 greater than the previous. Decide what is true about the sum of the first 100 entries of this sequence:
   (a) It is less than 10,000.
   (b) It is less than 20,000.
   (c) It is greater than 20,000.
   (d) It is even.
   (e) It is divisible by 3.

2. The pairwise sums of a certain set of three numbers are 5, 10 and 25. Decide which of the following statements about these three numbers are true:
   (a) The numbers are not uniquely determined.
   (b) The smallest of them is positive.
   (c) All of them are nonzero.
   (d) The sum of them is 15.
   (e) The largest of them is equal to 15.

3. Sixteen soccer teams took part in a tournament, each pair of teams playing one match. In the whole tournament 420 goals were scored. What holds for the average number of goals in one match?
   (a) It is an integer.
   (b) It is more than 2.
   (c) It is more than 3.
   (d) It is more than 5.
   (e) It is not uniquely determined by the given data.

4. If \(M\) is the set of all solutions to the equation \(1 + \cos x = 2 \sin^2 x\) in the real domain then:
   (a) \(\frac{199\pi}{3} \in M\).
   (b) \(M \cap \left(-\frac{\pi}{2}, \frac{\pi}{3}\right) = \emptyset\).
   (c) \(M \cap \left(-\frac{\pi}{3}, \frac{\pi}{3}\right) = \emptyset\).
   (d) The set \(M\) has two elements.
   (e) The set \(M \cap [0, \pi]\) has two elements.

5. Four semicircles are inscribed in a square (see the figure). Let \(S\) be the area of the square. What holds for the area of the grey region?
   (a) It equals \(S/3\).
   (b) It equals \(S/2\).
(c) It is larger than $S/2$.
(d) It equals $\frac{2}{3}S$.
(e) It is larger than $\frac{2}{3}S$.

6. Which of the following are true about the set $M$ of all solutions to the inequality $|x + 2| - |2x - 2| \leq -8$ in the real domain?

(a) All solutions are positive.
(b) $(-\infty, -5) \subset M$.
(c) $[-4, 4] \cap M \neq \emptyset$.
(d) $(5, +\infty) \subset M$.
(e) $(-1, 10) \cap M \neq \emptyset$.

7. The points $A = (0, 3), B = (6, 0), C = (4, 2), D = (2, 0)$ are given in the plane. The point $E$ is the intersection of lines $AB$ and $CD$. Answer which of the following are true:

(a) Lines $AD, BC$ are parallel.
(b) Lines $AB, CD$ are perpendicular.
(c) The point $E$ has integer coordinates.
(d) The distance from $A$ to $E$ equals $5\sqrt{3}$.
(e) The distance from $A$ to $E$ is less than 5.

8. In an algebrogram each letter stands for one of the digits 0, 1, ..., 9. Different letters stand for different digits. In the following algebrogram neither $C$ nor $E$ is zero.

\[
\begin{align*}
A \times B &= CA \\
+ &-+ \\
A \times C &= D \\
D \times A &= EC
\end{align*}
\]

Decide which of the following statements about the letters in this algebrogram are true:

(a) The digit corresponding to the letter $C$ must be less than 5.
(b) The digits corresponding to letters $C$ and $E$ can be both even.
(c) The digit corresponding to the letter $B$ must equal 6.
(d) There is more than one way to assign digits to the letters in order to make all the equalities hold.
(e) The task has no solution.

9. In the box in the figure, each of the three terminals (inputs) on the left is connected to one terminal (output) on the right, and every terminal on the right is used exactly once. There are six ways to connect inputs with outputs observing these conditions; one of them is depicted.
Now we take three boxes of the same type (call this type T) and connect them together: outputs of the first box are connected to inputs of the second one, outputs of the second box to inputs of the third one, as shown in the diagram.

Decide the validity or otherwise of the following statements:
(a) For every type T, the terminals $a_1$ and $b_1$ are connected.
(b) For at least four types T it is the case that terminals $a_2$ and $b_2$ are connected.
(c) For every type T, $a_1$ is connected with $b_1$ or $a_2$ with $b_2$ or $a_3$ with $b_3$.
(d) For every type T it holds that for every $i = 1, 2, 3$ terminal $a_i$ is connected to $b_i$.
(e) For at least three types T it is the case that for every $i = 1, 2, 3$ terminal $a_i$ is connected to $b_i$.

10. Alice and Bob are playing a game with bonbons. They put some bonbons on the table in several heaps, these heaps being ordered from left to right. A position in the game is a sequence of sizes of these heaps.

In a move a player takes one whole heap, but can only take the leftmost or rightmost heap. Alice starts, then the players move alternately until there are no more heaps on the table. The winner is the player who gets more bonbons; if each has the same number of bonbons at the end then Alice wins.

In the figure we see the position 1, 2, 3. Alice can take the heap with one bonbon or the one with three bonbons, but she cannot take the one with two. In the next move, Bob can take any of the remaining two heaps, and then Alice will take the remaining heap.

We say a position is won for Alice if Alice can ensure her victory, regardless of how Bob plays. Answer which of the following are true and which false:
(a) The position 3, 6, 4, 2 is won for Alice.
(b) The position 3, 4, 3, 4, 3 is won for Alice.
(c) Every position is won for Alice.
(d) Every position with four heaps is won for Alice.
(e) Every position with three heaps is won for Alice.