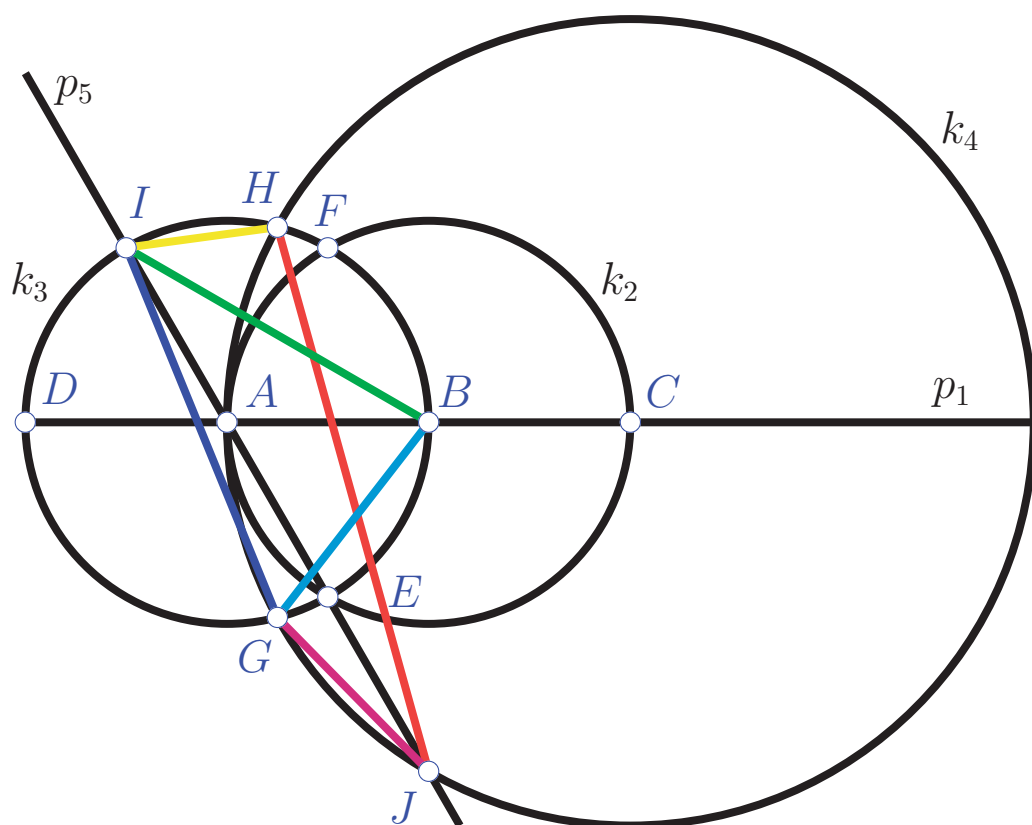


# The Golden Ratio Determined Using a Ruler and Compass



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Charles University

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Šárka Gergelitsová  
Tomáš Holan

**matfyz**press

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# 1 Introduction and Definitions

## 1.1 What This Book Is About

This book focuses on how to construct the golden ratio

$$\phi = \frac{\sqrt{5} + 1}{2}$$

using a ruler (straightedge) and a compass. This book will not discuss the history or occurrences of the golden ratio in nature, arts, architecture, etc.; numerous other books are devoted to such topics.

## 1.2 Why We Are Writing This Book

The golden ratio has been known for more than two thousand years, yet new ways to construct and describe the ratio are still discovered (e.g., [7]).

Therefore, we have decided to determine *the smallest number of steps* required to construct the golden ratio using different types of tools and to compile a *complete list* of these short constructions.

The construction of the golden ratio, the number of steps, the shortest construction and the tools utilized will be described later; in this section, note the following notation:

”Golden section” refers to either the number  $\phi$  or the division of a given segment into two parts with lengths in this ratio.

In this book, we use the name *golden ratio* to indicate the ratio  $\phi$ .

## 1.3 Terms

First, let us define the terms we will use:

**Step** is a construction (drawing) of a single line (straight line or circle) and its corresponding intersection points (required, but not always all) with already existing lines.

The definition for one step allows us to consider the length of the construction.

**Types of lines** are the following:

$p(A, B)$  a straight line passing through the given points;

$k(A, B)$  a circle with a given center and passing through the given point;  
and

$k(A, |BC|)$  a circle with a given center and a radius that is a distance between two points.

**Types of steps** corresponds to drawing the line of the given type (therefore, three total) and we will denote them identically to the type of the line, e.g., step of the type  $p(A, B)$ .

**Construction process** is a sequence of steps.

**Construction** is a set of lines and points that is a result of the construction process.

**Length of the construction** is the number of lines that equals the number of steps in every construction process of this construction.

**Different processes** Two processes are different if they are not permutations of each other, varying in more than the order of steps.

The identical construction (set of straight lines and circles) can sometimes be created by different processes by varying e.g., the position the pair of points forming the radius of the circle was selected from. Because we are describing the list of constructions, not the list of processes, from these processes we describe always one.

**Similar constructions** are constructions sets of lines (not points or names of lines) of which can be mapped onto each other by translation, scaling, rotation and/or reflection.

**Distance contained in the construction** is the distance between two points in the construction.

**Occurrence of the golden ratio in the construction** is the couple of distances contained in the construction, ratio of which (distances) equals to  $\phi$ .

**Construction of the golden ratio** is the construction containing the occurrence of the golden ratio.

This book contains *a list of all of the shortest constructions of the golden ratio*; these constructions are divided into *classes* to organize *similar constructions*. For every construction, we show *one process of construction*.

## 1.4 Types of Constructions

We previously mentioned construction by ruler and compass and we defined three types of steps: one step draws a straight line and two steps draw a circle.

Euclid in [1] uses a compass that collapses when lifted from the page. This compass we consider *a collapsible compass*, and a collapsible compass cannot be used for the steps  $k(A, |BC|)$ .

Mascheroni ([2]) and Mohr ([3]) stated that every ruler and compass construction can be constructed (without straight lines) by a compass only. These so-called Mohr-Mascheroni constructions do not use the steps of the type  $p(A, B)$ .

Other authors use other tools such as toothpicks or rusty compasses ([9], [10]). We distinguish four types of constructions, and we indicate them using the following two- or three-letter abbreviations:

**RC** ruler and compass constructions, can exploit all types of steps

**RCC** ruler and collapsible compass constructions, can use steps of type  $p(A, B)$  and  $k(A, B)$ , do not use steps of type  $k(A|BC|)$

**MM** Mohr-Mascheroni constructions use only a compass, can use steps of type  $k(A, B)$  and  $k(A, |AB|)$ , do not use steps of type  $p(A, B)$

**CCO** collapsible compass only constructions, can use only steps of type  $k(A, B)$

## 1.5 The Shortest Constructions

The goal of this book is to determine the minimal length required to construct the golden ratio for all four types of constructions. Additionally, this book presents the complete list of minimal length constructions of the golden ratio for each type.

In a construction, the golden ratio can be found at multiple different places. We will present only different constructions, and for every construction, we will show all possibilities how to find the golden ratio.

## 1.6 Naming the Constructions

To identify individual constructions, we label the constructions with a *prefix* indicating the type of construction (RC, RCC, MM or CCO; see above) and a *serial number*.

## 1.7 Content of This Book

After this introduction, a list of all shortest constructions will follow. The list is divided into four parts for the four types of construction. Following the list, the ideas of the proofs of the constructions are provided.

## 1.8 Number of the Shortest Constructions

For individual types of constructions, the following shortest constructions are found:

RC minimal length: 4 steps

number of golden ratio constructions for the minimal length: 4

number of processes of constructions for the minimal length: 8

number of classes for similar constructions: 1

RCC minimal length: 5 steps

number of golden ratio constructions for the minimal length: 457

number of processes of constructions for the minimal length: 2266

number of classes for similar constructions: 69

MM minimal length: 5 steps

number of golden ratio constructions for the minimal length: 368

number of processes of constructions for the minimal length: 988

number of classes for similar constructions: 80

CCO minimal length: 5 steps

number of golden ratio constructions for the minimal length: 114

number of processes of constructions for the minimal length: 1048

number of classes for similar constructions: 21

## 1.9 Relations between the Lists of Constructions of the Individual Types

Individual lists of the shortest constructions, except for constructions of the kind RC, that are one step shorter, are not mutually disjoint.

Every construction that can be drawn by a collapsible compass also belongs to constructions that can be created using a collapsible compass and a ruler (that will not be used).

The list of the shortest constructions of CCO is therefore (a proper) subset of the list of the shortest constructions of RC.

Similarly, every construction of CCO is also a construction of MM because constructions of both types do not use a ruler and can be constructed by a collapsible compass or an ordinary compass.

The list of the shortest constructions of CCO is therefore (also a proper) subset of the list of the shortest constructions of MM. Additionally, the list of the shortest constructions of CCO is the intersection of the lists of the shortest constructions of RC and MM.

Thus, in the list of constructions of RCC, only constructions that are not found in the constructions of CCO will be shown. Additionally, we do not show constructions of CCO in the list of the shortest constructions of MM. The numbering of constructions is not affected, and only some constructions from the lists for RCC and MM are omitted and are mentioned later with the corresponding construction in the list for CCO.

## 2 All of the Shortest Constructions

### 2.1 Description of a Construction

The description of every construction has identical format. For every construction, we present the following:

- name
- drawing in which the given point  $B$  is to the right of the given point  $A$
- the one process of construction creating this construction
- list of distances belonging to any of couples forming the golden ratio in this construction
- list of couples of distances (occurrences of the golden ratio) in this construction.

### 2.2 Description of a Process of Construction

The process of construction is described as a list of steps using lines and intersections. We use the following notation:

- $k_1 = k(A, B)$  circle with the center  $A$  passing through the point  $B$ ;
- $k_2 = k(A, |BC|)$  circle with the center  $A$  and the radius equal to the distance of the points  $B$  and  $C$ ;

- $p_3 = p(A, B)$  straight line passing through points  $A$  and  $B$ ; and
- $E \in k_4 \cap p_5$  point of intersection of two lines.

Individual lines in the description of the process of construction are marked  $p_i$  (straight lines) or  $k_i$  (circles), where index  $i$  is the number of the line in the construction. The points are indicated by the capitalized letters of the alphabet in ascending order.

### 2.3 Overview of All Classes of Similarity

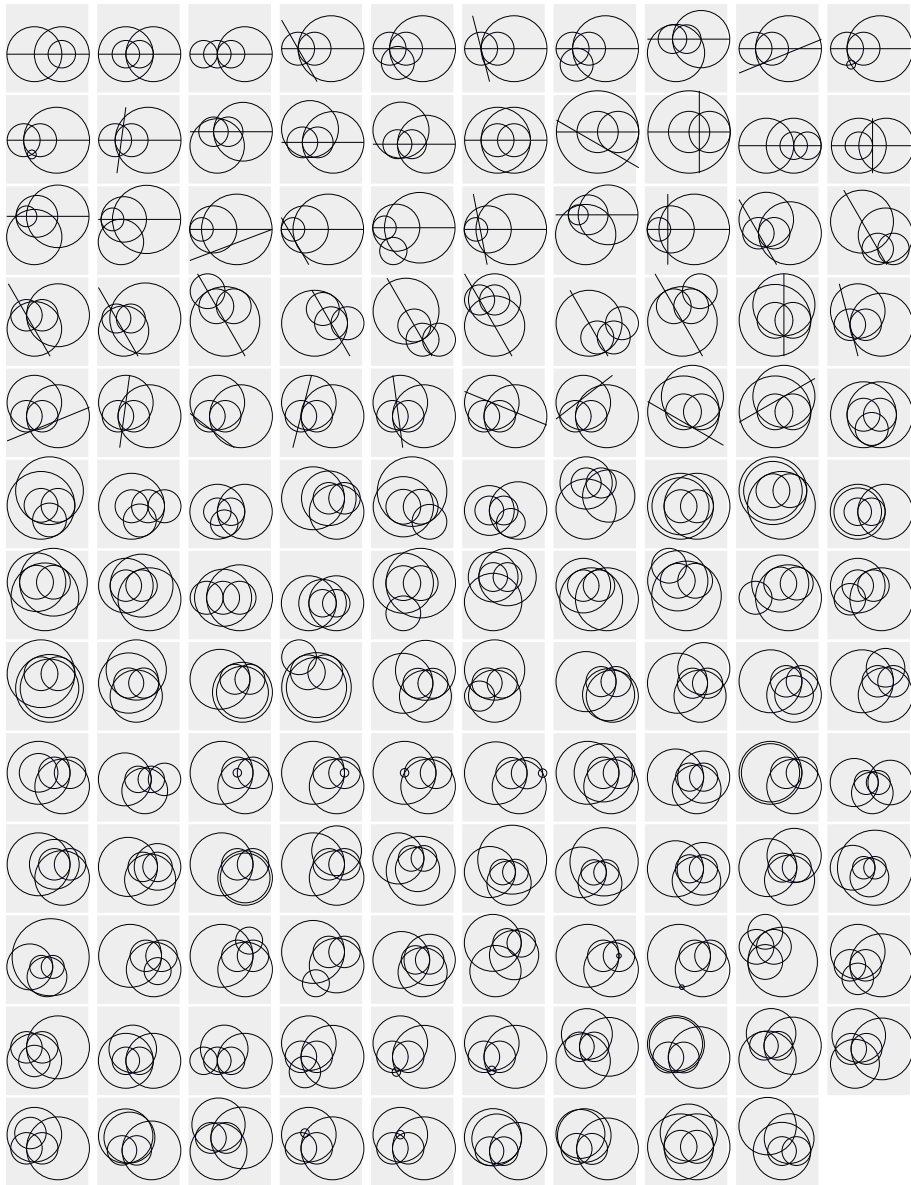
All constructions form classes of mutually similar constructions. In total, 715 different constructions are noted; however, only 129 similar classes are described. To easily find any specific construction, on the following page we present a table of representative images for all classes.

In these drawings, we do not indicate names of points or lines because of the readability. Additionally, the similarity of constructions concerns lines, not naming.

The 129 images must be small to be displayed on one page. Nevertheless, electronic versions of this book allow zooming and magnifying. However, to search the specific construction, visualizing the small pictures on one page before the list of bigger images on multiple pages allows for easier access.

Drawings have no numbers of classes or sections, but individual pictures have links embedded. Clicking on the picture transfers the reader to the appropriate section.

## 2.4 Overview of Classes of Similarity — Images



## 2.5 RC

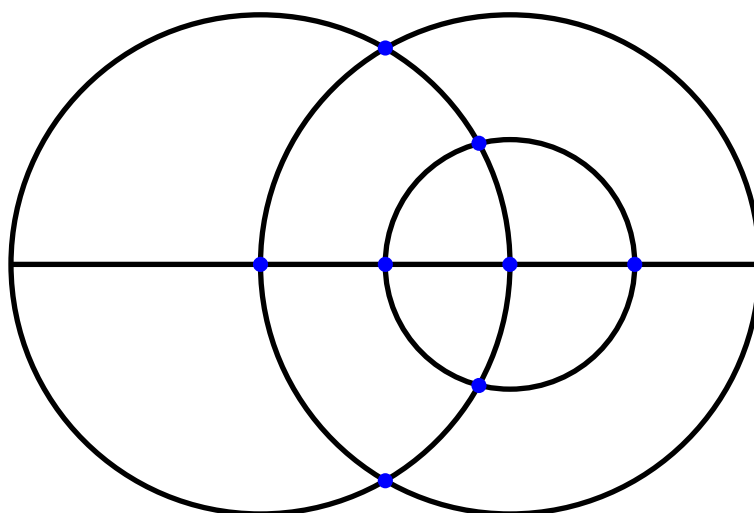
Construction processes presented in this section use the strongest tools of all four types: ruler (straight line passing through the given points) and a compass that can measure the distance between two arbitrary points and use this distance as a radius (a circle with a given center and radius).

Accordingly, constructions from this part are the shortest; to construct the golden ratio, they require just four steps.

Only four shortest constructions of the golden ratio are available (resulting from eight different processes). These constructions differ only by the roles of points  $A$  and  $B$  and by selecting one of the two points as the intersection of the circle and the straight line for the center of the large circle. Therefore, all constructions of this type are similar.

The constructions are named from RC1 to RC4; construction RC1 is identical with the construction described by K. Hofstetter [7].

## 2.6 Class of Similar Constructions No. 1

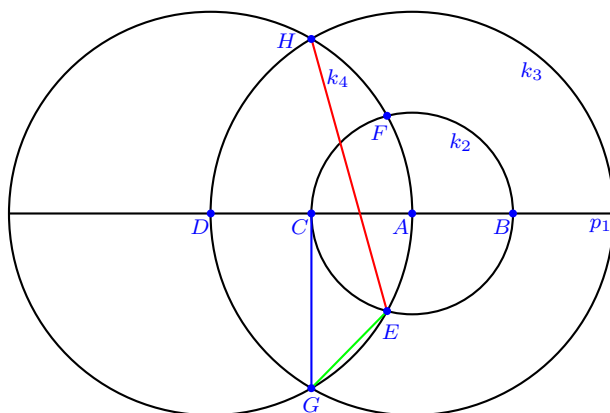


### Contained Constructions

RC1, RC2, RC3, RC4



## 2.6.1 Construction RC1



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(A, |AB|)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(A, |BC|)$   
 $D \in p_1 \cap k_3$
  4.  $k_4 = k(D, |BC|)$   
 $E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4$

## Distances

$$d_0 = |EH| = |FG| \text{ (red) } \dots 2$$

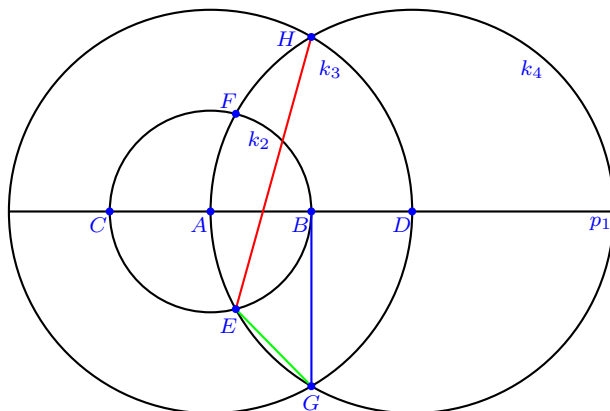
$$d_1 = |CG| = |CH| \text{ (blue) } \dots 2$$

$$d_2 = |EG| = |FH| \text{ (green) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)}$$

## 2.6.2 Construction RC2



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(A, |AB|)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(A, |BC|)$   
 $D \in p_1 \cap k_3$
  4.  $k_4 = k(D, |BC|)$   
 $E \in k_2 \cap k_4$ ,  $F \in k_2 \cap k_3$ ,  $G \in k_3 \cap k_4$ ,  $H \in k_3 \cap k_4$

## Distances

$$d_0 = |EH| = |FG| \text{ (red) } \dots 2$$

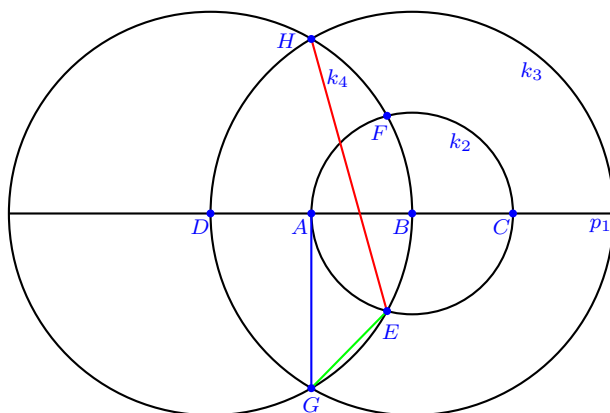
$$d_1 = |BG| = |BH| \text{ (blue) } \dots 2$$

$$d_2 = |EG| = |FH| \text{ (green) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)}$$

## 2.6.3 Construction RC3



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(B, |AC|)$   
 $D \in p_1 \cap k_3$
  4.  $k_4 = k(D, |AC|)$   
 $E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4$

## Distances

$$d_0 = |EH| = |FG| \text{ (red) } \dots 2$$

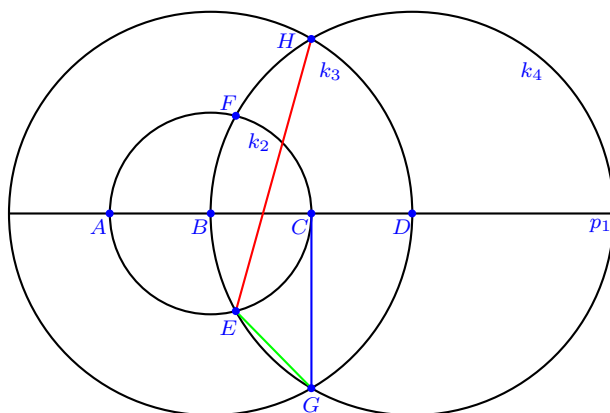
$$d_1 = |AG| = |AH| \text{ (blue) } \dots 2$$

$$d_2 = |EG| = |FH| \text{ (green) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)}$$

## 2.6.4 Construction RC4



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(B, |AC|)$   
 $D \in p_1 \cap k_3$
  4.  $k_4 = k(D, |AC|)$   
 $E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4$

## Distances

$$d_0 = |EH| = |FG| \text{ (red) } \dots 2$$

$$d_1 = |CG| = |CH| \text{ (blue) } \dots 2$$

$$d_2 = |EG| = |FH| \text{ (green) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)}$$

## 2.7 RCC

The construction processes presented in this section use more limited tools than processes in the previous section: a ruler (straight line passing through the given points) and a collapsible compass that cannot draw a circle with an arbitrary measured diameter and only allows drawing circles with a given center passing by another given point.

A compass drawing only circles  $k(A, B)$  cannot construct more ratios than a compass creating circles with an arbitrary measured diameter  $k(A, |BC|)$ . Fewer are constructed; therefore, the construction of the golden ratio by these tools requires at least 5 steps.

In total, 457 shortest constructions can be constructed by 2266 different processes forming 69 classes of mutually similar constructions.

The constructions are labeled RCC1 to RCC457.

From this list (the initial numbering is retained), we omit 114 constructions that do not require a ruler; These constructions are mentioned in the list of the constructions of CCO.

The presented constructions are numbered by the similarity of their process and divided into classes by similarity of the construction. Similarity between two constructions indicates that the straight lines and circles of the one construction can be mapped onto those of other by translation, scaling, rotation and reflection.

Construction RCC27 is identical to the construction described in [4] by K. Hofstetter; construction RCC52 to the construction of the same author described in [6].

### Remarks

From the constructions RC3 and RCC35, in which are constructed (apart from the names of lines and points) ratios of equally placed couples of points, we display the limited possibilities of the constructions using collapsible compasses. In RC3, we construct point  $D$  together with the large circle  $k_3 = k(B, |AC|)$  concentric with circle  $k_2$  because we can measure and transfer the distance  $AC$ . In RCC35, however, we have to first construct the point  $D$  using the circle (here  $k_3 = k(A, B)$ ), and thereafter construct the large circles (the circle concentric with the circle  $k_2$  passing through this point  $D$  ( $k_5 = k(B, D)$ ) and the circle with the center in the point  $D$  ( $k_4 = k(D, B)$ ).

(The order of constructing circles  $k_3$  and  $k_4$  in RC3 and circles  $k_4$  and  $k_5$  in RCC35 does not matter; the order can be reversed).

Therefore, RCC35 requires one more line than RC3.

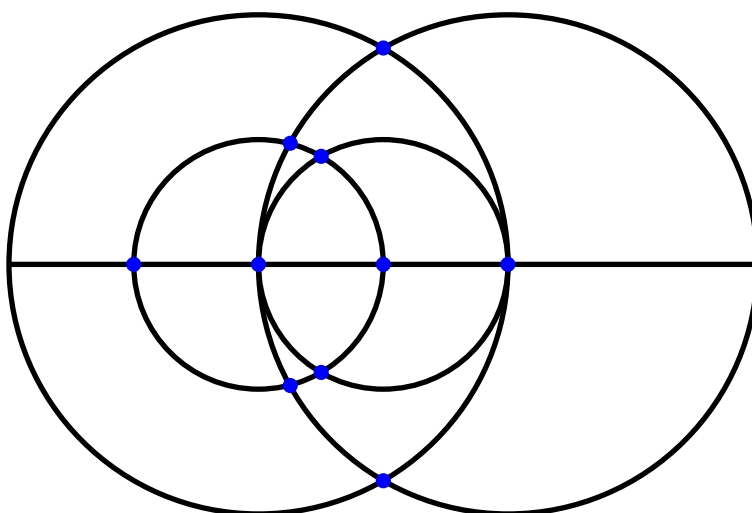
## 2.8 Constructions Omitted Due to Identity

RCC266 ( MM2 CCO1), RCC267 ( MM1 CCO2), RCC268 ( MM4 CCO3),  
 RCC269 ( MM3 CCO4), RCC270 ( MM12 CCO5), RCC271 ( MM11 CCO6),  
 RCC272 ( MM10 CCO7), RCC273 ( MM9 CCO8), RCC274 ( MM23 CCO9),  
 RCC275 ( MM24 CCO10), RCC276 ( MM5 CCO11), RCC277 ( MM6 CCO12),  
 RCC278 ( MM19 CCO13), RCC279 ( MM20 CCO14), RCC280 ( MM21 CCO15),  
 RCC281 ( MM22 CCO16), RCC282 ( MM14 CCO17), RCC283 ( MM13 CCO18),  
 RCC284 ( MM15 CCO19), RCC285 ( MM16 CCO20), RCC286 ( MM17 CCO21),  
 RCC287 ( MM18 CCO22), RCC288 ( MM7 CCO23), RCC289 ( MM8 CCO24),  
 RCC314 ( MM53 CCO25), RCC315 ( MM52 CCO26), RCC316 ( MM51 CCO27),  
 RCC317 ( MM50 CCO28), RCC318 ( MM64 CCO29), RCC319 ( MM65 CCO30),  
 RCC320 ( MM46 CCO31), RCC321 ( MM47 CCO32), RCC322 ( MM60 CCO33),  
 RCC323 ( MM61 CCO34), RCC324 ( MM62 CCO35), RCC325 ( MM63 CCO36),  
 RCC326 ( MM55 CCO37), RCC327 ( MM54 CCO38), RCC328 ( MM56 CCO39),  
 RCC329 ( MM57 CCO40), RCC330 ( MM58 CCO41), RCC331 ( MM59 CCO42),  
 RCC332 ( MM48 CCO43), RCC333 ( MM49 CCO44), RCC348 ( MM263 CCO45),  
 RCC349 ( MM258 CCO46), RCC351 ( MM323 CCO47), RCC353 ( MM339 CCO48),  
 RCC354 ( MM335 CCO49), RCC356 ( MM347 CCO50), RCC357 ( MM344 CCO51),  
 RCC358 ( MM325 CCO52), RCC360 ( MM353 CCO53), RCC362 ( MM326 CCO54),  
 RCC364 ( MM348 CCO55), RCC365 ( MM343 CCO56), RCC367 ( MM340 CCO57),  
 RCC368 ( MM336 CCO58), RCC369 ( MM328 CCO59), RCC371 ( MM354 CCO60),  
 RCC372 ( MM260 CCO61), RCC375 ( MM264 CCO62), RCC376 ( MM259 CCO63),  
 RCC379 ( MM267 CCO64), RCC381 ( MM277 CCO65), RCC383 ( MM293 CCO66),  
 RCC384 ( MM289 CCO67), RCC386 ( MM301 CCO68), RCC387 ( MM298 CCO69),  
 RCC389 ( MM307 CCO70), RCC390 ( MM279 CCO71), RCC392 ( MM280 CCO72),  
 RCC394 ( MM302 CCO73), RCC395 ( MM297 CCO74), RCC397 ( MM294 CCO75),  
 RCC398 ( MM290 CCO76), RCC400 ( MM308 CCO77), RCC401 ( MM282 CCO78),  
 RCC402 ( MM268 CCO79), RCC403 ( MM152 CCO80), RCC404 ( MM147 CCO81),  
 RCC406 ( MM212 CCO82), RCC408 ( MM228 CCO83), RCC409 ( MM224 CCO84),  
 RCC411 ( MM236 CCO85), RCC412 ( MM233 CCO86), RCC413 ( MM214 CCO87),  
 RCC415 ( MM242 CCO88), RCC417 ( MM215 CCO89), RCC419 ( MM237 CCO90),  
 RCC420 ( MM232 CCO91), RCC422 ( MM229 CCO92), RCC423 ( MM225 CCO93),  
 RCC424 ( MM217 CCO94), RCC426 ( MM243 CCO95), RCC427 ( MM149 CCO96),  
 RCC430 ( MM153 CCO97), RCC431 ( MM148 CCO98), RCC434 ( MM156 CCO99),  
 RCC436 ( MM166 CCO100), RCC438 ( MM182 CCO101), RCC439 ( MM178 CCO102),  
 RCC441 ( MM190 CCO103), RCC442 ( MM187 CCO104), RCC444 ( MM196 CCO105),  
 RCC445 ( MM168 CCO106), RCC447 ( MM169 CCO107), RCC449 ( MM191 CCO108),  
 RCC450 ( MM186 CCO109), RCC452 ( MM183 CCO110), RCC453 ( MM179 CCO111),  
 RCC455 ( MM197 CCO112), RCC456 ( MM171 CCO113), RCC457 ( MM157 CCO114),

Omitted 114 constructions.

Contains 343 constructions.

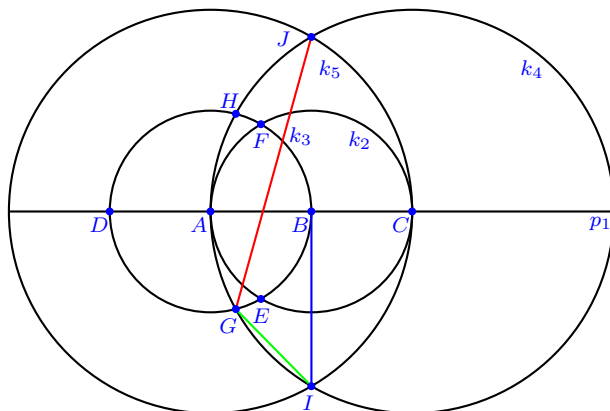
## 2.9 Class of Similar Constructions No. 2



### Contained Constructions

RCC1, RCC35, RCC66, RCC127, RCC159, RCC190

## 2.9.1 Construction RCC1



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(A, B)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(C, A)$   
 $G \in k_3 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(A, C)$   
 $I \in k_4 \cap k_5, J \in k_4 \cap k_5$

## Distances

$$d_0 = |GJ| = |HI| \text{ (red)} \dots 2$$

$$d_1 = |BI| = |BJ| = |CE| = |CF| = |DE| = |DF| = |EF| \text{ (blue)} \dots 7$$

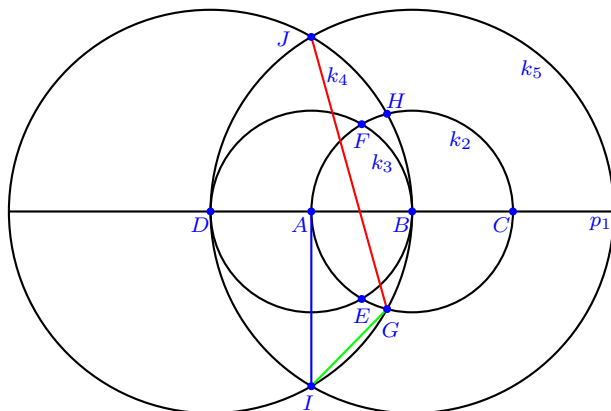
$$d_2 = |GI| = |HJ| \text{ (green)} \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (28 occurrences)}$$



## 2.9.2 Construction RCC35



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(A, B)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(D, B)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
  5.  $k_5 = k(B, D)$   
 $I \in k_4 \cap k_5, J \in k_4 \cap k_5$

## Distances

$$d_0 = |GJ| = |HI| \text{ (red) } \dots 2$$

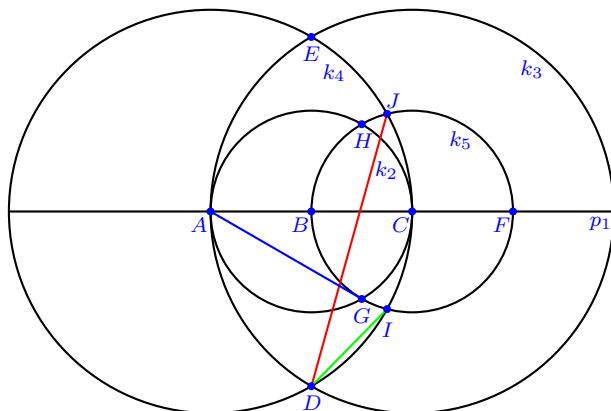
$$d_1 = |AI| = |AJ| = |CE| = |CF| = |DE| = |DF| = |EF| \text{ (blue) } \dots 7$$

$$d_2 = |GI| = |HJ| \text{ (green) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (28 occurrences)}$$

## 2.9.3 Construction RCC66



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, A)$
  4.  $k_4 = k(A, C)$   
 $D \in k_3 \cap k_4, E \in k_3 \cap k_4$
  5.  $k_5 = k(C, B)$   
 $F \in p_1 \cap k_5, G \in k_2 \cap k_5, H \in k_2 \cap k_5, I \in k_4 \cap k_5, J \in k_4 \cap k_5$

## Distances

$$d_0 = |DJ| = |EI| \text{ (red) } \dots 2$$

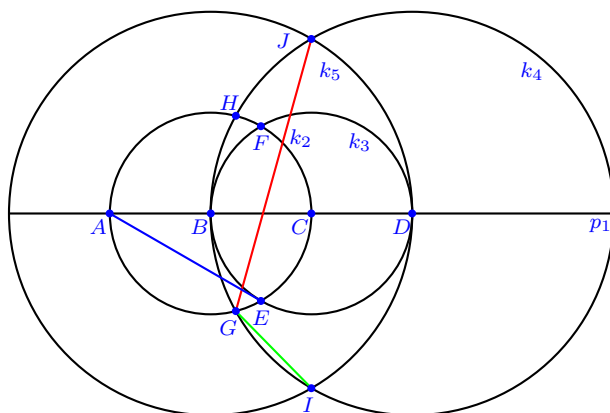
$$d_1 = |AG| = |AH| = |BD| = |BE| = |FG| = |FH| = |GH| \text{ (blue) } \dots 7$$

$$d_2 = |DI| = |EJ| \text{ (green) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (28 occurrences)}$$

## 2.9.4 Construction RCC127



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, B)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(D, B)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
  5.  $k_5 = k(B, D)$   
 $I \in k_4 \cap k_5, J \in k_4 \cap k_5$

## Distances

$$d_0 = |GJ| = |HI| \text{ (red)} \dots 2$$

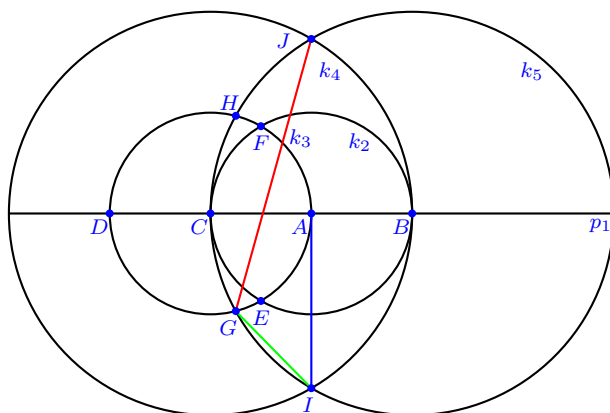
$$d_1 = |AE| = |AF| = |CI| = |CJ| = |DE| = |DF| = |EF| \text{ (blue)} \dots 7$$

$$d_2 = |GI| = |HJ| \text{ (green)} \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (28 occurrences)}$$

## 2.9.5 Construction RCC159



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, A)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(C, B)$
  5.  $k_5 = k(B, C)$   
 $G \in k_3 \cap k_5, H \in k_3 \cap k_5, I \in k_4 \cap k_5, J \in k_4 \cap k_5$

## Distances

$$d_0 = |GJ| = |HI| \text{ (red)} \dots 2$$

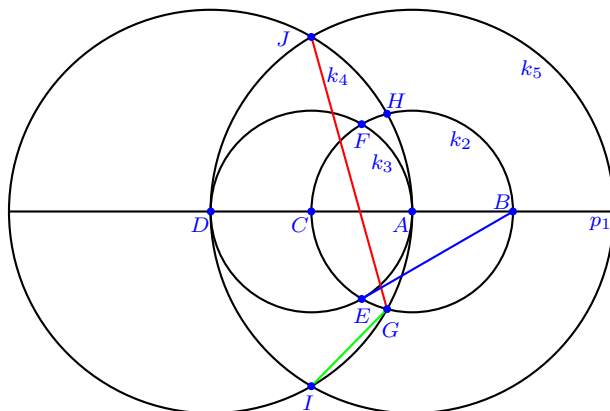
$$d_1 = |AI| = |AJ| = |BE| = |BF| = |DE| = |DF| = |EF| \text{ (blue)} \dots 7$$

$$d_2 = |GI| = |HJ| \text{ (green)} \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (28 occurrences)}$$

## 2.9.6 Construction RCC190



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, A)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(D, A)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
  5.  $k_5 = k(A, D)$   
 $I \in k_4 \cap k_5, J \in k_4 \cap k_5$

## Distances

$$d_0 = |GJ| = |HI| \text{ (red) } \dots 2$$

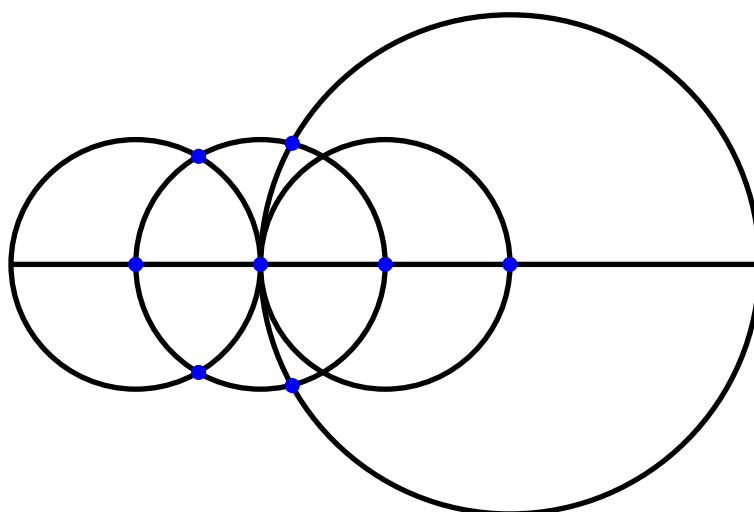
$$d_1 = |BE| = |BF| = |CI| = |CJ| = |DE| = |DF| = |EF| \text{ (blue) } \dots 7$$

$$d_2 = |GI| = |HJ| \text{ (green) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (28 occurrences)}$$

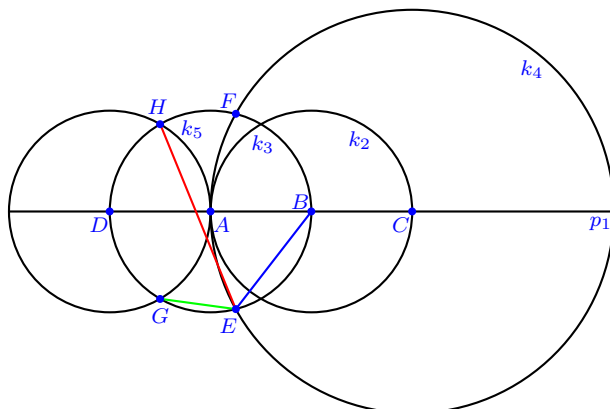
## 2.10 Class of Similar Constructions No. 3



### Contained Constructions

RCC2, RCC32, RCC33, RCC34, RCC87, RCC156, RCC165, RCC219

## 2.10.1 Construction RCC2



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(A, B)$   
 $D \in p_1 \cap k_3$
  4.  $k_4 = k(C, A)$   
 $E \in k_3 \cap k_4, F \in k_3 \cap k_4$
  5.  $k_5 = k(D, A)$   
 $G \in k_3 \cap k_5, H \in k_3 \cap k_5$

## Distances

$$d_0 = |EH| = |FG| \text{ (red) } \dots 2$$

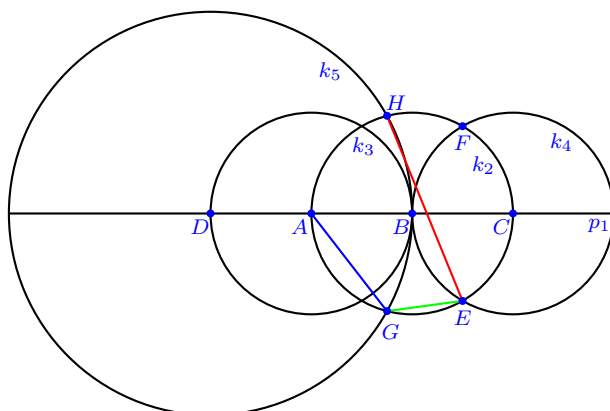
$$d_1 = |BE| = |BF| \text{ (blue) } \dots 2$$

$$d_2 = |EG| = |FH| \text{ (green) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)}$$

## 2.10.2 Construction RCC32



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(A, B)$   
 $D \in p_1 \cap k_3$
  4.  $k_4 = k(C, B)$   
 $E \in k_2 \cap k_4, F \in k_2 \cap k_4$
  5.  $k_5 = k(D, B)$   
 $G \in k_2 \cap k_5, H \in k_2 \cap k_5$

## Distances

$$d_0 = |EH| = |FG| \text{ (red) } \dots 2$$

$$d_1 = |AG| = |AH| \text{ (blue) } \dots 2$$

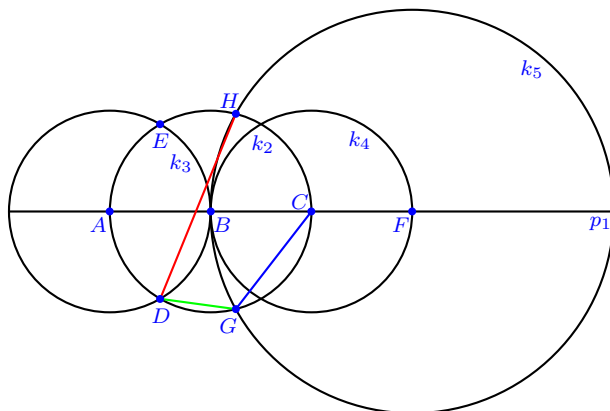
$$d_2 = |EG| = |FH| \text{ (green) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)}$$



## 2.10.3 Construction RCC33



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(A, B)$   
 $D \in k_2 \cap k_3, E \in k_2 \cap k_3$
  4.  $k_4 = k(C, B)$   
 $F \in p_1 \cap k_4$
  5.  $k_5 = k(F, B)$   
 $G \in k_2 \cap k_5, H \in k_2 \cap k_5$

## Distances

$$d_0 = |DH| = |EG| \text{ (red) } \dots 2$$

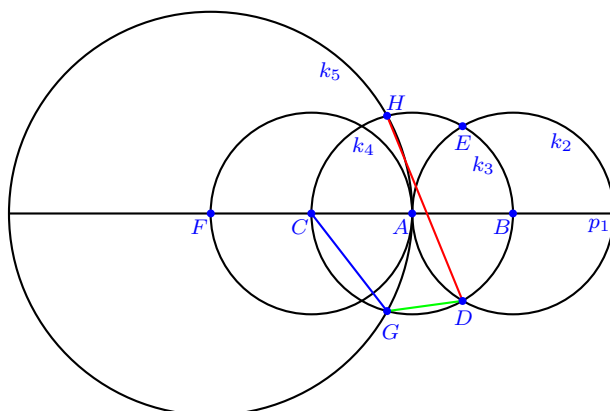
$$d_1 = |CG| = |CH| \text{ (blue) } \dots 2$$

$$d_2 = |DG| = |EH| \text{ (green) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)}$$

## 2.10.4 Construction RCC34



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$
  3.  $k_3 = k(A, B)$   
 $C \in p_1 \cap k_3, D \in k_2 \cap k_3, E \in k_2 \cap k_3$
  4.  $k_4 = k(C, A)$   
 $F \in p_1 \cap k_4$
  5.  $k_5 = k(F, A)$   
 $G \in k_3 \cap k_5, H \in k_3 \cap k_5$

## Distances

$$d_0 = |DH| = |EG| \text{ (red) } \dots 2$$

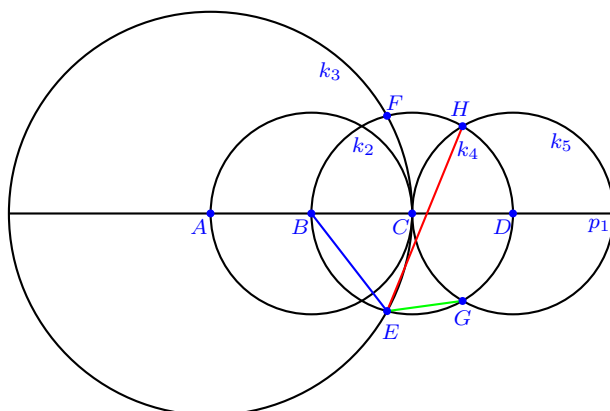
$$d_1 = |CG| = |CH| \text{ (blue) } \dots 2$$

$$d_2 = |DG| = |EH| \text{ (green) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)}$$

## 2.10.5 Construction RCC87



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(A, C)$
  4.  $k_4 = k(C, B)$   
 $D \in p_1 \cap k_4, E \in k_3 \cap k_4, F \in k_3 \cap k_4$
  5.  $k_5 = k(D, C)$   
 $G \in k_4 \cap k_5, H \in k_4 \cap k_5$

## Distances

$$d_0 = |EH| = |FG| \text{ (red) } \dots 2$$

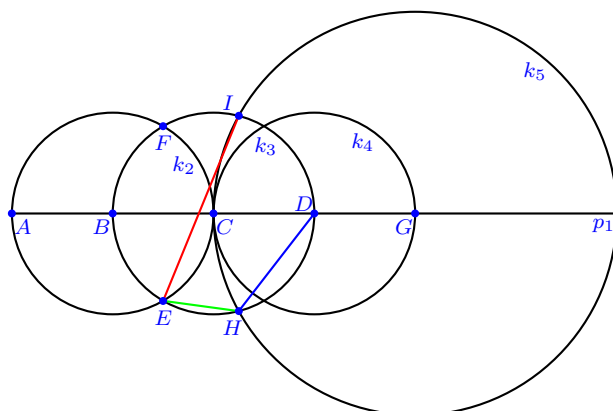
$$d_1 = |BE| = |BF| \text{ (blue) } \dots 2$$

$$d_2 = |EG| = |FH| \text{ (green) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)}$$

## 2.10.6 Construction RCC156



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, B)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(D, C)$   
 $G \in p_1 \cap k_4$
  5.  $k_5 = k(G, C)$   
 $H \in k_3 \cap k_5, I \in k_3 \cap k_5$

## Distances

$$d_0 = |EI| = |FH| \text{ (red) } \dots 2$$

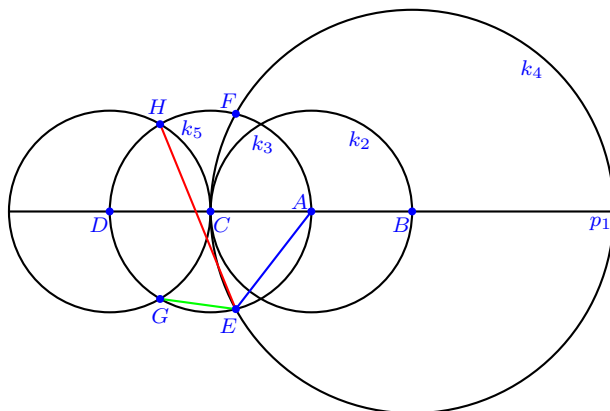
$$d_1 = |DH| = |DI| \text{ (blue) } \dots 2$$

$$d_2 = |EH| = |FI| \text{ (green) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)}$$

## 2.10.7 Construction RCC165



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, A)$   
 $D \in p_1 \cap k_3$
  4.  $k_4 = k(B, C)$   
 $E \in k_3 \cap k_4, F \in k_3 \cap k_4$
  5.  $k_5 = k(D, C)$   
 $G \in k_3 \cap k_5, H \in k_3 \cap k_5$

