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The Story of a Right Wavelet Conoid

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Abstract. Conoids are a special kind of warped ruled surfaces. This paper presents one of them, its geometric creation and motivation for using it. The original idea of utilizing right wavelet conoid comes from Gaudí, it was then continued in the work of Santiago Calatrava and there is one implementation of the surface in the Czech Republic with a Dutch inspiration.

Introduction

A conoid is a ruled surface specified by three control elements: plane ρ, straight line a called the axis of the conoid, and curve c. The conoid is then formed by all straight lines (rulings) which are parallel to control plane ρ and intersect axis a and control curve c (Figure 1a).

If $a \perp \rho$, then the conoid is called the right conoid (Figure 1b). Therefore, all forming rulings of the surface are perpendicular to axis a. It also means that if axis a and control curve c have been specified, there is no need to specify control plane ρ, because it must be perpendicular to axis a and its absolute position is not important.

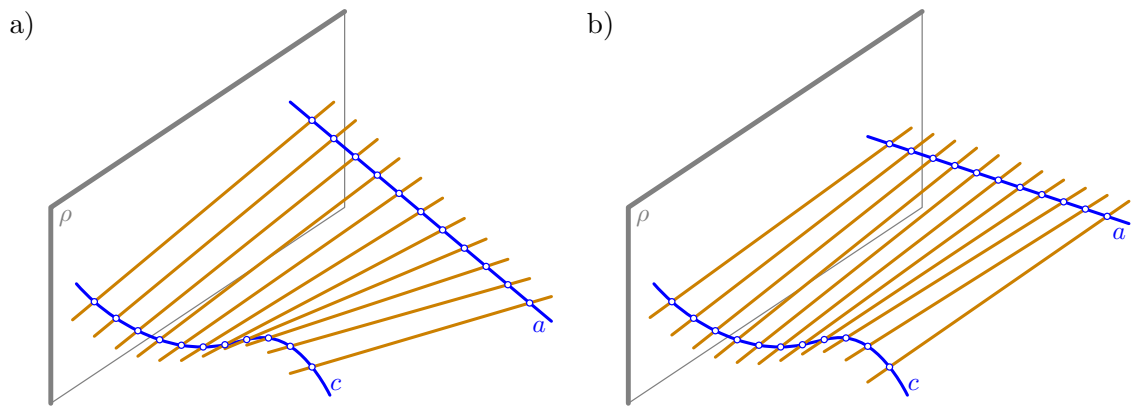

Figure 1. A general (a) and a right (b) conoid.

Right conoids (or their parts) are used in civil engineering. For example, a right conoid of a circle or a right parabolic conoid can be found as roofing constructions. A right conoid of a helix is often used for a spiral staircase. In this case an axis of the control helix is also the axis of the conoid. It is usually called a helicoid.

We will focus on one particular right conoid – a right wavelet conoid.

Right wavelet conoid

The control curve of this right conoid is a wavelet. For the description of the wavelet a sine curve will be chosen.

This surface is one of the first conoidal structures used for practical purposes. In the next three subsections we will describe its geometric creation, show a video tutorial and give a historical overview of its usage. All this is presented on a web page: `http://mdg.vsb.cz/jdolezal/StudOpory/Geometrie/Plochy/ZborcenePlochy/Konoidy/Realizace/PrimyVlnkovyKonoid.html`

Virtual animated model

The model of the surface and of its sequential creation is designed with the help of VRML (Virtual Reality Modeling Language), so that it can also become an element of the web page (to display it a relevant plug-in is needed). This model is conceived as a series of animations. In the model, orientation of the axes is as follows: x-axis from left to right, z-axis from bottom to top and y-axis from front to back.

In the first animation of the model, the sine curve is geometrically created. Let's have an initial circle with a given radius $r > 0$ and with its center at point $[-r; 0; 0]$ (Figure 2a). A copy of the initial circle then moves to a new position centered at point $[0; 0; r]$ (Figure 2b). In the next step the moved circle starts to roll along the x-axis. During this movement it straightens up (Figure 2c), simultaneously, the sine curve is drawn (Figure 2d). The wavelet-sine curve is described by the following parametric equations:

$$\begin{aligned} x &= ru \\ y &= 0 \\ z &= r \sin u, \end{aligned}$$

where $r > 0$ and $u \in \langle 0; 2\pi \rangle$.