## Distances

$$d_0 = |EH| = |FG| \text{ (red) } \dots 2$$

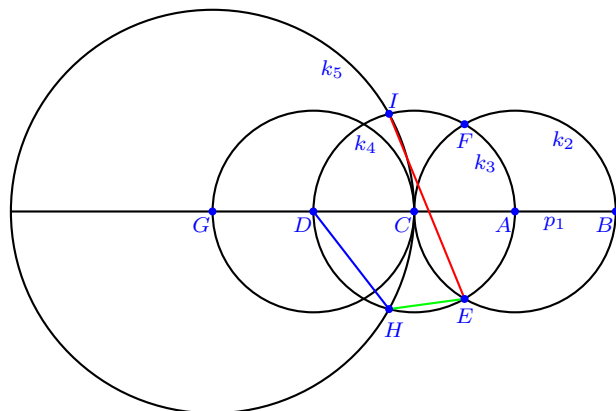
$$d_1 = |AE| = |AF| \text{ (blue) } \dots 2$$

$$d_2 = |EG| = |FH| \text{ (green) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)}$$

## 2.10.8 Construction RCC219



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, A)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(D, C)$   
 $G \in p_1 \cap k_4$
  5.  $k_5 = k(G, C)$   
 $H \in k_3 \cap k_5, I \in k_3 \cap k_5$

## Distances

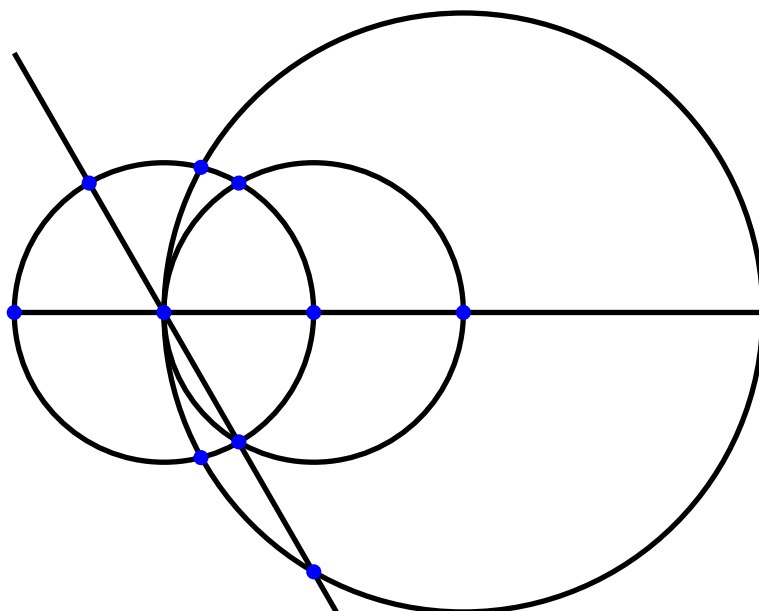
$$d_0 = |EI| = |FH| \text{ (red) } \dots 2$$

$$d_1 = |DH| = |DI| \text{ (blue) } \dots 2$$

$$d_2 = |EH| = |FI| \text{ (green) } \dots 2$$

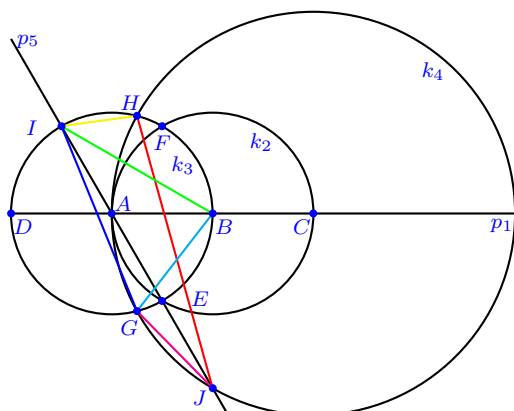
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)}$$

**2.11 Class of Similar Constructions No. 4****Contained Constructions**

RCC3, RCC5, RCC37, RCC39, RCC89, RCC91, RCC128, RCC130, RCC167,  
RCC169, RCC191, RCC193

## 2.11.1 Construction RCC3



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(A, B)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(C, A)$   
 $G \in k_3 \cap k_4, H \in k_3 \cap k_4$
  5.  $p_5 = p(A, E)$   
 $I \in p_5 \cap k_3, J \in p_5 \cap k_4$

## Distances

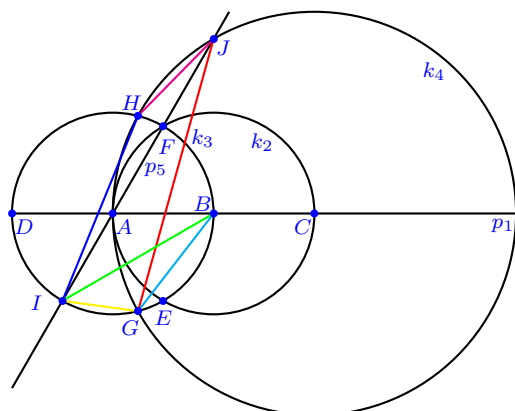
- $d_0 = |HJ|$  (red) ... 1  
 $d_1 = |GI|$  (blue) ... 1  
 $d_2 = |BI| = |BJ| = |CE| = |CF| = |DE| = |DF| = |EF|$  (green) ... 7  
 $d_3 = |BG| = |BH|$  (cyan) ... 2  
 $d_4 = |GJ|$  (magenta) ... 1  
 $d_5 = |HI|$  (yellow) ... 1

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (18 occurrences)}$$



## 2.11.2 Construction RCC5



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(A, B)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(C, A)$   
 $G \in k_3 \cap k_4, H \in k_3 \cap k_4$
  5.  $p_5 = p(A, F)$   
 $I \in p_5 \cap k_3, J \in p_5 \cap k_4$

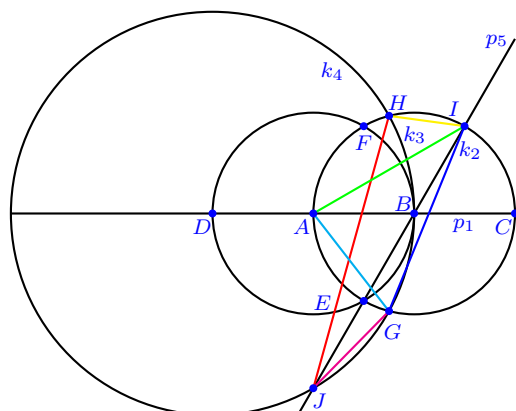
## Distances

- $d_0 = |GJ|$  (red) ... 1  
 $d_1 = |HI|$  (blue) ... 1  
 $d_2 = |BI| = |BJ| = |CE| = |CF| = |DE| = |DF| = |EF|$  (green) ... 7  
 $d_3 = |BG| = |BH|$  (cyan) ... 2  
 $d_4 = |HJ|$  (magenta) ... 1  
 $d_5 = |GI|$  (yellow) ... 1

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (18 occurrences)}$$

## 2.11.3 Construction RCC37



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(A, B)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(D, B)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
  5.  $p_5 = p(B, E)$   
 $I \in p_5 \cap k_2, J \in p_5 \cap k_4$

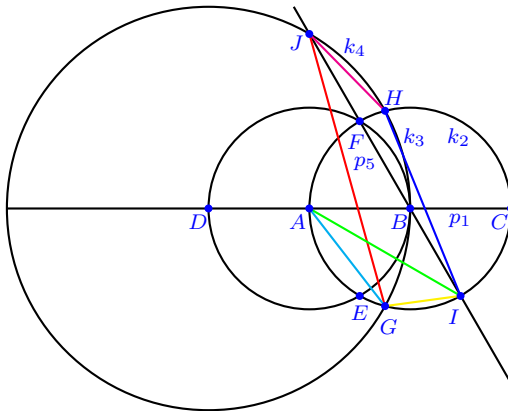
## Distances

- $d_0 = |HJ|$  (red) ... 1  
 $d_1 = |GI|$  (blue) ... 1  
 $d_2 = |AI| = |AJ| = |CE| = |CF| = |DE| = |DF| = |EF|$  (green) ... 7  
 $d_3 = |AG| = |AH|$  (cyan) ... 2  
 $d_4 = |GJ|$  (magenta) ... 1  
 $d_5 = |HI|$  (yellow) ... 1

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (18 occurrences)}$$

## 2.11.4 Construction RCC39



## Construction Process

- $A, B$  given initial points
- $p_1 = p(A, B)$
  - $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  - $k_3 = k(A, B)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  - $k_4 = k(D, B)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
  - $p_5 = p(B, F)$   
 $I \in p_5 \cap k_2, J \in p_5 \cap k_4$

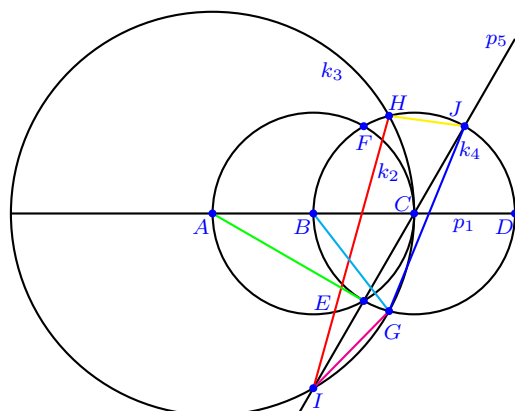
## Distances

$$\begin{aligned}
 d_0 &= |GJ| \text{ (red)} \dots 1 \\
 d_1 &= |HI| \text{ (blue)} \dots 1 \\
 d_2 &= |AI| = |AJ| = |CE| = |CF| = |DE| = |DF| = |EF| \text{ (green)} \dots 7 \\
 d_3 &= |AG| = |AH| \text{ (cyan)} \dots 2 \\
 d_4 &= |HJ| \text{ (magenta)} \dots 1 \\
 d_5 &= |GI| \text{ (yellow)} \dots 1
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (18 occurrences)}$$

## 2.11.5 Construction RCC89



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(A, C)$
  4.  $k_4 = k(C, B)$   
 $D \in p_1 \cap k_4, E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4$
  5.  $p_5 = p(C, E)$   
 $I \in p_5 \cap k_3, J \in p_5 \cap k_4$

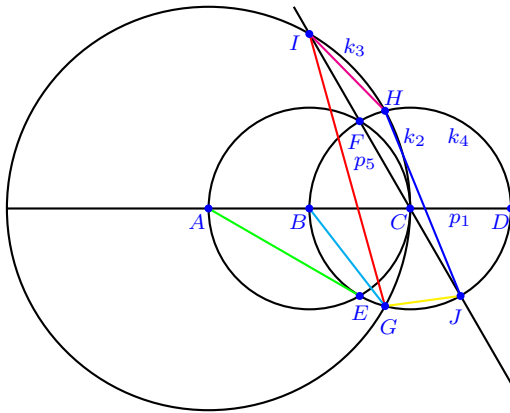
## Distances

- $d_0 = |HI|$  (red) ... 1  
 $d_1 = |GJ|$  (blue) ... 1  
 $d_2 = |AE| = |AF| = |BI| = |BJ| = |DE| = |DF| = |EF|$  (green) ... 7  
 $d_3 = |BG| = |BH|$  (cyan) ... 2  
 $d_4 = |GI|$  (magenta) ... 1  
 $d_5 = |HJ|$  (yellow) ... 1

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \quad (18 \text{ occurrences})$$

## 2.11.6 Construction RCC91



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(A, C)$
  4.  $k_4 = k(C, B)$   
 $D \in p_1 \cap k_4, E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4$
  5.  $p_5 = p(C, F)$   
 $I \in p_5 \cap k_3, J \in p_5 \cap k_4$

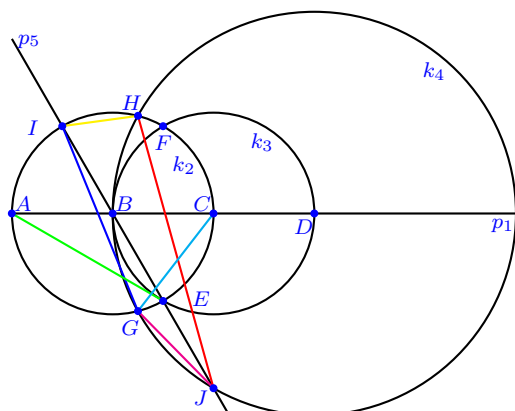
## Distances

- $d_0 = |GI|$  (red) ... 1  
 $d_1 = |HJ|$  (blue) ... 1  
 $d_2 = |AE| = |AF| = |BI| = |BJ| = |DE| = |DF| = |EF|$  (green) ... 7  
 $d_3 = |BC| = |BH|$  (cyan) ... 2  
 $d_4 = |HI|$  (magenta) ... 1  
 $d_5 = |GJ|$  (yellow) ... 1

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (18 occurrences)}$$

## 2.11.7 Construction RCC128



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, B)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(D, B)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
  5.  $p_5 = p(B, E)$   
 $I \in p_5 \cap k_2, J \in p_5 \cap k_4$

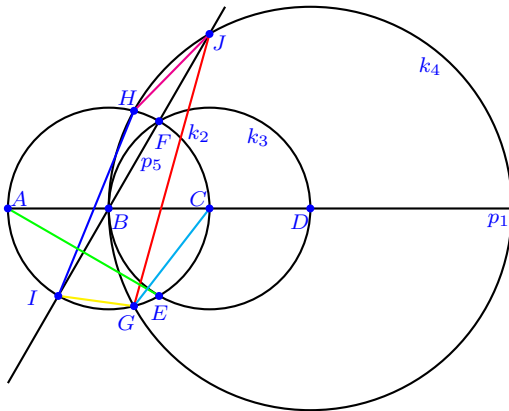
## Distances

- $d_0 = |HJ|$  (red) ... 1  
 $d_1 = |GI|$  (blue) ... 1  
 $d_2 = |AE| = |AF| = |CI| = |CJ| = |DE| = |DF| = |EF|$  (green) ... 7  
 $d_3 = |CG| = |CH|$  (cyan) ... 2  
 $d_4 = |GJ|$  (magenta) ... 1  
 $d_5 = |HI|$  (yellow) ... 1

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (18 occurrences)}$$

## 2.11.8 Construction RCC130



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, B)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(D, B)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
  5.  $p_5 = p(B, F)$   
 $I \in p_5 \cap k_2, J \in p_5 \cap k_4$

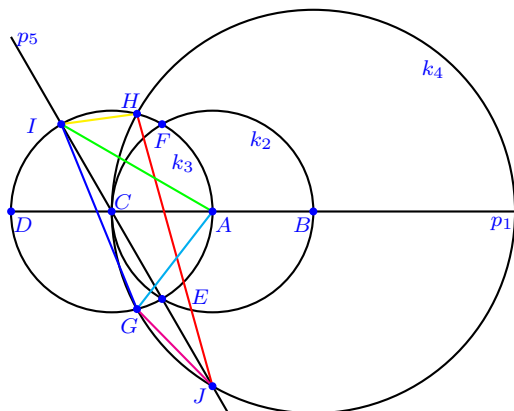
## Distances

$$\begin{aligned}
 d_0 &= |GJ| \text{ (red)} \dots 1 \\
 d_1 &= |HI| \text{ (blue)} \dots 1 \\
 d_2 &= |AE| = |AF| = |CI| = |CJ| = |DE| = |DF| = |EF| \text{ (green)} \dots 7 \\
 d_3 &= |CG| = |CH| \text{ (cyan)} \dots 2 \\
 d_4 &= |HJ| \text{ (magenta)} \dots 1 \\
 d_5 &= |GI| \text{ (yellow)} \dots 1
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (18 occurrences)}$$

## 2.11.9 Construction RCC167



## Construction Process

- $A, B$  given initial points
- $p_1 = p(A, B)$
  - $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
  - $k_3 = k(C, A)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  - $k_4 = k(B, C)$   
 $G \in k_3 \cap k_4, H \in k_3 \cap k_4$
  - $p_5 = p(C, E)$   
 $I \in p_5 \cap k_3, J \in p_5 \cap k_4$

## Distances

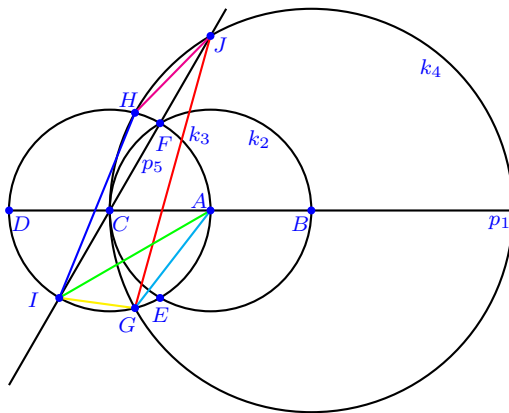
- $d_0 = |HJ|$  (red) ... 1  
 $d_1 = |GI|$  (blue) ... 1  
 $d_2 = |AI| = |AJ| = |BE| = |BF| = |DE| = |DF| = |EF|$  (green) ... 7  
 $d_3 = |AG| = |AH|$  (cyan) ... 2  
 $d_4 = |GJ|$  (magenta) ... 1  
 $d_5 = |HI|$  (yellow) ... 1

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \quad (18 \text{ occurrences})$$



## 2.11.10 Construction RCC169



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, A)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(B, C)$   
 $G \in k_3 \cap k_4, H \in k_3 \cap k_4$
  5.  $p_5 = p(C, F)$   
 $I \in p_5 \cap k_3, J \in p_5 \cap k_4$

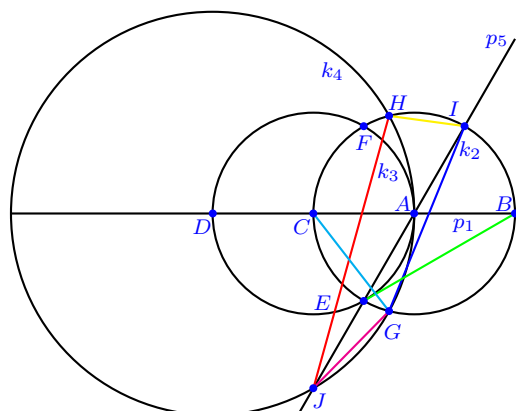
## Distances

- $d_0 = |GJ|$  (red) ... 1  
 $d_1 = |HI|$  (blue) ... 1  
 $d_2 = |AI| = |AJ| = |BE| = |BF| = |DE| = |DF| = |EF|$  (green) ... 7  
 $d_3 = |AG| = |AH|$  (cyan) ... 2  
 $d_4 = |HJ|$  (magenta) ... 1  
 $d_5 = |GI|$  (yellow) ... 1

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (18 occurrences)}$$

## 2.11.11 Construction RCC191



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, A)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(D, A)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
  5.  $p_5 = p(A, E)$   
 $I \in p_5 \cap k_2, J \in p_5 \cap k_4$

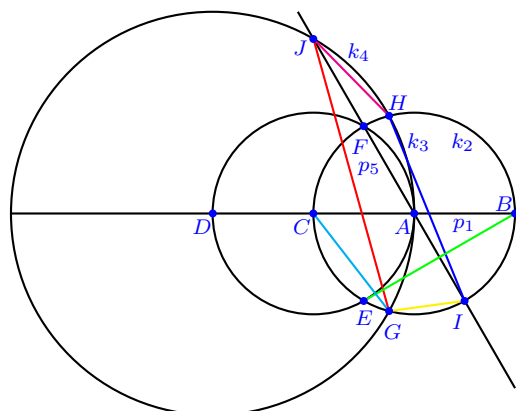
## Distances

- $d_0 = |HJ|$  (red) ... 1  
 $d_1 = |GI|$  (blue) ... 1  
 $d_2 = |BE| = |BF| = |CI| = |CJ| = |DE| = |DF| = |EF|$  (green) ... 7  
 $d_3 = |CG| = |CH|$  (cyan) ... 2  
 $d_4 = |GJ|$  (magenta) ... 1  
 $d_5 = |HI|$  (yellow) ... 1

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (18 occurrences)}$$

## 2.11.12 Construction RCC193



## Construction Process

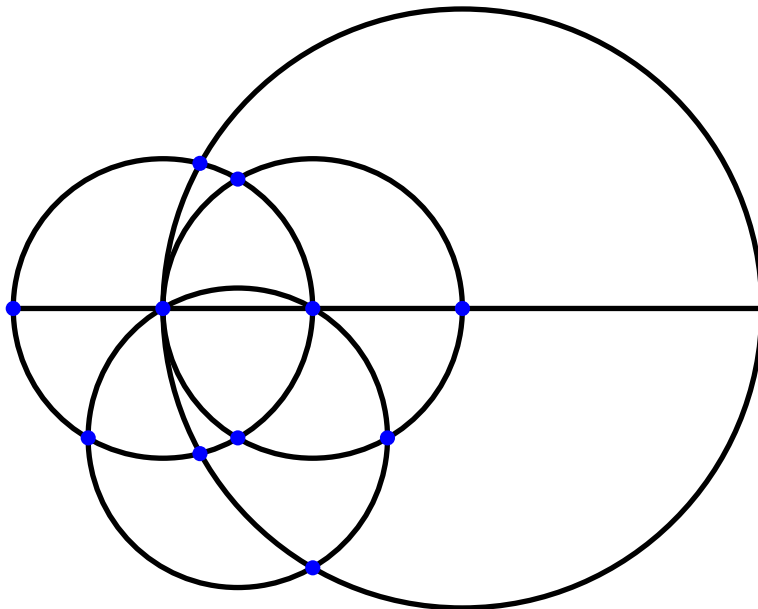
- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, A)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(D, A)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
  5.  $p_5 = p(A, F)$   
 $I \in p_5 \cap k_2, J \in p_5 \cap k_4$

## Distances

- $d_0 = |GJ|$  (red) ... 1  
 $d_1 = |HI|$  (blue) ... 1  
 $d_2 = |BE| = |BF| = |CI| = |CJ| = |DE| = |DF| = |EF|$  (green) ... 7  
 $d_3 = |CG| = |CH|$  (cyan) ... 2  
 $d_4 = |HJ|$  (magenta) ... 1  
 $d_5 = |GI|$  (yellow) ... 1

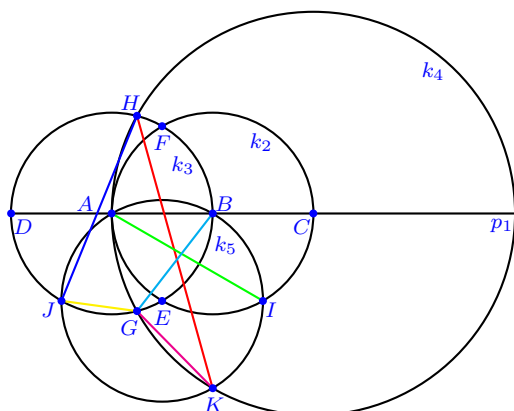
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (18 occurrences)}$$

**2.12 Class of Similar Constructions No. 5****Contained Constructions**

RCC4, RCC6, RCC36, RCC38, RCC88, RCC90, RCC129, RCC131, RCC166,  
RCC168, RCC192, RCC194, RCC252, RCC253, RCC264, RCC265, RCC300,  
RCC301, RCC312, RCC313

## 2.12.1 Construction RCC4



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(A, B)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(C, A)$   
 $G \in k_3 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(E, A)$   
 $I \in k_2 \cap k_5, J \in k_3 \cap k_5, K \in k_4 \cap k_5$

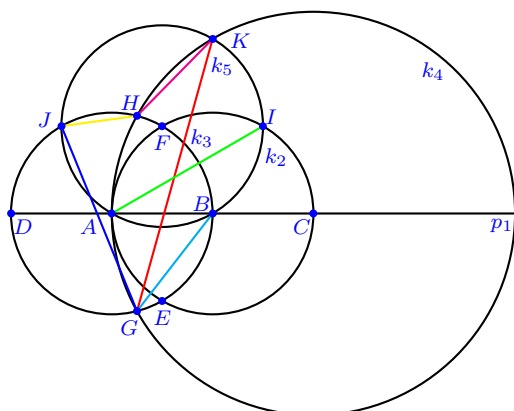
## Distances

$$\begin{aligned}
 d_0 &= |HK| \text{ (red)} \dots 1 \\
 d_1 &= |HJ| \text{ (blue)} \dots 1 \\
 d_2 &= |AI| = |BJ| = |BK| = |CE| = |CF| = |DE| = |DF| = |EF| = \\
 &= |JK| \text{ (green)} \dots 9 \\
 d_3 &= |BG| = |BH| \text{ (cyan)} \dots 2 \\
 d_4 &= |GK| \text{ (magenta)} \dots 1 \\
 d_5 &= |GJ| \text{ (yellow)} \dots 1
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (22 occurrences)}$$

## 2.12.2 Construction RCC6



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(A, B)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(C, A)$   
 $G \in k_3 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(F, A)$   
 $I \in k_2 \cap k_5, J \in k_3 \cap k_5, K \in k_4 \cap k_5$

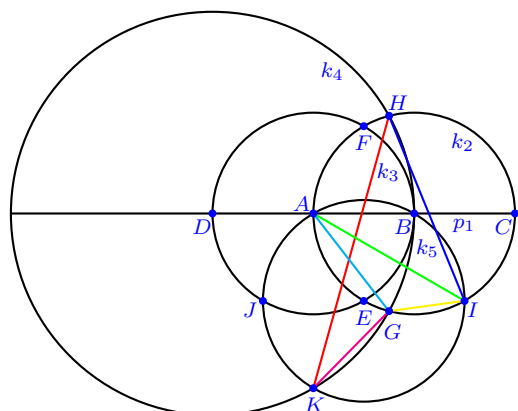
## Distances

$$\begin{aligned}
 d_0 &= |GK| \text{ (red)} \dots 1 \\
 d_1 &= |GJ| \text{ (blue)} \dots 1 \\
 d_2 &= |AI| = |BJ| = |BK| = |CE| = |CF| = |DE| = |DF| = |EF| = |JK| \text{ (green)} \dots 9 \\
 d_3 &= |BG| = |BH| \text{ (cyan)} \dots 2 \\
 d_4 &= |HK| \text{ (magenta)} \dots 1 \\
 d_5 &= |HJ| \text{ (yellow)} \dots 1
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (22 occurrences)}$$

## 2.12.3 Construction RCC36



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(A, B)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(D, B)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
  5.  $k_5 = k(E, A)$   
 $I \in k_2 \cap k_5, J \in k_3 \cap k_5, K \in k_4 \cap k_5$

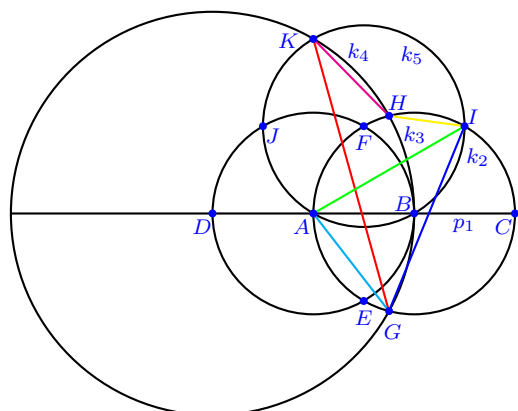
## Distances

$$\begin{aligned}
 d_0 &= |HK| \text{ (red)} \dots 1 \\
 d_1 &= |HI| \text{ (blue)} \dots 1 \\
 d_2 &= |AI| = |AK| = |BJ| = |CE| = |CF| = |DE| = |DF| = |EF| = |IK| \\
 &\text{ (green)} \dots 9 \\
 d_3 &= |AG| = |AH| \text{ (cyan)} \dots 2 \\
 d_4 &= |GK| \text{ (magenta)} \dots 1 \\
 d_5 &= |GI| \text{ (yellow)} \dots 1
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (22 occurrences)}$$

## 2.12.4 Construction RCC38



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(A, B)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(D, B)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
  5.  $k_5 = k(F, A)$   
 $I \in k_2 \cap k_5, J \in k_3 \cap k_5, K \in k_4 \cap k_5$

## Distances

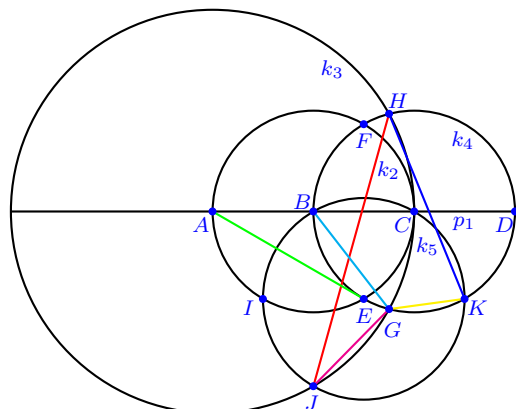
$$\begin{aligned}
 d_0 &= |GK| \text{ (red)} \dots 1 \\
 d_1 &= |GI| \text{ (blue)} \dots 1 \\
 d_2 &= |AI| = |AK| = |BJ| = |CE| = |CF| = |DE| = |DF| = |EF| = |IK| \\
 &\text{ (green)} \dots 9 \\
 d_3 &= |AG| = |AH| \text{ (cyan)} \dots 2 \\
 d_4 &= |HK| \text{ (magenta)} \dots 1 \\
 d_5 &= |HI| \text{ (yellow)} \dots 1
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (22 occurrences)}$$



## 2.12.5 Construction RCC88



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(A, C)$
  4.  $k_4 = k(C, B)$   
 $D \in p_1 \cap k_4, E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(E, B)$   
 $I \in k_2 \cap k_5, J \in k_3 \cap k_5, K \in k_4 \cap k_5$

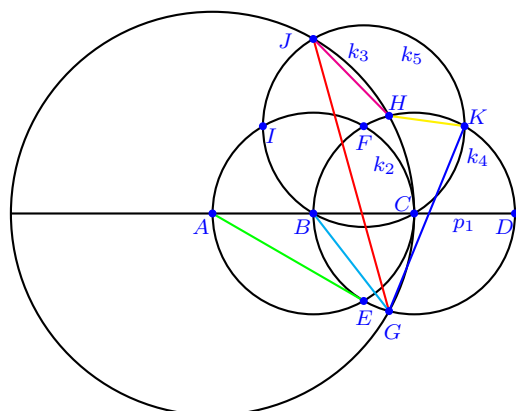
## Distances

- $d_0 = |HJ|$  (red) ... 1  
 $d_1 = |HK|$  (blue) ... 1  
 $d_2 = |AE| = |AF| = |BJ| = |BK| = |CI| = |DE| = |DF| = |EF| = |JK|$  (green) ... 9  
 $d_3 = |BG| = |BH|$  (cyan) ... 2  
 $d_4 = |GJ|$  (magenta) ... 1  
 $d_5 = |GK|$  (yellow) ... 1

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (22 occurrences)}$$

## 2.12.6 Construction RCC90



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(A, C)$
  4.  $k_4 = k(C, B)$   
 $D \in p_1 \cap k_4, E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(F, B)$   
 $I \in k_2 \cap k_5, J \in k_3 \cap k_5, K \in k_4 \cap k_5$

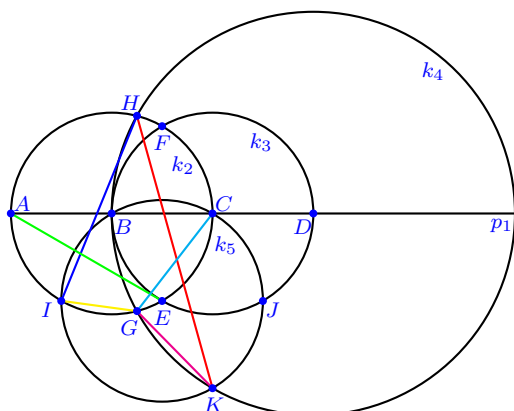
## Distances

- $d_0 = |GJ|$  (red) ... 1  
 $d_1 = |GK|$  (blue) ... 1  
 $d_2 = |AE| = |AF| = |BJ| = |BK| = |CI| = |DE| = |DF| = |EF| = |JK|$  (green) ... 9  
 $d_3 = |BG| = |BH|$  (cyan) ... 2  
 $d_4 = |HJ|$  (magenta) ... 1  
 $d_5 = |HK|$  (yellow) ... 1

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (22 occurrences)}$$

## 2.12.7 Construction RCC129



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, B)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(D, B)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
  5.  $k_5 = k(E, B)$   
 $I \in k_2 \cap k_5, J \in k_3 \cap k_5, K \in k_4 \cap k_5$

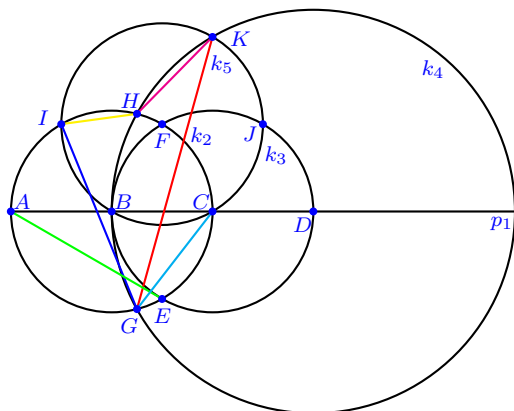
## Distances

$$\begin{aligned}
 d_0 &= |HK| \text{ (red)} \dots 1 \\
 d_1 &= |HI| \text{ (blue)} \dots 1 \\
 d_2 &= |AE| = |AF| = |BJ| = |CI| = |CK| = |DE| = |DF| = |EF| = |IK| \\
 &\text{ (green)} \dots 9 \\
 d_3 &= |CG| = |CH| \text{ (cyan)} \dots 2 \\
 d_4 &= |GK| \text{ (magenta)} \dots 1 \\
 d_5 &= |GI| \text{ (yellow)} \dots 1
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (22 occurrences)}$$

## 2.12.8 Construction RCC131



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, B)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(D, B)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
  5.  $k_5 = k(F, B)$   
 $I \in k_2 \cap k_5, J \in k_3 \cap k_5, K \in k_4 \cap k_5$

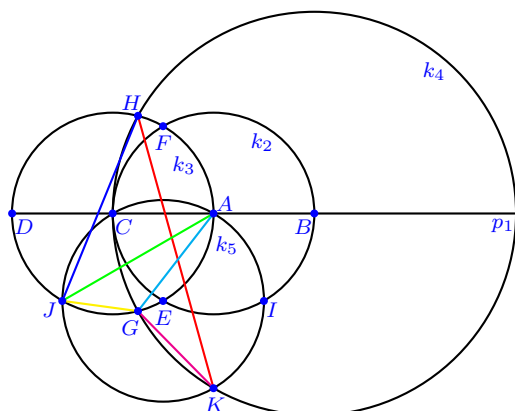
## Distances

$$\begin{aligned}
 d_0 &= |GK| \text{ (red)} \dots 1 \\
 d_1 &= |GI| \text{ (blue)} \dots 1 \\
 d_2 &= |AE| = |AF| = |BJ| = |CI| = |CK| = |DE| = |DF| = |EF| = |IK| \\
 &\text{ (green)} \dots 9 \\
 d_3 &= |CG| = |CH| \text{ (cyan)} \dots 2 \\
 d_4 &= |HK| \text{ (magenta)} \dots 1 \\
 d_5 &= |HI| \text{ (yellow)} \dots 1
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (22 occurrences)}$$

## 2.12.9 Construction RCC166



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, A)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(B, C)$   
 $G \in k_3 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(E, A)$   
 $I \in k_2 \cap k_5, J \in k_3 \cap k_5, K \in k_4 \cap k_5$

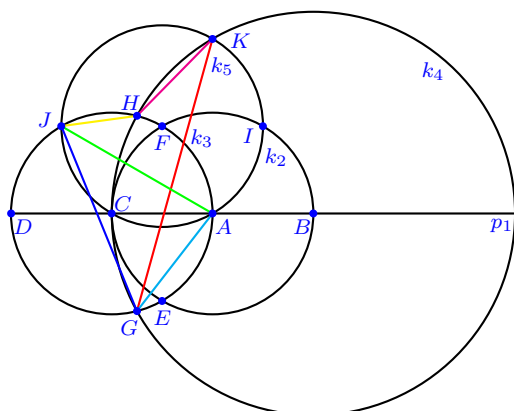
## Distances

$$\begin{aligned}
 d_0 &= |HK| \text{ (red)} \dots 1 \\
 d_1 &= |HJ| \text{ (blue)} \dots 1 \\
 d_2 &= |AJ| = |AK| = |BE| = |BF| = |CI| = |DE| = |DF| = |EF| = |JK| \\
 &\text{ (green)} \dots 9 \\
 d_3 &= |AG| = |AH| \text{ (cyan)} \dots 2 \\
 d_4 &= |GK| \text{ (magenta)} \dots 1 \\
 d_5 &= |GJ| \text{ (yellow)} \dots 1
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (22 occurrences)}$$

## 2.12.10 Construction RCC168



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, A)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(B, C)$   
 $G \in k_3 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(F, A)$   
 $I \in k_2 \cap k_5, J \in k_3 \cap k_5, K \in k_4 \cap k_5$

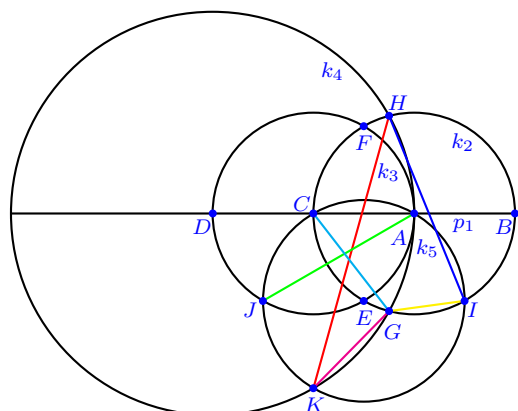
## Distances

$$\begin{aligned}
 d_0 &= |GK| \text{ (red)} \dots 1 \\
 d_1 &= |GJ| \text{ (blue)} \dots 1 \\
 d_2 &= |AJ| = |AK| = |BE| = |BF| = |CI| = |DE| = |DF| = |EF| = |JK| \\
 &\text{ (green)} \dots 9 \\
 d_3 &= |AG| = |AH| \text{ (cyan)} \dots 2 \\
 d_4 &= |HK| \text{ (magenta)} \dots 1 \\
 d_5 &= |HJ| \text{ (yellow)} \dots 1
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (22 occurrences)}$$

## 2.12.11 Construction RCC192



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, A)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(D, A)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
  5.  $k_5 = k(E, A)$   
 $I \in k_2 \cap k_5, J \in k_3 \cap k_5, K \in k_4 \cap k_5$

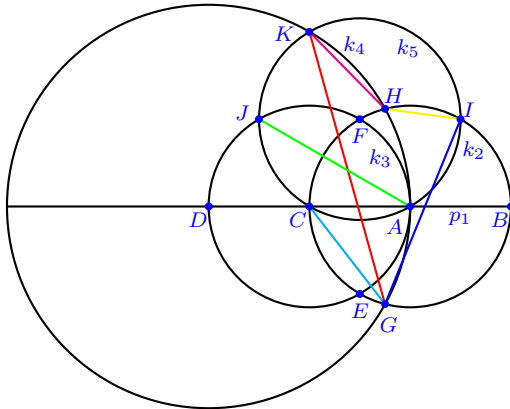
## Distances

$$\begin{aligned}
 d_0 &= |HK| \text{ (red)} \dots 1 \\
 d_1 &= |HI| \text{ (blue)} \dots 1 \\
 d_2 &= |AJ| = |BE| = |BF| = |CI| = |CK| = |DE| = |DF| = |EF| = |IK| \\
 &\text{ (green)} \dots 9 \\
 d_3 &= |CG| = |CH| \text{ (cyan)} \dots 2 \\
 d_4 &= |GK| \text{ (magenta)} \dots 1 \\
 d_5 &= |GI| \text{ (yellow)} \dots 1
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (22 occurrences)}$$

## 2.12.12 Construction RCC194



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, A)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(D, A)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
  5.  $k_5 = k(F, A)$   
 $I \in k_2 \cap k_5, J \in k_3 \cap k_5, K \in k_4 \cap k_5$

## Distances

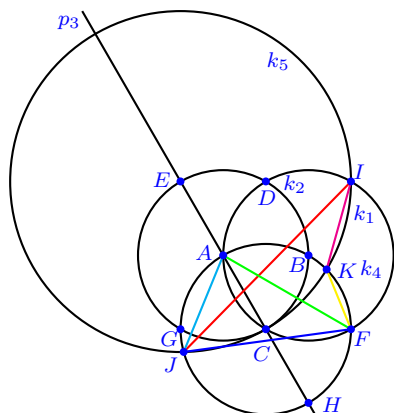
$$\begin{aligned}
 d_0 &= |GK| \text{ (red)} \dots 1 \\
 d_1 &= |GI| \text{ (blue)} \dots 1 \\
 d_2 &= |AJ| = |BE| = |BF| = |CI| = |CK| = |DE| = |DF| = |EF| = |IK| \\
 &\text{ (green)} \dots 9 \\
 d_3 &= |CG| = |CH| \text{ (cyan)} \dots 2 \\
 d_4 &= |HK| \text{ (magenta)} \dots 1 \\
 d_5 &= |HI| \text{ (yellow)} \dots 1
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (22 occurrences)}$$



## 2.12.13 Construction RCC252



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $p_3 = p(A, C)$   
 $E \in p_3 \cap k_2$
  4.  $k_4 = k(C, A)$   
 $F \in k_1 \cap k_4, G \in k_2 \cap k_4, H \in p_3 \cap k_4$
  5.  $k_5 = k(E, C)$   
 $I \in k_1 \cap k_5, J \in k_4 \cap k_5, K \in k_4 \cap k_5$

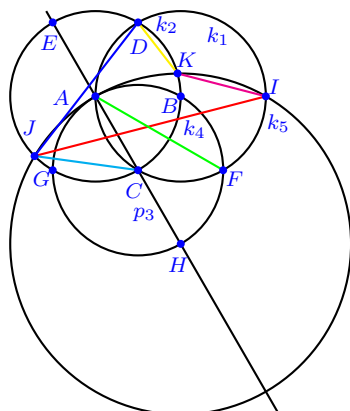
## Distances

$$\begin{aligned}
 d_0 &= |IJ| \text{ (red)} \dots 1 \\
 d_1 &= |FJ| \text{ (blue)} \dots 1 \\
 d_2 &= |AF| = |AI| = |BE| = |BG| = |BH| = |CD| = |EG| = |FI| = |GH| \\
 &\text{ (green)} \dots 9 \\
 d_3 &= |AJ| = |AK| \text{ (cyan)} \dots 2 \\
 d_4 &= |IK| \text{ (magenta)} \dots 1 \\
 d_5 &= |FK| \text{ (yellow)} \dots 1
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (22 occurrences)}$$

### 2.12.14 Construction RCC253



#### Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $p_3 = p(A, C)$   
 $E \in p_3 \cap k_2$
  4.  $k_4 = k(C, A)$   
 $F \in k_1 \cap k_4, G \in k_2 \cap k_4, H \in p_3 \cap k_4$
  5.  $k_5 = k(H, A)$   
 $I \in k_1 \cap k_5, J \in k_2 \cap k_5, K \in k_2 \cap k_5$

#### Distances

$$d_0 = |IJ| \text{ (red)} \dots 1$$

$$d_1 = |DJ| \text{ (blue)} \dots 1$$

$$d_2 = |AF| = |BE| = |BG| = |BH| = |CD| = |CI| = |DI| = |EG| = |GH| \text{ (green)} \dots 9$$

$$d_3 = |CJ| = |CK| \text{ (cyan)} \dots 2$$

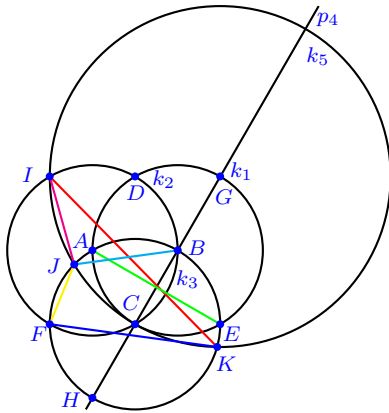
$$d_4 = |IK| \text{ (magenta)} \dots 1$$

$$d_5 = |DK| \text{ (yellow)} \dots 1$$

#### Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (22 occurrences)}$$

## 2.12.15 Construction RCC264



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, A)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $p_4 = p(B, C)$   
 $G \in p_4 \cap k_1, H \in p_4 \cap k_3$
  5.  $k_5 = k(G, C)$   
 $I \in k_2 \cap k_5, J \in k_3 \cap k_5, K \in k_3 \cap k_5$

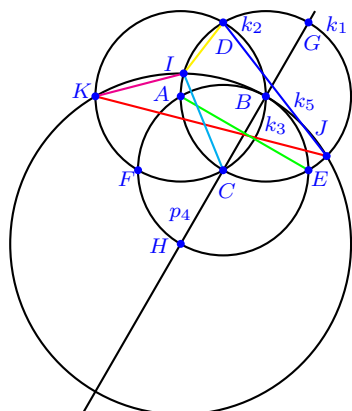
## Distances

$$\begin{aligned}
 d_0 &= |IK| \text{ (red)} \dots 1 \\
 d_1 &= |FK| \text{ (blue)} \dots 1 \\
 d_2 &= |AE| = |AG| = |AH| = |BF| = |BI| = |CD| = |EG| = |EH| = |FI| \\
 &\text{ (green)} \dots 9 \\
 d_3 &= |BJ| = |BK| \text{ (cyan)} \dots 2 \\
 d_4 &= |IJ| \text{ (magenta)} \dots 1 \\
 d_5 &= |FJ| \text{ (yellow)} \dots 1
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (22 occurrences)}$$

## 2.12.16 Construction RCC265



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, A)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $p_4 = p(B, C)$   
 $G \in p_4 \cap k_1, H \in p_4 \cap k_3$
  5.  $k_5 = k(H, B)$   
 $I \in k_1 \cap k_5, J \in k_1 \cap k_5, K \in k_2 \cap k_5$

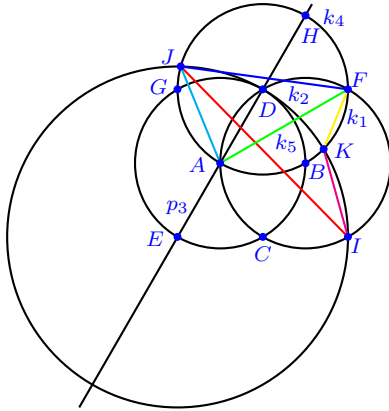
## Distances

$$\begin{aligned}
 d_0 &= |JK| \text{ (red)} \dots 1 \\
 d_1 &= |DJ| \text{ (blue)} \dots 1 \\
 d_2 &= |AE| = |AG| = |AH| = |BF| = |CD| = |CK| = |DK| = |EG| = |EH| \text{ (green)} \dots 9 \\
 d_3 &= |CI| = |CJ| \text{ (cyan)} \dots 2 \\
 d_4 &= |IK| \text{ (magenta)} \dots 1 \\
 d_5 &= |DI| \text{ (yellow)} \dots 1
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (22 occurrences)}$$

2.12.17 Construction RCC300



Construction Process

- $A, B$  given initial points
- 1.  $k_1 = k(B, A)$
- 2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
- 3.  $p_3 = p(A, D)$   
 $E \in p_3 \cap k_2$
- 4.  $k_4 = k(D, A)$   
 $F \in k_1 \cap k_4, G \in k_2 \cap k_4, H \in p_3 \cap k_4$
- 5.  $k_5 = k(E, D)$   
 $I \in k_1 \cap k_5, J \in k_4 \cap k_5, K \in k_4 \cap k_5$

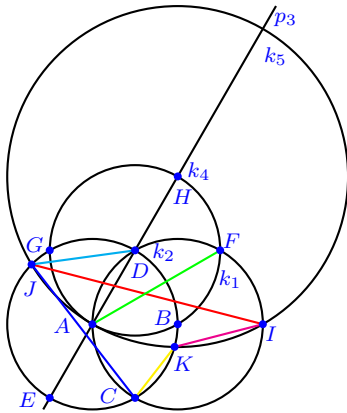
Distances

- $d_0 = |IJ|$  (red) ... 1
- $d_1 = |FJ|$  (blue) ... 1
- $d_2 = |AF| = |AI| = |BE| = |BG| = |BH| = |CD| = |EG| = |FI| = |GH|$  (green) ... 9
- $d_3 = |AJ| = |AK|$  (cyan) ... 2
- $d_4 = |IK|$  (magenta) ... 1
- $d_5 = |FK|$  (yellow) ... 1

Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (22 occurrences)}$$

**2.12.18 Construction RCC301**



**Construction Process**

- \$A, B\$ given initial points
- 1. \$k\_1 = k(B, A)\$
- 2. \$k\_2 = k(A, B)\$  
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
- 3. \$p\_3 = p(A, D)\$  
 $E \in p_3 \cap k_2$
- 4. \$k\_4 = k(D, A)\$  
 $F \in k_1 \cap k_4, G \in k_2 \cap k_4, H \in p_3 \cap k_4$
- 5. \$k\_5 = k(H, A)\$  
 $I \in k_1 \cap k_5, J \in k_2 \cap k_5, K \in k_2 \cap k_5$