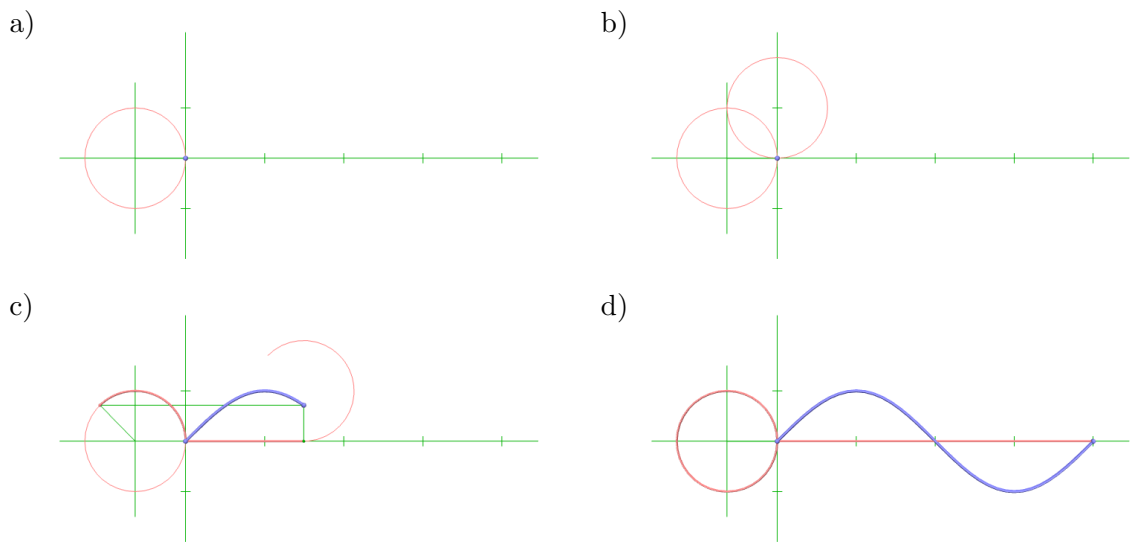

Figure 2. Individual steps of creating the wavelet-sine curve.

In the second animation, the created sine curve runs into the depth away from us along the y-axis (Figure 3a). We obtain a developable cylindrical surface with the control sine curve. All its rulings are parallel to the y-axis. They correspond to a new parameter v, where $v \in \langle 0; 1 \rangle$ and $a \neq 0$ is a constant:

$$\begin{aligned} x &= ru \\ y &= 2av \\ z &= r \sin u. \end{aligned}$$

For the third animation, parameter s is important. It controls flipping of the rear sine curve upside down with respect to the horizontal xy-plane (Figure 3b). As we can see in new corresponding equations

$$\begin{aligned} x &= ru \\ y &= 2av \\ z &= r(1 - 2sv) \sin u, \end{aligned}$$

the rulings of the surface remain rulings, only now they are not parallel but skew – the cylindrical surface has changed into a warped one.

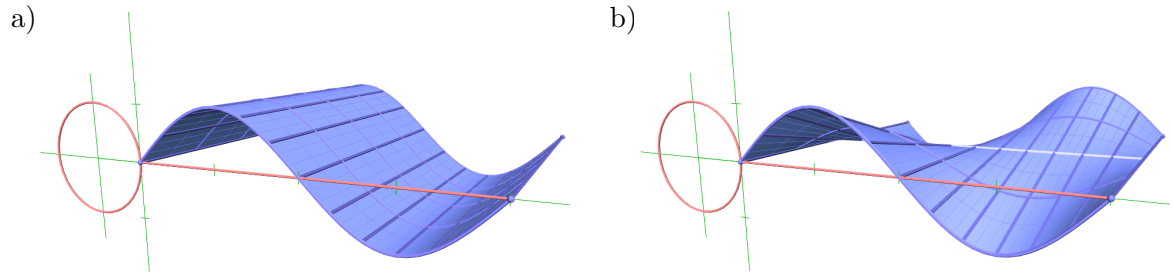

Figure 3. A cylindrical (a) and a conoidal (b) wavelet-sine surface.

The fourth animation is only a phase shift of the created conoid, so the last parameter p gets involved, where $p \in \langle 0; 2\pi \rangle$:

$$\begin{aligned} x &= ru \\ y &= 2av \\ z &= r(1 - 2sv)\sin(u + p). \end{aligned} \tag{1}$$

During this animation we can watch undulating of the surface. We may also change the appearance of the conoid into a form of beams of the same length. These beams are harmonically rotating around the axis of the conoid (Figure 4).