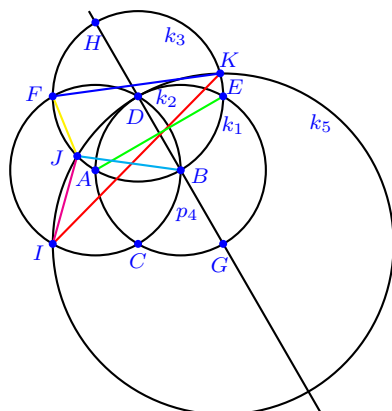
**Distances**

- \$d\_0 = |IJ|\$ (red) ... 1
- \$d\_1 = |CJ|\$ (blue) ... 1
- \$d\_2 = |AF| = |BE| = |BG| = |BH| = |CD| = |CI| = |DI| = |EG| = |GH|\$ (green) ... 9
- \$d\_3 = |DJ| = |DK|\$ (cyan) ... 2
- \$d\_4 = |IK|\$ (magenta) ... 1
- \$d\_5 = |CK|\$ (yellow) ... 1

**Occurrences of the Golden Ratio**

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (22 occurrences)}$$

## 2.12.19 Construction RCC312



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, A)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $p_4 = p(B, D)$   
 $G \in p_4 \cap k_1, H \in p_4 \cap k_3$
  5.  $k_5 = k(G, D)$   
 $I \in k_2 \cap k_5, J \in k_3 \cap k_5, K \in k_3 \cap k_5$

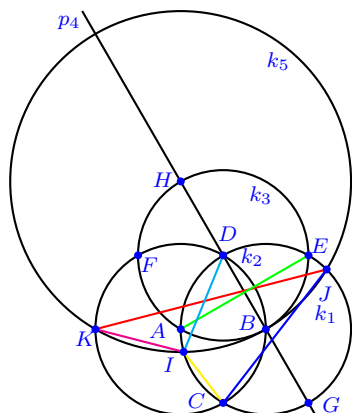
## Distances

$$\begin{aligned}
 d_0 &= |IK| \text{ (red) } \dots 1 \\
 d_1 &= |FK| \text{ (blue) } \dots 1 \\
 d_2 &= |AE| = |AG| = |AH| = |BF| = |BI| = |CD| = |EG| = |EH| = |FI| \\
 &\text{ (green) } \dots 9 \\
 d_3 &= |BJ| = |BK| \text{ (cyan) } \dots 2 \\
 d_4 &= |IJ| \text{ (magenta) } \dots 1 \\
 d_5 &= |FJ| \text{ (yellow) } \dots 1
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (22 occurrences)}$$

## 2.12.20 Construction RCC313



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, A)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $p_4 = p(B, D)$   
 $G \in p_4 \cap k_1, H \in p_4 \cap k_3$
  5.  $k_5 = k(H, B)$   
 $I \in k_1 \cap k_5, J \in k_1 \cap k_5, K \in k_2 \cap k_5$

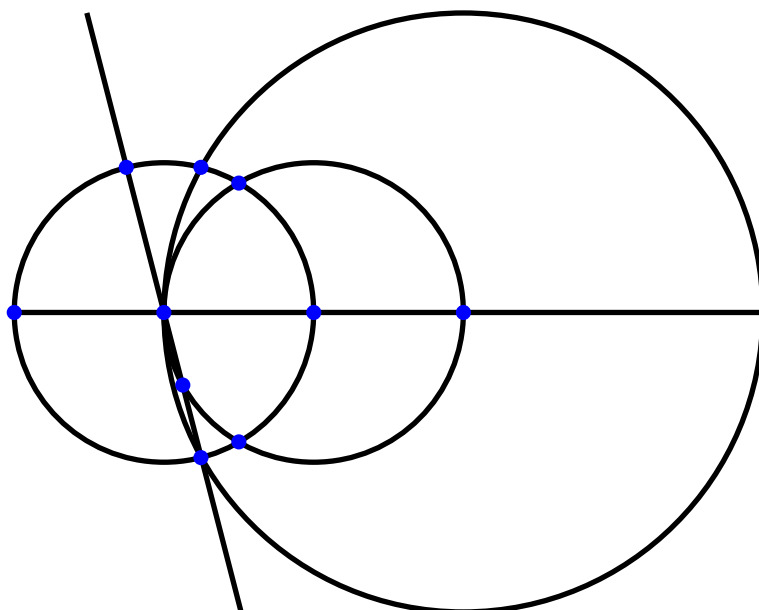
## Distances

$$\begin{aligned}
 d_0 &= |JK| \text{ (red)} \dots 1 \\
 d_1 &= |CJ| \text{ (blue)} \dots 1 \\
 d_2 &= |AE| = |AG| = |AH| = |BF| = |CD| = |CK| = |DK| = |EG| = |EH| \text{ (green)} \dots 9 \\
 d_3 &= |DI| = |DJ| \text{ (cyan)} \dots 2 \\
 d_4 &= |IK| \text{ (magenta)} \dots 1 \\
 d_5 &= |CI| \text{ (yellow)} \dots 1
 \end{aligned}$$

## Occurrences of the Golden Ratio

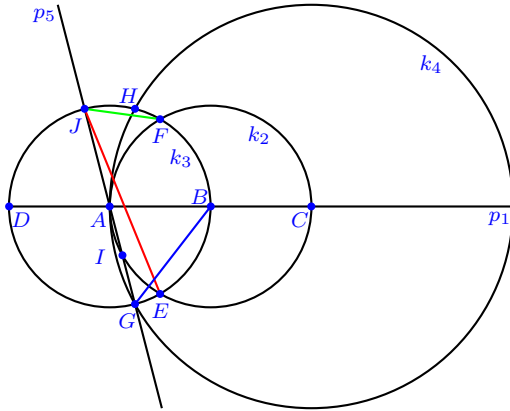
$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (22 occurrences)}$$



**2.13 Class of Similar Constructions No. 6****Contained Constructions**

RCC7, RCC16, RCC40, RCC49, RCC93, RCC102, RCC132, RCC141, RCC171,  
RCC180, RCC195, RCC204

**2.13.1 Construction RCC7**



**Construction Process**

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(A, B)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(C, A)$   
 $G \in k_3 \cap k_4, H \in k_3 \cap k_4$
  5.  $p_5 = p(A, G)$   
 $I \in p_5 \cap k_2, J \in p_5 \cap k_3$

**Distances**

$$d_0 = |EJ| \text{ (red)} \dots 1$$

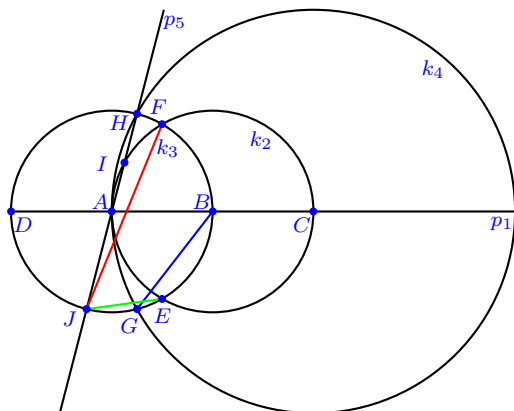
$$d_1 = |BG| = |BH| = |DI| = |DJ| \text{ (blue)} \dots 4$$

$$d_2 = |FJ| \text{ (green)} \dots 1$$

**Occurrences of the Golden Ratio**

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)}$$

## 2.13.2 Construction RCC16



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(A, B)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(C, A)$   
 $G \in k_3 \cap k_4, H \in k_3 \cap k_4$
  5.  $p_5 = p(A, H)$   
 $I \in p_5 \cap k_2, J \in p_5 \cap k_3$

## Distances

$$d_0 = |FJ| \text{ (red) } \dots 1$$

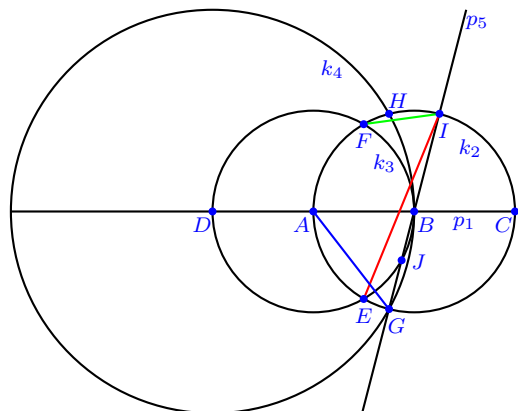
$$d_1 = |BG| = |BH| = |DI| = |DJ| \text{ (blue) } \dots 4$$

$$d_2 = |EJ| \text{ (green) } \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)}$$

## 2.13.3 Construction RCC40



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(A, B)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(D, B)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
  5.  $p_5 = p(B, G)$   
 $I \in p_5 \cap k_2, J \in p_5 \cap k_3$

## Distances

$$d_0 = |EI| \text{ (red)} \dots 1$$

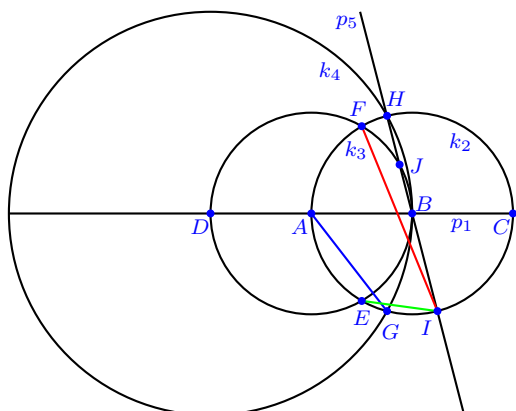
$$d_1 = |AG| = |AH| = |CI| = |CJ| \text{ (blue)} \dots 4$$

$$d_2 = |FI| \text{ (green)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)}$$

## 2.13.4 Construction RCC49



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(A, B)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(D, B)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
  5.  $p_5 = p(B, H)$   
 $I \in p_5 \cap k_2, J \in p_5 \cap k_3$

## Distances

$$d_0 = |FI| \text{ (red)} \dots 1$$

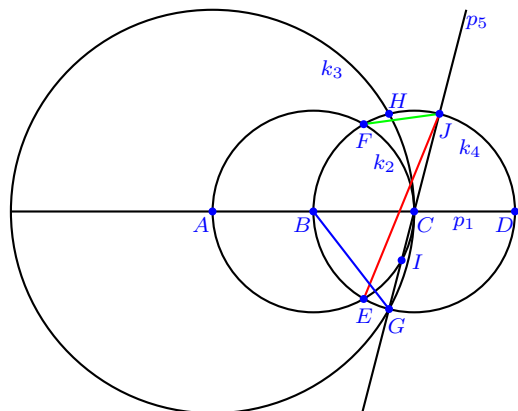
$$d_1 = |AG| = |AH| = |CI| = |CJ| \text{ (blue)} \dots 4$$

$$d_2 = |EI| \text{ (green)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)}$$

## 2.13.5 Construction RCC93



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(A, C)$
  4.  $k_4 = k(C, B)$   
 $D \in p_1 \cap k_4, E \in k_2 \cap k_4, F \in k_2 \cap k_3, G \in k_3 \cap k_4, H \in k_3 \cap k_4$
  5.  $p_5 = p(C, G)$   
 $I \in p_5 \cap k_2, J \in p_5 \cap k_4$

## Distances

$$d_0 = |EJ| \text{ (red) } \dots 1$$

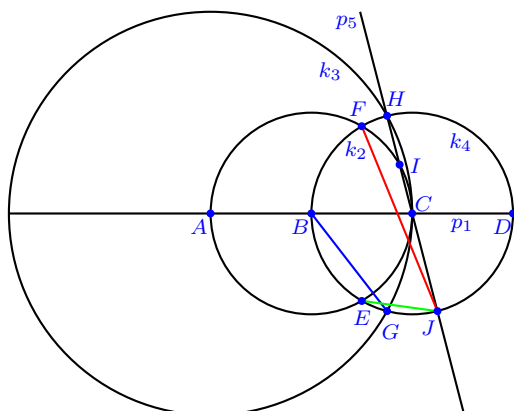
$$d_1 = |BG| = |BH| = |DI| = |DJ| \text{ (blue) } \dots 4$$

$$d_2 = |FJ| \text{ (green) } \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)}$$

## 2.13.6 Construction RCC102



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(A, C)$
  4.  $k_4 = k(C, B)$   
 $D \in p_1 \cap k_4, E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4$
  5.  $p_5 = p(C, H)$   
 $I \in p_5 \cap k_2, J \in p_5 \cap k_4$

## Distances

$$d_0 = |FJ| \text{ (red) } \dots 1$$

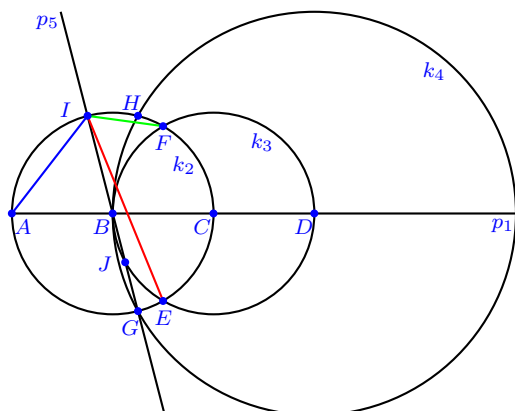
$$d_1 = |BG| = |BH| = |DI| = |DJ| \text{ (blue) } \dots 4$$

$$d_2 = |EJ| \text{ (green) } \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)}$$

## 2.13.7 Construction RCC132



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, B)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(D, B)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
  5.  $p_5 = p(B, G)$   
 $I \in p_5 \cap k_2, J \in p_5 \cap k_3$

## Distances

$$d_0 = |EI| \text{ (red)} \dots 1$$

$$d_1 = |AI| = |AJ| = |CG| = |CH| \text{ (blue)} \dots 4$$

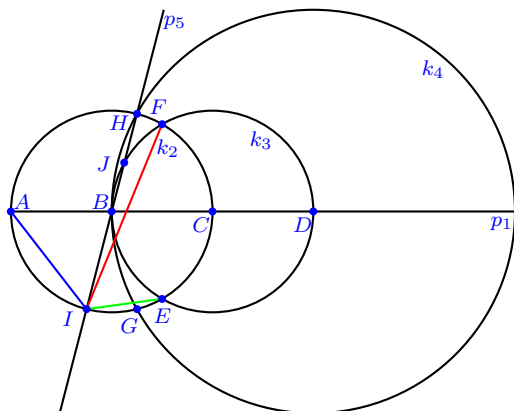
$$d_2 = |FI| \text{ (green)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)}$$



## 2.13.8 Construction RCC141



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, B)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(D, B)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
  5.  $p_5 = p(B, H)$   
 $I \in p_5 \cap k_2, J \in p_5 \cap k_3$

## Distances

$$d_0 = |FI| \text{ (red)} \dots 1$$

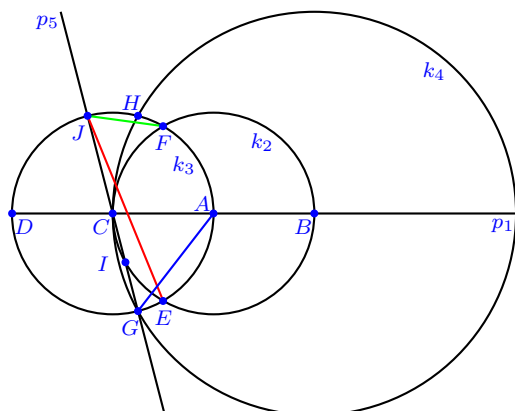
$$d_1 = |AI| = |AJ| = |CG| = |CH| \text{ (blue)} \dots 4$$

$$d_2 = |EI| \text{ (green)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)}$$

## 2.13.9 Construction RCC171



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, A)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(B, C)$   
 $G \in k_3 \cap k_4, H \in k_3 \cap k_4$
  5.  $p_5 = p(C, G)$   
 $I \in p_5 \cap k_2, J \in p_5 \cap k_3$

## Distances

$$d_0 = |EJ| \text{ (red) } \dots 1$$

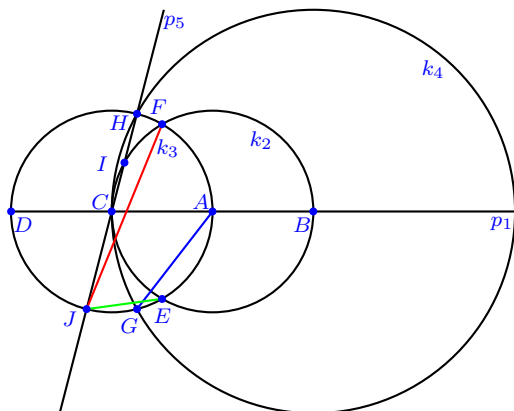
$$d_1 = |AG| = |AH| = |DI| = |DJ| \text{ (blue) } \dots 4$$

$$d_2 = |FJ| \text{ (green) } \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)}$$

## 2.13.10 Construction RCC180



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, A)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(B, C)$   
 $G \in k_3 \cap k_4, H \in k_3 \cap k_4$
  5.  $p_5 = p(C, H)$   
 $I \in p_5 \cap k_2, J \in p_5 \cap k_3$

## Distances

$$d_0 = |FJ| \text{ (red) } \dots 1$$

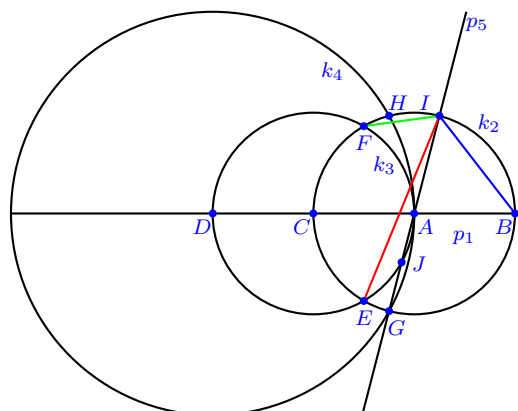
$$d_1 = |AG| = |AH| = |DI| = |DJ| \text{ (blue) } \dots 4$$

$$d_2 = |EJ| \text{ (green) } \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)}$$

## 2.13.11 Construction RCC195



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, A)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(D, A)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
  5.  $p_5 = p(A, G)$   
 $I \in p_5 \cap k_2, J \in p_5 \cap k_3$

## Distances

$$d_0 = |EI| \text{ (red)} \dots 1$$

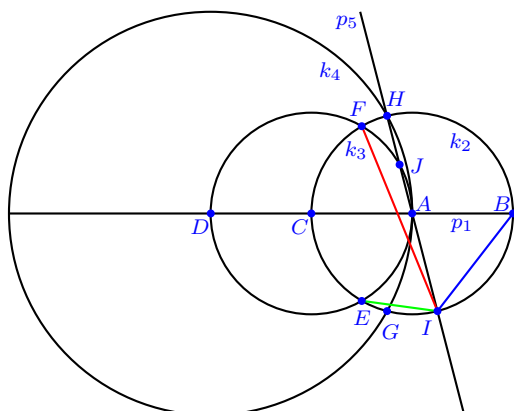
$$d_1 = |BI| = |BJ| = |CG| = |CH| \text{ (blue)} \dots 4$$

$$d_2 = |FI| \text{ (green)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)}$$

## 2.13.12 Construction RCC204



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, A)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(D, A)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
  5.  $p_5 = p(A, H)$   
 $I \in p_5 \cap k_2, J \in p_5 \cap k_3$

## Distances

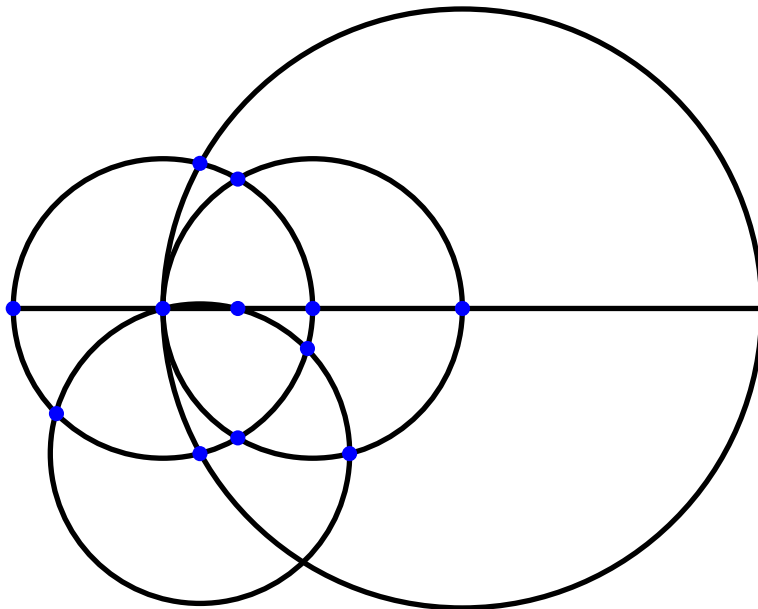
$$d_0 = |FI| \text{ (red)} \dots 1$$

$$d_1 = |BI| = |BJ| = |CG| = |CH| \text{ (blue)} \dots 4$$

$$d_2 = |EI| \text{ (green)} \dots 1$$

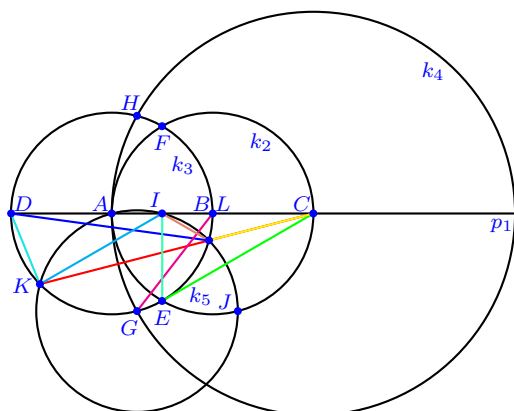
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)}$$

**2.14 Class of Similar Constructions No. 7****Contained Constructions**

RCC8, RCC17, RCC41, RCC50, RCC94, RCC103, RCC133, RCC142, RCC172,  
RCC181, RCC196, RCC205

## 2.14.1 Construction RCC8



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(A, B)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(C, A)$   
 $G \in k_3 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(G, A)$   
 $I \in p_1 \cap k_5, J \in k_2 \cap k_5, K \in k_3 \cap k_5, L \in k_3 \cap k_5$

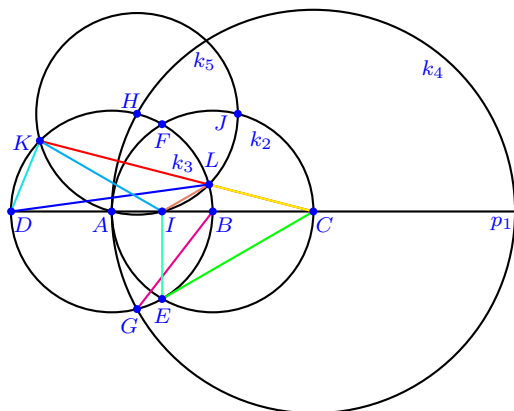
## Distances

- $d_0 = |CK|$  (red) ... 1  
 $d_1 = |DL| = |FJ| = |FK| = |JK|$  (blue) ... 4  
 $d_2 = |CE| = |CF| = |DE| = |DF| = |EF| = |KL|$  (green) ... 6  
 $d_3 = |IK|$  (cyan) ... 1  
 $d_4 = |BG| = |BH| = |CJ| = |EK| = |FL| = |IJ|$  (magenta) ... 6  
 $d_5 = |CL|$  (yellow) ... 1  
 $d_6 = |EI| = |FI|$  (grass) ... 2  
 $d_7 = |DK| = |EJ| = |EL| = |JL|$  (sea) ... 4  
 $d_8 = |IL|$  (darkorange) ... 1

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_4} = \frac{d_2}{d_5} = \frac{d_3}{d_6} = \frac{d_4}{d_7} = \frac{d_5}{d_8} \quad (64 \text{ occurrences})$$

## 2.14.2 Construction RCC17



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(A, B)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(C, A)$   
 $G \in k_3 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(H, A)$   
 $I \in p_1 \cap k_5, J \in k_2 \cap k_5, K \in k_3 \cap k_5, L \in k_3 \cap k_5$

## Distances

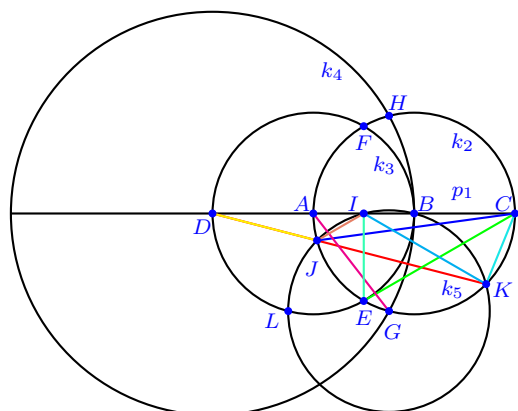
$$\begin{aligned}
 d_0 &= |CK| \text{ (red)} \dots 1 \\
 d_1 &= |DL| = |EJ| = |EK| = |JK| \text{ (blue)} \dots 4 \\
 d_2 &= |CE| = |CF| = |DE| = |DF| = |EF| = |KL| \text{ (green)} \dots 6 \\
 d_3 &= |IK| \text{ (cyan)} \dots 1 \\
 d_4 &= |BG| = |BH| = |CJ| = |EL| = |FK| = |IJ| \text{ (magenta)} \dots 6 \\
 d_5 &= |CL| \text{ (yellow)} \dots 1 \\
 d_6 &= |EI| = |FI| \text{ (grass)} \dots 2 \\
 d_7 &= |DK| = |FJ| = |FL| = |JL| \text{ (sea)} \dots 4 \\
 d_8 &= |IL| \text{ (darkorange)} \dots 1
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_4} = \frac{d_2}{d_5} = \frac{d_3}{d_6} = \frac{d_4}{d_7} = \frac{d_5}{d_8} \text{ (64 occurrences)}$$



## 2.14.3 Construction RCC41



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(A, B)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(D, B)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
  5.  $k_5 = k(G, B)$   
 $I \in p_1 \cap k_5, J \in k_2 \cap k_5, K \in k_2 \cap k_5, L \in k_3 \cap k_5$

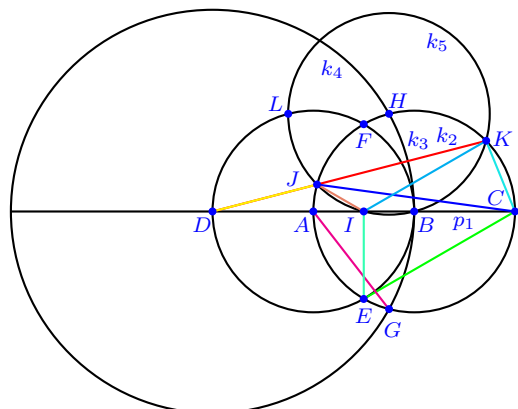
## Distances

- $d_0 = |DK|$  (red) ... 1
- $d_1 = |CJ| = |FK| = |FL| = |KL|$  (blue) ... 4
- $d_2 = |CE| = |CF| = |DE| = |DF| = |EF| = |JK|$  (green) ... 6
- $d_3 = |IK|$  (cyan) ... 1
- $d_4 = |AG| = |AH| = |DL| = |EK| = |FJ| = |IL|$  (magenta) ... 6
- $d_5 = |DJ|$  (yellow) ... 1
- $d_6 = |EI| = |FI|$  (grass) ... 2
- $d_7 = |CK| = |EJ| = |EL| = |JL|$  (sea) ... 4
- $d_8 = |IJ|$  (darkorange) ... 1

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_4} = \frac{d_2}{d_5} = \frac{d_3}{d_6} = \frac{d_4}{d_7} = \frac{d_6}{d_8} \text{ (64 occurrences)}$$

## 2.14.4 Construction RCC50



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(A, B)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(D, B)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
  5.  $k_5 = k(H, B)$   
 $I \in p_1 \cap k_5, J \in k_2 \cap k_5, K \in k_2 \cap k_5, L \in k_3 \cap k_5$

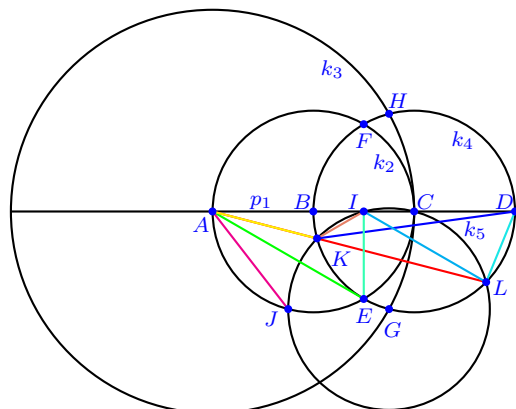
## Distances

- $d_0 = |DK|$  (red) ... 1
- $d_1 = |CJ| = |EK| = |EL| = |KL|$  (blue) ... 4
- $d_2 = |CE| = |CF| = |DE| = |DF| = |EF| = |JK|$  (green) ... 6
- $d_3 = |IK|$  (cyan) ... 1
- $d_4 = |AG| = |AH| = |DL| = |EJ| = |FK| = |IL|$  (magenta) ... 6
- $d_5 = |DJ|$  (yellow) ... 1
- $d_6 = |EI| = |FI|$  (grass) ... 2
- $d_7 = |CK| = |FJ| = |FL| = |JL|$  (sea) ... 4
- $d_8 = |IJ|$  (darkorange) ... 1

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_4} = \frac{d_2}{d_5} = \frac{d_3}{d_6} = \frac{d_4}{d_7} = \frac{d_6}{d_8} \quad (64 \text{ occurrences})$$

## 2.14.5 Construction RCC94



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(A, C)$
  4.  $k_4 = k(C, B)$   
 $D \in p_1 \cap k_4, E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(G, C)$   
 $I \in p_1 \cap k_5, J \in k_2 \cap k_5, K \in k_4 \cap k_5, L \in k_4 \cap k_5$

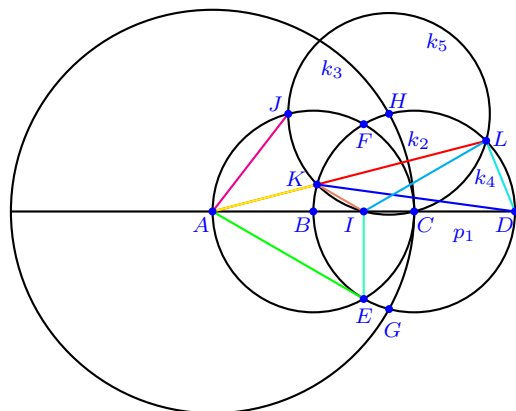
## Distances

- $d_0 = |AL|$  (red) ... 1  
 $d_1 = |DK| = |FJ| = |FL| = |JL|$  (blue) ... 4  
 $d_2 = |AE| = |AF| = |DE| = |DF| = |EF| = |KL|$  (green) ... 6  
 $d_3 = |IL|$  (cyan) ... 1  
 $d_4 = |AJ| = |BG| = |BH| = |EL| = |FK| = |IJ|$  (magenta) ... 6  
 $d_5 = |AK|$  (yellow) ... 1  
 $d_6 = |EI| = |FI|$  (grass) ... 2  
 $d_7 = |DL| = |EJ| = |EK| = |JK|$  (sea) ... 4  
 $d_8 = |IK|$  (darkorange) ... 1

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_4} = \frac{d_2}{d_5} = \frac{d_3}{d_6} = \frac{d_4}{d_7} = \frac{d_6}{d_8} \quad (64 \text{ occurrences})$$

## 2.14.6 Construction RCC103



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(A, C)$
  4.  $k_4 = k(C, B)$   
 $D \in p_1 \cap k_4, E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(H, C)$   
 $I \in p_1 \cap k_5, J \in k_2 \cap k_5, K \in k_4 \cap k_5, L \in k_4 \cap k_5$

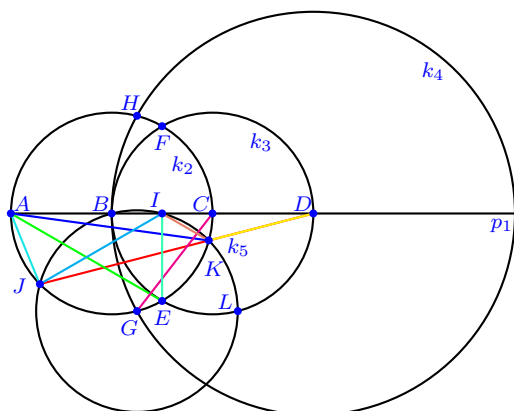
## Distances

- $d_0 = |AL|$  (red) ... 1  
 $d_1 = |DK| = |EJ| = |EL| = |JL|$  (blue) ... 4  
 $d_2 = |AE| = |AF| = |DE| = |DF| = |EF| = |KL|$  (green) ... 6  
 $d_3 = |IL|$  (cyan) ... 1  
 $d_4 = |AJ| = |BG| = |BH| = |EK| = |FL| = |IJ|$  (magenta) ... 6  
 $d_5 = |AK|$  (yellow) ... 1  
 $d_6 = |EI| = |FI|$  (grass) ... 2  
 $d_7 = |DL| = |FJ| = |FK| = |JK|$  (sea) ... 4  
 $d_8 = |IK|$  (darkorange) ... 1

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_4} = \frac{d_2}{d_5} = \frac{d_3}{d_6} = \frac{d_4}{d_7} = \frac{d_6}{d_8} \quad (64 \text{ occurrences})$$

## 2.14.7 Construction RCC133



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, B)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(D, B)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
  5.  $k_5 = k(G, B)$   
 $I \in p_1 \cap k_5, J \in k_2 \cap k_5, K \in k_2 \cap k_5, L \in k_3 \cap k_5$

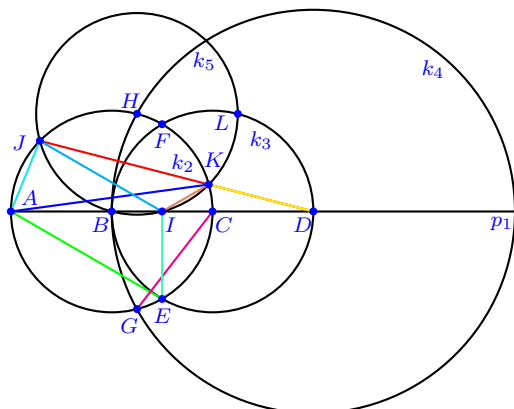
## Distances

- $d_0 = |DJ|$  (red) ... 1
- $d_1 = |AK| = |FJ| = |FL| = |JL|$  (blue) ... 4
- $d_2 = |AE| = |AF| = |DE| = |DF| = |EF| = |JK|$  (green) ... 6
- $d_3 = |IJ|$  (cyan) ... 1
- $d_4 = |CG| = |CH| = |DL| = |EJ| = |FK| = |IL|$  (magenta) ... 6
- $d_5 = |DK|$  (yellow) ... 1
- $d_6 = |EI| = |FI|$  (grass) ... 2
- $d_7 = |AJ| = |EK| = |EL| = |KL|$  (sea) ... 4
- $d_8 = |IK|$  (darkorange) ... 1

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_4} = \frac{d_2}{d_5} = \frac{d_3}{d_6} = \frac{d_4}{d_7} = \frac{d_6}{d_8} \text{ (64 occurrences)}$$

## 2.14.8 Construction RCC142



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, B)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(D, B)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
  5.  $k_5 = k(H, B)$   
 $I \in p_1 \cap k_5, J \in k_2 \cap k_5, K \in k_2 \cap k_5, L \in k_3 \cap k_5$

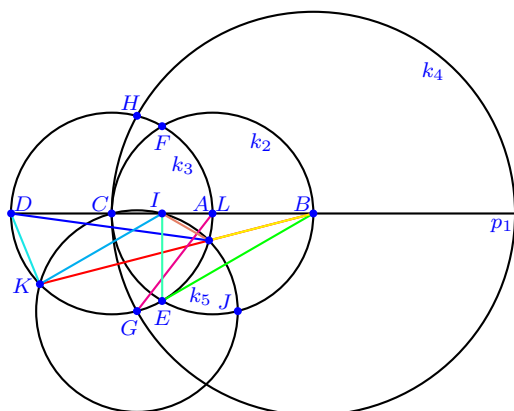
## Distances

- $d_0 = |DJ|$  (red) ... 1
- $d_1 = |AK| = |EJ| = |EL| = |JL|$  (blue) ... 4
- $d_2 = |AE| = |AF| = |DE| = |DF| = |EF| = |JK|$  (green) ... 6
- $d_3 = |IJ|$  (cyan) ... 1
- $d_4 = |CG| = |CH| = |DL| = |EK| = |FJ| = |IL|$  (magenta) ... 6
- $d_5 = |DK|$  (yellow) ... 1
- $d_6 = |EI| = |FI|$  (grass) ... 2
- $d_7 = |AJ| = |FK| = |FL| = |KL|$  (sea) ... 4
- $d_8 = |IK|$  (darkorange) ... 1

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_4} = \frac{d_2}{d_5} = \frac{d_3}{d_6} = \frac{d_4}{d_7} = \frac{d_6}{d_8} \text{ (64 occurrences)}$$

## 2.14.9 Construction RCC172



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, A)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(B, C)$   
 $G \in k_3 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(G, C)$   
 $I \in p_1 \cap k_5, J \in k_2 \cap k_5, K \in k_3 \cap k_5, L \in k_3 \cap k_5$

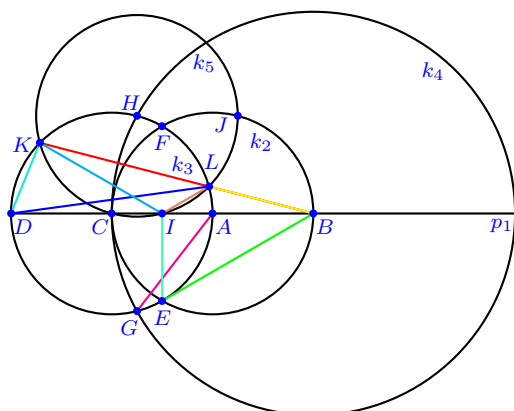
## Distances

- $$d_0 = |BK| \text{ (red)} \dots 1$$
- $$d_1 = |DL| = |FJ| = |FK| = |JK| \text{ (blue)} \dots 4$$
- $$d_2 = |BE| = |BF| = |DE| = |DF| = |EF| = |KL| \text{ (green)} \dots 6$$
- $$d_3 = |IK| \text{ (cyan)} \dots 1$$
- $$d_4 = |AG| = |AH| = |BJ| = |EK| = |FL| = |IJ| \text{ (magenta)} \dots 6$$
- $$d_5 = |BL| \text{ (yellow)} \dots 1$$
- $$d_6 = |EI| = |FI| \text{ (grass)} \dots 2$$
- $$d_7 = |DK| = |EJ| = |EL| = |JL| \text{ (sea)} \dots 4$$
- $$d_8 = |IL| \text{ (darkorange)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_4} = \frac{d_2}{d_5} = \frac{d_3}{d_6} = \frac{d_4}{d_7} = \frac{d_5}{d_8} \text{ (64 occurrences)}$$

## 2.14.10 Construction RCC181



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, A)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(B, C)$   
 $G \in k_3 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(H, C)$   
 $I \in p_1 \cap k_5, J \in k_2 \cap k_5, K \in k_3 \cap k_5, L \in k_3 \cap k_5$

## Distances

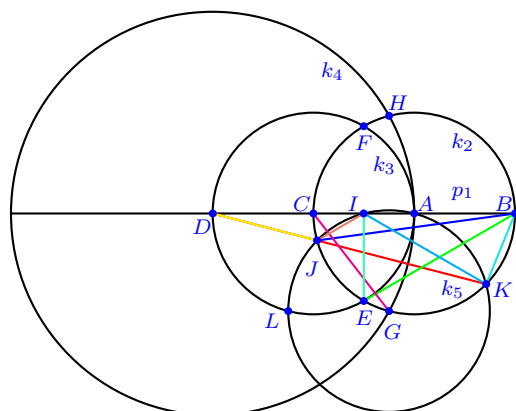
- $$d_0 = |BK| \text{ (red)} \dots 1$$
- $$d_1 = |DL| = |EJ| = |EK| = |JK| \text{ (blue)} \dots 4$$
- $$d_2 = |BE| = |BF| = |DE| = |DF| = |EF| = |KL| \text{ (green)} \dots 6$$
- $$d_3 = |IK| \text{ (cyan)} \dots 1$$
- $$d_4 = |AG| = |AH| = |BJ| = |EL| = |FK| = |IJ| \text{ (magenta)} \dots 6$$
- $$d_5 = |BL| \text{ (yellow)} \dots 1$$
- $$d_6 = |EI| = |FI| \text{ (grass)} \dots 2$$
- $$d_7 = |DK| = |FJ| = |FL| = |JL| \text{ (sea)} \dots 4$$
- $$d_8 = |IL| \text{ (darkorange)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_4} = \frac{d_2}{d_5} = \frac{d_3}{d_6} = \frac{d_4}{d_7} = \frac{d_6}{d_8} \text{ (64 occurrences)}$$



## 2.14.11 Construction RCC196



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, A)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(D, A)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
  5.  $k_5 = k(G, A)$   
 $I \in p_1 \cap k_5, J \in k_2 \cap k_5, K \in k_2 \cap k_5, L \in k_3 \cap k_5$

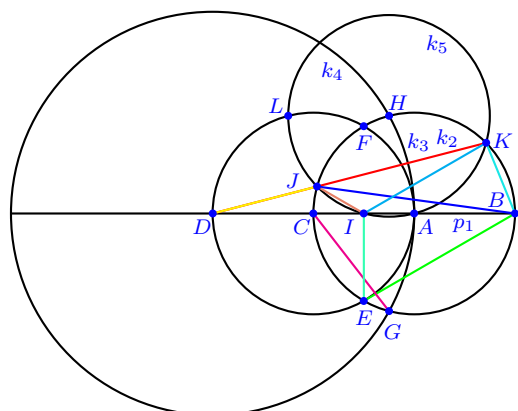
## Distances

- $d_0 = |DK|$  (red) ... 1
- $d_1 = |BJ| = |FK| = |FL| = |KL|$  (blue) ... 4
- $d_2 = |BE| = |BF| = |DE| = |DF| = |EF| = |JK|$  (green) ... 6
- $d_3 = |IK|$  (cyan) ... 1
- $d_4 = |CG| = |CH| = |DL| = |EK| = |FJ| = |IL|$  (magenta) ... 6
- $d_5 = |DJ|$  (yellow) ... 1
- $d_6 = |EI| = |FI|$  (grass) ... 2
- $d_7 = |BK| = |EJ| = |EL| = |JL|$  (sea) ... 4
- $d_8 = |IJ|$  (darkorange) ... 1

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_4} = \frac{d_2}{d_5} = \frac{d_3}{d_6} = \frac{d_4}{d_7} = \frac{d_6}{d_8} \text{ (64 occurrences)}$$

## 2.14.12 Construction RCC205



## Construction Process

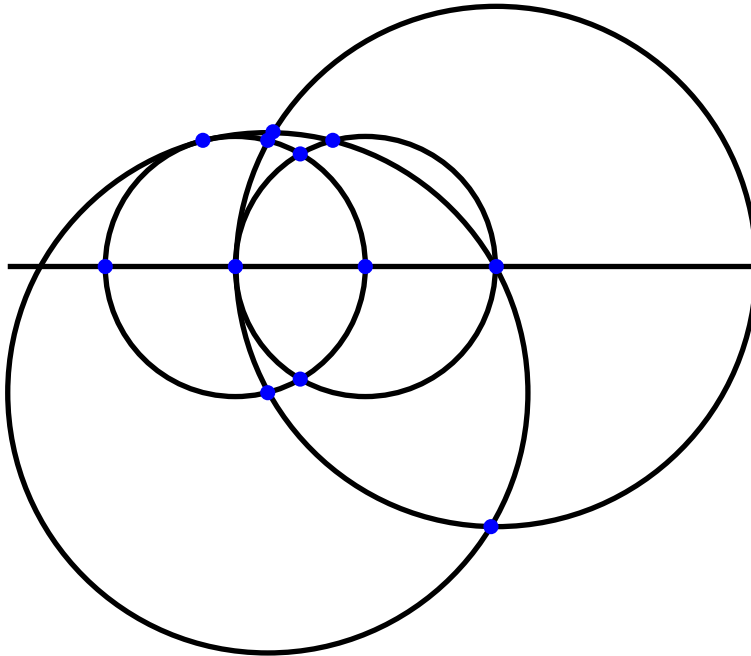
- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, A)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(D, A)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
  5.  $k_5 = k(H, A)$   
 $I \in p_1 \cap k_5, J \in k_2 \cap k_5, K \in k_2 \cap k_5, L \in k_3 \cap k_5$

## Distances

- $d_0 = |DK|$  (red) ... 1  
 $d_1 = |BJ| = |EK| = |EL| = |KL|$  (blue) ... 4  
 $d_2 = |BE| = |BF| = |DE| = |DF| = |EF| = |JK|$  (green) ... 6  
 $d_3 = |IK|$  (cyan) ... 1  
 $d_4 = |CG| = |CH| = |DL| = |EJ| = |FK| = |IL|$  (magenta) ... 6  
 $d_5 = |DJ|$  (yellow) ... 1  
 $d_6 = |EI| = |FI|$  (grass) ... 2  
 $d_7 = |BK| = |FJ| = |FL| = |JL|$  (sea) ... 4  
 $d_8 = |IJ|$  (darkorange) ... 1

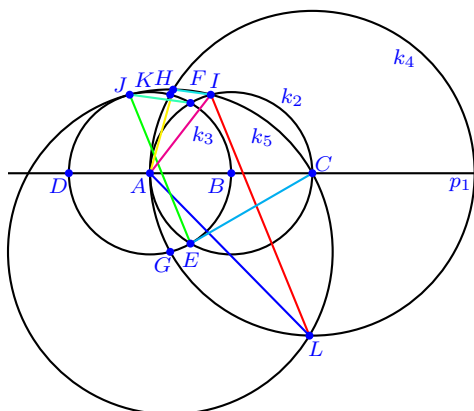
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_4} = \frac{d_2}{d_5} = \frac{d_3}{d_6} = \frac{d_4}{d_7} = \frac{d_6}{d_8} \text{ (64 occurrences)}$$

**2.15 Class of Similar Constructions No. 8****Contained Constructions**

RCC9, RCC18, RCC42, RCC51, RCC92, RCC101, RCC134, RCC143, RCC170,  
RCC179, RCC197, RCC206

### 2.15.1 Construction RCC9



#### Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(A, B)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(C, A)$   
 $G \in k_3 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(G, C)$   
 $I \in k_2 \cap k_5, J \in k_3 \cap k_5, K \in k_4 \cap k_5, L \in k_4 \cap k_5$

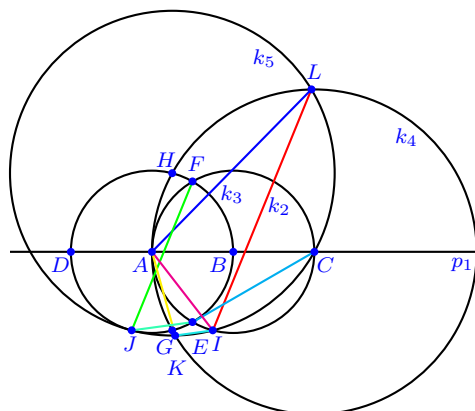
#### Distances

- $d_0 = |IL|$  (red) ... 1  
 $d_1 = |AL|$  (blue) ... 1  
 $d_2 = |EJ|$  (green) ... 1  
 $d_3 = |CE| = |CF| = |DE| = |DF| = |EF|$  (cyan) ... 5  
 $d_4 = |AI| = |BG| = |BH| = |DJ|$  (magenta) ... 4  
 $d_5 = |AK|$  (yellow) ... 1  
 $d_6 = |FJ|$  (grass) ... 1  
 $d_7 = |IK|$  (sea) ... 1

#### Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} = \frac{d_4}{d_6} = \frac{d_6}{d_7} \text{ (20 occurrences)}$$

## 2.15.2 Construction RCC18



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(A, B)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(C, A)$   
 $G \in k_3 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(H, C)$   
 $I \in k_2 \cap k_5, J \in k_3 \cap k_5, K \in k_4 \cap k_5, L \in k_4 \cap k_5$