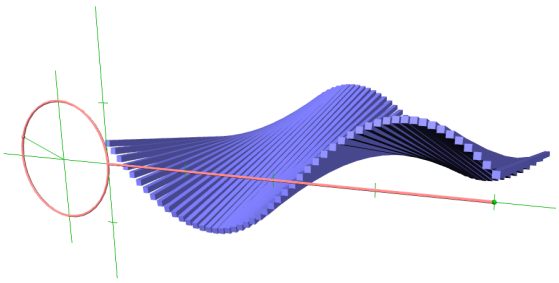

Figure 4. Screenshot of the model shaped to undulating form of rotating beams.

When the fourth animation is over, it is possible to display equations (1) in the model with six parametric sliders – changing the values of r, a, s and p parameters will allow you to watch the appropriate forming of the surface, and by setting the values of parameters u and v you can change corresponding u-ruling and v-wavelet curve (Figure 5).

Video tutorial

The above described model provides a lot of options allowing alteration of the model but in fact it is not a creative tool. Therefore, a learning video was created as a possibility for anyone who would like to design this conoidal surface on his own.

The first and longest part of the video was made as a screen record of step by step modeling of the surface with Google SketchUp, a free application (Figure 6a). The second part shows how previous modeling can be extended to the geolocated model of a real building with realistic photo textures (Figure 6b) and in the third part the whole model can be viewed in Google Earth application (Figure 6c).

The entire video is uploaded to YouTube server and as such is a part of the above mentioned web page devoted to this interesting right conoid.

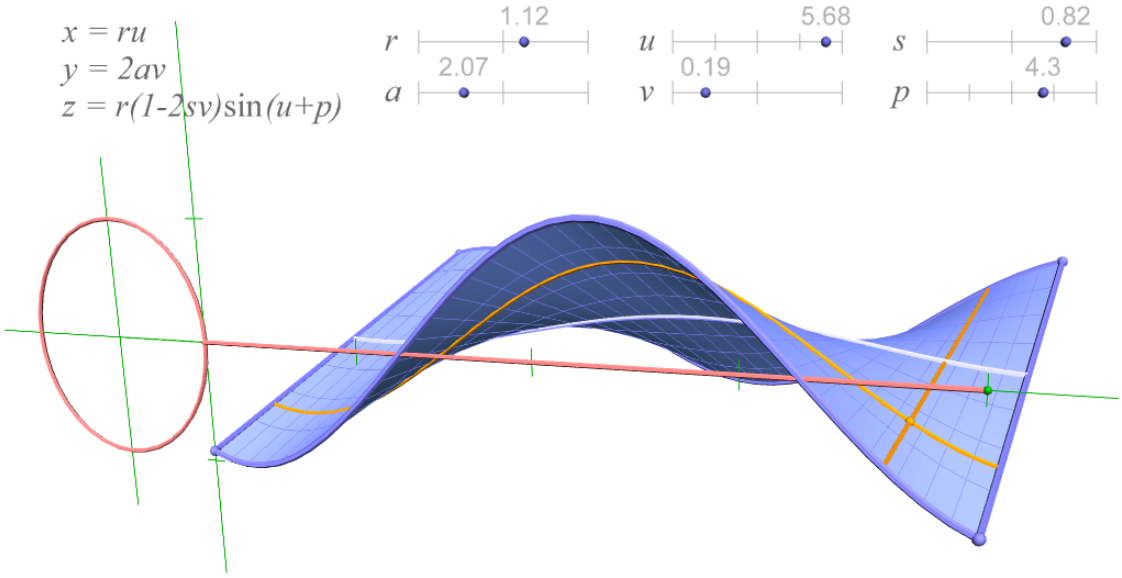

Figure 5. Parametric sliders used for better understanding of the relationship between the shaping of the surface and its mathematical expression.

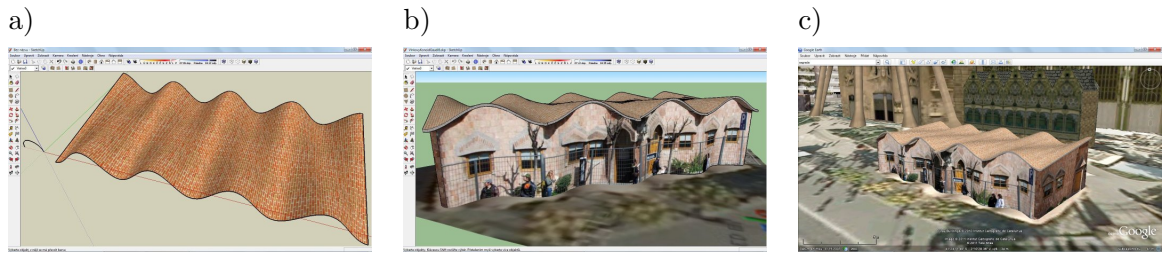

Figure 6. A model of the surface (a), its implementation in a real building using Google SketchUp (b) and sharing of the model in Google Earth application (c).