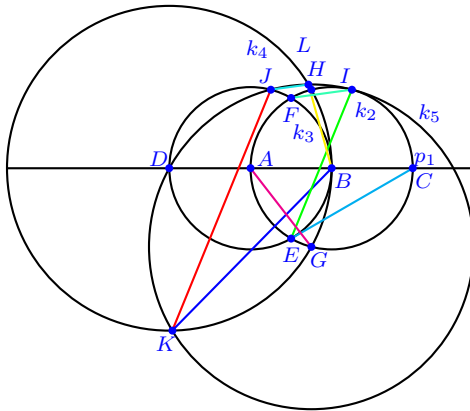
## Distances

- $d_0 = |IL|$  (red) ... 1  
 $d_1 = |AL|$  (blue) ... 1  
 $d_2 = |FJ|$  (green) ... 1  
 $d_3 = |CE| = |CF| = |DE| = |DF| = |EF|$  (cyan) ... 5  
 $d_4 = |AI| = |BG| = |BH| = |DJ|$  (magenta) ... 4  
 $d_5 = |AK|$  (yellow) ... 1  
 $d_6 = |EJ|$  (grass) ... 1  
 $d_7 = |IK|$  (sea) ... 1

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} = \frac{d_4}{d_6} = \frac{d_6}{d_7} \quad (20 \text{ occurrences})$$

### 2.15.3 Construction RCC42



#### Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(A, B)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(D, B)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
  5.  $k_5 = k(G, D)$   
 $I \in k_2 \cap k_5, J \in k_3 \cap k_5, K \in k_4 \cap k_5, L \in k_4 \cap k_5$

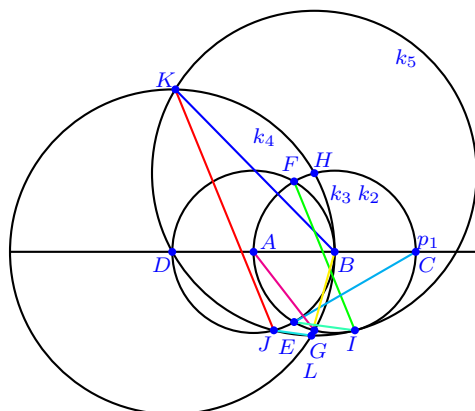
#### Distances

- $d_0 = |JK|$  (red) ... 1
- $d_1 = |BK|$  (blue) ... 1
- $d_2 = |EI|$  (green) ... 1
- $d_3 = |CE| = |CF| = |DE| = |DF| = |EF|$  (cyan) ... 5
- $d_4 = |AG| = |AH| = |BJ| = |CI|$  (magenta) ... 4
- $d_5 = |BL|$  (yellow) ... 1
- $d_6 = |FI|$  (grass) ... 1
- $d_7 = |JL|$  (sea) ... 1

#### Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} = \frac{d_4}{d_6} = \frac{d_6}{d_7} \quad (20 \text{ occurrences})$$

## 2.15.4 Construction RCC51



## Construction Process

$A, B$  given initial points

1.  $p_1 = p(A, B)$
2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
3.  $k_3 = k(A, B)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
4.  $k_4 = k(D, B)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
5.  $k_5 = k(H, D)$   
 $I \in k_2 \cap k_5, J \in k_3 \cap k_5, K \in k_4 \cap k_5, L \in k_4 \cap k_5$

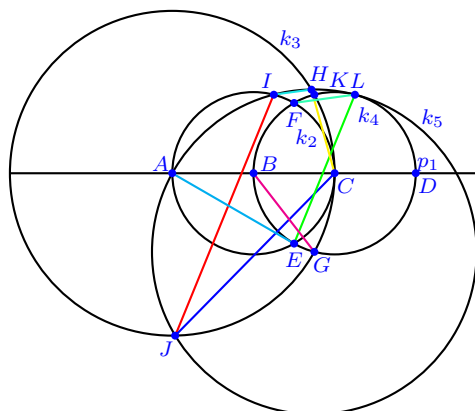
## Distances

$$\begin{aligned}
 d_0 &= |JK| \text{ (red)} \dots 1 \\
 d_1 &= |BK| \text{ (blue)} \dots 1 \\
 d_2 &= |FI| \text{ (green)} \dots 1 \\
 d_3 &= |CE| = |CF| = |DE| = |DF| = |EF| \text{ (cyan)} \dots 5 \\
 d_4 &= |AG| = |AH| = |BJ| = |CI| \text{ (magenta)} \dots 4 \\
 d_5 &= |BL| \text{ (yellow)} \dots 1 \\
 d_6 &= |EI| \text{ (grass)} \dots 1 \\
 d_7 &= |JL| \text{ (sea)} \dots 1
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} = \frac{d_4}{d_6} = \frac{d_6}{d_7} \text{ (20 occurrences)}$$

## 2.15.5 Construction RCC92



## Construction Process

$A, B$  given initial points

1.  $p_1 = p(A, B)$
2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
3.  $k_3 = k(A, C)$
4.  $k_4 = k(C, B)$   
 $D \in p_1 \cap k_4, E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4$
5.  $k_5 = k(G, A)$   
 $I \in k_2 \cap k_5, J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5$

## Distances

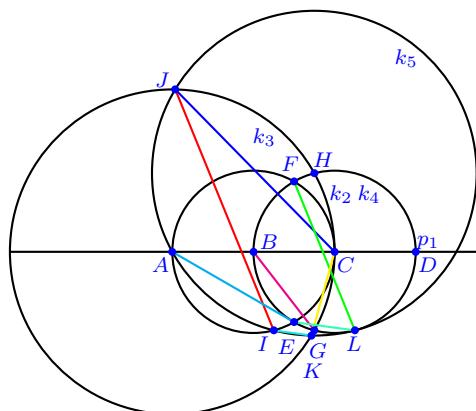
- $d_0 = |IJ|$  (red) ... 1  
 $d_1 = |CJ|$  (blue) ... 1  
 $d_2 = |EL|$  (green) ... 1  
 $d_3 = |AE| = |AF| = |DE| = |DF| = |EF|$  (cyan) ... 5  
 $d_4 = |BG| = |BH| = |CI| = |DL|$  (magenta) ... 4  
 $d_5 = |CK|$  (yellow) ... 1  
 $d_6 = |FL|$  (grass) ... 1  
 $d_7 = |IK|$  (sea) ... 1

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} = \frac{d_4}{d_6} = \frac{d_6}{d_7} \quad (20 \text{ occurrences})$$



## 2.15.6 Construction RCC101



## Construction Process

$A, B$  given initial points

1.  $p_1 = p(A, B)$
2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
3.  $k_3 = k(A, C)$
4.  $k_4 = k(C, B)$   
 $D \in p_1 \cap k_4, E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4$
5.  $k_5 = k(H, A)$   
 $I \in k_2 \cap k_5, J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5$

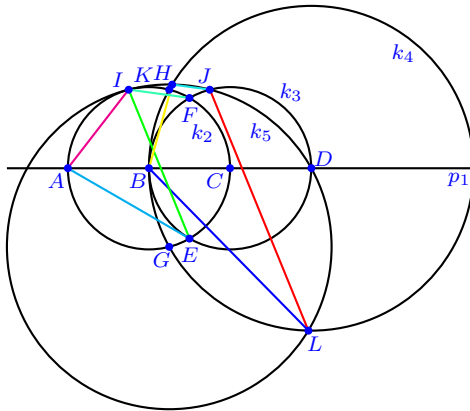
## Distances

- $d_0 = |IJ|$  (red) ... 1  
 $d_1 = |CJ|$  (blue) ... 1  
 $d_2 = |FL|$  (green) ... 1  
 $d_3 = |AE| = |AF| = |DE| = |DF| = |EF|$  (cyan) ... 5  
 $d_4 = |BG| = |BH| = |CI| = |DL|$  (magenta) ... 4  
 $d_5 = |CK|$  (yellow) ... 1  
 $d_6 = |EL|$  (grass) ... 1  
 $d_7 = |IK|$  (sea) ... 1

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} = \frac{d_4}{d_6} = \frac{d_6}{d_7} \quad (20 \text{ occurrences})$$

2.15.7 Construction RCC134



Construction Process

- $A, B$  given initial points
- 1.  $p_1 = p(A, B)$
- 2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
- 3.  $k_3 = k(C, B)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
- 4.  $k_4 = k(D, B)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
- 5.  $k_5 = k(G, D)$   
 $I \in k_2 \cap k_5, J \in k_3 \cap k_5, K \in k_4 \cap k_5, L \in k_4 \cap k_5$

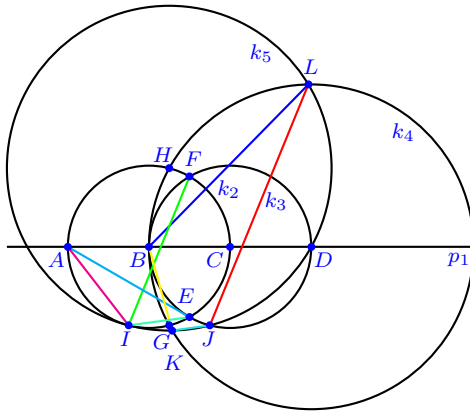
Distances

- $d_0 = |JL|$  (red) ... 1
- $d_1 = |BL|$  (blue) ... 1
- $d_2 = |EI|$  (green) ... 1
- $d_3 = |AE| = |AF| = |DE| = |DF| = |EF|$  (cyan) ... 5
- $d_4 = |AI| = |BJ| = |CG| = |CH|$  (magenta) ... 4
- $d_5 = |BK|$  (yellow) ... 1
- $d_6 = |FI|$  (grass) ... 1
- $d_7 = |JK|$  (sea) ... 1

Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} = \frac{d_4}{d_6} = \frac{d_6}{d_7} \text{ (20 occurrences)}$$

**2.15.8 Construction RCC143**



**Construction Process**

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, B)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(D, B)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
  5.  $k_5 = k(H, D)$   
 $I \in k_2 \cap k_5, J \in k_3 \cap k_5, K \in k_4 \cap k_5, L \in k_4 \cap k_5$

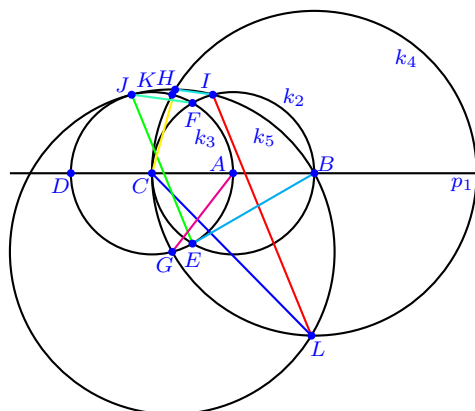
**Distances**

- $d_0 = |JL|$  (red) ... 1
- $d_1 = |BL|$  (blue) ... 1
- $d_2 = |FI|$  (green) ... 1
- $d_3 = |AE| = |AF| = |DE| = |DF| = |EF|$  (cyan) ... 5
- $d_4 = |AI| = |BJ| = |CG| = |CH|$  (magenta) ... 4
- $d_5 = |BK|$  (yellow) ... 1
- $d_6 = |EI|$  (grass) ... 1
- $d_7 = |JK|$  (sea) ... 1

**Occurrences of the Golden Ratio**

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} = \frac{d_4}{d_6} = \frac{d_6}{d_7} \text{ (20 occurrences)}$$

## 2.15.9 Construction RCC170



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, A)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(B, C)$   
 $G \in k_3 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(G, B)$   
 $I \in k_2 \cap k_5, J \in k_3 \cap k_5, K \in k_4 \cap k_5, L \in k_4 \cap k_5$

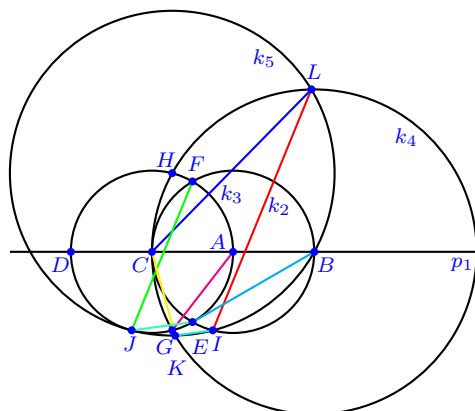
## Distances

$$\begin{aligned}
 d_0 &= |IL| \text{ (red)} \dots 1 \\
 d_1 &= |CL| \text{ (blue)} \dots 1 \\
 d_2 &= |EJ| \text{ (green)} \dots 1 \\
 d_3 &= |BE| = |BF| = |DE| = |DF| = |EF| \text{ (cyan)} \dots 5 \\
 d_4 &= |AG| = |AH| = |CI| = |DJ| \text{ (magenta)} \dots 4 \\
 d_5 &= |CK| \text{ (yellow)} \dots 1 \\
 d_6 &= |FJ| \text{ (grass)} \dots 1 \\
 d_7 &= |IK| \text{ (sea)} \dots 1
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} = \frac{d_4}{d_6} = \frac{d_6}{d_7} \text{ (20 occurrences)}$$

## 2.15.10 Construction RCC179



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, A)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(B, C)$   
 $G \in k_3 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(H, B)$   
 $I \in k_2 \cap k_5, J \in k_3 \cap k_5, K \in k_4 \cap k_5, L \in k_4 \cap k_5$

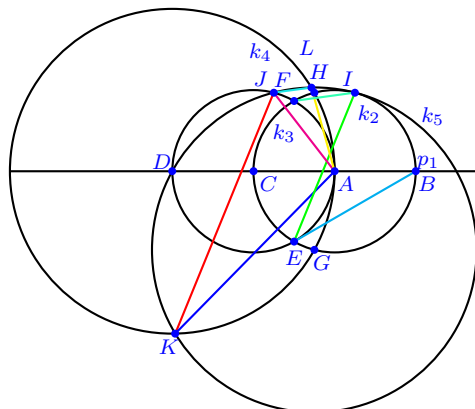
## Distances

- $d_0 = |IL|$  (red) ... 1  
 $d_1 = |CL|$  (blue) ... 1  
 $d_2 = |FJ|$  (green) ... 1  
 $d_3 = |BE| = |BF| = |DE| = |DF| = |EF|$  (cyan) ... 5  
 $d_4 = |AG| = |AH| = |CI| = |DJ|$  (magenta) ... 4  
 $d_5 = |CK|$  (yellow) ... 1  
 $d_6 = |EJ|$  (grass) ... 1  
 $d_7 = |IK|$  (sea) ... 1

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} = \frac{d_4}{d_6} = \frac{d_6}{d_7} \quad (20 \text{ occurrences})$$

## 2.15.11 Construction RCC197



## Construction Process

$A, B$  given initial points

1.  $p_1 = p(A, B)$
2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
3.  $k_3 = k(C, A)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
4.  $k_4 = k(D, A)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
5.  $k_5 = k(G, D)$   
 $I \in k_2 \cap k_5, J \in k_3 \cap k_5, K \in k_4 \cap k_5, L \in k_4 \cap k_5$

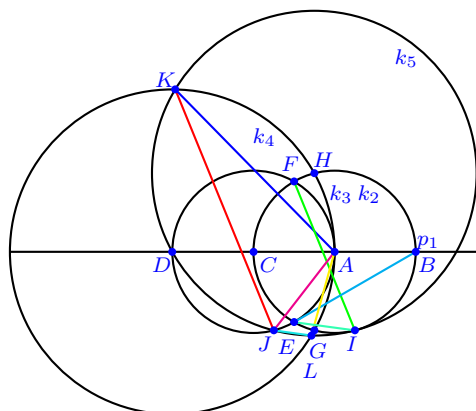
## Distances

- $d_0 = |JK|$  (red) ... 1  
 $d_1 = |AK|$  (blue) ... 1  
 $d_2 = |EI|$  (green) ... 1  
 $d_3 = |BE| = |BF| = |DE| = |DF| = |EF|$  (cyan) ... 5  
 $d_4 = |AJ| = |BI| = |CG| = |CH|$  (magenta) ... 4  
 $d_5 = |AL|$  (yellow) ... 1  
 $d_6 = |FI|$  (grass) ... 1  
 $d_7 = |JL|$  (sea) ... 1

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} = \frac{d_4}{d_6} = \frac{d_6}{d_7} \quad (20 \text{ occurrences})$$

## 2.15.12 Construction RCC206



## Construction Process

$A, B$  given initial points

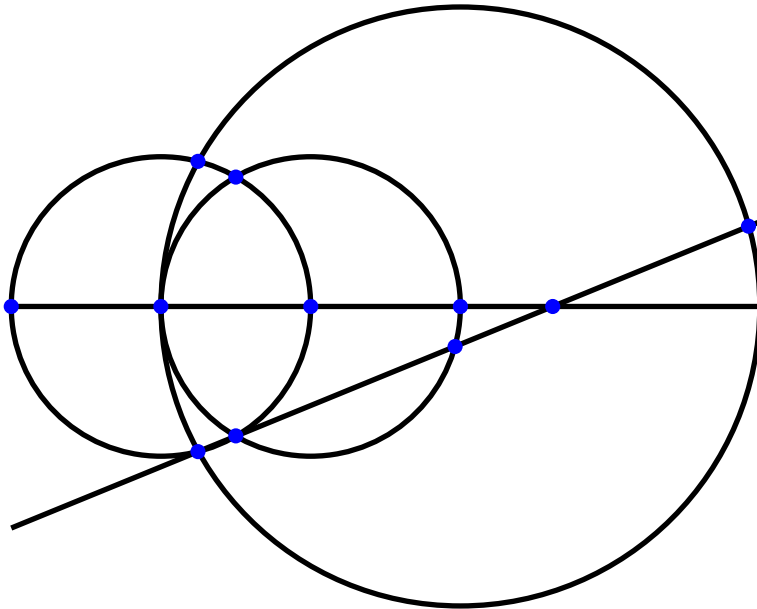
1.  $p_1 = p(A, B)$
2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
3.  $k_3 = k(C, A)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
4.  $k_4 = k(D, A)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
5.  $k_5 = k(H, D)$   
 $I \in k_2 \cap k_5, J \in k_3 \cap k_5, K \in k_4 \cap k_5, L \in k_4 \cap k_5$

## Distances

- $d_0 = |JK|$  (red) ... 1  
 $d_1 = |AK|$  (blue) ... 1  
 $d_2 = |FI|$  (green) ... 1  
 $d_3 = |BE| = |BF| = |DE| = |DF| = |EF|$  (cyan) ... 5  
 $d_4 = |AJ| = |BI| = |CG| = |CH|$  (magenta) ... 4  
 $d_5 = |AL|$  (yellow) ... 1  
 $d_6 = |EI|$  (grass) ... 1  
 $d_7 = |JL|$  (sea) ... 1

## Occurrences of the Golden Ratio

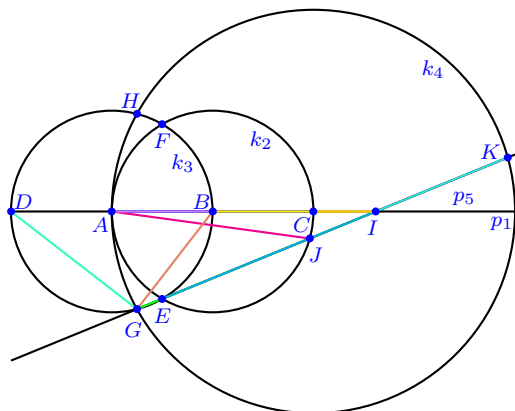
$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} = \frac{d_4}{d_6} = \frac{d_5}{d_7} \quad (20 \text{ occurrences})$$

**2.16 Class of Similar Constructions No. 9****Contained Constructions**

RCC10, RCC22, RCC43, RCC55, RCC95, RCC107, RCC135, RCC147, RCC173,  
RCC185, RCC198, RCC210



## 2.16.1 Construction RCC10



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(A, B)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(C, A)$   
 $G \in k_3 \cap k_4, H \in k_3 \cap k_4$
  5.  $p_5 = p(E, G)$   
 $I \in p_1 \cap p_5, J \in p_5 \cap k_2, K \in p_5 \cap k_4$

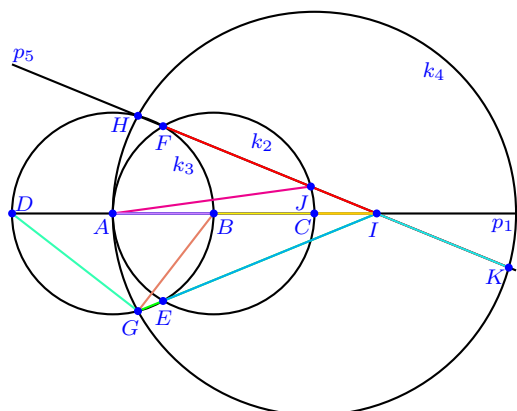
## Distances

$$\begin{aligned}
 d_0 &= |EK| = |HK| \text{ (red)} \dots 2 \\
 d_1 &= |AI| \text{ (blue)} \dots 1 \\
 d_2 &= |GI| = |HI| \text{ (green)} \dots 2 \\
 d_3 &= |EI| = |FI| \text{ (cyan)} \dots 2 \\
 d_4 &= |AJ| \text{ (magenta)} \dots 1 \\
 d_5 &= |BI| \text{ (yellow)} \dots 1 \\
 d_6 &= |DG| = |DH| = |EJ| \text{ (grass)} \dots 3 \\
 d_7 &= |IK| \text{ (sea)} \dots 1 \\
 d_8 &= |BG| = |BH| \text{ (darkorange)} \dots 2 \\
 d_9 &= |AB| = |AD| = |AE| = |AF| = |AG| = |AH| = |BC| = |BE| = \\
 &= |BF| = |BJ| \text{ (violet)} \dots 10 \\
 d_{10} &= |CI| \text{ (myorange)} \dots 1
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_3} = \frac{d_1}{d_5} = \frac{d_2}{d_6} = \frac{d_3}{d_7} = \frac{d_4}{d_8} = \frac{d_5}{d_9} = \frac{d_9}{d_{10}} \text{ (35 occurrences)}$$

### 2.16.2 Construction RCC22



#### Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(A, B)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(C, A)$   
 $G \in k_3 \cap k_4, H \in k_3 \cap k_4$
  5.  $p_5 = p(F, H)$   
 $I \in p_1 \cap p_5, J \in p_5 \cap k_2, K \in p_5 \cap k_4$

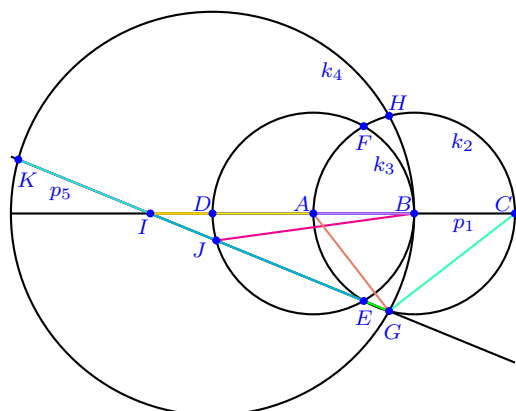
#### Distances

- $d_0 = |FK| = |GK|$  (red) ... 2
- $d_1 = |AI|$  (blue) ... 1
- $d_2 = |GI| = |HI|$  (green) ... 2
- $d_3 = |EI| = |FI|$  (cyan) ... 2
- $d_4 = |AJ|$  (magenta) ... 1
- $d_5 = |BI|$  (yellow) ... 1
- $d_6 = |DG| = |DH| = |FJ|$  (grass) ... 3
- $d_7 = |IK|$  (sea) ... 1
- $d_8 = |BG| = |BH|$  (darkorange) ... 2
- $d_9 = |AB| = |AD| = |AE| = |AF| = |AG| = |AH| = |BC| = |BE| = |BF| = |BJ|$  (violet) ... 10
- $d_{10} = |CI|$  (myorange) ... 1

#### Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_3} = \frac{d_1}{d_5} = \frac{d_2}{d_6} = \frac{d_3}{d_7} = \frac{d_4}{d_8} = \frac{d_5}{d_9} = \frac{d_9}{d_{10}} \text{ (35 occurrences)}$$

## 2.16.3 Construction RCC43



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(A, B)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(D, B)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
  5.  $p_5 = p(E, G)$   
 $I \in p_1 \cap p_5, J \in p_5 \cap k_3, K \in p_5 \cap k_4$

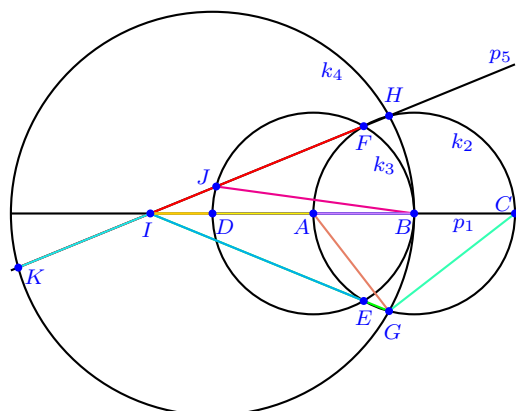
## Distances

$$\begin{aligned}
 d_0 &= |EK| = |HK| \text{ (red)} \dots 2 \\
 d_1 &= |BI| \text{ (blue)} \dots 1 \\
 d_2 &= |GI| = |HI| \text{ (green)} \dots 2 \\
 d_3 &= |EI| = |FI| \text{ (cyan)} \dots 2 \\
 d_4 &= |BJ| \text{ (magenta)} \dots 1 \\
 d_5 &= |AI| \text{ (yellow)} \dots 1 \\
 d_6 &= |CG| = |CH| = |EJ| \text{ (grass)} \dots 3 \\
 d_7 &= |IK| \text{ (sea)} \dots 1 \\
 d_8 &= |AG| = |AH| \text{ (darkorange)} \dots 2 \\
 d_9 &= |AB| = |AD| = |AE| = |AF| = |AJ| = |BC| = |BE| = |BF| = \\
 &|BG| = |BH| \text{ (violet)} \dots 10 \\
 d_{10} &= |DI| \text{ (myorange)} \dots 1
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_3} = \frac{d_1}{d_5} = \frac{d_2}{d_6} = \frac{d_3}{d_7} = \frac{d_4}{d_8} = \frac{d_5}{d_9} = \frac{d_9}{d_{10}} \text{ (35 occurrences)}$$

## 2.16.4 Construction RCC55



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(A, B)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(D, B)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
  5.  $p_5 = p(F, H)$   
 $I \in p_1 \cap p_5, J \in p_5 \cap k_3, K \in p_5 \cap k_4$

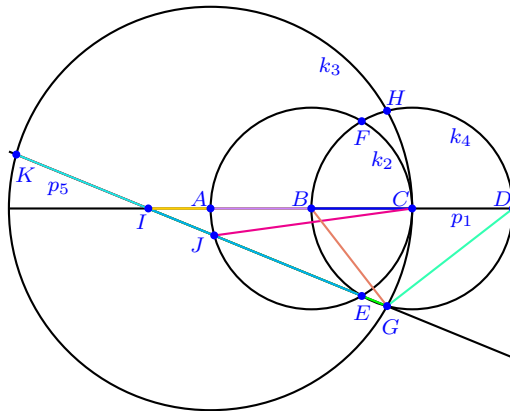
## Distances

$$\begin{aligned}
 d_0 &= |FK| = |GK| \text{ (red)} \dots 2 \\
 d_1 &= |BI| \text{ (blue)} \dots 1 \\
 d_2 &= |GI| = |HI| \text{ (green)} \dots 2 \\
 d_3 &= |EI| = |FI| \text{ (cyan)} \dots 2 \\
 d_4 &= |BJ| \text{ (magenta)} \dots 1 \\
 d_5 &= |AI| \text{ (yellow)} \dots 1 \\
 d_6 &= |CG| = |CH| = |FJ| \text{ (grass)} \dots 3 \\
 d_7 &= |IK| \text{ (sea)} \dots 1 \\
 d_8 &= |AG| = |AH| \text{ (darkorange)} \dots 2 \\
 d_9 &= |AB| = |AD| = |AE| = |AF| = |AJ| = |BC| = |BE| = |BF| = \\
 &|BG| = |BH| \text{ (violet)} \dots 10 \\
 d_{10} &= |DI| \text{ (myorange)} \dots 1
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_3} = \frac{d_1}{d_5} = \frac{d_2}{d_6} = \frac{d_3}{d_7} = \frac{d_4}{d_8} = \frac{d_5}{d_9} = \frac{d_9}{d_{10}} \text{ (35 occurrences)}$$

## 2.16.5 Construction RCC95



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(A, C)$
  4.  $k_4 = k(C, B)$   
 $D \in p_1 \cap k_4, E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4$
  5.  $p_5 = p(E, G)$   
 $I \in p_1 \cap p_5, J \in p_5 \cap k_2, K \in p_5 \cap k_3$

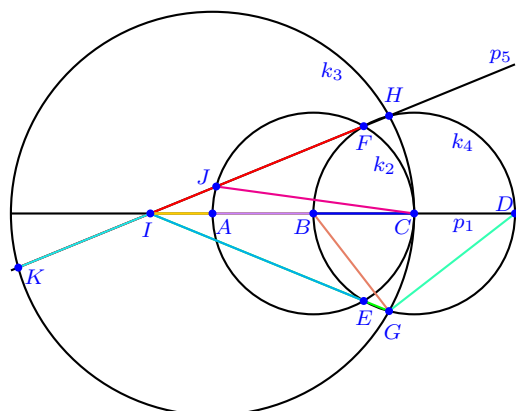
## Distances

$$\begin{aligned}
 d_0 &= |EK| = |HK| \text{ (red)} \dots 2 \\
 d_1 &= |CI| \text{ (blue)} \dots 1 \\
 d_2 &= |GI| = |HI| \text{ (green)} \dots 2 \\
 d_3 &= |EI| = |FI| \text{ (cyan)} \dots 2 \\
 d_4 &= |CJ| \text{ (magenta)} \dots 1 \\
 d_5 &= |BI| \text{ (yellow)} \dots 1 \\
 d_6 &= |DG| = |DH| = |EJ| \text{ (darkorange)} \dots 3 \\
 d_7 &= |IK| \text{ (sea)} \dots 1 \\
 d_8 &= |BG| = |BH| \text{ (darkorange)} \dots 2 \\
 d_9 &= |AB| = |BC| = |BE| = |BF| = |BJ| = |CD| = |CE| = |CF| = \\
 &|CG| = |CH| \text{ (violet)} \dots 10 \\
 d_{10} &= |AI| \text{ (myorange)} \dots 1
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_3} = \frac{d_1}{d_5} = \frac{d_2}{d_6} = \frac{d_3}{d_7} = \frac{d_4}{d_8} = \frac{d_5}{d_9} = \frac{d_9}{d_{10}} \text{ (35 occurrences)}$$

## 2.16.6 Construction RCC107



## Construction Process

- $A, B$  given initial points
- $p_1 = p(A, B)$
  - $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  - $k_3 = k(A, C)$
  - $k_4 = k(C, B)$   
 $D \in p_1 \cap k_4, E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4$
  - $p_5 = p(F, H)$   
 $I \in p_1 \cap p_5, J \in p_5 \cap k_2, K \in p_5 \cap k_3$

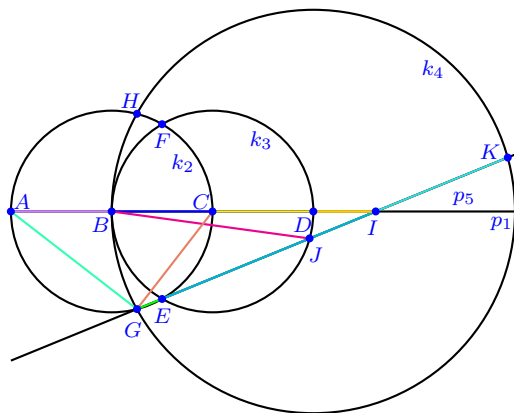
## Distances

$$\begin{aligned}
 d_0 &= |FK| = |GK| \text{ (red)} \dots 2 \\
 d_1 &= |CI| \text{ (blue)} \dots 1 \\
 d_2 &= |GI| = |HI| \text{ (green)} \dots 2 \\
 d_3 &= |EI| = |FI| \text{ (cyan)} \dots 2 \\
 d_4 &= |CJ| \text{ (magenta)} \dots 1 \\
 d_5 &= |BI| \text{ (yellow)} \dots 1 \\
 d_6 &= |DG| = |DH| = |FJ| \text{ (grass)} \dots 3 \\
 d_7 &= |IK| \text{ (sea)} \dots 1 \\
 d_8 &= |BG| = |BH| \text{ (darkorange)} \dots 2 \\
 d_9 &= |AB| = |BC| = |BE| = |BF| = |BJ| = |CD| = |CE| = |CF| = \\
 &|CG| = |CH| \text{ (violet)} \dots 10 \\
 d_{10} &= |AI| \text{ (myorange)} \dots 1
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_3} = \frac{d_1}{d_5} = \frac{d_2}{d_6} = \frac{d_3}{d_7} = \frac{d_4}{d_8} = \frac{d_5}{d_9} = \frac{d_9}{d_{10}} \text{ (35 occurrences)}$$

## 2.16.7 Construction RCC135



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, B)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(D, B)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
  5.  $p_5 = p(E, G)$   
 $I \in p_1 \cap p_5, J \in p_5 \cap k_3, K \in p_5 \cap k_4$

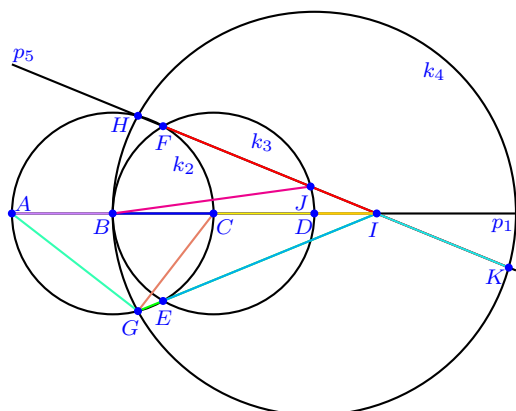
## Distances

$$\begin{aligned}
 d_0 &= |EK| = |HK| \text{ (red)} \dots 2 \\
 d_1 &= |BI| \text{ (blue)} \dots 1 \\
 d_2 &= |GI| = |HI| \text{ (green)} \dots 2 \\
 d_3 &= |EI| = |FI| \text{ (cyan)} \dots 2 \\
 d_4 &= |BJ| \text{ (magenta)} \dots 1 \\
 d_5 &= |CI| \text{ (yellow)} \dots 1 \\
 d_6 &= |AG| = |AH| = |EJ| \text{ (grass)} \dots 3 \\
 d_7 &= |IK| \text{ (sea)} \dots 1 \\
 d_8 &= |CG| = |CH| \text{ (darkorange)} \dots 2 \\
 d_9 &= |AB| = |BC| = |BE| = |BF| = |BG| = |BH| = |CD| = |CE| = \\
 &|CF| = |CJ| \text{ (violet)} \dots 10 \\
 d_{10} &= |DI| \text{ (myorange)} \dots 1
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_3} = \frac{d_1}{d_5} = \frac{d_2}{d_6} = \frac{d_3}{d_7} = \frac{d_4}{d_8} = \frac{d_5}{d_9} = \frac{d_9}{d_{10}} \text{ (35 occurrences)}$$

## 2.16.8 Construction RCC147



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, B)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(D, B)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
  5.  $p_5 = p(F, H)$   
 $I \in p_1 \cap p_5, J \in p_5 \cap k_3, K \in p_5 \cap k_4$

## Distances

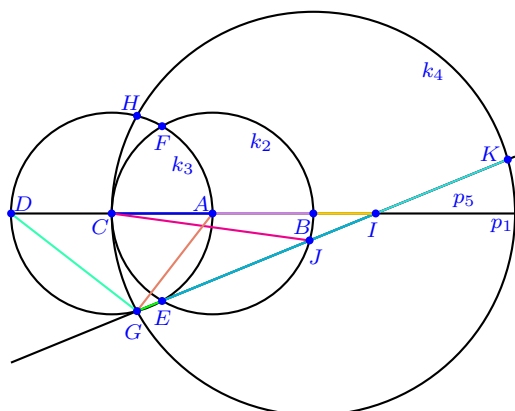
$$\begin{aligned}
 d_0 &= |FK| = |GK| \text{ (red)} \dots 2 \\
 d_1 &= |BI| \text{ (blue)} \dots 1 \\
 d_2 &= |GI| = |HI| \text{ (green)} \dots 2 \\
 d_3 &= |EI| = |FI| \text{ (cyan)} \dots 2 \\
 d_4 &= |BJ| \text{ (magenta)} \dots 1 \\
 d_5 &= |CI| \text{ (yellow)} \dots 1 \\
 d_6 &= |AG| = |AH| = |FJ| \text{ (grass)} \dots 3 \\
 d_7 &= |IK| \text{ (sea)} \dots 1 \\
 d_8 &= |CG| = |CH| \text{ (darkorange)} \dots 2 \\
 d_9 &= |AB| = |BC| = |BE| = |BF| = |BG| = |BH| = |CD| = |CE| = \\
 &|CF| = |CJ| \text{ (violet)} \dots 10 \\
 d_{10} &= |DI| \text{ (myorange)} \dots 1
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_3} = \frac{d_1}{d_5} = \frac{d_2}{d_6} = \frac{d_3}{d_7} = \frac{d_4}{d_8} = \frac{d_5}{d_9} = \frac{d_9}{d_{10}} \text{ (35 occurrences)}$$



## 2.16.9 Construction RCC173



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, A)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(B, C)$   
 $G \in k_3 \cap k_4, H \in k_3 \cap k_4$
  5.  $p_5 = p(E, G)$   
 $I \in p_1 \cap p_5, J \in p_5 \cap k_2, K \in p_5 \cap k_4$

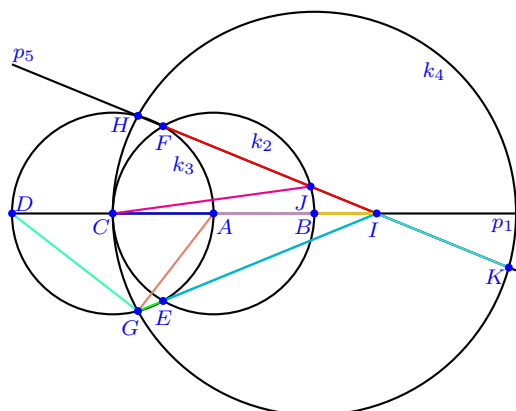
## Distances

$$\begin{aligned}
 d_0 &= |EK| = |HK| \text{ (red)} \dots 2 \\
 d_1 &= |CI| \text{ (blue)} \dots 1 \\
 d_2 &= |GI| = |HI| \text{ (green)} \dots 2 \\
 d_3 &= |EI| = |FI| \text{ (cyan)} \dots 2 \\
 d_4 &= |CJ| \text{ (magenta)} \dots 1 \\
 d_5 &= |AI| \text{ (yellow)} \dots 1 \\
 d_6 &= |DG| = |DH| = |EJ| \text{ (grass)} \dots 3 \\
 d_7 &= |IK| \text{ (sea)} \dots 1 \\
 d_8 &= |AG| = |AH| \text{ (darkorange)} \dots 2 \\
 d_9 &= |AB| = |AC| = |AE| = |AF| = |AJ| = |CD| = |CE| = |CF| = \\
 &|CG| = |CH| \text{ (violet)} \dots 10 \\
 d_{10} &= |BI| \text{ (myorange)} \dots 1
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_3} = \frac{d_1}{d_5} = \frac{d_2}{d_6} = \frac{d_3}{d_7} = \frac{d_4}{d_8} = \frac{d_5}{d_9} = \frac{d_9}{d_{10}} \text{ (35 occurrences)}$$

## 2.16.10 Construction RCC185



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, A)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(B, C)$   
 $G \in k_3 \cap k_4, H \in k_3 \cap k_4$
  5.  $p_5 = p(F, H)$   
 $I \in p_1 \cap p_5, J \in p_5 \cap k_2, K \in p_5 \cap k_4$

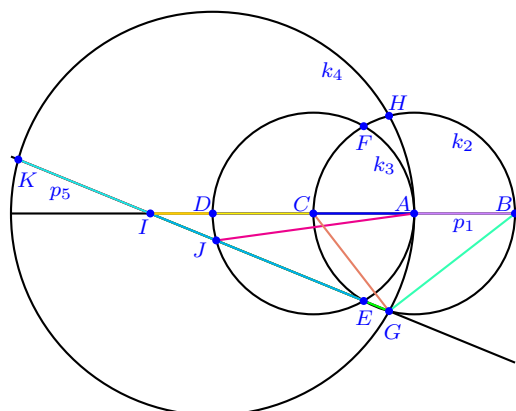
## Distances

$$\begin{aligned}
 d_0 &= |FK| = |GK| \text{ (red)} \dots 2 \\
 d_1 &= |CI| \text{ (blue)} \dots 1 \\
 d_2 &= |GI| = |HI| \text{ (green)} \dots 2 \\
 d_3 &= |EI| = |FI| \text{ (cyan)} \dots 2 \\
 d_4 &= |CJ| \text{ (magenta)} \dots 1 \\
 d_5 &= |AI| \text{ (yellow)} \dots 1 \\
 d_6 &= |DG| = |DH| = |FJ| \text{ (grass)} \dots 3 \\
 d_7 &= |IK| \text{ (sea)} \dots 1 \\
 d_8 &= |AG| = |AH| \text{ (darkorange)} \dots 2 \\
 d_9 &= |AB| = |AC| = |AE| = |AF| = |AJ| = |CD| = |CE| = |CF| = \\
 &|CG| = |CH| \text{ (violet)} \dots 10 \\
 d_{10} &= |BI| \text{ (myorange)} \dots 1
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_3} = \frac{d_1}{d_5} = \frac{d_2}{d_6} = \frac{d_3}{d_7} = \frac{d_4}{d_8} = \frac{d_5}{d_9} = \frac{d_9}{d_{10}} \text{ (35 occurrences)}$$

## 2.16.11 Construction RCC198



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, A)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(D, A)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
  5.  $p_5 = p(E, G)$   
 $I \in p_1 \cap p_5, J \in p_5 \cap k_3, K \in p_5 \cap k_4$

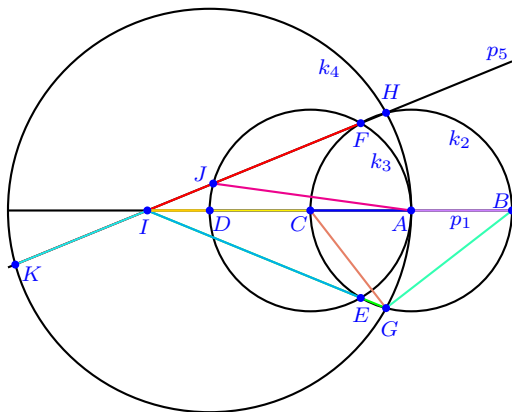
## Distances

$$\begin{aligned}
 d_0 &= |EK| = |HK| \text{ (red)} \dots 2 \\
 d_1 &= |AI| \text{ (blue)} \dots 1 \\
 d_2 &= |GI| = |HI| \text{ (green)} \dots 2 \\
 d_3 &= |EI| = |FI| \text{ (cyan)} \dots 2 \\
 d_4 &= |AJ| \text{ (magenta)} \dots 1 \\
 d_5 &= |CI| \text{ (yellow)} \dots 1 \\
 d_6 &= |BG| = |BH| = |EJ| \text{ (grass)} \dots 3 \\
 d_7 &= |IK| \text{ (sea)} \dots 1 \\
 d_8 &= |CG| = |CH| \text{ (darkorange)} \dots 2 \\
 d_9 &= |AB| = |AC| = |AE| = |AF| = |AG| = |AH| = |CD| = |CE| = \\
 &= |CF| = |CJ| \text{ (violet)} \dots 10 \\
 d_{10} &= |DI| \text{ (myorange)} \dots 1
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_3} = \frac{d_1}{d_5} = \frac{d_2}{d_6} = \frac{d_3}{d_7} = \frac{d_4}{d_8} = \frac{d_5}{d_9} = \frac{d_9}{d_{10}} \text{ (35 occurrences)}$$

## 2.16.12 Construction RCC210



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, A)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(D, A)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
  5.  $p_5 = p(F, H)$   
 $I \in p_1 \cap p_5, J \in p_5 \cap k_3, K \in p_5 \cap k_4$

## Distances

$$d_0 = |FK| = |GK| \text{ (red) } \dots 2$$

$$d_1 = |AI| \text{ (blue) } \dots 1$$

$$d_2 = |GI| = |HI| \text{ (green) } \dots 2$$

$$d_3 = |EI| = |FI| \text{ (cyan) } \dots 2$$

$$d_4 = |AJ| \text{ (magenta) } \dots 1$$

$$d_5 = |CI| \text{ (yellow) } \dots 1$$

$$d_6 = |BG| = |BH| = |FJ| \text{ (grass) } \dots 3$$

$$d_7 = |IK| \text{ (sea) } \dots 1$$

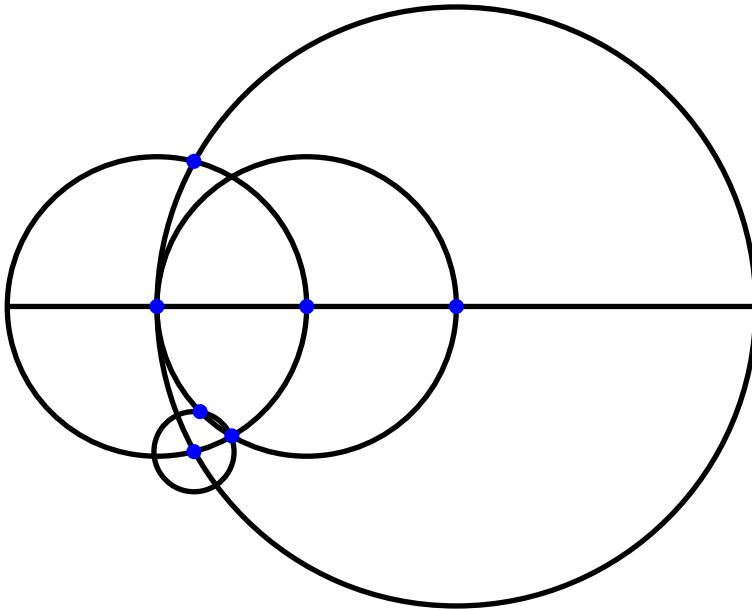
$$d_8 = |CG| = |CH| \text{ (darkorange) } \dots 2$$

$$d_9 = |AB| = |AC| = |AE| = |AF| = |AG| = |AH| = |CD| = |CE| = |CF| = |CJ| \text{ (violet) } \dots 10$$

$$d_{10} = |DI| \text{ (myorange) } \dots 1$$

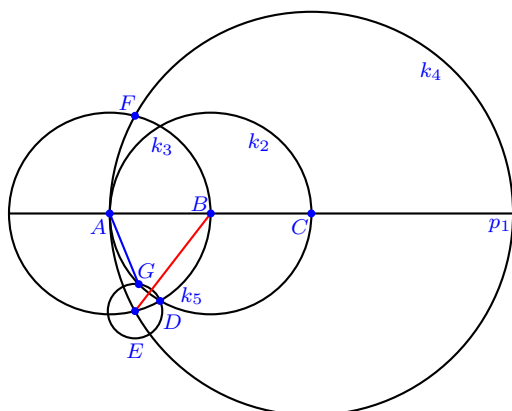
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_3} = \frac{d_1}{d_5} = \frac{d_2}{d_6} = \frac{d_3}{d_7} = \frac{d_4}{d_8} = \frac{d_5}{d_9} = \frac{d_9}{d_{10}} \text{ (35 occurrences)}$$

**2.17 Class of Similar Constructions No. 10****Contained Constructions**

RCC11, RCC23, RCC44, RCC56, RCC96, RCC108, RCC136, RCC148, RCC174,  
RCC186, RCC199, RCC211

## 2.17.1 Construction RCC11



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(A, B)$   
 $D \in k_2 \cap k_3$
  4.  $k_4 = k(C, A)$   
 $E \in k_3 \cap k_4, F \in k_3 \cap k_4$
  5.  $k_5 = k(E, D)$   
 $G \in k_2 \cap k_5$

## Distances

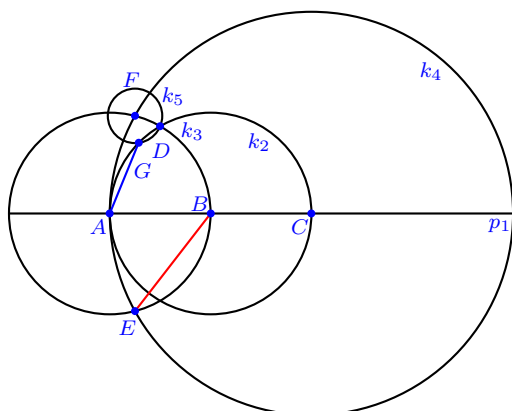
$$d_0 = |BE| = |BF| \text{ (red)} \dots 2$$

$$d_1 = |AG| \text{ (blue)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (2 occurrences)}$$