History of the surface

The right wavelet conoid was first used by the great Catalan architect Antoni Gaudí *(1852-1926)* and this is why the surface is sometimes called Gaudí's surface. The Temple of the Holy Family in Barcelona, called Sagrada Família in Spanish, is a famous work of Gaudí and several different kinds of ruled surfaces are implemented in this temple.

In 1906 a roof of a storehouse for plaster models was built near the Sagrada Família building site. The shape of the roof was a sine conoid (Figure 7a and in the background of Figure 7b). At the beginning of the Spanish Civil War in 1936 the whole complex of the buildings burned out and never been reconstructed to its original form.

Gaudí's temporary school building is much more famous – it was intended for children of Sagrada Família workers (Figure 7b). It was built in 1909, burned out in 1936, was reconstructed afterwards, burned out again in 1939 and was reconstructed again. Due to the proceeding work on the temple it was decided to remove and completely rebuild the school building – this was done in 2002 (Figure 7c and the model in Figures 6bc). There was also constructed a replica of the school building in a nearby town of Badalona.

What is interesting is that Gaudí used the conoidal shape not only for the roof but for walls, too. This allowed him to reduce the building costs to a great extent – it cost about 9000 pesetas at that time; current price of the building is estimated at 60 millions of pesetas.

Another world-renowned Spanish architect Santiago Calatrava *(*1951)* extended Gaudí's

a) b) c)

Figure 7. Gaudí's storehouse, studio and workshop (a); the original (b) and reconstructed (c) school building.

original idea into the motion using the sine phase shift. In 2000 he installed a moving sculpture called "Wave" in front of the Meadows Museum in Dallas (Figure 8a) and in 2004 he used the same idea as the "Nations Wall" for the Olympic Sports Complex in Athens (Figure 8b). In 2001 Calatrava designed a static variant of the right wavelet conoid as a roofing of Bodegas Ysios winery building in northern Spain (Figure 8c).

a) b) c)

Figure 8. The moving sculpture "Wave" in Dallas (a), the "Nations Wall" at the Olympic Sports Complex in Athens (b); the Bodegas Ysios winery building in northern Spain (c).

We can also find one implementation of this surface in the Czech Republic: a roofing of several buildings in Landal Marina holiday resort near the Lipno nad Vltavou village. In front of the central building the wavelet-sine curves coming from both sides rise to form a traditional Dutch female hat, a signature of Dutch studio Factor Architecten, designer of the building (designed in 2003, Figure 9).

Conclusion

This paper aims to show a complex illustration of using efficient geometric experience for effective technical practice, e.g. for magnificent architecture. Also it is a resume of the most relevant resources – historical, structural and purely mathematical.

All these results were compiled in the form of an attractive web presentation including the virtual 3D model, the guidance video and the list of realizations with many useful links for further study.

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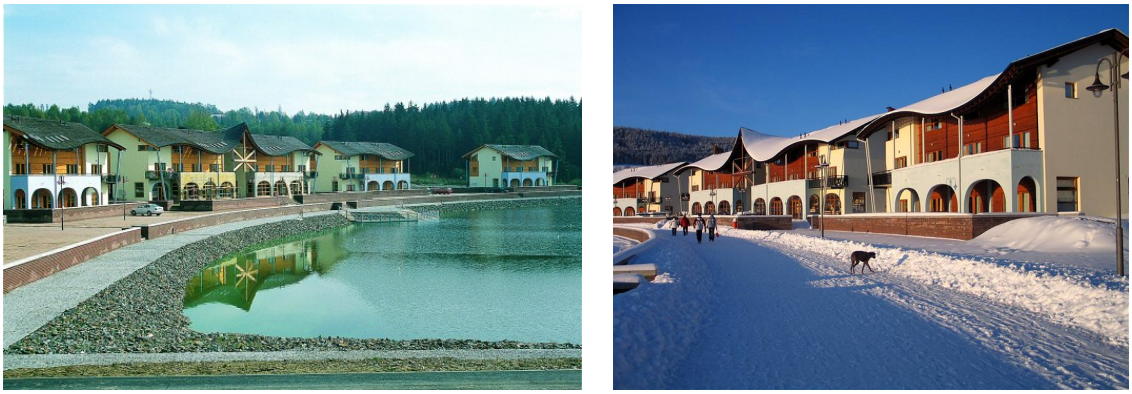

Figure 9. Summer and winter photos of conoidally roofed buildings near Lipno nad Vltavou.

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