## 2.17.2 Construction RCC23



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(A, B)$   
 $D \in k_2 \cap k_3$
  4.  $k_4 = k(C, A)$   
 $E \in k_3 \cap k_4, F \in k_3 \cap k_4$
  5.  $k_5 = k(F, D)$   
 $G \in k_2 \cap k_5$

## Distances

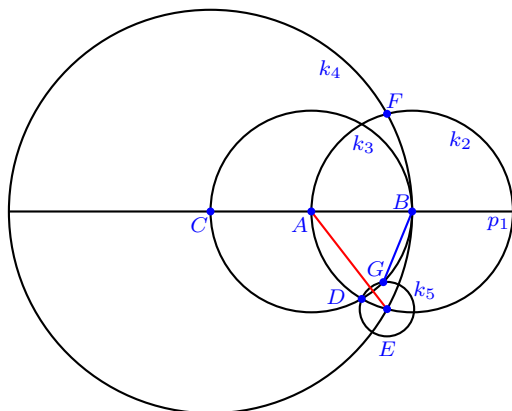
$$d_0 = |BE| = |BF| \text{ (red)} \dots 2$$

$$d_1 = |AG| \text{ (blue)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (2 occurrences)}$$

## 2.17.3 Construction RCC44



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$
  3.  $k_3 = k(A, B)$   
 $C \in p_1 \cap k_3, D \in k_2 \cap k_3$
  4.  $k_4 = k(C, B)$   
 $E \in k_2 \cap k_4, F \in k_2 \cap k_4$
  5.  $k_5 = k(E, D)$   
 $G \in k_3 \cap k_5$

## Distances

$$d_0 = |AE| = |AF| \text{ (red) } \dots 2$$

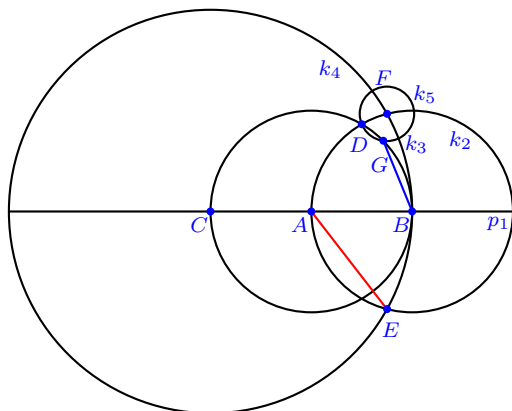
$$d_1 = |BG| \text{ (blue) } \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (2 occurrences)}$$



## 2.17.4 Construction RCC56



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$
  3.  $k_3 = k(A, B)$   
 $C \in p_1 \cap k_3, D \in k_2 \cap k_3$
  4.  $k_4 = k(C, B)$   
 $E \in k_2 \cap k_4, F \in k_2 \cap k_4$
  5.  $k_5 = k(F, D)$   
 $G \in k_3 \cap k_5$

## Distances

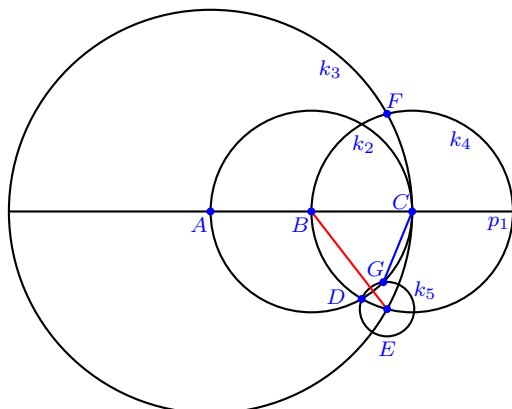
$$d_0 = |AE| = |AF| \text{ (red) } \dots 2$$

$$d_1 = |BG| \text{ (blue) } \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (2 occurrences)}$$

## 2.17.5 Construction RCC96



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(A, C)$
  4.  $k_4 = k(C, B)$   
 $D \in k_2 \cap k_4, E \in k_3 \cap k_4, F \in k_3 \cap k_4$
  5.  $k_5 = k(E, D)$   
 $G \in k_2 \cap k_5$

## Distances

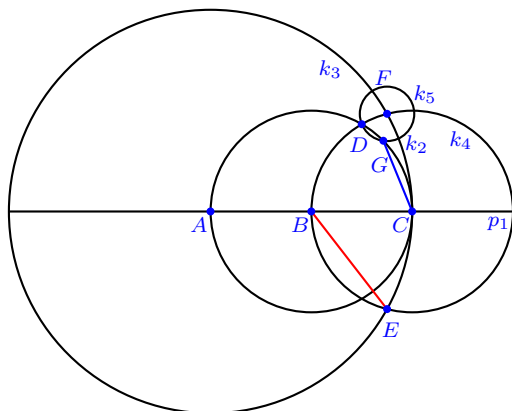
$$d_0 = |BE| = |BF| \text{ (red) } \dots 2$$

$$d_1 = |CG| \text{ (blue) } \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (2 occurrences)}$$

## 2.17.6 Construction RCC108



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(A, C)$
  4.  $k_4 = k(C, B)$   
 $D \in k_2 \cap k_4, E \in k_3 \cap k_4, F \in k_3 \cap k_4$
  5.  $k_5 = k(F, D)$   
 $G \in k_2 \cap k_5$

## Distances

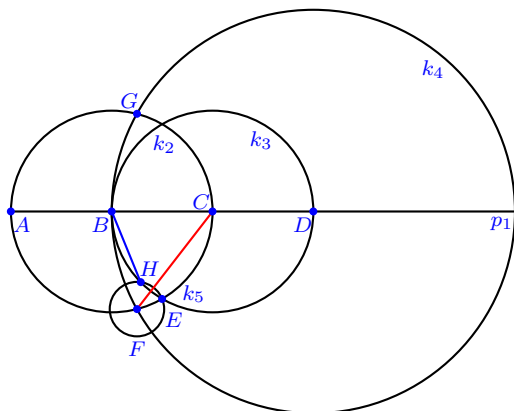
$$d_0 = |BE| = |BF| \text{ (red)} \dots 2$$

$$d_1 = |CG| \text{ (blue)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (2 occurrences)}$$

## 2.17.7 Construction RCC136



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, B)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3$
  4.  $k_4 = k(D, B)$   
 $F \in k_2 \cap k_4, G \in k_2 \cap k_4$
  5.  $k_5 = k(F, E)$   
 $H \in k_3 \cap k_5$

## Distances

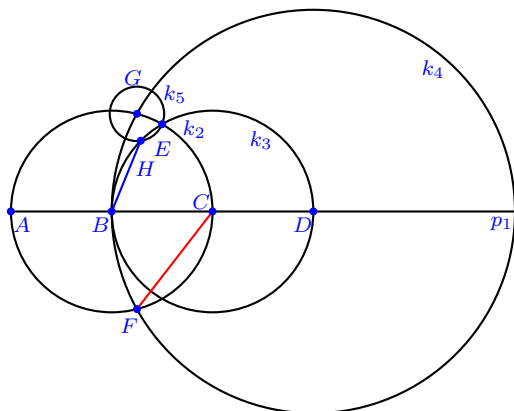
$$d_0 = |CF| = |CG| \text{ (red) } \dots 2$$

$$d_1 = |BH| \text{ (blue) } \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (2 occurrences)}$$

## 2.17.8 Construction RCC148



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, B)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3$
  4.  $k_4 = k(D, B)$   
 $F \in k_2 \cap k_4, G \in k_2 \cap k_4$
  5.  $k_5 = k(G, E)$   
 $H \in k_3 \cap k_5$

## Distances

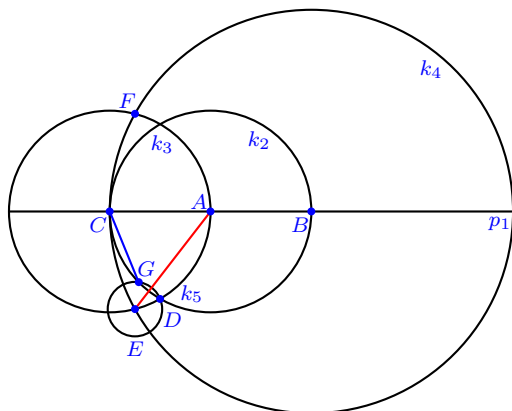
$$d_0 = |CF| = |CG| \text{ (red) } \dots 2$$

$$d_1 = |BH| \text{ (blue) } \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (2 occurrences)}$$

## 2.17.9 Construction RCC174



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, A)$   
 $D \in k_2 \cap k_3$
  4.  $k_4 = k(B, C)$   
 $E \in k_3 \cap k_4, F \in k_3 \cap k_4$
  5.  $k_5 = k(E, D)$   
 $G \in k_2 \cap k_5$

## Distances

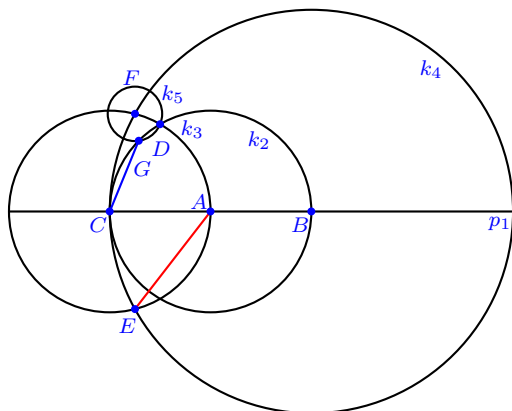
$$d_0 = |AE| = |AF| \text{ (red)} \dots 2$$

$$d_1 = |CG| \text{ (blue)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (2 occurrences)}$$

## 2.17.10 Construction RCC186



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, A)$   
 $D \in k_2 \cap k_3$
  4.  $k_4 = k(B, C)$   
 $E \in k_3 \cap k_4, F \in k_3 \cap k_4$
  5.  $k_5 = k(F, D)$   
 $G \in k_2 \cap k_5$

## Distances

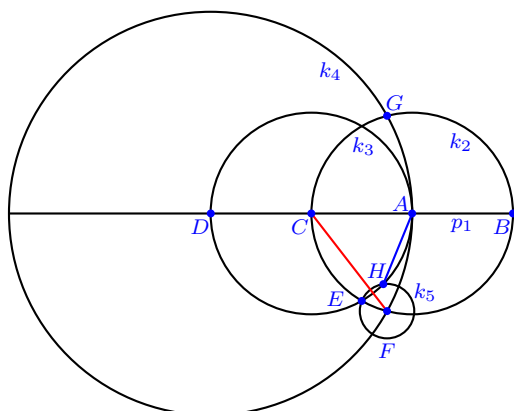
$$d_0 = |AE| = |AF| \text{ (red) } \dots 2$$

$$d_1 = |CG| \text{ (blue) } \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (2 occurrences)}$$

## 2.17.11 Construction RCC199



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, A)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3$
  4.  $k_4 = k(D, A)$   
 $F \in k_2 \cap k_4, G \in k_2 \cap k_4$
  5.  $k_5 = k(F, E)$   
 $H \in k_3 \cap k_5$

## Distances

$$d_0 = |CF| = |CG| \text{ (red) } \dots 2$$

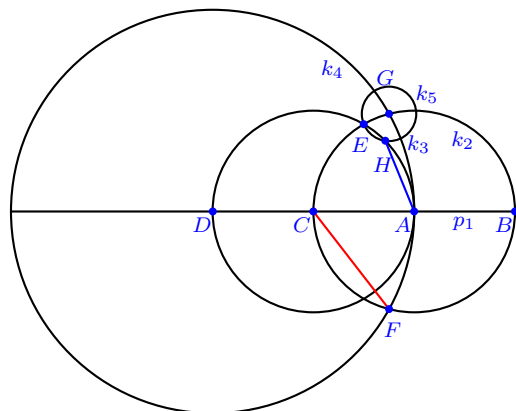
$$d_1 = |AH| \text{ (blue) } \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (2 occurrences)}$$



## 2.17.12 Construction RCC211



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, A)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3$
  4.  $k_4 = k(D, A)$   
 $F \in k_2 \cap k_4, G \in k_2 \cap k_4$
  5.  $k_5 = k(G, E)$   
 $H \in k_3 \cap k_5$

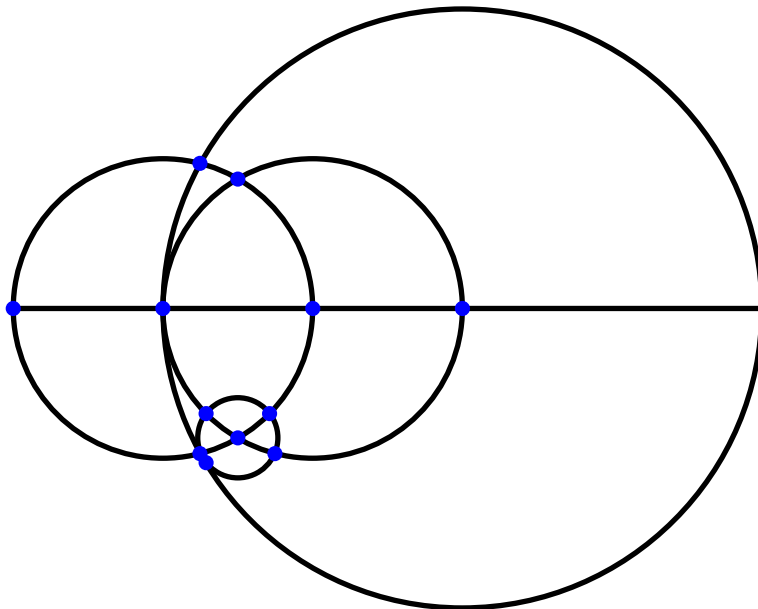
## Distances

$$d_0 = |CF| = |CG| \text{ (red) } \dots 2$$

$$d_1 = |AH| \text{ (blue) } \dots 1$$

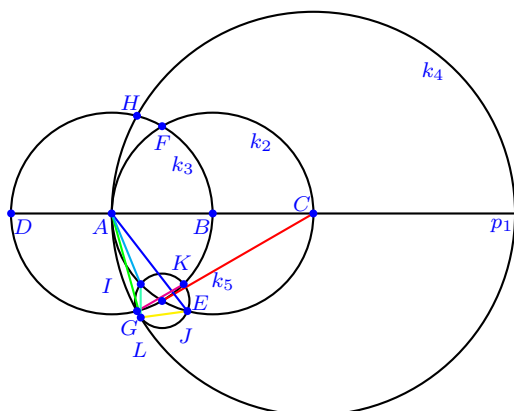
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (2 occurrences)}$$

**2.18 Class of Similar Constructions No. 11****Contained Constructions**

RCC12, RCC24, RCC45, RCC57, RCC97, RCC109, RCC137, RCC149, RCC175,  
RCC187, RCC200, RCC212

## 2.18.1 Construction RCC12



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(A, B)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(C, A)$   
 $G \in k_3 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(E, G)$   
 $I \in k_2 \cap k_5, J \in k_2 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5$

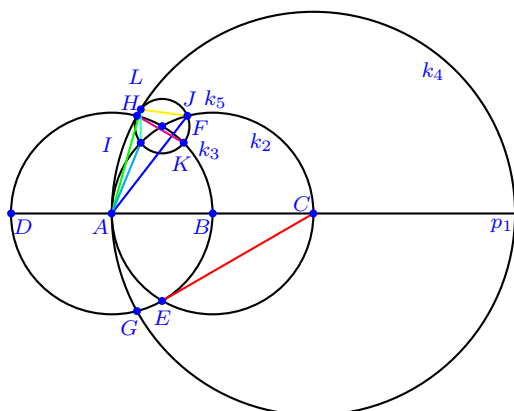
## Distances

$$\begin{aligned}
 d_0 &= |CE| = |CF| = |DE| = |DF| = |EF| = |HK| \text{ (red)} \dots 6 \\
 d_1 &= |AJ| = |BG| = |BH| \text{ (blue)} \dots 3 \\
 d_2 &= |AL| \text{ (green)} \dots 1 \\
 d_3 &= |AI| = |BK| \text{ (cyan)} \dots 2 \\
 d_4 &= |GK| = |IJ| \text{ (magenta)} \dots 2 \\
 d_5 &= |JL| \text{ (yellow)} \dots 1 \\
 d_6 &= |IL| \text{ (grass)} \dots 1
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_3}{d_5} = \frac{d_4}{d_6} \text{ (16 occurrences)}$$

## 2.18.2 Construction RCC24



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(A, B)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(C, A)$   
 $G \in k_3 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(F, H)$   
 $I \in k_2 \cap k_5, J \in k_2 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5$

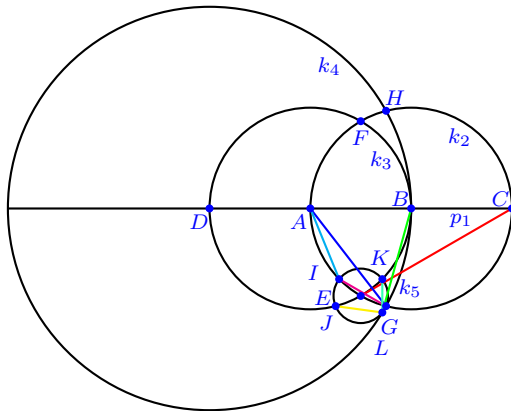
## Distances

$$\begin{aligned}
 d_0 &= |CE| = |CF| = |DE| = |DF| = |EF| = |GK| \text{ (red)} \dots 6 \\
 d_1 &= |AJ| = |BG| = |BH| \text{ (blue)} \dots 3 \\
 d_2 &= |AL| \text{ (green)} \dots 1 \\
 d_3 &= |AI| = |BK| \text{ (cyan)} \dots 2 \\
 d_4 &= |HK| = |IJ| \text{ (magenta)} \dots 2 \\
 d_5 &= |JL| \text{ (yellow)} \dots 1 \\
 d_6 &= |IL| \text{ (grass)} \dots 1
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_3}{d_5} = \frac{d_4}{d_6} \text{ (16 occurrences)}$$

2.18.3 Construction RCC45



Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(A, B)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(D, B)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
  5.  $k_5 = k(E, G)$   
 $I \in k_2 \cap k_5, J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5$

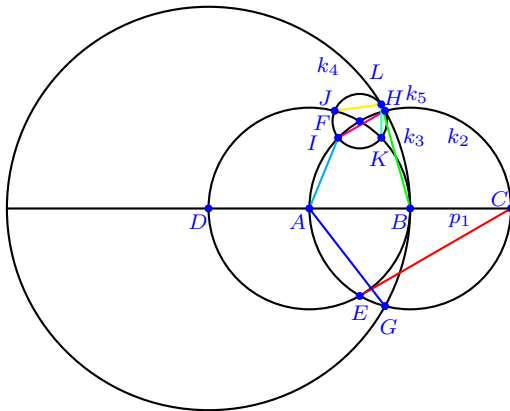
Distances

- $d_0 = |CE| = |CF| = |DE| = |DF| = |EF| = |HI|$  (red) ... 6  
 $d_1 = |AG| = |AH| = |BJ|$  (blue) ... 3  
 $d_2 = |BL|$  (green) ... 1  
 $d_3 = |AI| = |BK|$  (cyan) ... 2  
 $d_4 = |GI| = |JK|$  (magenta) ... 2  
 $d_5 = |JL|$  (yellow) ... 1  
 $d_6 = |KL|$  (grass) ... 1

Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_3}{d_5} = \frac{d_4}{d_6} \text{ (16 occurrences)}$$

2.18.4 Construction RCC57



Construction Process

- $A, B$  given initial points
- 1.  $p_1 = p(A, B)$
- 2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
- 3.  $k_3 = k(A, B)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
- 4.  $k_4 = k(D, B)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
- 5.  $k_5 = k(F, H)$   
 $I \in k_2 \cap k_5, J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5$

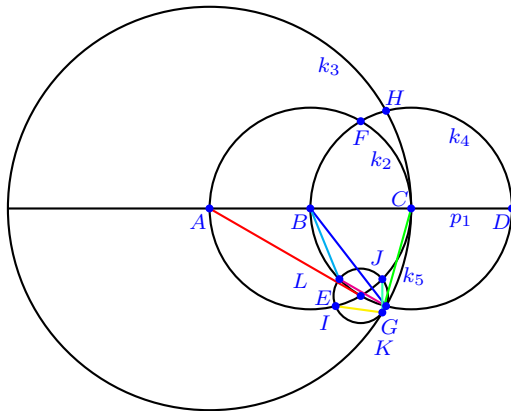
Distances

- $d_0 = |CE| = |CF| = |DE| = |DF| = |EF| = |GI|$  (red) ... 6
- $d_1 = |AG| = |AH| = |BJ|$  (blue) ... 3
- $d_2 = |BL|$  (green) ... 1
- $d_3 = |AI| = |BK|$  (cyan) ... 2
- $d_4 = |HI| = |JK|$  (magenta) ... 2
- $d_5 = |JL|$  (yellow) ... 1
- $d_6 = |KL|$  (grass) ... 1

Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_3}{d_5} = \frac{d_4}{d_6} \text{ (16 occurrences)}$$

## 2.18.5 Construction RCC97



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(A, C)$
  4.  $k_4 = k(C, B)$   
 $D \in p_1 \cap k_4, E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(E, G)$   
 $I \in k_2 \cap k_5, J \in k_2 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5$

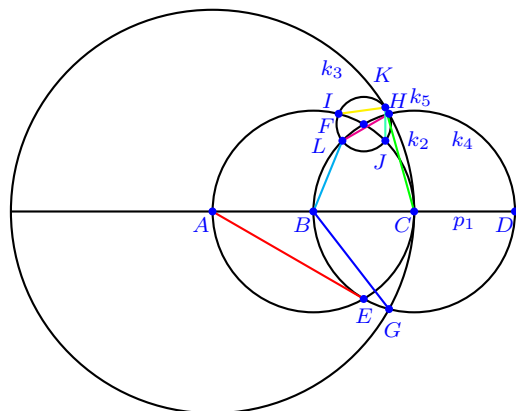
## Distances

$$\begin{aligned}
 d_0 &= |AE| = |AF| = |DE| = |DF| = |EF| = |HL| \text{ (red)} \dots 6 \\
 d_1 &= |BG| = |BH| = |CI| \text{ (blue)} \dots 3 \\
 d_2 &= |CK| \text{ (green)} \dots 1 \\
 d_3 &= |BL| = |CJ| \text{ (cyan)} \dots 2 \\
 d_4 &= |GL| = |IJ| \text{ (magenta)} \dots 2 \\
 d_5 &= |IK| \text{ (yellow)} \dots 1 \\
 d_6 &= |JK| \text{ (grass)} \dots 1
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_3}{d_5} = \frac{d_4}{d_6} \text{ (16 occurrences)}$$

## 2.18.6 Construction RCC109



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(A, C)$
  4.  $k_4 = k(C, B)$   
 $D \in p_1 \cap k_4, E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(F, H)$   
 $I \in k_2 \cap k_5, J \in k_2 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5$

## Distances

$$d_0 = |AE| = |AF| = |DE| = |DF| = |EF| = |GL| \text{ (red) } \dots 6$$

$$d_1 = |BG| = |BH| = |CI| \text{ (blue) } \dots 3$$

$$d_2 = |CK| \text{ (green) } \dots 1$$

$$d_3 = |BL| = |CJ| \text{ (cyan) } \dots 2$$

$$d_4 = |HL| = |IJ| \text{ (magenta) } \dots 2$$

$$d_5 = |IK| \text{ (yellow) } \dots 1$$

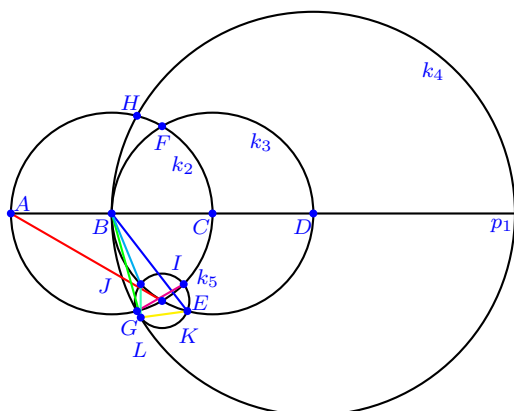
$$d_6 = |JK| \text{ (grass) } \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_3}{d_5} = \frac{d_4}{d_6} \text{ (16 occurrences)}$$



## 2.18.7 Construction RCC137



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, B)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(D, B)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
  5.  $k_5 = k(E, G)$   
 $I \in k_2 \cap k_5, J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5$

## Distances

$$d_0 = |AE| = |AF| = |DE| = |DF| = |EF| = |HI| \text{ (red)} \dots 6$$

$$d_1 = |BK| = |CG| = |CH| \text{ (blue)} \dots 3$$

$$d_2 = |BL| \text{ (green)} \dots 1$$

$$d_3 = |BJ| = |CI| \text{ (cyan)} \dots 2$$

$$d_4 = |GI| = |JK| \text{ (magenta)} \dots 2$$

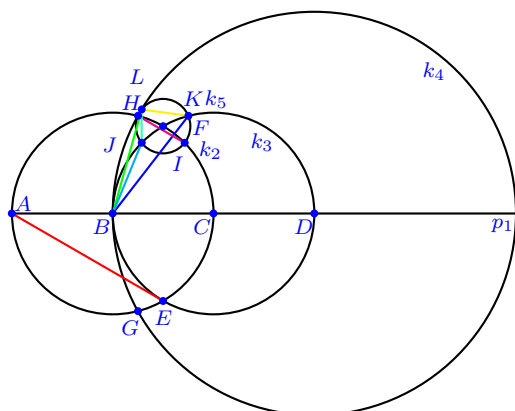
$$d_5 = |KL| \text{ (yellow)} \dots 1$$

$$d_6 = |JL| \text{ (grass)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_3}{d_5} = \frac{d_4}{d_6} \text{ (16 occurrences)}$$

## 2.18.8 Construction RCC149



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, B)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(D, B)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
  5.  $k_5 = k(F, H)$   
 $I \in k_2 \cap k_5, J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5$

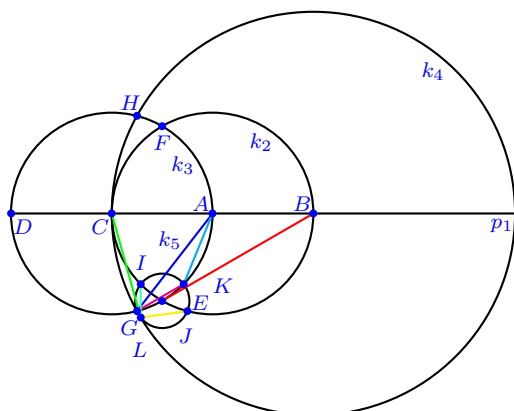
## Distances

$$\begin{aligned}
 d_0 &= |AE| = |AF| = |DE| = |DF| = |EF| = |GI| \text{ (red)} \dots 6 \\
 d_1 &= |BK| = |CG| = |CH| \text{ (blue)} \dots 3 \\
 d_2 &= |BL| \text{ (green)} \dots 1 \\
 d_3 &= |BJ| = |CI| \text{ (cyan)} \dots 2 \\
 d_4 &= |HI| = |JK| \text{ (magenta)} \dots 2 \\
 d_5 &= |KL| \text{ (yellow)} \dots 1 \\
 d_6 &= |JL| \text{ (grass)} \dots 1
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_3}{d_5} = \frac{d_4}{d_6} \text{ (16 occurrences)}$$

## 2.18.9 Construction RCC175



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, A)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(B, C)$   
 $G \in k_3 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(E, G)$   
 $I \in k_2 \cap k_5, J \in k_2 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5$

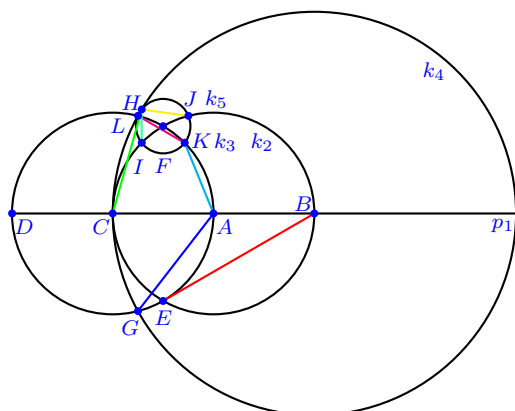
## Distances

$$\begin{aligned}
 d_0 &= |BE| = |BF| = |DE| = |DF| = |EF| = |HK| \text{ (red)} \dots 6 \\
 d_1 &= |AG| = |AH| = |CJ| \text{ (blue)} \dots 3 \\
 d_2 &= |CL| \text{ (green)} \dots 1 \\
 d_3 &= |AK| = |CI| \text{ (cyan)} \dots 2 \\
 d_4 &= |GK| = |IJ| \text{ (magenta)} \dots 2 \\
 d_5 &= |JL| \text{ (yellow)} \dots 1 \\
 d_6 &= |IL| \text{ (grass)} \dots 1
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_3}{d_5} = \frac{d_4}{d_6} \text{ (16 occurrences)}$$

## 2.18.10 Construction RCC187



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, A)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(B, C)$   
 $G \in k_3 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(F, H)$   
 $I \in k_2 \cap k_5, J \in k_2 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5$

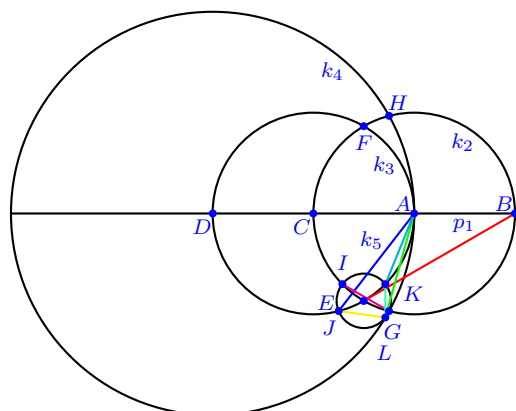
## Distances

$$\begin{aligned}
 d_0 &= |BE| = |BF| = |DE| = |DF| = |EF| = |GK| \text{ (red)} \dots 6 \\
 d_1 &= |AG| = |AH| = |CJ| \text{ (blue)} \dots 3 \\
 d_2 &= |CL| \text{ (green)} \dots 1 \\
 d_3 &= |AK| = |CI| \text{ (cyan)} \dots 2 \\
 d_4 &= |HK| = |IJ| \text{ (magenta)} \dots 2 \\
 d_5 &= |JL| \text{ (yellow)} \dots 1 \\
 d_6 &= |IL| \text{ (grass)} \dots 1
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_3}{d_5} = \frac{d_4}{d_6} \text{ (16 occurrences)}$$

## 2.18.11 Construction RCC200



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, A)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(D, A)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
  5.  $k_5 = k(E, G)$   
 $I \in k_2 \cap k_5, J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5$

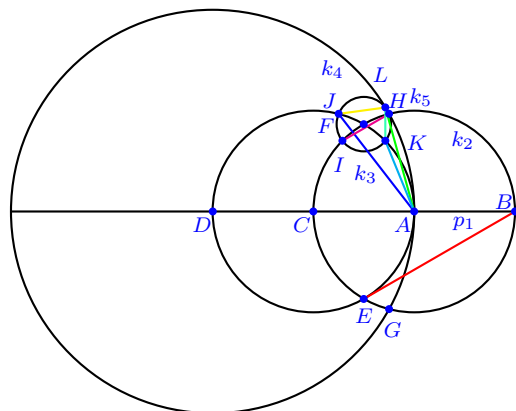
## Distances

$$\begin{aligned}
 d_0 &= |BE| = |BF| = |DE| = |DF| = |EF| = |HI| \text{ (red)} \dots 6 \\
 d_1 &= |AJ| = |CG| = |CH| \text{ (blue)} \dots 3 \\
 d_2 &= |AL| \text{ (green)} \dots 1 \\
 d_3 &= |AK| = |CI| \text{ (cyan)} \dots 2 \\
 d_4 &= |GI| = |JK| \text{ (magenta)} \dots 2 \\
 d_5 &= |JL| \text{ (yellow)} \dots 1 \\
 d_6 &= |KL| \text{ (grass)} \dots 1
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_3}{d_5} = \frac{d_4}{d_6} \text{ (16 occurrences)}$$

## 2.18.12 Construction RCC212



## Construction Process

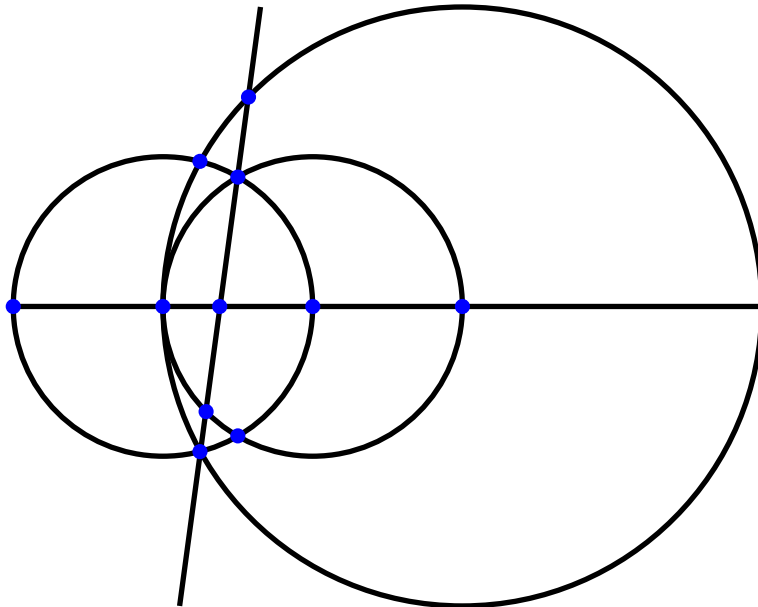
- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, A)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(D, A)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
  5.  $k_5 = k(F, H)$   
 $I \in k_2 \cap k_5, J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5$

## Distances

$$\begin{aligned}
 d_0 &= |BE| = |BF| = |DE| = |DF| = |EF| = |GI| \text{ (red)} \dots 6 \\
 d_1 &= |AJ| = |CG| = |CH| \text{ (blue)} \dots 3 \\
 d_2 &= |AL| \text{ (green)} \dots 1 \\
 d_3 &= |AK| = |CI| \text{ (cyan)} \dots 2 \\
 d_4 &= |HI| = |JK| \text{ (magenta)} \dots 2 \\
 d_5 &= |JL| \text{ (yellow)} \dots 1 \\
 d_6 &= |KL| \text{ (grass)} \dots 1
 \end{aligned}$$

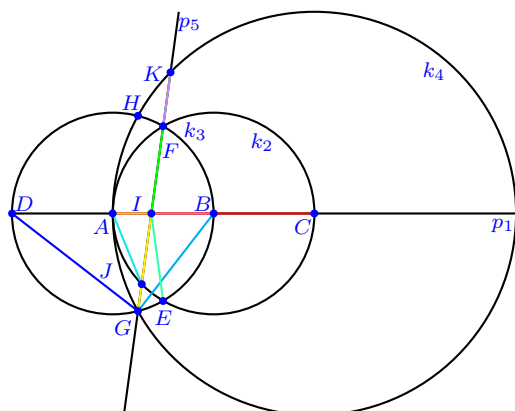
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_3}{d_5} = \frac{d_4}{d_6} \text{ (16 occurrences)}$$

**2.19 Class of Similar Constructions No. 12****Contained Constructions**

RCC13, RCC19, RCC46, RCC52, RCC98, RCC104, RCC138, RCC144, RCC176,  
RCC182, RCC201, RCC207

## 2.19.1 Construction RCC13



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(A, B)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(C, A)$   
 $G \in k_3 \cap k_4, H \in k_3 \cap k_4$
  5.  $p_5 = p(F, G)$   
 $I \in p_1 \cap p_5, J \in p_5 \cap k_2, K \in p_5 \cap k_4$

## Distances

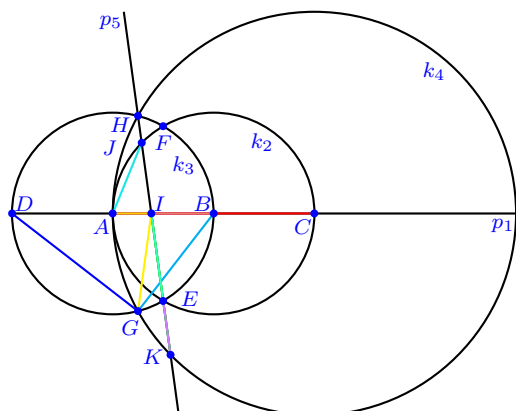
- $d_0 = |CI|$  (red) ... 1  
 $d_1 = |DG| = |DH| = |FJ|$  (blue) ... 3  
 $d_2 = |IK|$  (green) ... 1  
 $d_3 = |BG| = |BH|$  (cyan) ... 2  
 $d_4 = |AB| = |AD| = |AE| = |AF| = |AG| = |AH| = |BC| = |BE| = |BF| = |BJ|$  (magenta) ... 10  
 $d_5 = |GI| = |HI|$  (yellow) ... 2  
 $d_6 = |EI| = |FI|$  (grass) ... 2  
 $d_7 = |AJ|$  (sea) ... 1  
 $d_8 = |BI|$  (darkorange) ... 1  
 $d_9 = |FK| = |HK|$  (violet) ... 2  
 $d_{10} = |AI|$  (myorange) ... 1

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_4} = \frac{d_1}{d_5} = \frac{d_2}{d_6} = \frac{d_3}{d_7} = \frac{d_4}{d_8} = \frac{d_6}{d_9} = \frac{d_8}{d_{10}} \quad (35 \text{ occurrences})$$



## 2.19.2 Construction RCC19



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(A, B)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(C, A)$   
 $G \in k_3 \cap k_4, H \in k_3 \cap k_4$
  5.  $p_5 = p(E, H)$   
 $I \in p_1 \cap p_5, J \in p_5 \cap k_2, K \in p_5 \cap k_4$

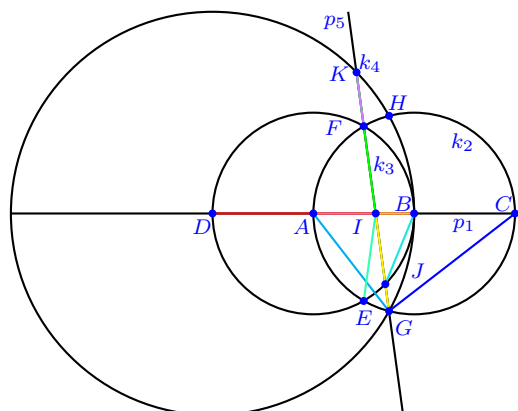
## Distances

$$\begin{aligned}
 d_0 &= |CI| \text{ (red)} \dots 1 \\
 d_1 &= |DG| = |DH| = |EJ| \text{ (blue)} \dots 3 \\
 d_2 &= |IK| \text{ (green)} \dots 1 \\
 d_3 &= |BG| = |BH| \text{ (cyan)} \dots 2 \\
 d_4 &= |AB| = |AD| = |AE| = |AF| = |AG| = |AH| = |BC| = |BE| = \\
 &= |BF| = |BJ| \text{ (magenta)} \dots 10 \\
 d_5 &= |GI| = |HI| \text{ (yellow)} \dots 2 \\
 d_6 &= |EI| = |FI| \text{ (grass)} \dots 2 \\
 d_7 &= |AJ| \text{ (sea)} \dots 1 \\
 d_8 &= |BI| \text{ (darkorange)} \dots 1 \\
 d_9 &= |EK| = |GK| \text{ (violet)} \dots 2 \\
 d_{10} &= |AI| \text{ (myorange)} \dots 1
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_4} = \frac{d_1}{d_5} = \frac{d_2}{d_6} = \frac{d_3}{d_7} = \frac{d_4}{d_8} = \frac{d_6}{d_9} = \frac{d_8}{d_{10}} \text{ (35 occurrences)}$$

## 2.19.3 Construction RCC46



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(A, B)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(D, B)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
  5.  $p_5 = p(F, G)$   
 $I \in p_1 \cap p_5, J \in p_5 \cap k_3, K \in p_5 \cap k_4$

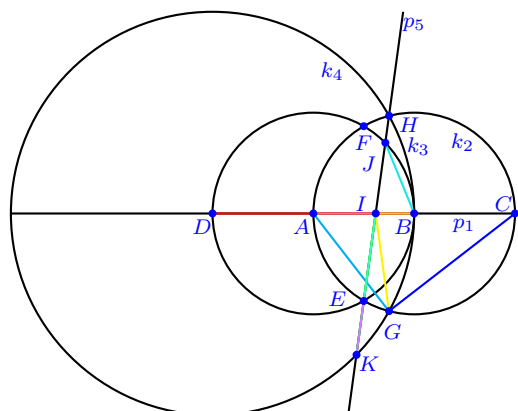
## Distances

- $d_0 = |DI|$  (red) ... 1  
 $d_1 = |CG| = |CH| = |FJ|$  (blue) ... 3  
 $d_2 = |IK|$  (green) ... 1  
 $d_3 = |AG| = |AH|$  (cyan) ... 2  
 $d_4 = |AB| = |AD| = |AE| = |AF| = |AJ| = |BC| = |BE| = |BF| = |BG| = |BH|$  (magenta) ... 10  
 $d_5 = |GI| = |HI|$  (yellow) ... 2  
 $d_6 = |EI| = |FI|$  (grass) ... 2  
 $d_7 = |BJ|$  (sea) ... 1  
 $d_8 = |AI|$  (darkorange) ... 1  
 $d_9 = |FK| = |HK|$  (violet) ... 2  
 $d_{10} = |BI|$  (myorange) ... 1

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_4} = \frac{d_1}{d_5} = \frac{d_2}{d_6} = \frac{d_3}{d_7} = \frac{d_4}{d_8} = \frac{d_6}{d_9} = \frac{d_8}{d_{10}} \quad (35 \text{ occurrences})$$

## 2.19.4 Construction RCC52



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(A, B)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(D, B)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
  5.  $p_5 = p(E, H)$   
 $I \in p_1 \cap p_5, J \in p_5 \cap k_3, K \in p_5 \cap k_4$

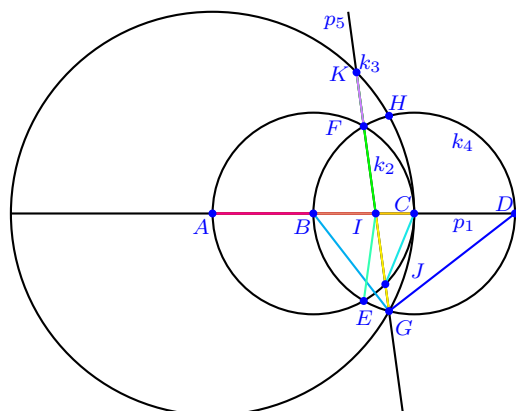
## Distances

- $d_0 = |DI|$  (red) ... 1  
 $d_1 = |CG| = |CH| = |EJ|$  (blue) ... 3  
 $d_2 = |IK|$  (green) ... 1  
 $d_3 = |AG| = |AH|$  (cyan) ... 2  
 $d_4 = |AB| = |AD| = |AE| = |AF| = |AJ| = |BC| = |BE| = |BF| = |BG| = |BH|$  (magenta) ... 10  
 $d_5 = |GI| = |HI|$  (yellow) ... 2  
 $d_6 = |EI| = |FI|$  (grass) ... 2  
 $d_7 = |BJ|$  (sea) ... 1  
 $d_8 = |AI|$  (darkorange) ... 1  
 $d_9 = |EK| = |GK|$  (violet) ... 2  
 $d_{10} = |BI|$  (myorange) ... 1

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_4} = \frac{d_1}{d_5} = \frac{d_2}{d_6} = \frac{d_3}{d_7} = \frac{d_4}{d_8} = \frac{d_6}{d_9} = \frac{d_8}{d_{10}} \quad (35 \text{ occurrences})$$

## 2.19.5 Construction RCC98



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(A, C)$
  4.  $k_4 = k(C, B)$   
 $D \in p_1 \cap k_4, E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4$
  5.  $p_5 = p(F, G)$   
 $I \in p_1 \cap p_5, J \in p_5 \cap k_2, K \in p_5 \cap k_3$

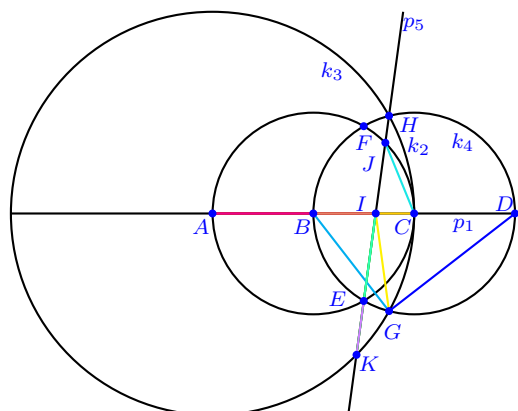
## Distances

- $d_0 = |AI|$  (red) ... 1  
 $d_1 = |DG| = |DH| = |FJ|$  (blue) ... 3  
 $d_2 = |IK|$  (green) ... 1  
 $d_3 = |BG| = |BH|$  (cyan) ... 2  
 $d_4 = |AB| = |BC| = |BE| = |BF| = |BJ| = |CD| = |CE| = |CF| = |CG| = |CH|$  (magenta) ... 10  
 $d_5 = |GI| = |HI|$  (yellow) ... 2  
 $d_6 = |EI| = |FI|$  (grass) ... 2  
 $d_7 = |CJ|$  (sea) ... 1  
 $d_8 = |BI|$  (darkorange) ... 1  
 $d_9 = |FK| = |HK|$  (violet) ... 2  
 $d_{10} = |CI|$  (myorange) ... 1

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_4} = \frac{d_1}{d_5} = \frac{d_2}{d_6} = \frac{d_3}{d_7} = \frac{d_4}{d_8} = \frac{d_6}{d_9} = \frac{d_8}{d_{10}} \quad (35 \text{ occurrences})$$

## 2.19.6 Construction RCC104



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(A, C)$
  4.  $k_4 = k(C, B)$   
 $D \in p_1 \cap k_4, E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4$
  5.  $p_5 = p(E, H)$   
 $I \in p_1 \cap p_5, J \in p_5 \cap k_2, K \in p_5 \cap k_3$

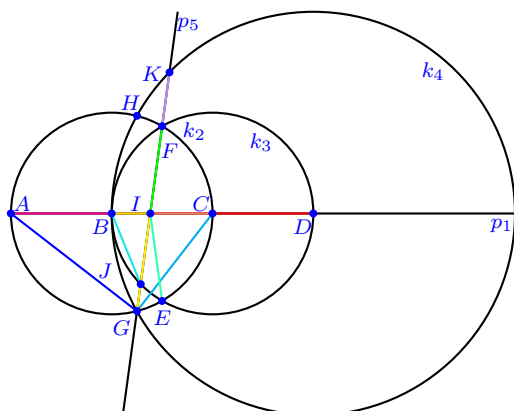
## Distances

- $d_0 = |AI|$  (red) ... 1  
 $d_1 = |DG| = |DH| = |EJ|$  (blue) ... 3  
 $d_2 = |IK|$  (green) ... 1  
 $d_3 = |BG| = |BH|$  (cyan) ... 2  
 $d_4 = |AB| = |BC| = |BE| = |BF| = |BJ| = |CD| = |CE| = |CF| = |CG| = |CH|$  (magenta) ... 10  
 $d_5 = |GI| = |HI|$  (yellow) ... 2  
 $d_6 = |EI| = |FI|$  (grass) ... 2  
 $d_7 = |CJ|$  (sea) ... 1  
 $d_8 = |BI|$  (darkorange) ... 1  
 $d_9 = |EK| = |GK|$  (violet) ... 2  
 $d_{10} = |CI|$  (myorange) ... 1

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_4} = \frac{d_1}{d_5} = \frac{d_2}{d_6} = \frac{d_3}{d_7} = \frac{d_4}{d_8} = \frac{d_6}{d_9} = \frac{d_8}{d_{10}} \quad (35 \text{ occurrences})$$

## 2.19.7 Construction RCC138



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, B)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(D, B)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
  5.  $p_5 = p(F, G)$   
 $I \in p_1 \cap p_5, J \in p_5 \cap k_3, K \in p_5 \cap k_4$

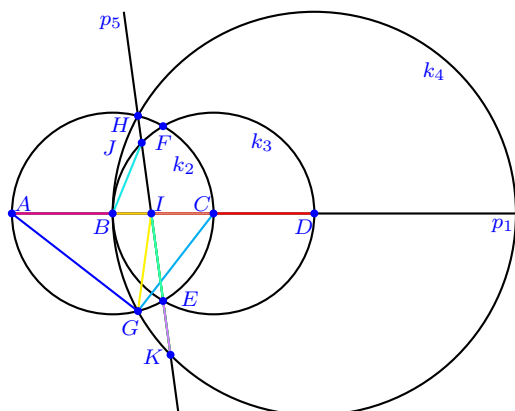
## Distances

- $d_0 = |DI|$  (red) ... 1  
 $d_1 = |AG| = |AH| = |FJ|$  (blue) ... 3  
 $d_2 = |IK|$  (green) ... 1  
 $d_3 = |CG| = |CH|$  (cyan) ... 2  
 $d_4 = |AB| = |BC| = |BE| = |BF| = |BG| = |BH| = |CD| = |CE| = |CF| = |CJ|$  (magenta) ... 10  
 $d_5 = |GI| = |HI|$  (yellow) ... 2  
 $d_6 = |EI| = |FI|$  (grass) ... 2  
 $d_7 = |BJ|$  (sea) ... 1  
 $d_8 = |CI|$  (darkorange) ... 1  
 $d_9 = |FK| = |HK|$  (violet) ... 2  
 $d_{10} = |BI|$  (myorange) ... 1

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_4} = \frac{d_1}{d_5} = \frac{d_2}{d_6} = \frac{d_3}{d_7} = \frac{d_4}{d_8} = \frac{d_6}{d_9} = \frac{d_8}{d_{10}} \quad (35 \text{ occurrences})$$

## 2.19.8 Construction RCC144



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, B)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(D, B)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
  5.  $p_5 = p(E, H)$   
 $I \in p_1 \cap p_5, J \in p_5 \cap k_3, K \in p_5 \cap k_4$

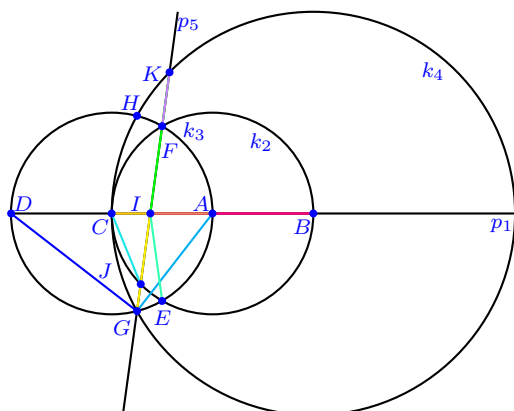
## Distances

- $d_0 = |DI|$  (red) ... 1  
 $d_1 = |AG| = |AH| = |EJ|$  (blue) ... 3  
 $d_2 = |IK|$  (green) ... 1  
 $d_3 = |CG| = |CH|$  (cyan) ... 2  
 $d_4 = |AB| = |BC| = |BE| = |BF| = |BG| = |BH| = |CD| = |CE| = |CF| = |CJ|$  (magenta) ... 10  
 $d_5 = |GI| = |HI|$  (yellow) ... 2  
 $d_6 = |EI| = |FI|$  (grass) ... 2  
 $d_7 = |BJ|$  (sea) ... 1  
 $d_8 = |CI|$  (darkorange) ... 1  
 $d_9 = |EK| = |GK|$  (violet) ... 2  
 $d_{10} = |BI|$  (myorange) ... 1

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_4} = \frac{d_1}{d_5} = \frac{d_2}{d_6} = \frac{d_3}{d_7} = \frac{d_4}{d_8} = \frac{d_6}{d_9} = \frac{d_8}{d_{10}} \quad (35 \text{ occurrences})$$

## 2.19.9 Construction RCC176



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, A)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(B, C)$   
 $G \in k_3 \cap k_4, H \in k_3 \cap k_4$
  5.  $p_5 = p(F, G)$   
 $I \in p_1 \cap p_5, J \in p_5 \cap k_2, K \in p_5 \cap k_4$

## Distances

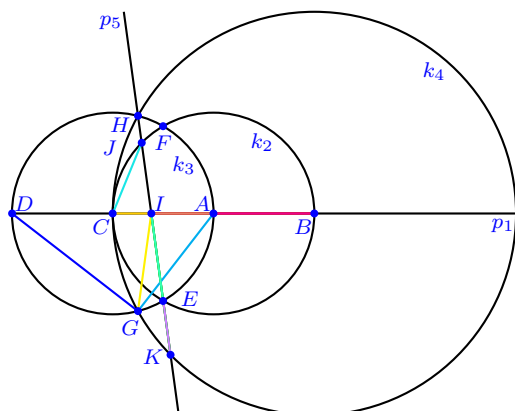
- $d_0 = |BI|$  (red) ... 1  
 $d_1 = |DG| = |DH| = |FJ|$  (blue) ... 3  
 $d_2 = |IK|$  (green) ... 1  
 $d_3 = |AG| = |AH|$  (cyan) ... 2  
 $d_4 = |AB| = |AC| = |AE| = |AF| = |AJ| = |CD| = |CE| = |CF| = |CG| = |CH|$  (magenta) ... 10  
 $d_5 = |GI| = |HI|$  (yellow) ... 2  
 $d_6 = |EI| = |FI|$  (grass) ... 2  
 $d_7 = |CJ|$  (sea) ... 1  
 $d_8 = |AI|$  (darkorange) ... 1  
 $d_9 = |FK| = |HK|$  (violet) ... 2  
 $d_{10} = |CI|$  (myorange) ... 1

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_4} = \frac{d_1}{d_5} = \frac{d_2}{d_6} = \frac{d_3}{d_7} = \frac{d_4}{d_8} = \frac{d_6}{d_9} = \frac{d_8}{d_{10}} \quad (35 \text{ occurrences})$$



## 2.19.10 Construction RCC182



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, A)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(B, C)$   
 $G \in k_3 \cap k_4, H \in k_3 \cap k_4$
  5.  $p_5 = p(E, H)$   
 $I \in p_1 \cap p_5, J \in p_5 \cap k_2, K \in p_5 \cap k_4$

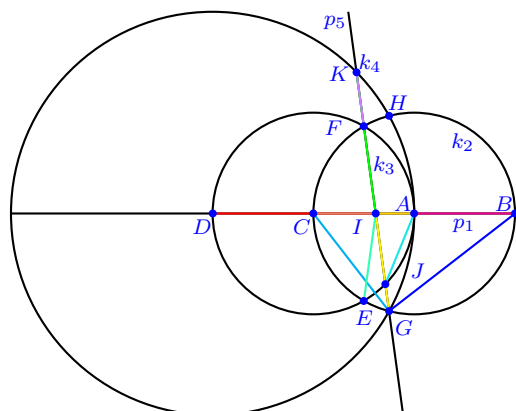
## Distances

- $d_0 = |BI|$  (red) ... 1  
 $d_1 = |DG| = |DH| = |EJ|$  (blue) ... 3  
 $d_2 = |IK|$  (green) ... 1  
 $d_3 = |AG| = |AH|$  (cyan) ... 2  
 $d_4 = |AB| = |AC| = |AE| = |AF| = |AJ| = |CD| = |CE| = |CF| = |CG| = |CH|$  (magenta) ... 10  
 $d_5 = |GI| = |HI|$  (yellow) ... 2  
 $d_6 = |EI| = |FI|$  (grass) ... 2  
 $d_7 = |CJ|$  (sea) ... 1  
 $d_8 = |AI|$  (darkorange) ... 1  
 $d_9 = |EK| = |GK|$  (violet) ... 2  
 $d_{10} = |CI|$  (myorange) ... 1

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_4} = \frac{d_1}{d_5} = \frac{d_2}{d_6} = \frac{d_3}{d_7} = \frac{d_4}{d_8} = \frac{d_6}{d_9} = \frac{d_8}{d_{10}} \quad (35 \text{ occurrences})$$

## 2.19.11 Construction RCC201



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, A)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(D, A)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
  5.  $p_5 = p(F, G)$   
 $I \in p_1 \cap p_5, J \in p_5 \cap k_3, K \in p_5 \cap k_4$

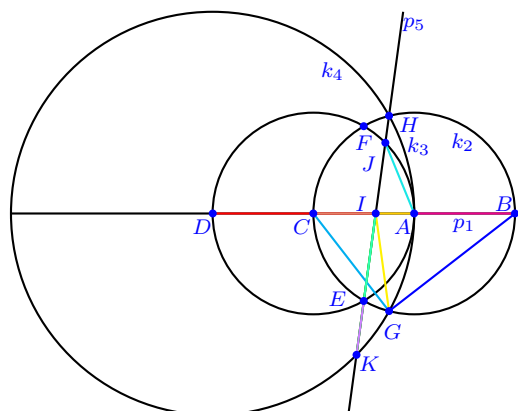
## Distances

- $d_0 = |DI|$  (red) ... 1  
 $d_1 = |BG| = |BH| = |FJ|$  (blue) ... 3  
 $d_2 = |IK|$  (green) ... 1  
 $d_3 = |CG| = |CH|$  (cyan) ... 2  
 $d_4 = |AB| = |AC| = |AE| = |AF| = |AG| = |AH| = |CD| = |CE| = |CF| = |CJ|$  (magenta) ... 10  
 $d_5 = |GI| = |HI|$  (yellow) ... 2  
 $d_6 = |EI| = |FI|$  (grass) ... 2  
 $d_7 = |AJ|$  (sea) ... 1  
 $d_8 = |CI|$  (darkorange) ... 1  
 $d_9 = |FK| = |HK|$  (violet) ... 2  
 $d_{10} = |AI|$  (myorange) ... 1

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_4} = \frac{d_1}{d_5} = \frac{d_2}{d_6} = \frac{d_3}{d_7} = \frac{d_4}{d_8} = \frac{d_6}{d_9} = \frac{d_8}{d_{10}} \quad (35 \text{ occurrences})$$

## 2.19.12 Construction RCC207



## Construction Process

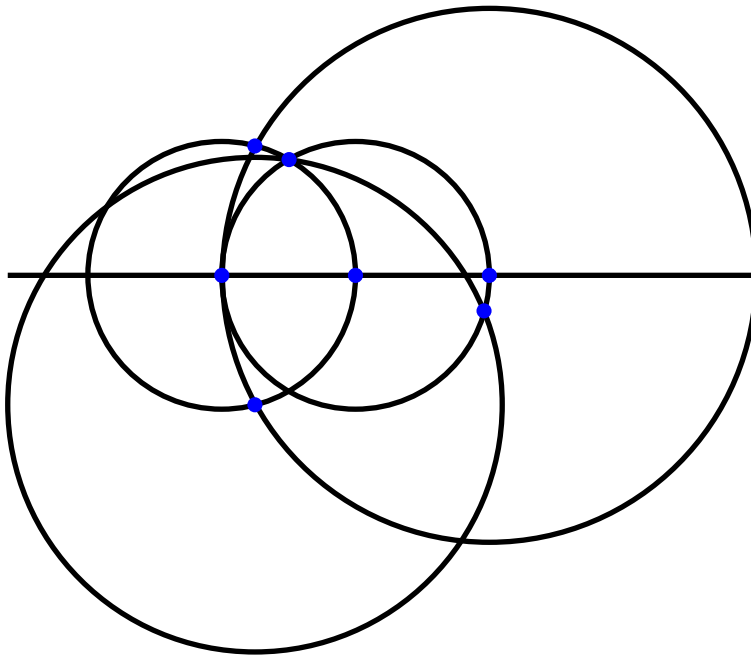
- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, A)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(D, A)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
  5.  $p_5 = p(E, H)$   
 $I \in p_1 \cap p_5, J \in p_5 \cap k_3, K \in p_5 \cap k_4$

## Distances

- $d_0 = |DI|$  (red) ... 1  
 $d_1 = |BG| = |BH| = |EJ|$  (blue) ... 3  
 $d_2 = |IK|$  (green) ... 1  
 $d_3 = |CG| = |CH|$  (cyan) ... 2  
 $d_4 = |AB| = |AC| = |AE| = |AF| = |AG| = |AH| = |CD| = |CE| = |CF| = |CJ|$  (magenta) ... 10  
 $d_5 = |GI| = |HI|$  (yellow) ... 2  
 $d_6 = |EI| = |FI|$  (grass) ... 2  
 $d_7 = |AJ|$  (sea) ... 1  
 $d_8 = |CI|$  (darkorange) ... 1  
 $d_9 = |EK| = |GK|$  (violet) ... 2  
 $d_{10} = |AI|$  (myorange) ... 1

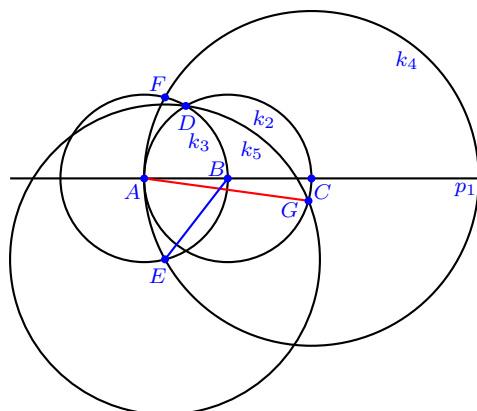
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_4} = \frac{d_1}{d_5} = \frac{d_2}{d_6} = \frac{d_3}{d_7} = \frac{d_4}{d_8} = \frac{d_6}{d_9} = \frac{d_8}{d_{10}} \quad (35 \text{ occurrences})$$

**2.20 Class of Similar Constructions No. 13****Contained Constructions**

RCC14, RCC20, RCC47, RCC53, RCC99, RCC105, RCC139, RCC145, RCC177,  
RCC183, RCC202, RCC208

## 2.20.1 Construction RCC14



## Construction Process

$A, B$  given initial points

1.  $p_1 = p(A, B)$
2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
3.  $k_3 = k(A, B)$   
 $D \in k_2 \cap k_3$
4.  $k_4 = k(C, A)$   
 $E \in k_3 \cap k_4, F \in k_3 \cap k_4$
5.  $k_5 = k(E, D)$   
 $G \in k_2 \cap k_5$

## Distances

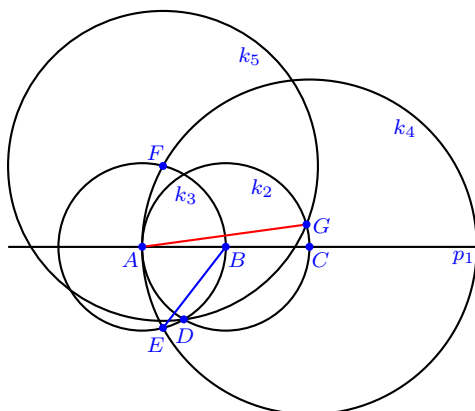
$$d_0 = |AG| \text{ (red)} \dots 1$$

$$d_1 = |BE| = |BF| \text{ (blue)} \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (2 occurrences)}$$

## 2.20.2 Construction RCC20



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(A, B)$   
 $D \in k_2 \cap k_3$
  4.  $k_4 = k(C, A)$   
 $E \in k_3 \cap k_4, F \in k_3 \cap k_4$
  5.  $k_5 = k(F, D)$   
 $G \in k_2 \cap k_5$

## Distances

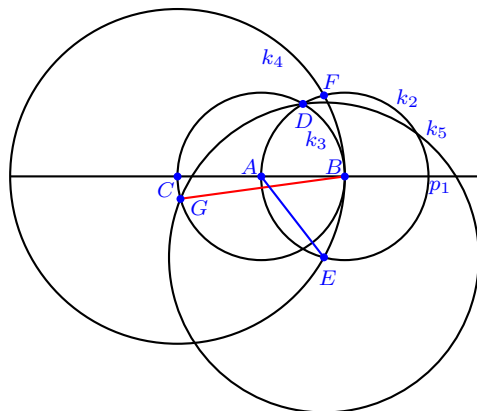
$$d_0 = |AG| \text{ (red)} \dots 1$$

$$d_1 = |BE| = |BF| \text{ (blue)} \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (2 occurrences)}$$

## 2.20.3 Construction RCC47



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$
  3.  $k_3 = k(A, B)$   
 $C \in p_1 \cap k_3, D \in k_2 \cap k_3$
  4.  $k_4 = k(C, B)$   
 $E \in k_2 \cap k_4, F \in k_2 \cap k_4$
  5.  $k_5 = k(E, D)$   
 $G \in k_3 \cap k_5$

## Distances

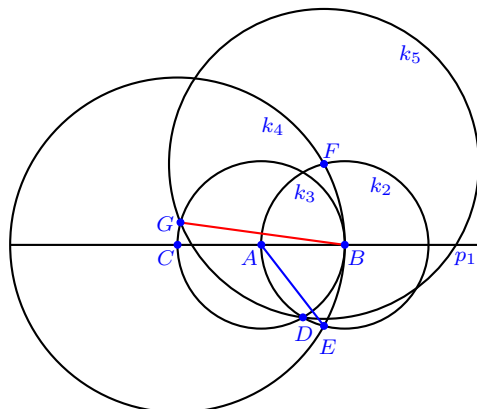
$$d_0 = |BG| \text{ (red)} \dots 1$$

$$d_1 = |AE| = |AF| \text{ (blue)} \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (2 occurrences)}$$

## 2.20.4 Construction RCC53



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$
  3.  $k_3 = k(A, B)$   
 $C \in p_1 \cap k_3, D \in k_2 \cap k_3$
  4.  $k_4 = k(C, B)$   
 $E \in k_2 \cap k_4, F \in k_2 \cap k_4$
  5.  $k_5 = k(F, D)$   
 $G \in k_3 \cap k_5$

## Distances

$$d_0 = |BG| \text{ (red) } \dots 1$$

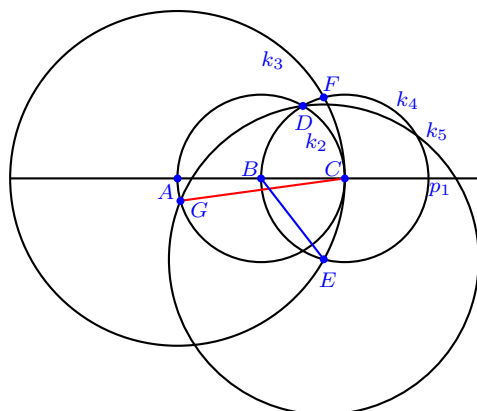
$$d_1 = |AE| = |AF| \text{ (blue) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (2 occurrences)}$$



## 2.20.5 Construction RCC99



## Construction Process

$A, B$  given initial points

1.  $p_1 = p(A, B)$
2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
3.  $k_3 = k(A, C)$
4.  $k_4 = k(C, B)$   
 $D \in k_2 \cap k_4, E \in k_3 \cap k_4, F \in k_3 \cap k_4$
5.  $k_5 = k(E, D)$   
 $G \in k_2 \cap k_5$

## Distances

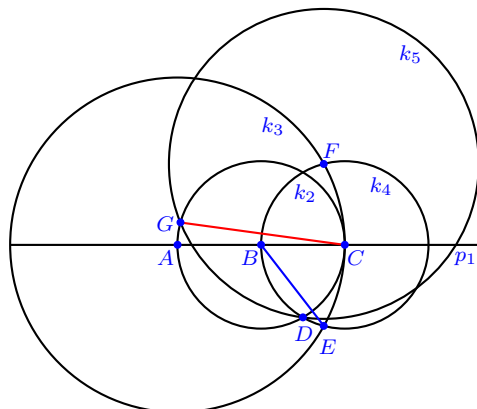
$$d_0 = |CG| \text{ (red)} \dots 1$$

$$d_1 = |BE| = |BF| \text{ (blue)} \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (2 occurrences)}$$

## 2.20.6 Construction RCC105



## Construction Process

$A, B$  given initial points

1.  $p_1 = p(A, B)$
2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
3.  $k_3 = k(A, C)$
4.  $k_4 = k(C, B)$   
 $D \in k_2 \cap k_4, E \in k_3 \cap k_4, F \in k_3 \cap k_4$
5.  $k_5 = k(F, D)$   
 $G \in k_2 \cap k_5$

## Distances

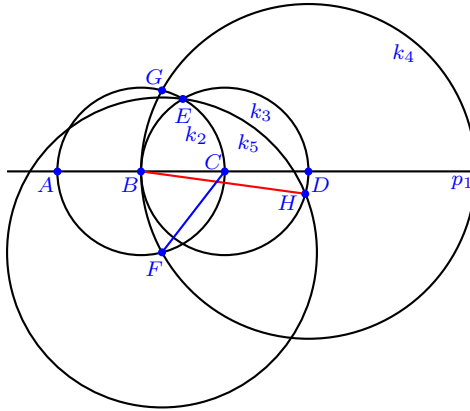
$$d_0 = |CG| \text{ (red)} \dots 1$$

$$d_1 = |BE| = |BF| \text{ (blue)} \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (2 occurrences)}$$

## 2.20.7 Construction RCC139



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, B)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3$
  4.  $k_4 = k(D, B)$   
 $F \in k_2 \cap k_4, G \in k_2 \cap k_4$
  5.  $k_5 = k(F, E)$   
 $H \in k_3 \cap k_5$

## Distances

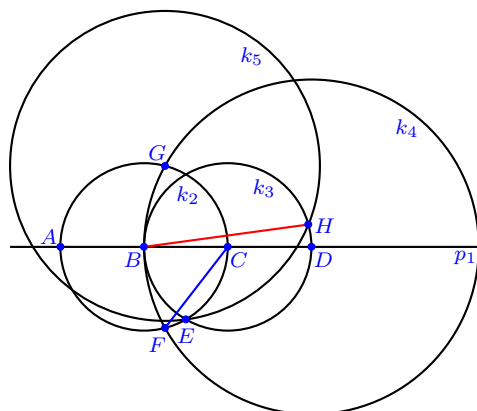
$$d_0 = |BH| \text{ (red)} \dots 1$$

$$d_1 = |CF| = |CG| \text{ (blue)} \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (2 occurrences)}$$

## 2.20.8 Construction RCC145



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, B)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3$
  4.  $k_4 = k(D, B)$   
 $F \in k_2 \cap k_4, G \in k_2 \cap k_4$
  5.  $k_5 = k(G, E)$   
 $H \in k_3 \cap k_5$

## Distances

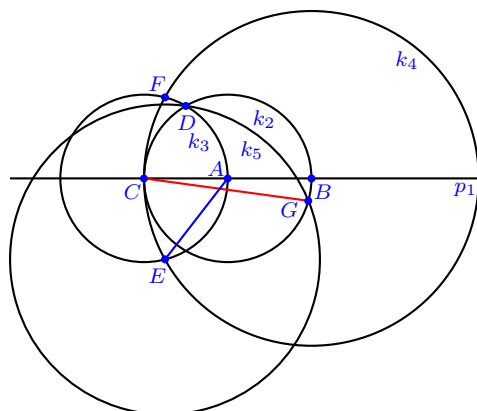
$$d_0 = |BH| \text{ (red)} \dots 1$$

$$d_1 = |CF| = |CG| \text{ (blue)} \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (2 occurrences)}$$

## 2.20.9 Construction RCC177



## Construction Process

$A, B$  given initial points

1.  $p_1 = p(A, B)$
2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
3.  $k_3 = k(C, A)$   
 $D \in k_2 \cap k_3$
4.  $k_4 = k(B, C)$   
 $E \in k_3 \cap k_4, F \in k_3 \cap k_4$
5.  $k_5 = k(E, D)$   
 $G \in k_2 \cap k_5$

## Distances

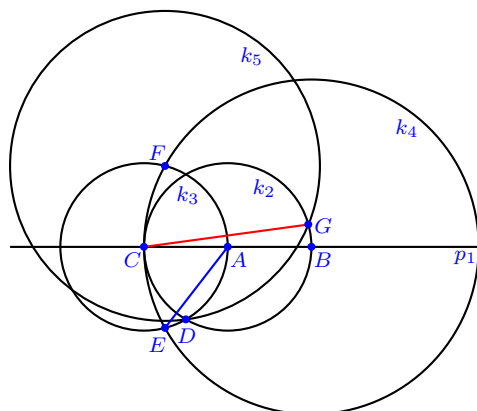
$$d_0 = |CG| \text{ (red)} \dots 1$$

$$d_1 = |AE| = |AF| \text{ (blue)} \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (2 occurrences)}$$

## 2.20.10 Construction RCC183



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, A)$   
 $D \in k_2 \cap k_3$
  4.  $k_4 = k(B, C)$   
 $E \in k_3 \cap k_4, F \in k_3 \cap k_4$
  5.  $k_5 = k(F, D)$   
 $G \in k_2 \cap k_5$

## Distances

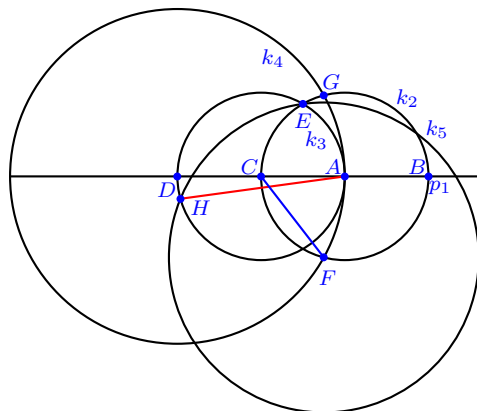
$$d_0 = |CG| \text{ (red)} \dots 1$$

$$d_1 = |AE| = |AF| \text{ (blue)} \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (2 occurrences)}$$

## 2.20.11 Construction RCC202



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, A)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3$
  4.  $k_4 = k(D, A)$   
 $F \in k_2 \cap k_4, G \in k_2 \cap k_4$
  5.  $k_5 = k(F, E)$   
 $H \in k_3 \cap k_5$

## Distances

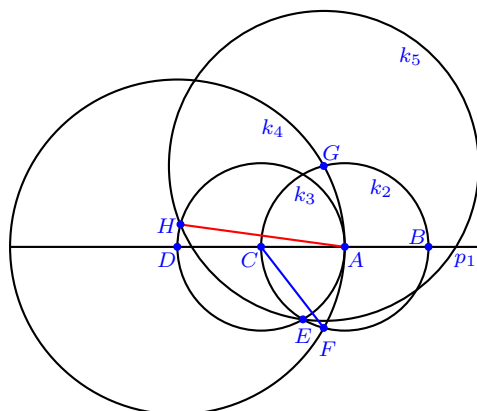
$$d_0 = |AH| \text{ (red) } \dots 1$$

$$d_1 = |CF| = |CG| \text{ (blue) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (2 occurrences)}$$

## 2.20.12 Construction RCC208



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, A)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3$
  4.  $k_4 = k(D, A)$   
 $F \in k_2 \cap k_4, G \in k_2 \cap k_4$
  5.  $k_5 = k(G, E)$   
 $H \in k_3 \cap k_5$

## Distances

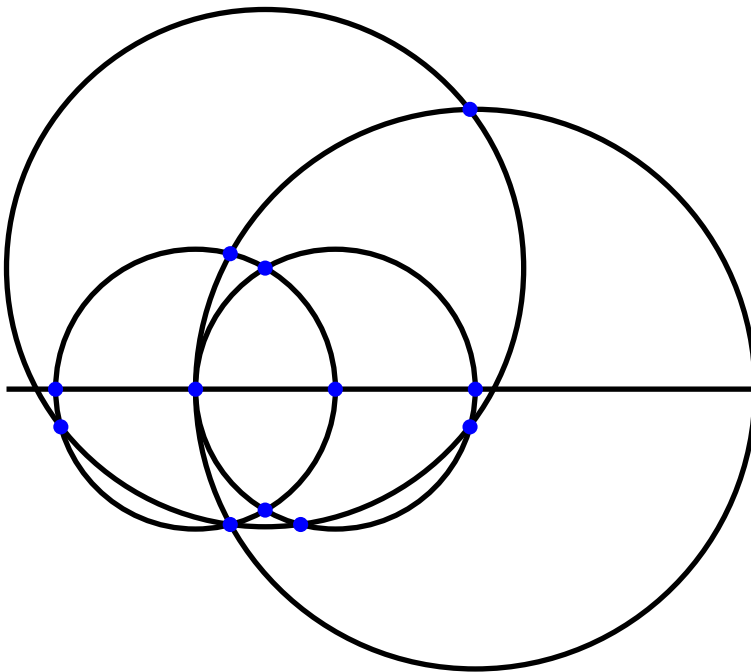
$$d_0 = |AH| \text{ (red) } \dots 1$$

$$d_1 = |CF| = |CG| \text{ (blue) } \dots 2$$

## Occurrences of the Golden Ratio

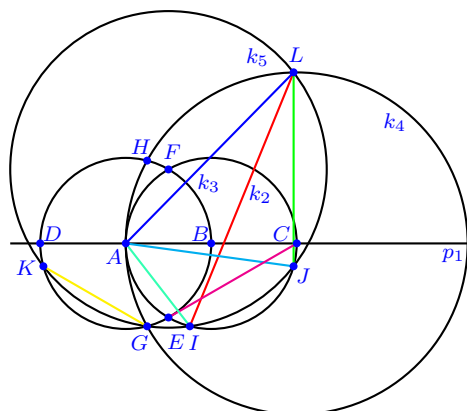
$$\phi = \frac{d_0}{d_1} \text{ (2 occurrences)}$$



**2.21 Class of Similar Constructions No. 14****Contained Constructions**

RCC15, RCC21, RCC48, RCC54, RCC100, RCC106, RCC140, RCC146, RCC178,  
RCC184, RCC203, RCC209

## 2.21.1 Construction RCC15



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(A, B)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(C, A)$   
 $G \in k_3 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(F, G)$   
 $I \in k_2 \cap k_5, J \in k_2 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5$

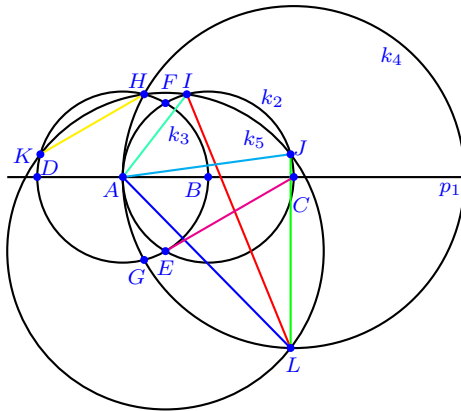
## Distances

- $d_0 = |IL|$  (red) ... 1  
 $d_1 = |AL|$  (blue) ... 1  
 $d_2 = |JL|$  (green) ... 1  
 $d_3 = |AJ| = |BK|$  (cyan) ... 2  
 $d_4 = |CE| = |CF| = |DE| = |DF| = |EF| = |HK|$  (magenta) ... 6  
 $d_5 = |GK| = |IJ|$  (yellow) ... 2  
 $d_6 = |AI| = |BG| = |BH|$  (grass) ... 3

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_3} = \frac{d_1}{d_4} = \frac{d_2}{d_5} = \frac{d_3}{d_6} \quad (16 \text{ occurrences})$$

2.21.2 Construction RCC21



Construction Process

- $A, B$  given initial points
- 1.  $p_1 = p(A, B)$
- 2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
- 3.  $k_3 = k(A, B)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
- 4.  $k_4 = k(C, A)$   
 $G \in k_3 \cap k_4, H \in k_3 \cap k_4$
- 5.  $k_5 = k(E, H)$   
 $I \in k_2 \cap k_5, J \in k_2 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5$

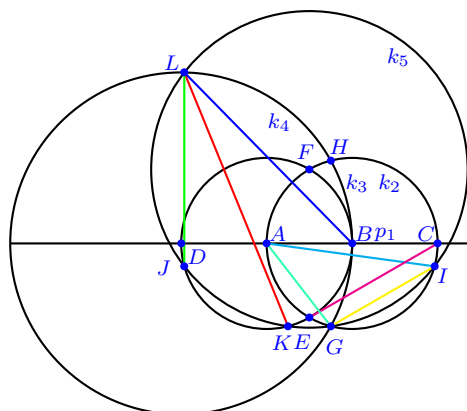
Distances

- $d_0 = |IL|$  (red) ... 1
- $d_1 = |AL|$  (blue) ... 1
- $d_2 = |JL|$  (green) ... 1
- $d_3 = |AJ| = |BK|$  (cyan) ... 2
- $d_4 = |CE| = |CF| = |DE| = |DF| = |EF| = |GK|$  (magenta) ... 6
- $d_5 = |HK| = |IJ|$  (yellow) ... 2
- $d_6 = |AI| = |BG| = |BH|$  (grass) ... 3

Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_3} = \frac{d_1}{d_4} = \frac{d_2}{d_5} = \frac{d_3}{d_6} \text{ (16 occurrences)}$$

## 2.21.3 Construction RCC48



## Construction Process

$A, B$  given initial points

1.  $p_1 = p(A, B)$
2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
3.  $k_3 = k(A, B)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
4.  $k_4 = k(D, B)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
5.  $k_5 = k(F, G)$   
 $I \in k_2 \cap k_5, J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5$

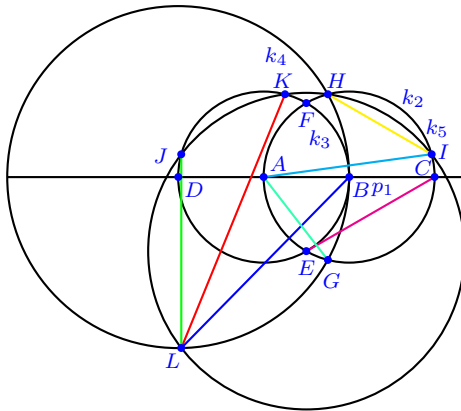
## Distances

$$\begin{aligned}
 d_0 &= |KL| \text{ (red)} \dots 1 \\
 d_1 &= |BL| \text{ (blue)} \dots 1 \\
 d_2 &= |JL| \text{ (green)} \dots 1 \\
 d_3 &= |AI| = |BJ| \text{ (cyan)} \dots 2 \\
 d_4 &= |CE| = |CF| = |DE| = |DF| = |EF| = |HI| \text{ (magenta)} \dots 6 \\
 d_5 &= |GI| = |JK| \text{ (yellow)} \dots 2 \\
 d_6 &= |AG| = |AH| = |BK| \text{ (grass)} \dots 3
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_3} = \frac{d_1}{d_4} = \frac{d_2}{d_5} = \frac{d_3}{d_6} \text{ (16 occurrences)}$$

## 2.21.4 Construction RCC54



## Construction Process

$A, B$  given initial points

1.  $p_1 = p(A, B)$
2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
3.  $k_3 = k(A, B)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
4.  $k_4 = k(D, B)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
5.  $k_5 = k(E, H)$   
 $I \in k_2 \cap k_5, J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5$

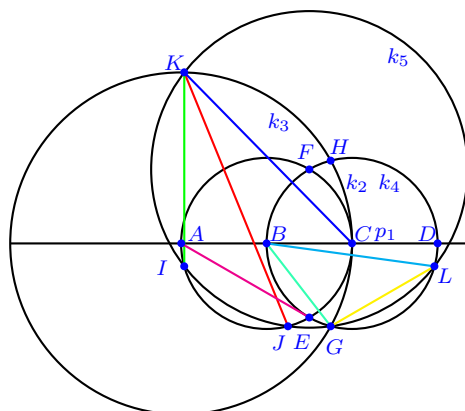
## Distances

- $d_0 = |KL|$  (red) ... 1  
 $d_1 = |BL|$  (blue) ... 1  
 $d_2 = |JL|$  (green) ... 1  
 $d_3 = |AI| = |BJ|$  (cyan) ... 2  
 $d_4 = |CE| = |CF| = |DE| = |DF| = |EF| = |GI|$  (magenta) ... 6  
 $d_5 = |HI| = |JK|$  (yellow) ... 2  
 $d_6 = |AG| = |AH| = |BK|$  (grass) ... 3

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_3} = \frac{d_1}{d_4} = \frac{d_2}{d_5} = \frac{d_3}{d_6} \quad (16 \text{ occurrences})$$

## 2.21.5 Construction RCC100



## Construction Process

$A, B$  given initial points

1.  $p_1 = p(A, B)$
2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
3.  $k_3 = k(A, C)$
4.  $k_4 = k(C, B)$   
 $D \in p_1 \cap k_4, E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4$
5.  $k_5 = k(F, G)$   
 $I \in k_2 \cap k_5, J \in k_2 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5$

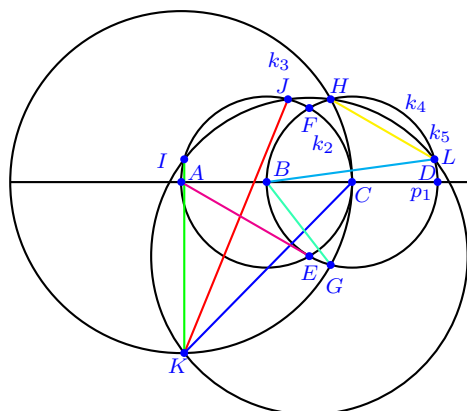
## Distances

- $$d_0 = |JK| \text{ (red) } \dots 1$$
- $$d_1 = |CK| \text{ (blue) } \dots 1$$
- $$d_2 = |IK| \text{ (green) } \dots 1$$
- $$d_3 = |BL| = |CI| \text{ (cyan) } \dots 2$$
- $$d_4 = |AE| = |AF| = |DE| = |DF| = |EF| = |HL| \text{ (magenta) } \dots 6$$
- $$d_5 = |GL| = |IJ| \text{ (yellow) } \dots 2$$
- $$d_6 = |BG| = |BH| = |CJ| \text{ (grass) } \dots 3$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_3} = \frac{d_1}{d_4} = \frac{d_2}{d_5} = \frac{d_3}{d_6} \text{ (16 occurrences)}$$

## 2.21.6 Construction RCC106



## Construction Process

$A, B$  given initial points

1.  $p_1 = p(A, B)$
2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
3.  $k_3 = k(A, C)$
4.  $k_4 = k(C, B)$   
 $D \in p_1 \cap k_4, E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4$
5.  $k_5 = k(E, H)$   
 $I \in k_2 \cap k_5, J \in k_2 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5$

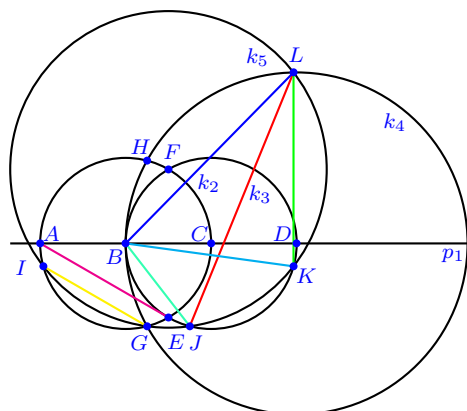
## Distances

- $$d_0 = |JK| \text{ (red) } \dots 1$$
- $$d_1 = |CK| \text{ (blue) } \dots 1$$
- $$d_2 = |IK| \text{ (green) } \dots 1$$
- $$d_3 = |BL| = |CI| \text{ (cyan) } \dots 2$$
- $$d_4 = |AE| = |AF| = |DE| = |DF| = |EF| = |GL| \text{ (magenta) } \dots 6$$
- $$d_5 = |HL| = |IJ| \text{ (yellow) } \dots 2$$
- $$d_6 = |BG| = |BH| = |CJ| \text{ (grass) } \dots 3$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_3} = \frac{d_1}{d_4} = \frac{d_2}{d_5} = \frac{d_3}{d_6} \text{ (16 occurrences)}$$

## 2.21.7 Construction RCC140



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, B)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(D, B)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
  5.  $k_5 = k(F, G)$   
 $I \in k_2 \cap k_5, J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5$

## Distances

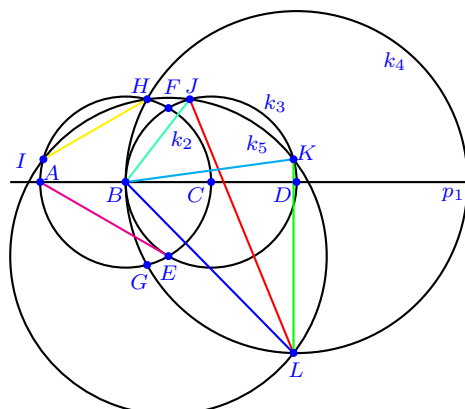
- $d_0 = |JL|$  (red) ... 1  
 $d_1 = |BL|$  (blue) ... 1  
 $d_2 = |KL|$  (green) ... 1  
 $d_3 = |BK| = |CI|$  (cyan) ... 2  
 $d_4 = |AE| = |AF| = |DE| = |DF| = |EF| = |HI|$  (magenta) ... 6  
 $d_5 = |GI| = |JK|$  (yellow) ... 2  
 $d_6 = |BJ| = |CG| = |CH|$  (grass) ... 3

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_3} = \frac{d_1}{d_4} = \frac{d_2}{d_5} = \frac{d_3}{d_6} \quad (16 \text{ occurrences})$$



## 2.21.8 Construction RCC146



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, B)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(D, B)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
  5.  $k_5 = k(E, H)$   
 $I \in k_2 \cap k_5, J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5$

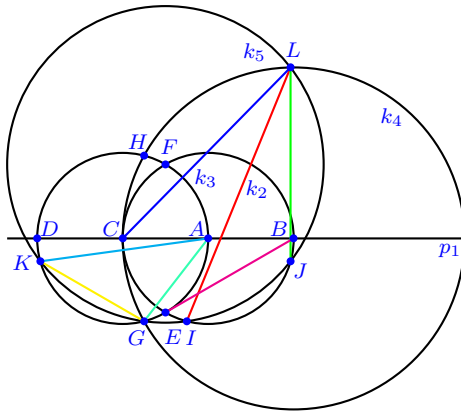
## Distances

- $d_0 = |JL|$  (red) ... 1  
 $d_1 = |BL|$  (blue) ... 1  
 $d_2 = |KL|$  (green) ... 1  
 $d_3 = |BK| = |CI|$  (cyan) ... 2  
 $d_4 = |AE| = |AF| = |DE| = |DF| = |EF| = |GI|$  (magenta) ... 6  
 $d_5 = |HI| = |JK|$  (yellow) ... 2  
 $d_6 = |BJ| = |CG| = |CH|$  (grass) ... 3

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_3} = \frac{d_1}{d_4} = \frac{d_2}{d_5} = \frac{d_3}{d_6} \quad (16 \text{ occurrences})$$

## 2.21.9 Construction RCC178



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, A)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(B, C)$   
 $G \in k_3 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(F, G)$   
 $I \in k_2 \cap k_5, J \in k_2 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5$

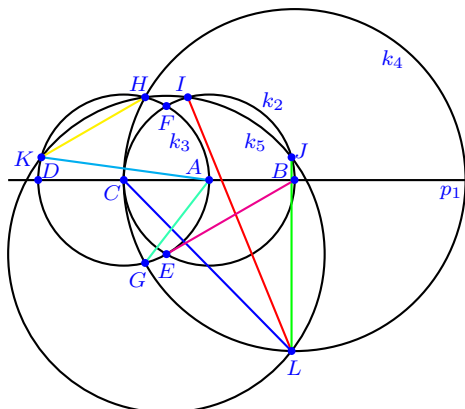
## Distances

- $d_0 = |IL|$  (red) ... 1  
 $d_1 = |CL|$  (blue) ... 1  
 $d_2 = |JL|$  (green) ... 1  
 $d_3 = |AK| = |CJ|$  (cyan) ... 2  
 $d_4 = |BE| = |BF| = |DE| = |DF| = |EF| = |HK|$  (magenta) ... 6  
 $d_5 = |GK| = |IJ|$  (yellow) ... 2  
 $d_6 = |AG| = |AH| = |CI|$  (grass) ... 3

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_3} = \frac{d_1}{d_4} = \frac{d_2}{d_5} = \frac{d_3}{d_6} \quad (16 \text{ occurrences})$$

## 2.21.10 Construction RCC184



## Construction Process

$A, B$  given initial points

1.  $p_1 = p(A, B)$
2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
3.  $k_3 = k(C, A)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
4.  $k_4 = k(B, C)$   
 $G \in k_3 \cap k_4, H \in k_3 \cap k_4$
5.  $k_5 = k(E, H)$   
 $I \in k_2 \cap k_5, J \in k_2 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5$

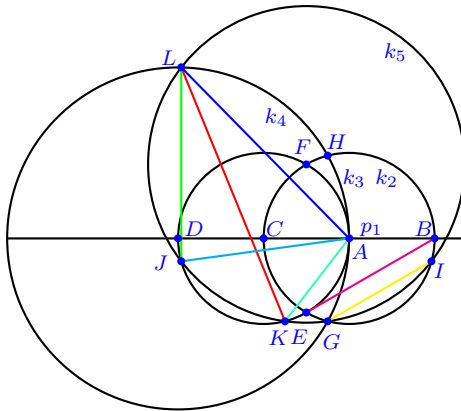
## Distances

- $d_0 = |IL|$  (red) ... 1  
 $d_1 = |CL|$  (blue) ... 1  
 $d_2 = |JL|$  (green) ... 1  
 $d_3 = |AK| = |CJ|$  (cyan) ... 2  
 $d_4 = |BE| = |BF| = |DE| = |DF| = |EF| = |GK|$  (magenta) ... 6  
 $d_5 = |HK| = |IJ|$  (yellow) ... 2  
 $d_6 = |AG| = |AH| = |CI|$  (grass) ... 3

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_3} = \frac{d_1}{d_4} = \frac{d_2}{d_5} = \frac{d_3}{d_6} \quad (16 \text{ occurrences})$$

2.21.11 Construction RCC203



Construction Process

- \$A, B\$ given initial points
- 1. \$p\_1 = p(A, B)\$
- 2. \$k\_2 = k(A, B)\$  
\$C \in p\_1 \cap k\_2\$
- 3. \$k\_3 = k(C, A)\$  
\$D \in p\_1 \cap k\_3, E \in k\_2 \cap k\_3, F \in k\_2 \cap k\_3\$
- 4. \$k\_4 = k(D, A)\$  
\$G \in k\_2 \cap k\_4, H \in k\_2 \cap k\_4\$
- 5. \$k\_5 = k(F, G)\$  
\$I \in k\_2 \cap k\_5, J \in k\_3 \cap k\_5, K \in k\_3 \cap k\_5, L \in k\_4 \cap k\_5\$

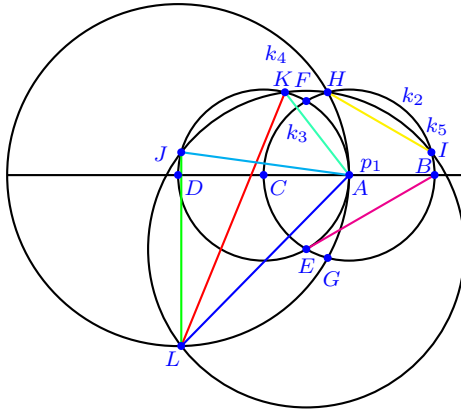
Distances

- \$d\_0 = |KL|\$ (red) ... 1
- \$d\_1 = |AL|\$ (blue) ... 1
- \$d\_2 = |JL|\$ (green) ... 1
- \$d\_3 = |AJ| = |CI|\$ (cyan) ... 2
- \$d\_4 = |BE| = |BF| = |DE| = |DF| = |EF| = |HI|\$ (magenta) ... 6
- \$d\_5 = |GI| = |JK|\$ (yellow) ... 2
- \$d\_6 = |AK| = |CG| = |CH|\$ (grass) ... 3

Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_3} = \frac{d_1}{d_4} = \frac{d_2}{d_5} = \frac{d_3}{d_6} \text{ (16 occurrences)}$$

## 2.21.12 Construction RCC209



## Construction Process

$A, B$  given initial points

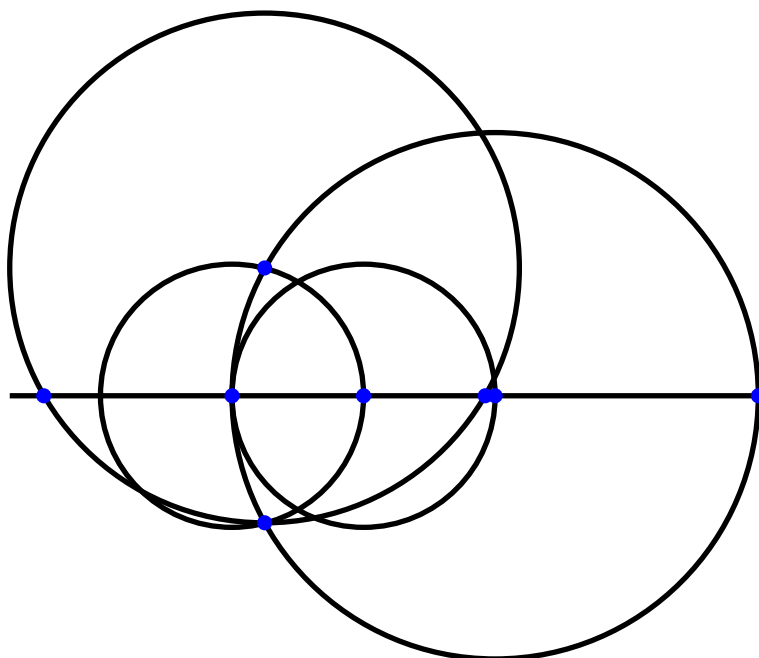
1.  $p_1 = p(A, B)$
2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
3.  $k_3 = k(C, A)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
4.  $k_4 = k(D, A)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
5.  $k_5 = k(E, H)$   
 $I \in k_2 \cap k_5, J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5$

## Distances

- $d_0 = |KL|$  (red) ... 1  
 $d_1 = |AL|$  (blue) ... 1  
 $d_2 = |JL|$  (green) ... 1  
 $d_3 = |AJ| = |CI|$  (cyan) ... 2  
 $d_4 = |BE| = |BF| = |DE| = |DF| = |EF| = |GI|$  (magenta) ... 6  
 $d_5 = |HI| = |JK|$  (yellow) ... 2  
 $d_6 = |AK| = |CG| = |CH|$  (grass) ... 3

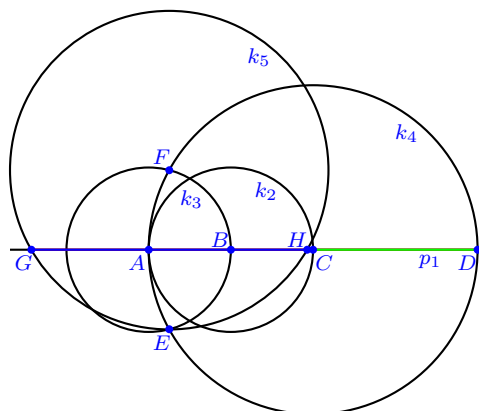
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_3} = \frac{d_1}{d_4} = \frac{d_2}{d_5} = \frac{d_3}{d_6} \quad (16 \text{ occurrences})$$

**2.22 Class of Similar Constructions No. 15****Contained Constructions**

RCC25, RCC26, RCC58, RCC59, RCC110, RCC111, RCC150, RCC151, RCC188,  
RCC189, RCC213, RCC214

## 2.22.1 Construction RCC25



## Construction Process

$A, B$  given initial points

1.  $p_1 = p(A, B)$
2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
3.  $k_3 = k(A, B)$
4.  $k_4 = k(C, A)$   
 $D \in p_1 \cap k_4, E \in k_3 \cap k_4, F \in k_3 \cap k_4$
5.  $k_5 = k(F, E)$   
 $G \in p_1 \cap k_5, H \in p_1 \cap k_5$

## Distances

$$d_0 = |DG| \text{ (red) } \dots 1$$

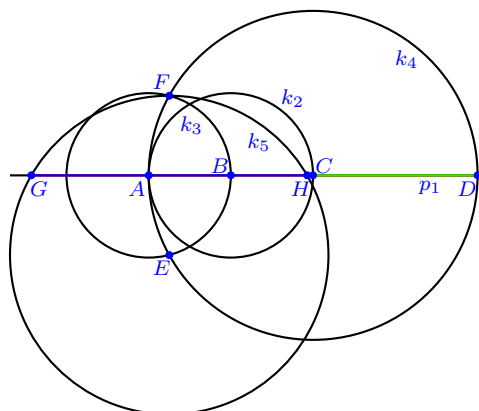
$$d_1 = |GH| \text{ (blue) } \dots 1$$

$$d_2 = |DH| \text{ (green) } \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (2 occurrences)}$$

## 2.22.2 Construction RCC26



## Construction Process

$A, B$  given initial points

1.  $p_1 = p(A, B)$
2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
3.  $k_3 = k(A, B)$
4.  $k_4 = k(C, A)$   
 $D \in p_1 \cap k_4, E \in k_3 \cap k_4, F \in k_3 \cap k_4$
5.  $k_5 = k(E, F)$   
 $G \in p_1 \cap k_5, H \in p_1 \cap k_5$

## Distances

$$d_0 = |DG| \text{ (red) } \dots 1$$

$$d_1 = |GH| \text{ (blue) } \dots 1$$

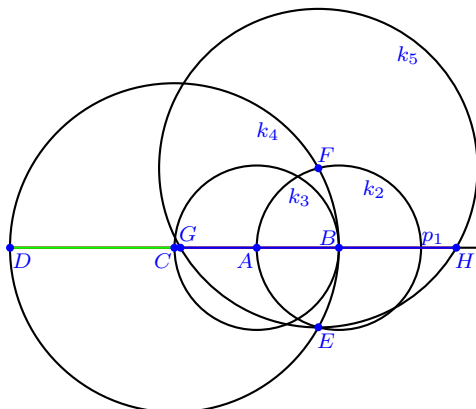
$$d_2 = |DH| \text{ (green) } \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (2 occurrences)}$$



## 2.22.3 Construction RCC58



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$
  3.  $k_3 = k(A, B)$   
 $C \in p_1 \cap k_3$
  4.  $k_4 = k(C, B)$   
 $D \in p_1 \cap k_4, E \in k_2 \cap k_4, F \in k_2 \cap k_4$
  5.  $k_5 = k(F, E)$   
 $G \in p_1 \cap k_5, H \in p_1 \cap k_5$

## Distances

$$d_0 = |DH| \text{ (red) } \dots 1$$

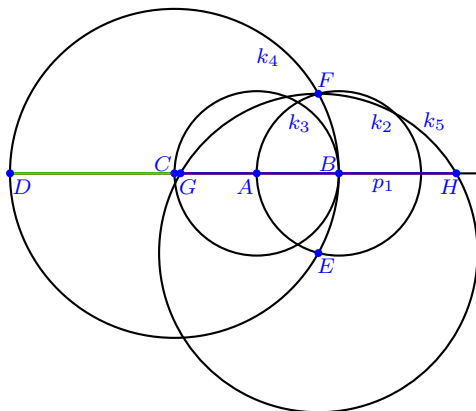
$$d_1 = |GH| \text{ (blue) } \dots 1$$

$$d_2 = |DG| \text{ (green) } \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (2 occurrences)}$$

## 2.22.4 Construction RCC59



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$
  3.  $k_3 = k(A, B)$   
 $C \in p_1 \cap k_3$
  4.  $k_4 = k(C, B)$   
 $D \in p_1 \cap k_4, E \in k_2 \cap k_4, F \in k_2 \cap k_4$
  5.  $k_5 = k(E, F)$   
 $G \in p_1 \cap k_5, H \in p_1 \cap k_5$

## Distances

$$d_0 = |DH| \text{ (red) } \dots 1$$

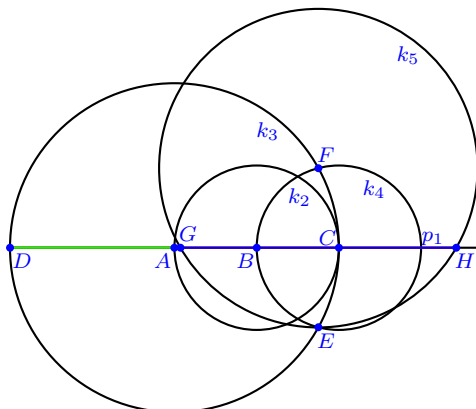
$$d_1 = |GH| \text{ (blue) } \dots 1$$

$$d_2 = |DG| \text{ (green) } \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (2 occurrences)}$$

## 2.22.5 Construction RCC110



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(A, C)$   
 $D \in p_1 \cap k_3$
  4.  $k_4 = k(C, B)$   
 $E \in k_3 \cap k_4, F \in k_3 \cap k_4$
  5.  $k_5 = k(F, E)$   
 $G \in p_1 \cap k_5, H \in p_1 \cap k_5$

## Distances

$$d_0 = |DH| \text{ (red) } \dots 1$$

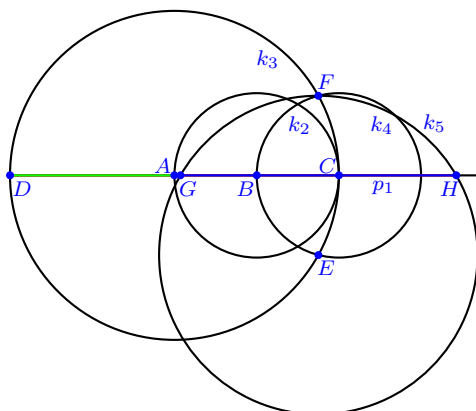
$$d_1 = |GH| \text{ (blue) } \dots 1$$

$$d_2 = |DG| \text{ (green) } \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (2 occurrences)}$$

## 2.22.6 Construction RCC111



## Construction Process

$A, B$  given initial points

1.  $p_1 = p(A, B)$
2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
3.  $k_3 = k(A, C)$   
 $D \in p_1 \cap k_3$
4.  $k_4 = k(C, B)$   
 $E \in k_3 \cap k_4, F \in k_3 \cap k_4$
5.  $k_5 = k(E, F)$   
 $G \in p_1 \cap k_5, H \in p_1 \cap k_5$

## Distances

$$d_0 = |DH| \text{ (red) } \dots 1$$

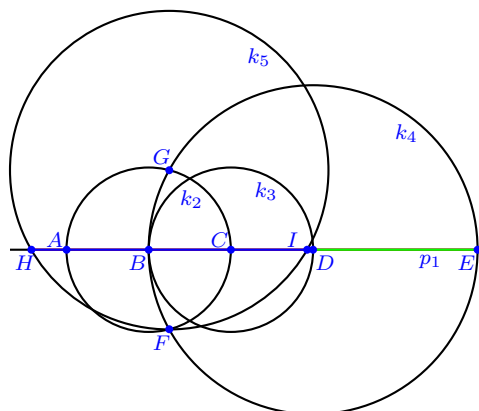
$$d_1 = |GH| \text{ (blue) } \dots 1$$

$$d_2 = |DG| \text{ (green) } \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (2 occurrences)}$$

## 2.22.7 Construction RCC150



## Construction Process

$A, B$  given initial points

1.  $p_1 = p(A, B)$
2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
3.  $k_3 = k(C, B)$   
 $D \in p_1 \cap k_3$
4.  $k_4 = k(D, B)$   
 $E \in p_1 \cap k_4, F \in k_2 \cap k_4, G \in k_2 \cap k_4$
5.  $k_5 = k(G, F)$   
 $H \in p_1 \cap k_5, I \in p_1 \cap k_5$

## Distances

$$d_0 = |EH| \text{ (red) } \dots 1$$

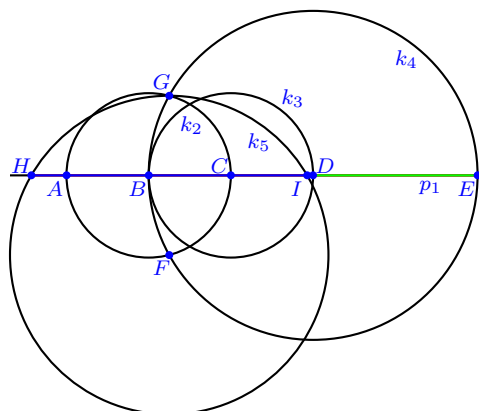
$$d_1 = |HI| \text{ (blue) } \dots 1$$

$$d_2 = |EI| \text{ (green) } \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (2 occurrences)}$$

## 2.22.8 Construction RCC151



## Construction Process

$A, B$  given initial points

1.  $p_1 = p(A, B)$
2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
3.  $k_3 = k(C, B)$   
 $D \in p_1 \cap k_3$
4.  $k_4 = k(D, B)$   
 $E \in p_1 \cap k_4, F \in k_2 \cap k_4, G \in k_2 \cap k_4$
5.  $k_5 = k(F, G)$   
 $H \in p_1 \cap k_5, I \in p_1 \cap k_5$

## Distances

$$d_0 = |EH| \text{ (red)} \dots 1$$

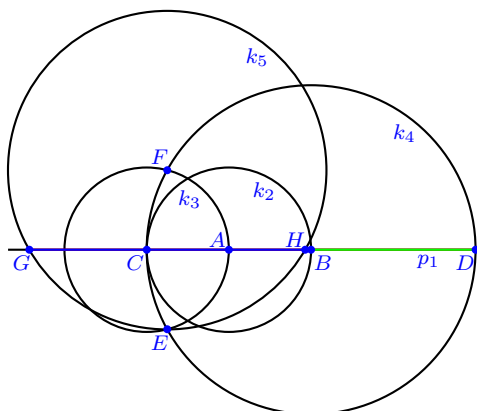
$$d_1 = |HI| \text{ (blue)} \dots 1$$

$$d_2 = |EI| \text{ (green)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (2 occurrences)}$$

## 2.22.9 Construction RCC188



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, A)$
  4.  $k_4 = k(B, C)$   
 $D \in p_1 \cap k_4, E \in k_3 \cap k_4, F \in k_3 \cap k_4$
  5.  $k_5 = k(F, E)$   
 $G \in p_1 \cap k_5, H \in p_1 \cap k_5$

## Distances

$$d_0 = |DG| \text{ (red) } \dots 1$$

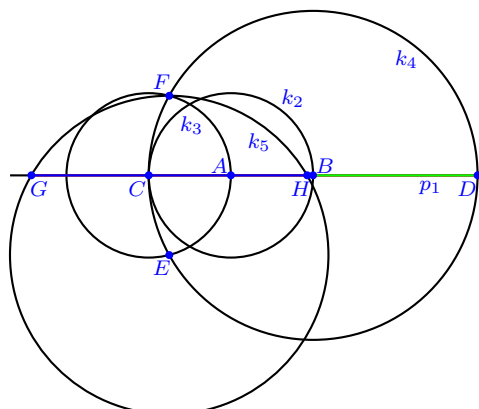
$$d_1 = |GH| \text{ (blue) } \dots 1$$

$$d_2 = |DH| \text{ (green) } \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (2 occurrences)}$$

## 2.22.10 Construction RCC189



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, A)$
  4.  $k_4 = k(B, C)$   
 $D \in p_1 \cap k_4, E \in k_3 \cap k_4, F \in k_3 \cap k_4$
  5.  $k_5 = k(E, F)$   
 $G \in p_1 \cap k_5, H \in p_1 \cap k_5$

## Distances

$$d_0 = |DG| \text{ (red) } \dots 1$$

$$d_1 = |GH| \text{ (blue) } \dots 1$$

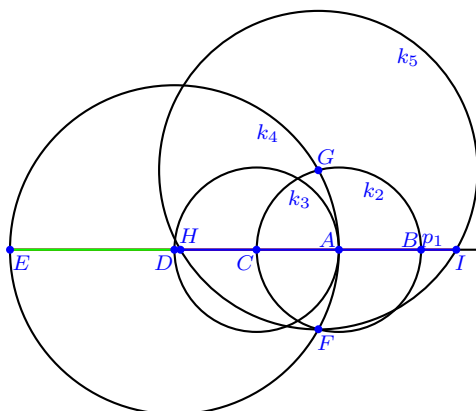
$$d_2 = |DH| \text{ (green) } \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (2 occurrences)}$$



## 2.22.11 Construction RCC213



## Construction Process

$A, B$  given initial points

1.  $p_1 = p(A, B)$
2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
3.  $k_3 = k(C, A)$   
 $D \in p_1 \cap k_3$
4.  $k_4 = k(D, A)$   
 $E \in p_1 \cap k_4, F \in k_2 \cap k_4, G \in k_2 \cap k_4$
5.  $k_5 = k(G, F)$   
 $H \in p_1 \cap k_5, I \in p_1 \cap k_5$

## Distances

$$d_0 = |EI| \text{ (red) } \dots 1$$

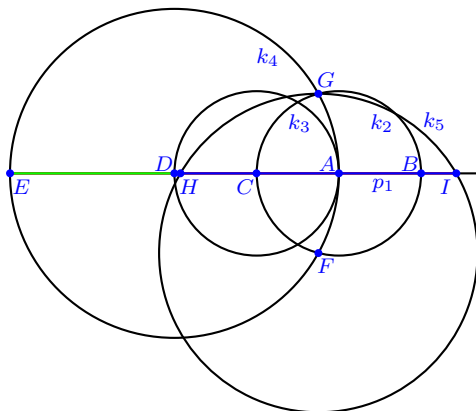
$$d_1 = |HI| \text{ (blue) } \dots 1$$

$$d_2 = |EH| \text{ (green) } \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (2 occurrences)}$$

## 2.22.12 Construction RCC214



## Construction Process

$A, B$  given initial points

1.  $p_1 = p(A, B)$
2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
3.  $k_3 = k(C, A)$   
 $D \in p_1 \cap k_3$
4.  $k_4 = k(D, A)$   
 $E \in p_1 \cap k_4, F \in k_2 \cap k_4, G \in k_2 \cap k_4$
5.  $k_5 = k(F, G)$   
 $H \in p_1 \cap k_5, I \in p_1 \cap k_5$

## Distances

$$d_0 = |EI| \text{ (red) } \dots 1$$

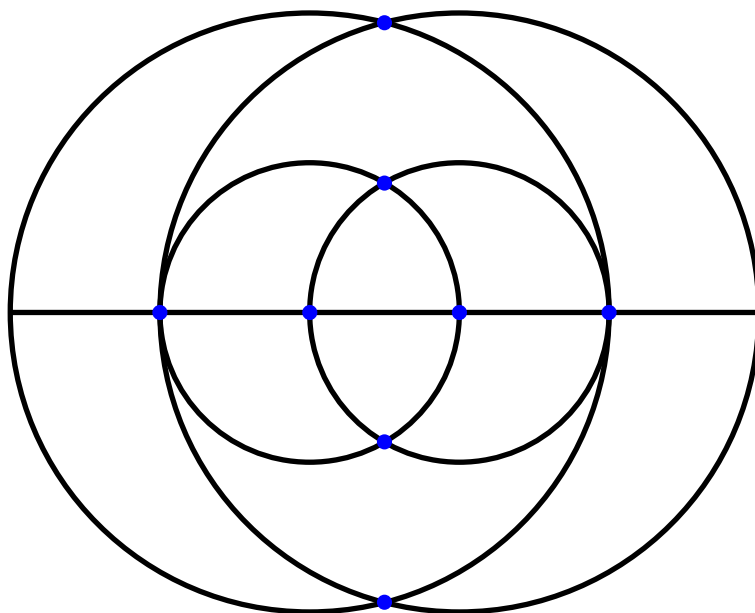
$$d_1 = |HI| \text{ (blue) } \dots 1$$

$$d_2 = |EH| \text{ (green) } \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (2 occurrences)}$$

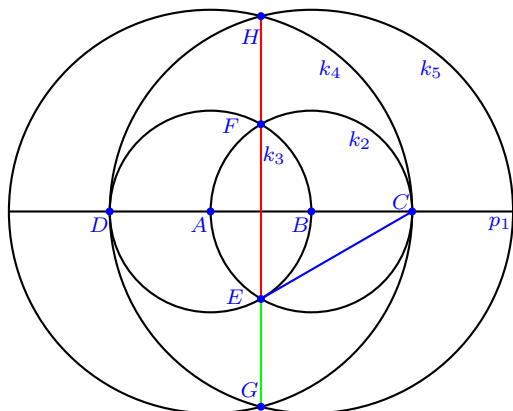
## 2.23 Class of Similar Constructions No. 16



### Contained Constructions

RCC27, RCC68, RCC160

## 2.23.1 Construction RCC27



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(A, B)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(A, C)$
  5.  $k_5 = k(B, D)$   
 $G \in k_4 \cap k_5, H \in k_4 \cap k_5$

## Distances

$$d_0 = |EH| = |FG| \text{ (red) } \dots 2$$

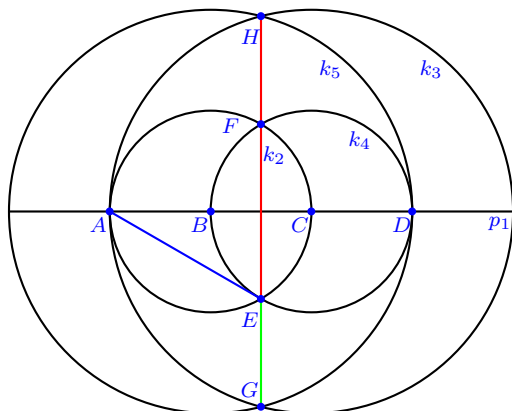
$$d_1 = |CE| = |CF| = |DE| = |DF| = |EF| \text{ (blue) } \dots 5$$

$$d_2 = |EG| = |FH| \text{ (green) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (20 occurrences)}$$

## 2.23.2 Construction RCC68



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, A)$
  4.  $k_4 = k(C, B)$   
 $D \in p_1 \cap k_4, E \in k_2 \cap k_4, F \in k_2 \cap k_4$
  5.  $k_5 = k(B, D)$   
 $G \in k_3 \cap k_5, H \in k_3 \cap k_5$

## Distances

$$d_0 = |EH| = |FG| \text{ (red) } \dots 2$$

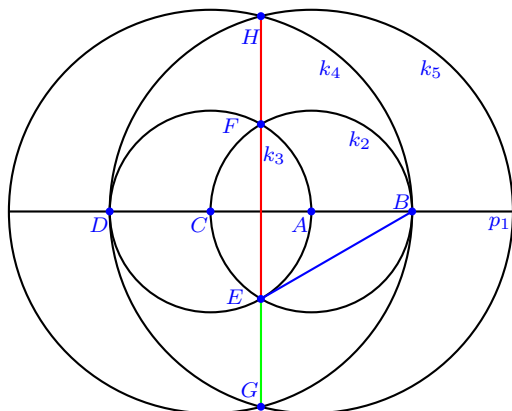
$$d_1 = |AE| = |AF| = |DE| = |DF| = |EF| \text{ (blue) } \dots 5$$

$$d_2 = |EG| = |FH| \text{ (green) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (20 occurrences)}$$

## 2.23.3 Construction RCC160



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, A)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(C, B)$
  5.  $k_5 = k(A, D)$   
 $G \in k_4 \cap k_5, H \in k_4 \cap k_5$

## Distances

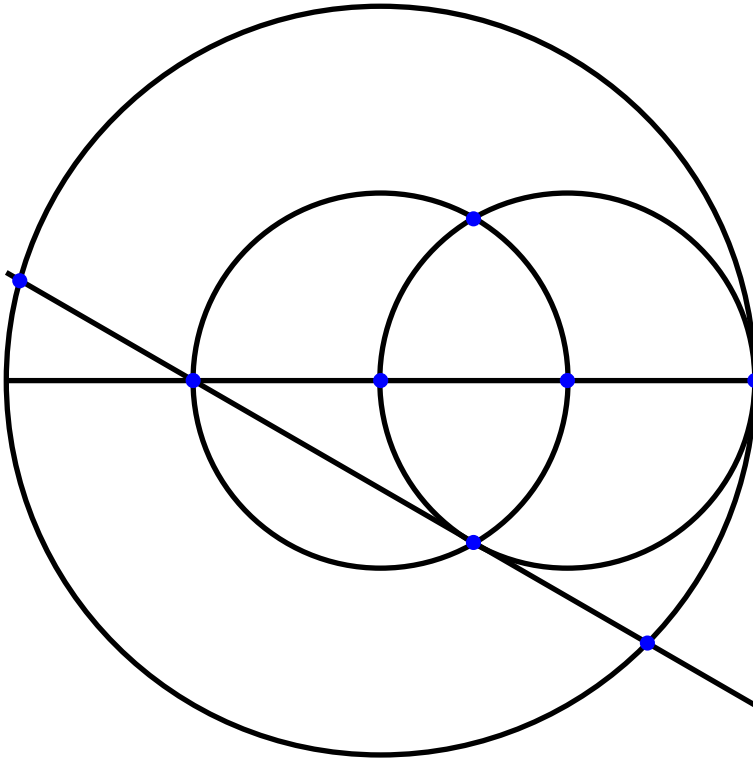
$$d_0 = |EH| = |FG| \text{ (red) } \dots 2$$

$$d_1 = |BE| = |BF| = |DE| = |DF| = |EF| \text{ (blue) } \dots 5$$

$$d_2 = |EG| = |FH| \text{ (green) } \dots 2$$

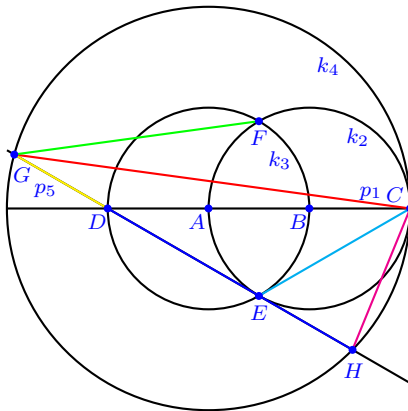
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (20 occurrences)}$$

**2.24 Class of Similar Constructions No. 17****Contained Constructions**

RCC28, RCC29, RCC60, RCC61, RCC69, RCC70, RCC152, RCC153, RCC161,  
RCC162, RCC215, RCC216

## 2.24.1 Construction RCC28



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(A, B)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(A, C)$
  5.  $p_5 = p(D, E)$   
 $G \in p_5 \cap k_4, H \in p_5 \cap k_4$

## Distances

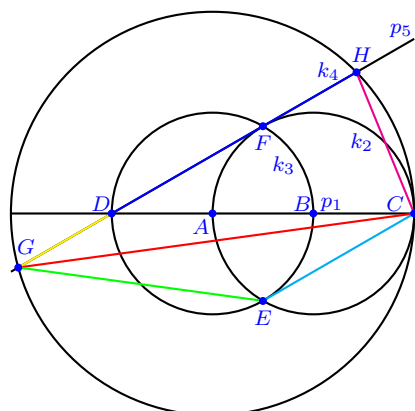
- $d_0 = |CG|$  (red) ... 1  
 $d_1 = |DH| = |EG|$  (blue) ... 2  
 $d_2 = |FG| = |FH|$  (green) ... 2  
 $d_3 = |CE| = |CF| = |DE| = |DF| = |EF|$  (cyan) ... 5  
 $d_4 = |CH|$  (magenta) ... 1  
 $d_5 = |DG| = |EH|$  (yellow) ... 2

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \quad (24 \text{ occurrences})$$



## 2.24.2 Construction RCC29



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(A, B)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(A, C)$
  5.  $p_5 = p(D, F)$   
 $G \in p_5 \cap k_4, H \in p_5 \cap k_4$

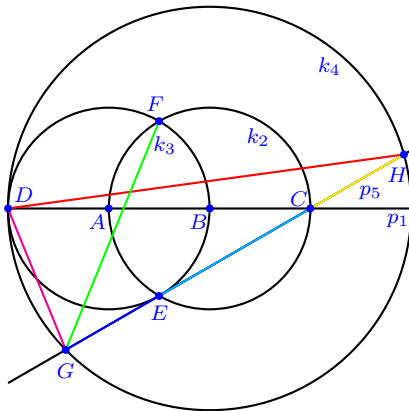
## Distances

$$\begin{aligned}
 d_0 &= |CG| \text{ (red)} \dots 1 \\
 d_1 &= |DH| = |FG| \text{ (blue)} \dots 2 \\
 d_2 &= |EG| = |EH| \text{ (green)} \dots 2 \\
 d_3 &= |CE| = |CF| = |DE| = |DF| = |EF| \text{ (cyan)} \dots 5 \\
 d_4 &= |CH| \text{ (magenta)} \dots 1 \\
 d_5 &= |DG| = |FH| \text{ (yellow)} \dots 2
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (24 occurrences)}$$

## 2.24.3 Construction RCC60



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(A, B)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(B, D)$
  5.  $p_5 = p(C, E)$   
 $G \in p_5 \cap k_4, H \in p_5 \cap k_4$

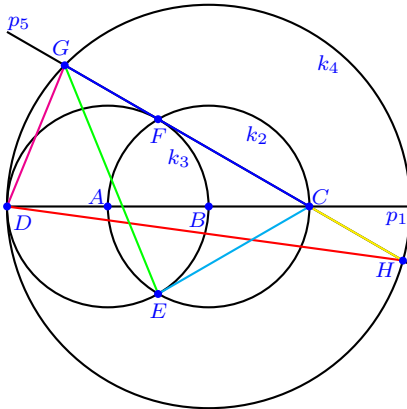
## Distances

$$\begin{aligned}
 d_0 &= |DH| \text{ (red)} \dots 1 \\
 d_1 &= |CG| = |EH| \text{ (blue)} \dots 2 \\
 d_2 &= |FG| = |FH| \text{ (green)} \dots 2 \\
 d_3 &= |CE| = |CF| = |DE| = |DF| = |EF| \text{ (cyan)} \dots 5 \\
 d_4 &= |DG| \text{ (magenta)} \dots 1 \\
 d_5 &= |CH| = |EG| \text{ (yellow)} \dots 2
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (24 occurrences)}$$

## 2.24.4 Construction RCC61

**Construction Process**

$A, B$  given initial points

1.  $p_1 = p(A, B)$
2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
3.  $k_3 = k(A, B)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
4.  $k_4 = k(B, D)$
5.  $p_5 = p(C, F)$   
 $G \in p_5 \cap k_4, H \in p_5 \cap k_4$

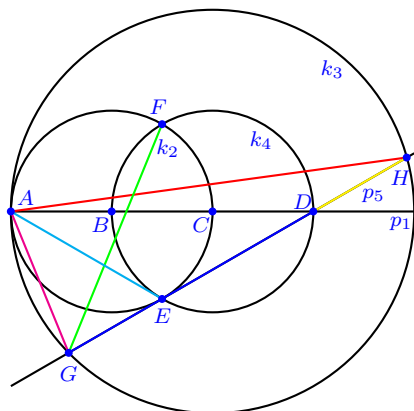
**Distances**

$$\begin{aligned}
 d_0 &= |DH| \text{ (red)} \dots 1 \\
 d_1 &= |CG| = |FH| \text{ (blue)} \dots 2 \\
 d_2 &= |EG| = |EH| \text{ (green)} \dots 2 \\
 d_3 &= |CE| = |CF| = |DE| = |DF| = |EF| \text{ (cyan)} \dots 5 \\
 d_4 &= |DG| \text{ (magenta)} \dots 1 \\
 d_5 &= |CH| = |FG| \text{ (yellow)} \dots 2
 \end{aligned}$$

**Occurrences of the Golden Ratio**

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (24 occurrences)}$$

## 2.24.5 Construction RCC69



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, A)$
  4.  $k_4 = k(C, B)$   
 $D \in p_1 \cap k_4, E \in k_2 \cap k_4, F \in k_2 \cap k_4$
  5.  $p_5 = p(D, E)$   
 $G \in p_5 \cap k_3, H \in p_5 \cap k_3$

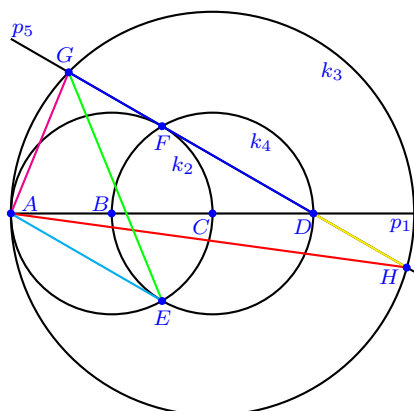
## Distances

$$\begin{aligned}
 d_0 &= |AH| \text{ (red)} \dots 1 \\
 d_1 &= |DG| = |EH| \text{ (blue)} \dots 2 \\
 d_2 &= |FG| = |FH| \text{ (green)} \dots 2 \\
 d_3 &= |AE| = |AF| = |DE| = |DF| = |EF| \text{ (cyan)} \dots 5 \\
 d_4 &= |AG| \text{ (magenta)} \dots 1 \\
 d_5 &= |DH| = |EG| \text{ (yellow)} \dots 2
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (24 occurrences)}$$

## 2.24.6 Construction RCC70



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, A)$
  4.  $k_4 = k(C, B)$   
 $D \in p_1 \cap k_4, E \in k_2 \cap k_4, F \in k_2 \cap k_4$
  5.  $p_5 = p(D, F)$   
 $G \in p_5 \cap k_3, H \in p_5 \cap k_3$

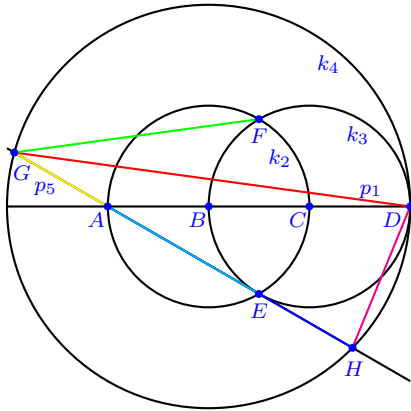
## Distances

$$\begin{aligned}
 d_0 &= |AH| \text{ (red)} \dots 1 \\
 d_1 &= |DG| = |FH| \text{ (blue)} \dots 2 \\
 d_2 &= |EG| = |EH| \text{ (green)} \dots 2 \\
 d_3 &= |AE| = |AF| = |DE| = |DF| = |EF| \text{ (cyan)} \dots 5 \\
 d_4 &= |AG| \text{ (magenta)} \dots 1 \\
 d_5 &= |DH| = |FG| \text{ (yellow)} \dots 2
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (24 occurrences)}$$

**2.24.7 Construction RCC152**



**Construction Process**

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, B)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(B, D)$
  5.  $p_5 = p(A, E)$   
 $G \in p_5 \cap k_4, H \in p_5 \cap k_4$

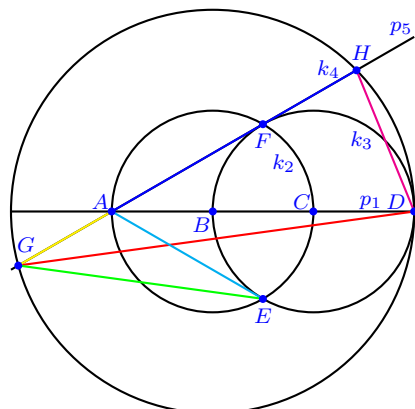
**Distances**

- $d_0 = |DG|$  (red) ... 1  
 $d_1 = |AH| = |EG|$  (blue) ... 2  
 $d_2 = |FG| = |FH|$  (green) ... 2  
 $d_3 = |AE| = |AF| = |DE| = |DF| = |EF|$  (cyan) ... 5  
 $d_4 = |DH|$  (magenta) ... 1  
 $d_5 = |AG| = |EH|$  (yellow) ... 2

**Occurrences of the Golden Ratio**

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (24 occurrences)}$$

## 2.24.8 Construction RCC153



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, B)$   
 $D \in p_1 \cap k_3$ ,  $E \in k_2 \cap k_3$ ,  $F \in k_2 \cap k_3$
  4.  $k_4 = k(B, D)$
  5.  $p_5 = p(A, F)$   
 $G \in p_5 \cap k_4$ ,  $H \in p_5 \cap k_4$

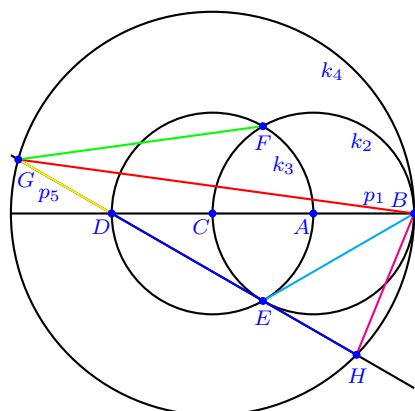
## Distances

$$\begin{aligned}
 d_0 &= |DG| \text{ (red)} \dots 1 \\
 d_1 &= |AH| = |FG| \text{ (blue)} \dots 2 \\
 d_2 &= |EG| = |EH| \text{ (green)} \dots 2 \\
 d_3 &= |AE| = |AF| = |DE| = |DF| = |EF| \text{ (cyan)} \dots 5 \\
 d_4 &= |DH| \text{ (magenta)} \dots 1 \\
 d_5 &= |AG| = |FH| \text{ (yellow)} \dots 2
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (24 occurrences)}$$

## 2.24.9 Construction RCC161



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, A)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(C, B)$
  5.  $p_5 = p(D, E)$   
 $G \in p_5 \cap k_4, H \in p_5 \cap k_4$

## Distances

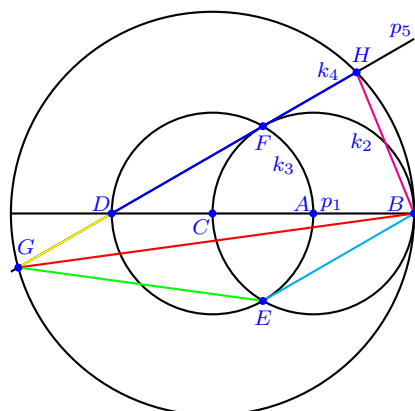
$$\begin{aligned}
 d_0 &= |BG| \text{ (red)} \dots 1 \\
 d_1 &= |DH| = |EG| \text{ (blue)} \dots 2 \\
 d_2 &= |FG| = |FH| \text{ (green)} \dots 2 \\
 d_3 &= |BE| = |BF| = |DE| = |DF| = |EF| \text{ (cyan)} \dots 5 \\
 d_4 &= |BH| \text{ (magenta)} \dots 1 \\
 d_5 &= |DG| = |EH| \text{ (yellow)} \dots 2
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (24 occurrences)}$$



## 2.24.10 Construction RCC162



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, A)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(C, B)$
  5.  $p_5 = p(D, F)$   
 $G \in p_5 \cap k_4, H \in p_5 \cap k_4$

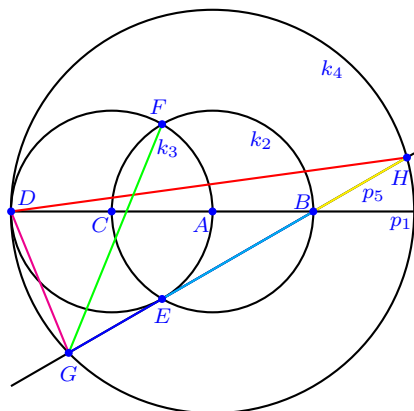
## Distances

- $d_0 = |BG|$  (red) ... 1  
 $d_1 = |DH| = |FG|$  (blue) ... 2  
 $d_2 = |EG| = |EH|$  (green) ... 2  
 $d_3 = |BE| = |BF| = |DE| = |DF| = |EF|$  (cyan) ... 5  
 $d_4 = |BH|$  (magenta) ... 1  
 $d_5 = |DG| = |FH|$  (yellow) ... 2

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (24 occurrences)}$$

## 2.24.11 Construction RCC215



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, A)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(A, D)$
  5.  $p_5 = p(B, E)$   
 $G \in p_5 \cap k_4, H \in p_5 \cap k_4$

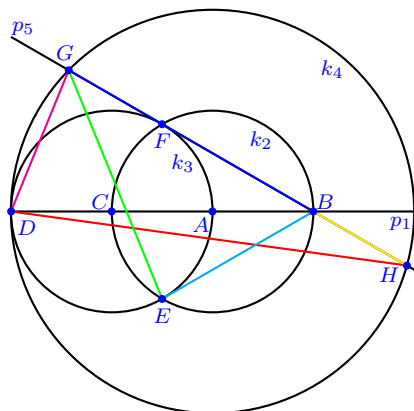
## Distances

$$\begin{aligned}
 d_0 &= |DH| \text{ (red)} \dots 1 \\
 d_1 &= |BG| = |EH| \text{ (blue)} \dots 2 \\
 d_2 &= |FG| = |FH| \text{ (green)} \dots 2 \\
 d_3 &= |BE| = |BF| = |DE| = |DF| = |EF| \text{ (cyan)} \dots 5 \\
 d_4 &= |DG| \text{ (magenta)} \dots 1 \\
 d_5 &= |BH| = |EG| \text{ (yellow)} \dots 2
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (24 occurrences)}$$

## 2.24.12 Construction RCC216



## Construction Process

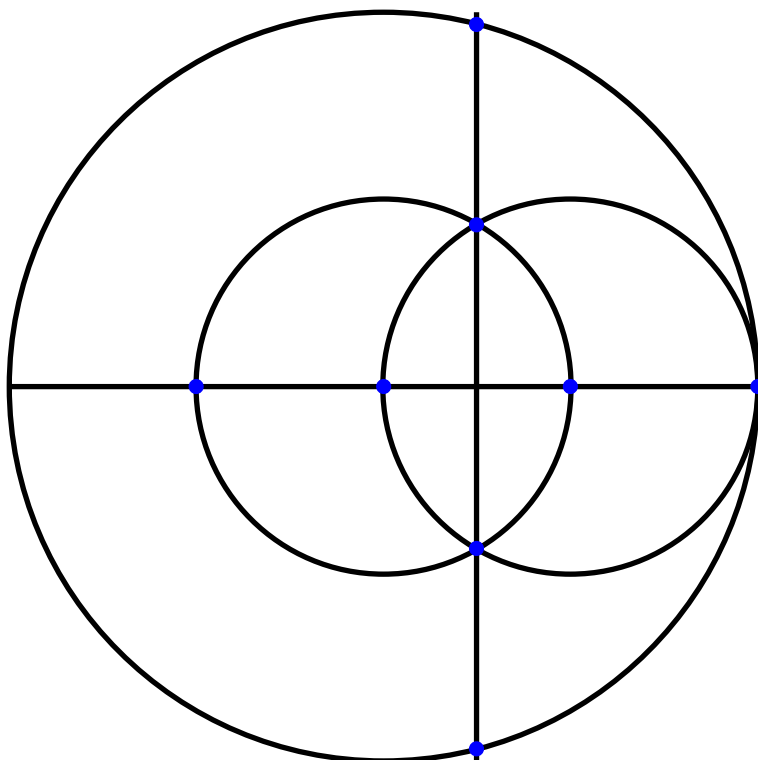
- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, A)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(A, D)$
  5.  $p_5 = p(B, F)$   
 $G \in p_5 \cap k_4, H \in p_5 \cap k_4$

## Distances

$$\begin{aligned}
 d_0 &= |DH| \text{ (red)} \dots 1 \\
 d_1 &= |BG| = |FH| \text{ (blue)} \dots 2 \\
 d_2 &= |EG| = |EH| \text{ (green)} \dots 2 \\
 d_3 &= |BE| = |BF| = |DE| = |DF| = |EF| \text{ (cyan)} \dots 5 \\
 d_4 &= |DG| \text{ (magenta)} \dots 1 \\
 d_5 &= |BH| = |FG| \text{ (yellow)} \dots 2
 \end{aligned}$$

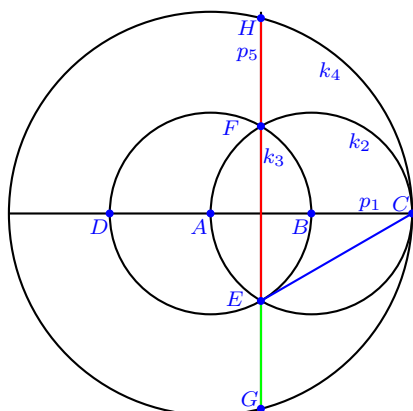
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (24 occurrences)}$$

**2.25 Class of Similar Constructions No. 18****Contained Constructions**

RCC30, RCC62, RCC71, RCC154, RCC163, RCC217

## 2.25.1 Construction RCC30



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(A, B)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(A, C)$
  5.  $p_5 = p(E, F)$   
 $G \in p_5 \cap k_4, H \in p_5 \cap k_4$

## Distances

$$d_0 = |EH| = |FG| \text{ (red)} \dots 2$$

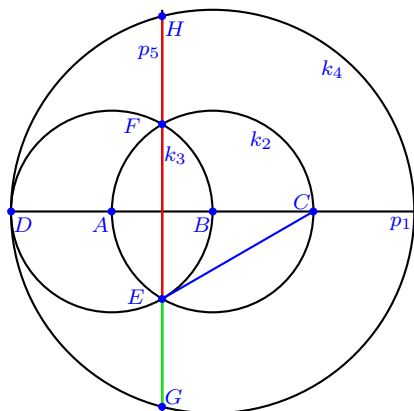
$$d_1 = |CE| = |CF| = |DE| = |DF| = |EF| \text{ (blue)} \dots 5$$

$$d_2 = |EG| = |FH| \text{ (green)} \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (20 occurrences)}$$

## 2.25.2 Construction RCC62



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(A, B)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(B, D)$
  5.  $p_5 = p(E, F)$   
 $G \in p_5 \cap k_4, H \in p_5 \cap k_4$

## Distances

$$d_0 = |EH| = |FG| \text{ (red) } \dots 2$$

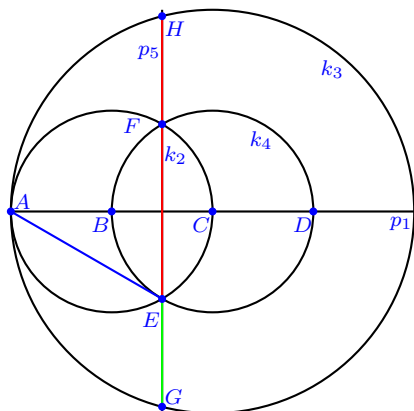
$$d_1 = |CE| = |CF| = |DE| = |DF| = |EF| \text{ (blue) } \dots 5$$

$$d_2 = |EG| = |FH| \text{ (green) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (20 occurrences)}$$

## 2.25.3 Construction RCC71



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, A)$
  4.  $k_4 = k(C, B)$   
 $D \in p_1 \cap k_4, E \in k_2 \cap k_4, F \in k_2 \cap k_4$
  5.  $p_5 = p(E, F)$   
 $G \in p_5 \cap k_3, H \in p_5 \cap k_3$

## Distances

$$d_0 = |EH| = |FG| \text{ (red) } \dots 2$$

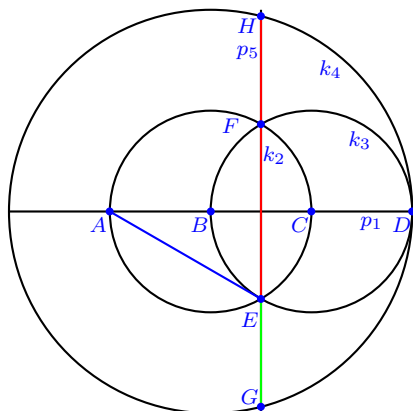
$$d_1 = |AE| = |AF| = |DE| = |DF| = |EF| \text{ (blue) } \dots 5$$

$$d_2 = |EG| = |FH| \text{ (green) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (20 occurrences)}$$

## 2.25.4 Construction RCC154



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, B)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(B, D)$
  5.  $p_5 = p(E, F)$   
 $G \in p_5 \cap k_4, H \in p_5 \cap k_4$

## Distances

$$d_0 = |EH| = |FG| \text{ (red) } \dots 2$$

$$d_1 = |AE| = |AF| = |DE| = |DF| = |EF| \text{ (blue) } \dots 5$$

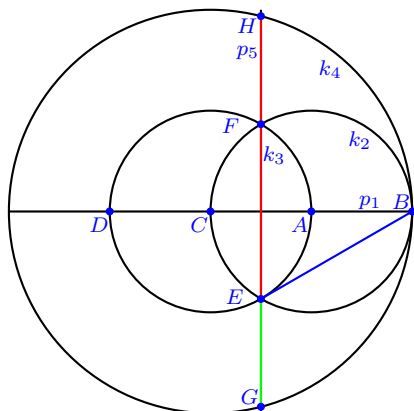
$$d_2 = |EG| = |FH| \text{ (green) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (20 occurrences)}$$



## 2.25.5 Construction RCC163



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, A)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(C, B)$
  5.  $p_5 = p(E, F)$   
 $G \in p_5 \cap k_4, H \in p_5 \cap k_4$

## Distances

$$d_0 = |EH| = |FG| \text{ (red) } \dots 2$$

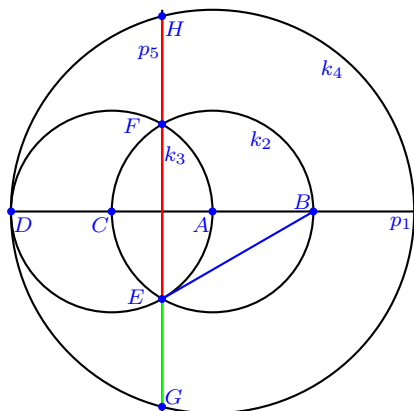
$$d_1 = |BE| = |BF| = |DE| = |DF| = |EF| \text{ (blue) } \dots 5$$

$$d_2 = |EG| = |FH| \text{ (green) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (20 occurrences)}$$

## 2.25.6 Construction RCC217



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, A)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(A, D)$
  5.  $p_5 = p(E, F)$   
 $G \in p_5 \cap k_4, H \in p_5 \cap k_4$

## Distances

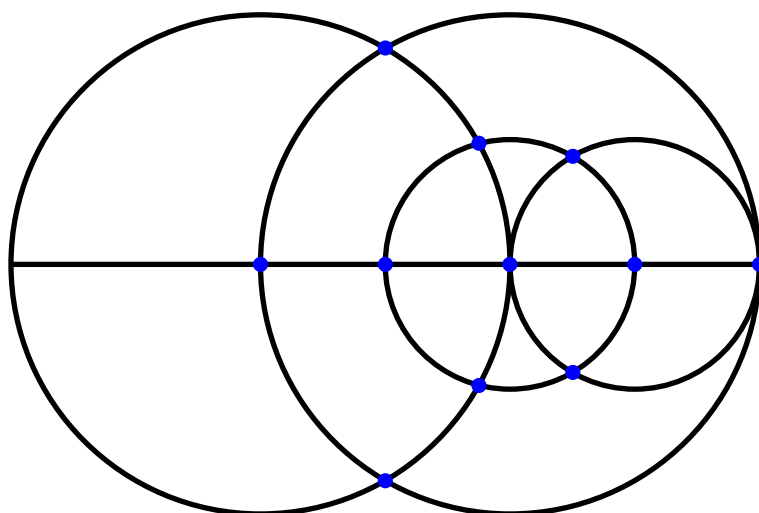
$$d_0 = |EH| = |FG| \text{ (red) } \dots 2$$

$$d_1 = |BE| = |BF| = |DE| = |DF| = |EF| \text{ (blue) } \dots 5$$

$$d_2 = |EG| = |FH| \text{ (green) } \dots 2$$

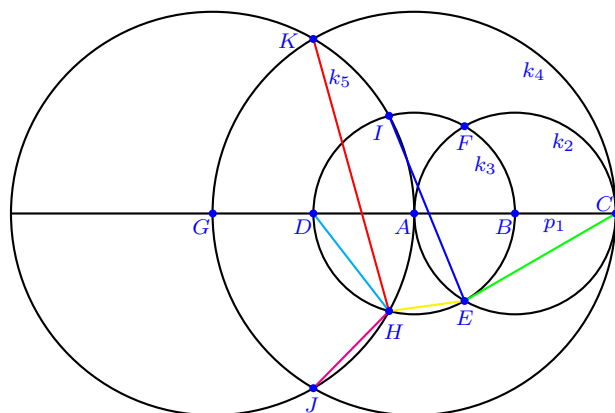
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (20 occurrences)}$$

**2.26 Class of Similar Constructions No. 19****Contained Constructions**

RCC31, RCC63, RCC67, RCC155, RCC164, RCC218

## 2.26.1 Construction RCC31



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(A, B)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(A, C)$   
 $G \in p_1 \cap k_4$
  5.  $k_5 = k(G, A)$   
 $H \in k_3 \cap k_5, I \in k_3 \cap k_5, J \in k_4 \cap k_5, K \in k_4 \cap k_5$

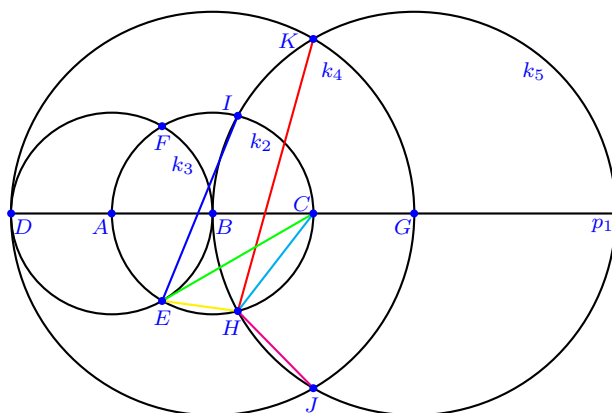
## Distances

$$\begin{aligned}
 d_0 &= |HK| = |IJ| \text{ (red)} \dots 2 \\
 d_1 &= |EI| = |FH| \text{ (blue)} \dots 2 \\
 d_2 &= |CE| = |CF| = |DE| = |DF| = |DJ| = |DK| = |EF| = |EJ| = \\
 &= |FK| \text{ (green)} \dots 9 \\
 d_3 &= |DH| = |DI| \text{ (cyan)} \dots 2 \\
 d_4 &= |HJ| = |IK| \text{ (magenta)} \dots 2 \\
 d_5 &= |EH| = |FI| \text{ (yellow)} \dots 2
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (44 occurrences)}$$

## 2.26.2 Construction RCC63



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(A, B)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(B, D)$   
 $G \in p_1 \cap k_4$
  5.  $k_5 = k(G, B)$   
 $H \in k_2 \cap k_5, I \in k_2 \cap k_5, J \in k_4 \cap k_5, K \in k_4 \cap k_5$

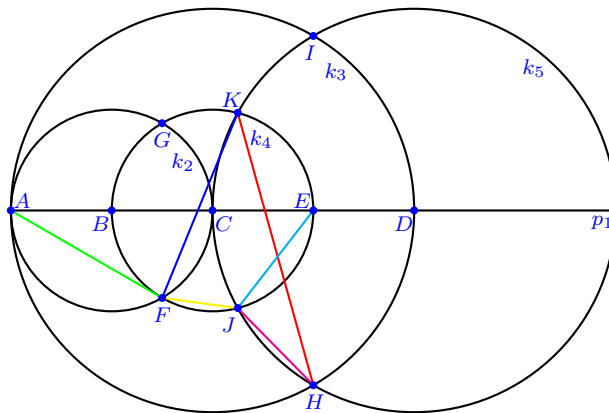
## Distances

$$\begin{aligned}
 d_0 &= |HK| = |IJ| \text{ (red)} \dots 2 \\
 d_1 &= |EI| = |FH| \text{ (blue)} \dots 2 \\
 d_2 &= |CE| = |CF| = |CJ| = |CK| = |DE| = |DF| = |EF| = |EJ| = |FK| \text{ (green)} \dots 9 \\
 d_3 &= |CH| = |CI| \text{ (cyan)} \dots 2 \\
 d_4 &= |HJ| = |IK| \text{ (magenta)} \dots 2 \\
 d_5 &= |EH| = |FI| \text{ (yellow)} \dots 2
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (44 occurrences)}$$

2.26.3 Construction RCC67



Construction Process

- A, B given initial points
- 1.  $p_1 = p(A, B)$
- 2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
- 3.  $k_3 = k(C, A)$   
 $D \in p_1 \cap k_3$
- 4.  $k_4 = k(C, B)$   
 $E \in p_1 \cap k_4, F \in k_2 \cap k_4, G \in k_2 \cap k_4$
- 5.  $k_5 = k(D, C)$   
 $H \in k_3 \cap k_5, I \in k_3 \cap k_5, J \in k_4 \cap k_5, K \in k_4 \cap k_5$

Distances

$$d_0 = |HK| = |IJ| \text{ (red) } \dots 2$$

$$d_1 = |FK| = |GJ| \text{ (blue) } \dots 2$$

$$d_2 = |AF| = |AG| = |EF| = |EG| = |EH| = |EI| = |FG| = |FH| = |GI| \text{ (green) } \dots 9$$

$$d_3 = |EJ| = |EK| \text{ (cyan) } \dots 2$$

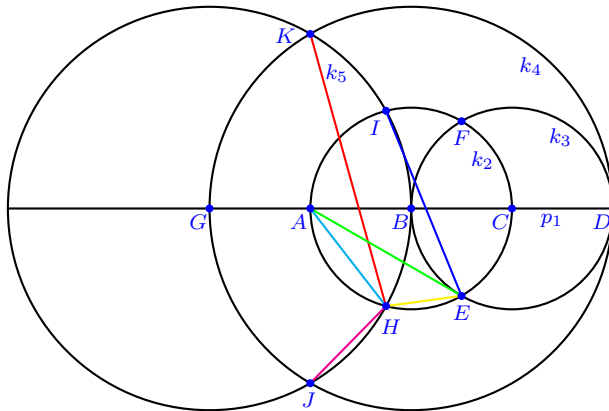
$$d_4 = |HJ| = |IK| \text{ (magenta) } \dots 2$$

$$d_5 = |FJ| = |GK| \text{ (yellow) } \dots 2$$

Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (44 occurrences)}$$

2.26.4 Construction RCC155



Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, B)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(B, D)$   
 $G \in p_1 \cap k_4$
  5.  $k_5 = k(G, B)$   
 $H \in k_2 \cap k_5, I \in k_2 \cap k_5, J \in k_4 \cap k_5, K \in k_4 \cap k_5$

Distances

$$d_0 = |HK| = |IJ| \text{ (red)} \dots 2$$

$$d_1 = |EI| = |FH| \text{ (blue)} \dots 2$$

$$d_2 = |AE| = |AF| = |AJ| = |AK| = |DE| = |DF| = |EF| = |EJ| = |FK| \text{ (green)} \dots 9$$

$$d_3 = |AH| = |AI| \text{ (cyan)} \dots 2$$

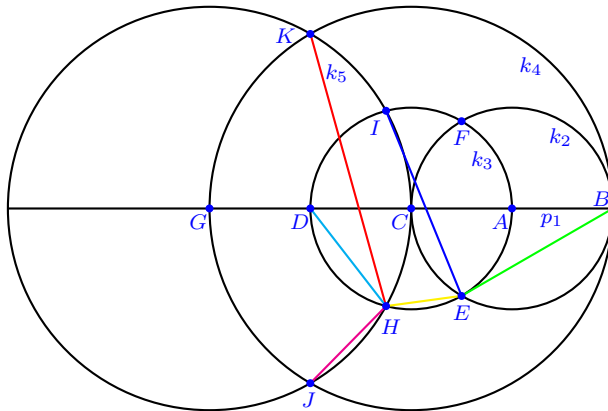
$$d_4 = |HJ| = |IK| \text{ (magenta)} \dots 2$$

$$d_5 = |EH| = |FI| \text{ (yellow)} \dots 2$$

Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (44 occurrences)}$$

2.26.5 Construction RCC164



Construction Process

- $A, B$  given initial points
- 1.  $p_1 = p(A, B)$
- 2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
- 3.  $k_3 = k(C, A)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
- 4.  $k_4 = k(C, B)$   
 $G \in p_1 \cap k_4$
- 5.  $k_5 = k(G, C)$   
 $H \in k_3 \cap k_5, I \in k_3 \cap k_5, J \in k_4 \cap k_5, K \in k_4 \cap k_5$

Distances

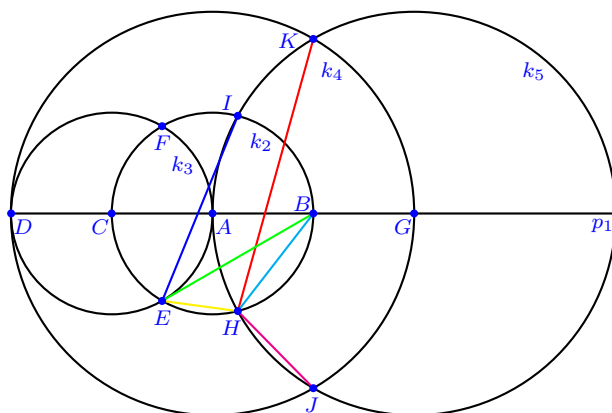
$$\begin{aligned}
 d_0 &= |HK| = |IJ| \text{ (red)} \dots 2 \\
 d_1 &= |EI| = |FH| \text{ (blue)} \dots 2 \\
 d_2 &= |BE| = |BF| = |DE| = |DF| = |DJ| = |DK| = |EF| = |EJ| = |FK| \text{ (green)} \dots 9 \\
 d_3 &= |DH| = |DI| \text{ (cyan)} \dots 2 \\
 d_4 &= |HJ| = |IK| \text{ (magenta)} \dots 2 \\
 d_5 &= |EH| = |FI| \text{ (yellow)} \dots 2
 \end{aligned}$$

Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (44 occurrences)}$$



## 2.26.6 Construction RCC218



## Construction Process

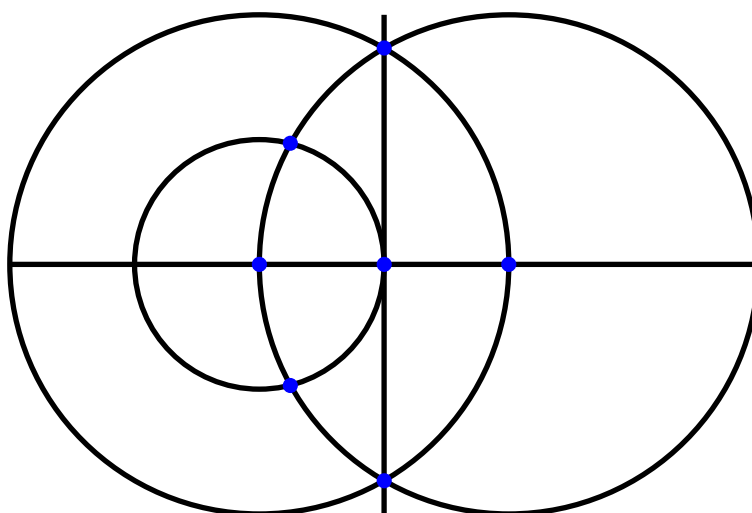
- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, A)$   
 $D \in p_1 \cap k_3, E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(A, D)$   
 $G \in p_1 \cap k_4$
  5.  $k_5 = k(G, A)$   
 $H \in k_2 \cap k_5, I \in k_2 \cap k_5, J \in k_4 \cap k_5, K \in k_4 \cap k_5$

## Distances

$$\begin{aligned}
 d_0 &= |HK| = |IJ| \text{ (red)} \dots 2 \\
 d_1 &= |EI| = |FH| \text{ (blue)} \dots 2 \\
 d_2 &= |BE| = |BF| = |BJ| = |BK| = |DE| = |DF| = |EF| = |EJ| = |FK| \text{ (green)} \dots 9 \\
 d_3 &= |BH| = |BI| \text{ (cyan)} \dots 2 \\
 d_4 &= |HJ| = |IK| \text{ (magenta)} \dots 2 \\
 d_5 &= |EH| = |FI| \text{ (yellow)} \dots 2
 \end{aligned}$$

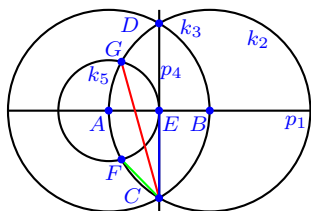
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (44 occurrences)}$$

**2.27 Class of Similar Constructions No. 20****Contained Constructions**

RCC64, RCC65, RCC157, RCC158, RCC220, RCC221

## 2.27.1 Construction RCC64



## Construction Process

$A, B$  given initial points

1.  $p_1 = p(A, B)$
2.  $k_2 = k(B, A)$
3.  $k_3 = k(A, B)$   
 $C \in k_2 \cap k_3, D \in k_2 \cap k_3$
4.  $p_4 = p(C, D)$   
 $E \in p_1 \cap p_4$
5.  $k_5 = k(A, E)$   
 $F \in k_2 \cap k_5, G \in k_2 \cap k_5$

## Distances

$$d_0 = |CG| = |DF| \text{ (red) } \dots 2$$

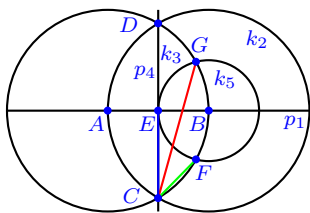
$$d_1 = |CE| = |DE| \text{ (blue) } \dots 2$$

$$d_2 = |CF| = |DG| \text{ (green) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)}$$

### 2.27.2 Construction RCC65



#### Construction Process

$A, B$  given initial points

1.  $p_1 = p(A, B)$
2.  $k_2 = k(B, A)$
3.  $k_3 = k(A, B)$   
 $C \in k_2 \cap k_3, D \in k_2 \cap k_3$
4.  $p_4 = p(C, D)$   
 $E \in p_1 \cap p_4$
5.  $k_5 = k(B, E)$   
 $F \in k_3 \cap k_5, G \in k_3 \cap k_5$

#### Distances

$$d_0 = |CG| = |DF| \text{ (red) } \dots 2$$

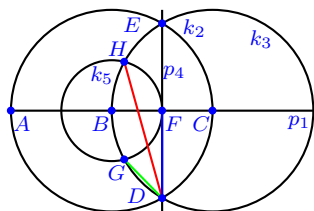
$$d_1 = |CE| = |DE| \text{ (blue) } \dots 2$$

$$d_2 = |CF| = |DG| \text{ (green) } \dots 2$$

#### Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)}$$

## 2.27.3 Construction RCC157



## Construction Process

$A, B$  given initial points

1.  $p_1 = p(A, B)$
2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
3.  $k_3 = k(C, B)$   
 $D \in k_2 \cap k_3, E \in k_2 \cap k_3$
4.  $p_4 = p(D, E)$   
 $F \in p_1 \cap p_4$
5.  $k_5 = k(B, F)$   
 $G \in k_3 \cap k_5, H \in k_3 \cap k_5$

## Distances

$$d_0 = |DH| = |EG| \text{ (red) } \dots 2$$

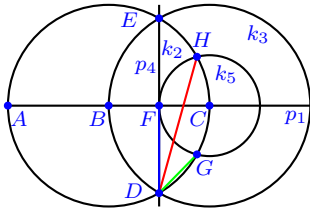
$$d_1 = |DF| = |EF| \text{ (blue) } \dots 2$$

$$d_2 = |DG| = |EH| \text{ (green) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)}$$

## 2.27.4 Construction RCC158



## Construction Process

$A, B$  given initial points

1.  $p_1 = p(A, B)$
2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
3.  $k_3 = k(C, B)$   
 $D \in k_2 \cap k_3, E \in k_2 \cap k_3$
4.  $p_4 = p(D, E)$   
 $F \in p_1 \cap p_4$
5.  $k_5 = k(C, F)$   
 $G \in k_2 \cap k_5, H \in k_2 \cap k_5$

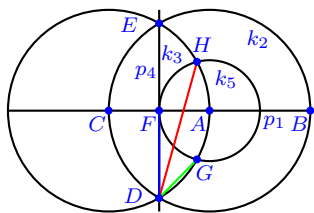
## Distances

$$\begin{aligned} d_0 &= |DH| = |EG| \text{ (red) } \dots 2 \\ d_1 &= |DF| = |EF| \text{ (blue) } \dots 2 \\ d_2 &= |DG| = |EH| \text{ (green) } \dots 2 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)}$$

## 2.27.5 Construction RCC220



## Construction Process

$A, B$  given initial points

1.  $p_1 = p(A, B)$
2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
3.  $k_3 = k(C, A)$   
 $D \in k_2 \cap k_3, E \in k_2 \cap k_3$
4.  $p_4 = p(D, E)$   
 $F \in p_1 \cap p_4$
5.  $k_5 = k(A, F)$   
 $G \in k_3 \cap k_5, H \in k_3 \cap k_5$

## Distances

$$d_0 = |DH| = |EG| \text{ (red) } \dots 2$$

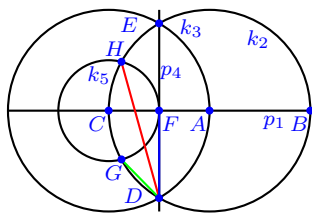
$$d_1 = |DF| = |EF| \text{ (blue) } \dots 2$$

$$d_2 = |DG| = |EH| \text{ (green) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)}$$

## 2.27.6 Construction RCC221



## Construction Process

$A, B$  given initial points

1.  $p_1 = p(A, B)$
2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
3.  $k_3 = k(C, A)$   
 $D \in k_2 \cap k_3, E \in k_2 \cap k_3$
4.  $p_4 = p(D, E)$   
 $F \in p_1 \cap p_4$
5.  $k_5 = k(C, F)$   
 $G \in k_2 \cap k_5, H \in k_2 \cap k_5$

## Distances

$$d_0 = |DH| = |EG| \text{ (red) } \dots 2$$

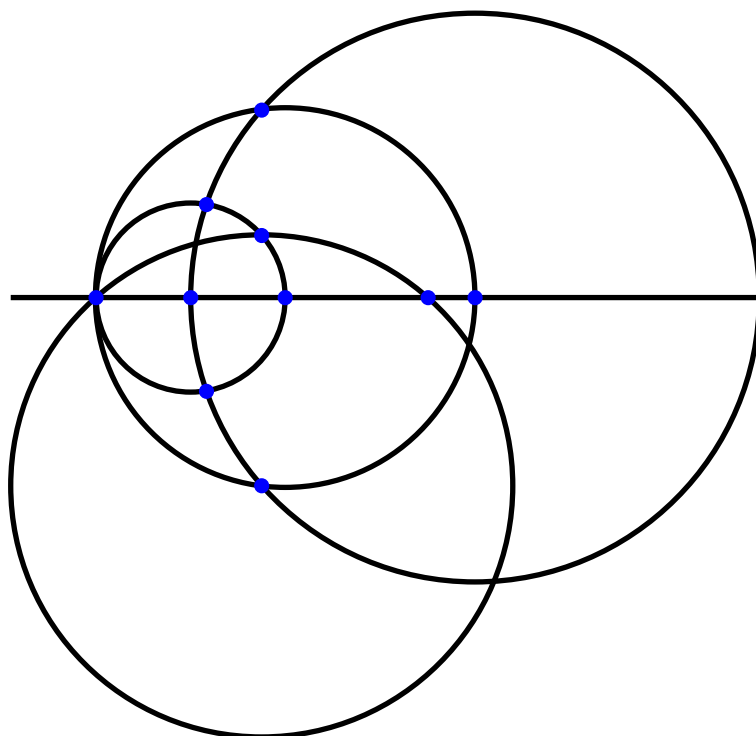
$$d_1 = |DF| = |EF| \text{ (blue) } \dots 2$$

$$d_2 = |DG| = |EH| \text{ (green) } \dots 2$$

## Occurrences of the Golden Ratio

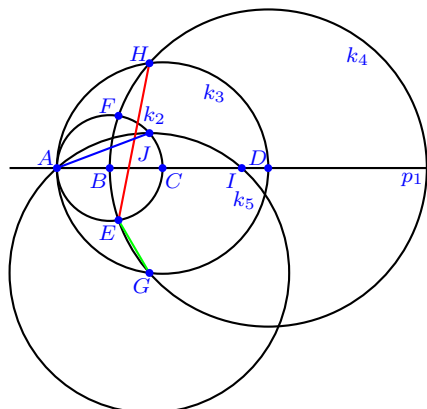
$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)}$$



**2.28 Class of Similar Constructions No. 21****Contained Constructions**

RCC72, RCC79, RCC113, RCC120, RCC222, RCC229, RCC238, RCC245

## 2.28.1 Construction RCC72



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, A)$   
 $D \in p_1 \cap k_3$
  4.  $k_4 = k(D, B)$   
 $E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(G, A)$   
 $I \in p_1 \cap k_5, J \in k_2 \cap k_5$

## Distances

$$d_0 = |EH| = |FG| \text{ (red) } \dots 2$$

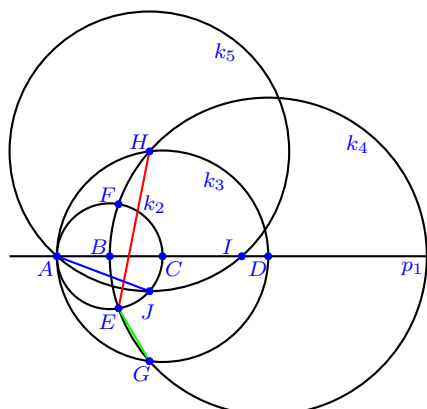
$$d_1 = |AJ| = |IJ| \text{ (blue) } \dots 2$$

$$d_2 = |EG| = |FH| \text{ (green) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)}$$

## 2.28.2 Construction RCC79



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, A)$   
 $D \in p_1 \cap k_3$
  4.  $k_4 = k(D, B)$   
 $E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(H, A)$   
 $I \in p_1 \cap k_5, J \in k_2 \cap k_5$

## Distances

$$d_0 = |EH| = |FG| \text{ (red) } \dots 2$$

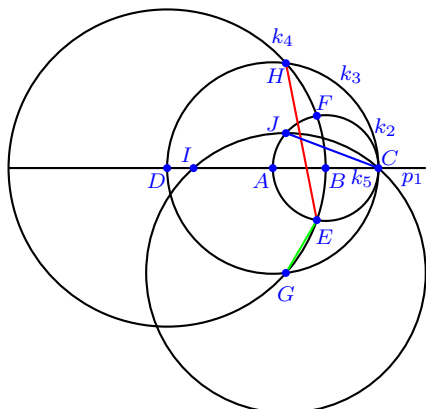
$$d_1 = |AJ| = |IJ| \text{ (blue) } \dots 2$$

$$d_2 = |EG| = |FH| \text{ (green) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)}$$

## 2.28.3 Construction RCC113



## Construction Process

$A, B$  given initial points

1.  $p_1 = p(A, B)$
2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
3.  $k_3 = k(A, C)$   
 $D \in p_1 \cap k_3$
4.  $k_4 = k(D, B)$   
 $E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4$
5.  $k_5 = k(G, C)$   
 $I \in p_1 \cap k_5, J \in k_2 \cap k_5$

## Distances

$$d_0 = |EH| = |FG| \text{ (red) } \dots 2$$

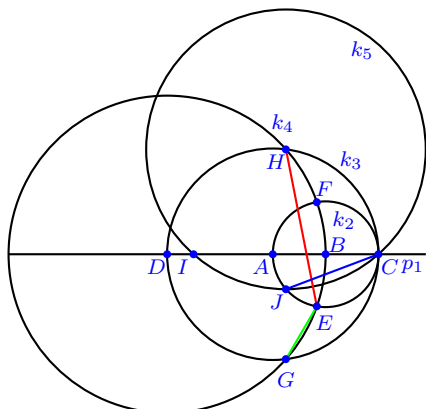
$$d_1 = |CJ| = |IJ| \text{ (blue) } \dots 2$$

$$d_2 = |EG| = |FH| \text{ (green) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)}$$

## 2.28.4 Construction RCC120



## Construction Process

$A, B$  given initial points

1.  $p_1 = p(A, B)$
2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
3.  $k_3 = k(A, C)$   
 $D \in p_1 \cap k_3$
4.  $k_4 = k(D, B)$   
 $E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4$
5.  $k_5 = k(H, C)$   
 $I \in p_1 \cap k_5, J \in k_2 \cap k_5$

## Distances

$$d_0 = |EH| = |FG| \text{ (red) } \dots 2$$

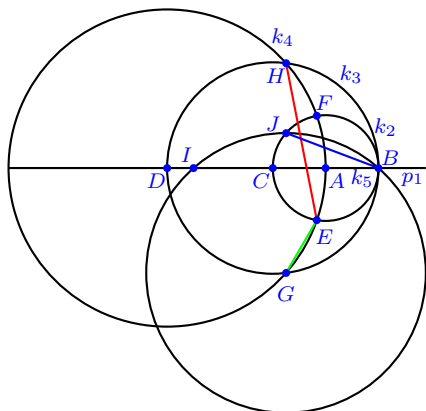
$$d_1 = |CJ| = |IJ| \text{ (blue) } \dots 2$$

$$d_2 = |EG| = |FH| \text{ (green) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)}$$

## 2.28.5 Construction RCC222



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, B)$   
 $D \in p_1 \cap k_3$
  4.  $k_4 = k(D, A)$   
 $E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(G, B)$   
 $I \in p_1 \cap k_5, J \in k_2 \cap k_5$

## Distances

$$d_0 = |EH| = |FG| \text{ (red) } \dots 2$$

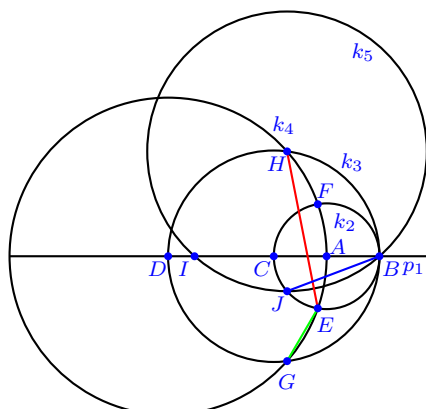
$$d_1 = |BJ| = |IJ| \text{ (blue) } \dots 2$$

$$d_2 = |EG| = |FH| \text{ (green) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)}$$

## 2.28.6 Construction RCC229



## Construction Process

$A, B$  given initial points

1.  $p_1 = p(A, B)$
2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
3.  $k_3 = k(C, B)$   
 $D \in p_1 \cap k_3$
4.  $k_4 = k(D, A)$   
 $E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4$
5.  $k_5 = k(H, B)$   
 $I \in p_1 \cap k_5, J \in k_2 \cap k_5$

## Distances

$$d_0 = |EH| = |FG| \text{ (red) } \dots 2$$

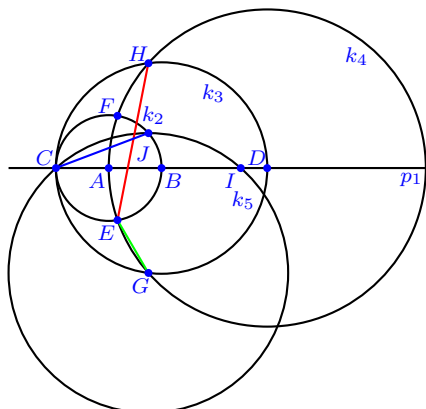
$$d_1 = |BJ| = |IJ| \text{ (blue) } \dots 2$$

$$d_2 = |EG| = |FH| \text{ (green) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)}$$

## 2.28.7 Construction RCC238



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(B, C)$   
 $D \in p_1 \cap k_3$
  4.  $k_4 = k(D, A)$   
 $E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(G, C)$   
 $I \in p_1 \cap k_5, J \in k_2 \cap k_5$

## Distances

$$d_0 = |EH| = |FG| \text{ (red) } \dots 2$$

$$d_1 = |CJ| = |IJ| \text{ (blue) } \dots 2$$

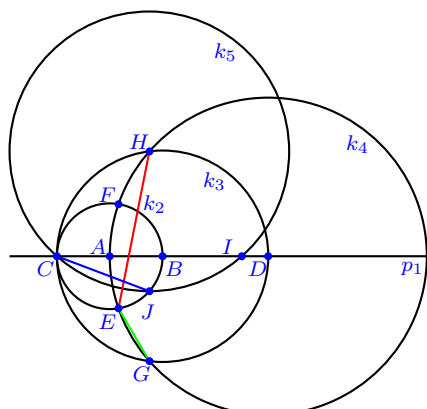
$$d_2 = |EG| = |FH| \text{ (green) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)}$$



## 2.28.8 Construction RCC245



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(B, C)$   
 $D \in p_1 \cap k_3$
  4.  $k_4 = k(D, A)$   
 $E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(H, C)$   
 $I \in p_1 \cap k_5, J \in k_2 \cap k_5$

## Distances

$$d_0 = |EH| = |FG| \text{ (red)} \dots 2$$

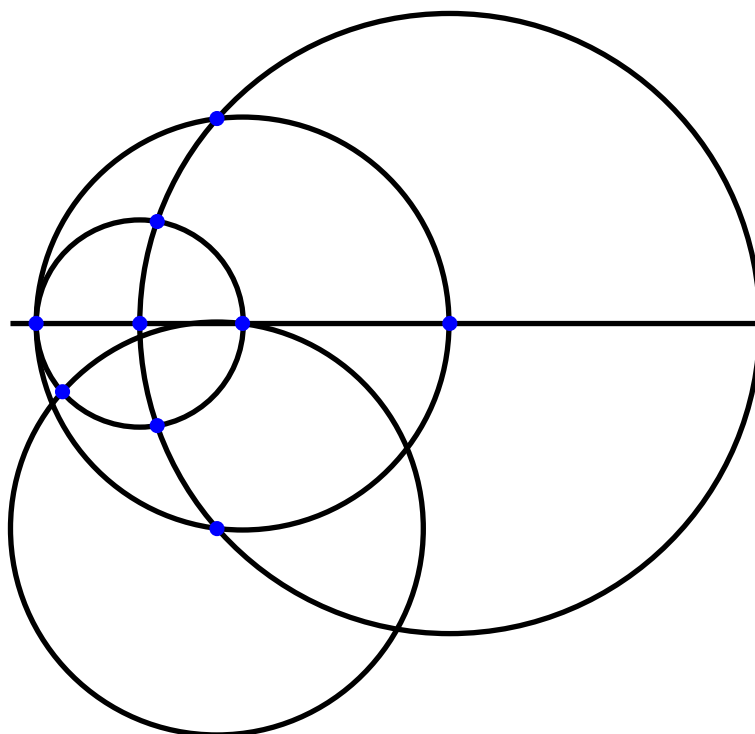
$$d_1 = |CJ| = |IJ| \text{ (blue)} \dots 2$$

$$d_2 = |EG| = |FH| \text{ (green)} \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)}$$

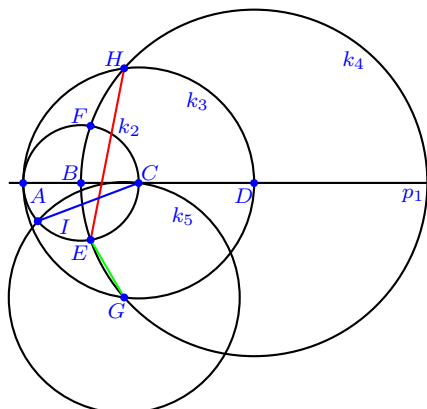
## 2.29 Class of Similar Constructions No. 22



### Contained Constructions

RCC73, RCC80, RCC112, RCC119, RCC223, RCC230, RCC237, RCC244

## 2.29.1 Construction RCC73



## Construction Process

$A, B$  given initial points

1.  $p_1 = p(A, B)$
2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
3.  $k_3 = k(C, A)$   
 $D \in p_1 \cap k_3$
4.  $k_4 = k(D, B)$   
 $E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4$
5.  $k_5 = k(G, C)$   
 $I \in k_2 \cap k_5$

## Distances

$$d_0 = |EH| = |FG| \text{ (red) } \dots 2$$

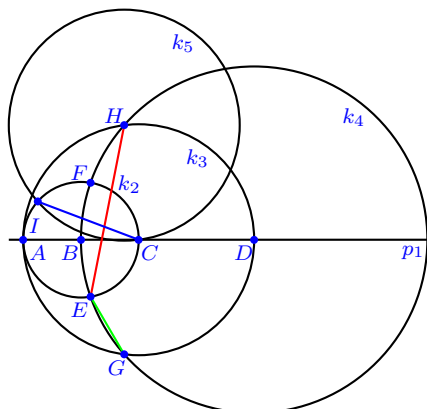
$$d_1 = |CI| \text{ (blue) } \dots 1$$

$$d_2 = |EG| = |FH| \text{ (green) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (4 occurrences)}$$

## 2.29.2 Construction RCC80



## Construction Process

$A, B$  given initial points

1.  $p_1 = p(A, B)$
2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
3.  $k_3 = k(C, A)$   
 $D \in p_1 \cap k_3$
4.  $k_4 = k(D, B)$   
 $E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4$
5.  $k_5 = k(H, C)$   
 $I \in k_2 \cap k_5$

## Distances

$$d_0 = |EH| = |FG| \text{ (red) } \dots 2$$

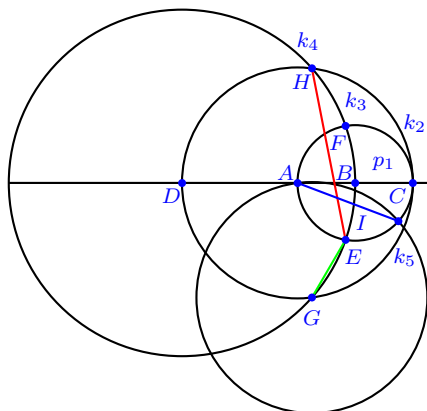
$$d_1 = |CI| \text{ (blue) } \dots 1$$

$$d_2 = |EG| = |FH| \text{ (green) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (4 occurrences)}$$

## 2.29.3 Construction RCC112



## Construction Process

$A, B$  given initial points

1.  $p_1 = p(A, B)$
2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
3.  $k_3 = k(A, C)$   
 $D \in p_1 \cap k_3$
4.  $k_4 = k(D, B)$   
 $E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4$
5.  $k_5 = k(G, A)$   
 $I \in k_2 \cap k_5$

## Distances

$$d_0 = |EH| = |FG| \text{ (red) } \dots 2$$

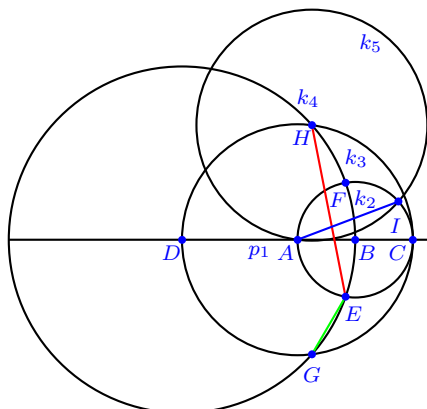
$$d_1 = |AI| \text{ (blue) } \dots 1$$

$$d_2 = |EG| = |FH| \text{ (green) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (4 occurrences)}$$

## 2.29.4 Construction RCC119



## Construction Process

$A, B$  given initial points

1.  $p_1 = p(A, B)$
2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
3.  $k_3 = k(A, C)$   
 $D \in p_1 \cap k_3$
4.  $k_4 = k(D, B)$   
 $E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4$
5.  $k_5 = k(H, A)$   
 $I \in k_2 \cap k_5$

## Distances

$$d_0 = |EH| = |FG| \text{ (red) } \dots 2$$

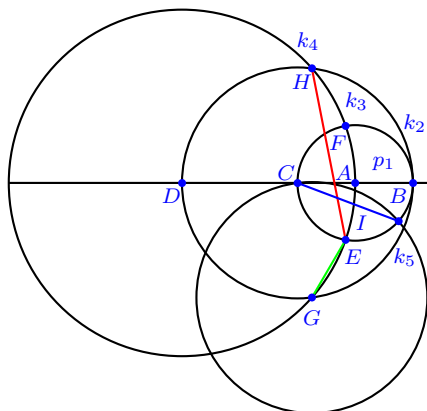
$$d_1 = |AI| \text{ (blue) } \dots 1$$

$$d_2 = |EG| = |FH| \text{ (green) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (4 occurrences)}$$

## 2.29.5 Construction RCC223



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, B)$   
 $D \in p_1 \cap k_3$
  4.  $k_4 = k(D, A)$   
 $E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(G, C)$   
 $I \in k_2 \cap k_5$

## Distances

$$d_0 = |EH| = |FG| \text{ (red) } \dots 2$$

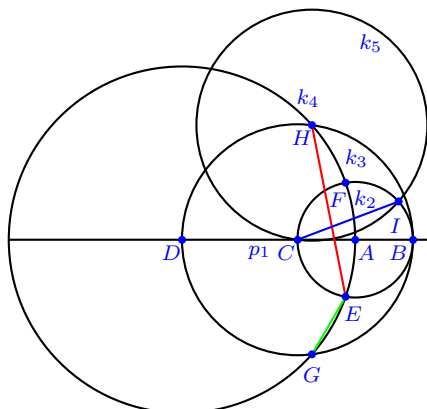
$$d_1 = |CI| \text{ (blue) } \dots 1$$

$$d_2 = |EG| = |FH| \text{ (green) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (4 occurrences)}$$

## 2.29.6 Construction RCC230



## Construction Process

$A, B$  given initial points

1.  $p_1 = p(A, B)$
2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
3.  $k_3 = k(C, B)$   
 $D \in p_1 \cap k_3$
4.  $k_4 = k(D, A)$   
 $E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4$
5.  $k_5 = k(H, C)$   
 $I \in k_2 \cap k_5$

## Distances

$$d_0 = |EH| = |FG| \text{ (red) } \dots 2$$

$$d_1 = |CI| \text{ (blue) } \dots 1$$

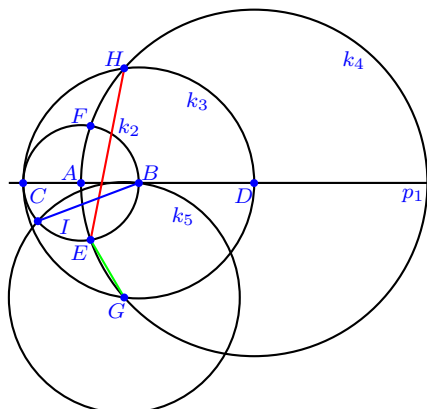
$$d_2 = |EG| = |FH| \text{ (green) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (4 occurrences)}$$



## 2.29.7 Construction RCC237



## Construction Process

$A, B$  given initial points

1.  $p_1 = p(A, B)$
2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
3.  $k_3 = k(B, C)$   
 $D \in p_1 \cap k_3$
4.  $k_4 = k(D, A)$   
 $E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4$
5.  $k_5 = k(G, B)$   
 $I \in k_2 \cap k_5$

## Distances

$$d_0 = |EH| = |FG| \text{ (red) } \dots 2$$

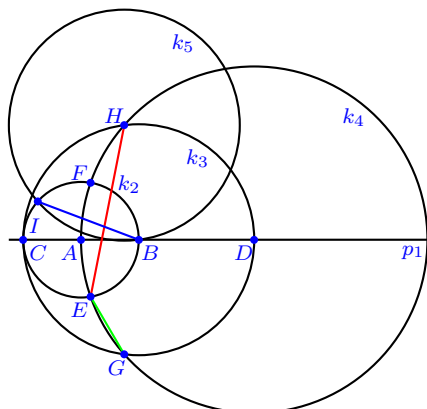
$$d_1 = |BI| \text{ (blue) } \dots 1$$

$$d_2 = |EG| = |FH| \text{ (green) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (4 occurrences)}$$

## 2.29.8 Construction RCC244



## Construction Process

$A, B$  given initial points

1.  $p_1 = p(A, B)$
2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
3.  $k_3 = k(B, C)$   
 $D \in p_1 \cap k_3$
4.  $k_4 = k(D, A)$   
 $E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4$
5.  $k_5 = k(H, B)$   
 $I \in k_2 \cap k_5$

## Distances

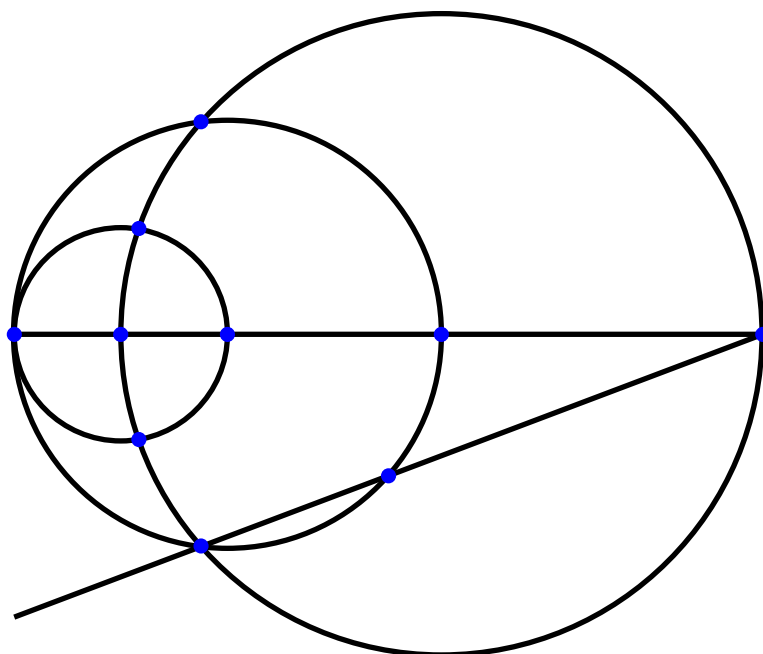
$$d_0 = |EH| = |FG| \text{ (red) } \dots 2$$

$$d_1 = |BI| \text{ (blue) } \dots 1$$

$$d_2 = |EG| = |FH| \text{ (green) } \dots 2$$

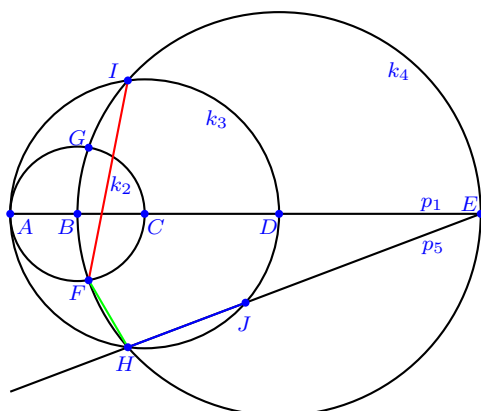
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (4 occurrences)}$$

**2.30 Class of Similar Constructions No. 23****Contained Constructions**

RCC74, RCC81, RCC114, RCC121, RCC224, RCC231, RCC239, RCC246

## 2.30.1 Construction RCC74



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, A)$   
 $D \in p_1 \cap k_3$
  4.  $k_4 = k(D, B)$   
 $E \in p_1 \cap k_4, F \in k_2 \cap k_4, G \in k_2 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  5.  $p_5 = p(E, H)$   
 $J \in p_5 \cap k_3$

## Distances

$$d_0 = |FI| = |GH| \text{ (red)} \dots 2$$

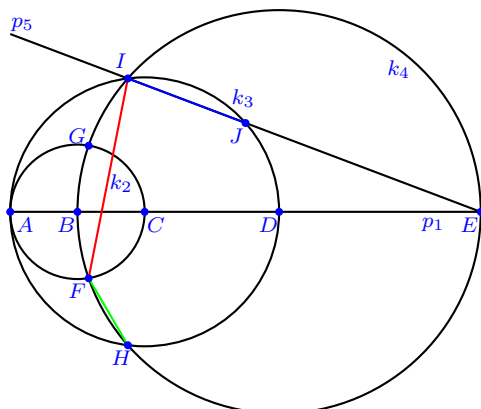
$$d_1 = |HJ| \text{ (blue)} \dots 1$$

$$d_2 = |FH| = |GI| \text{ (green)} \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (4 occurrences)}$$

## 2.30.2 Construction RCC81



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, A)$   
 $D \in p_1 \cap k_3$
  4.  $k_4 = k(D, B)$   
 $E \in p_1 \cap k_4, F \in k_2 \cap k_4, G \in k_2 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  5.  $p_5 = p(E, I)$   
 $J \in p_5 \cap k_3$

## Distances

$$d_0 = |FI| = |GH| \text{ (red)} \dots 2$$

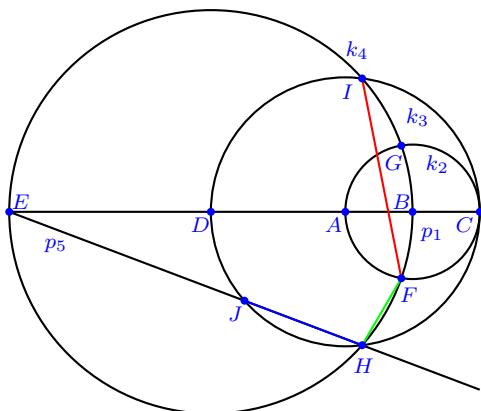
$$d_1 = |IJ| \text{ (blue)} \dots 1$$

$$d_2 = |FH| = |GI| \text{ (green)} \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (4 occurrences)}$$

## 2.30.3 Construction RCC114



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(A, C)$   
 $D \in p_1 \cap k_3$
  4.  $k_4 = k(D, B)$   
 $E \in p_1 \cap k_4, F \in k_2 \cap k_4, G \in k_2 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  5.  $p_5 = p(E, H)$   
 $J \in p_5 \cap k_3$

## Distances

$$d_0 = |FI| = |GH| \text{ (red)} \dots 2$$

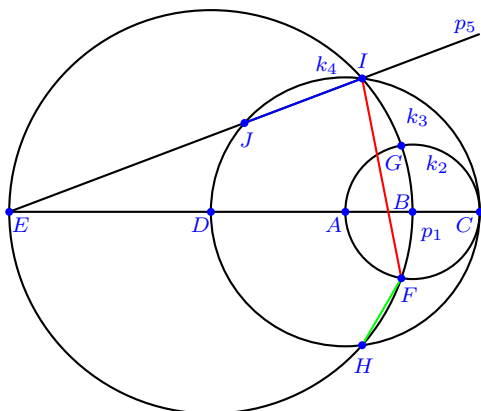
$$d_1 = |HJ| \text{ (blue)} \dots 1$$

$$d_2 = |FH| = |GI| \text{ (green)} \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (4 occurrences)}$$

## 2.30.4 Construction RCC121



## Construction Process

$A, B$  given initial points

1.  $p_1 = p(A, B)$
2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
3.  $k_3 = k(A, C)$   
 $D \in p_1 \cap k_3$
4.  $k_4 = k(D, B)$   
 $E \in p_1 \cap k_4, F \in k_2 \cap k_4, G \in k_2 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
5.  $p_5 = p(E, I)$   
 $J \in p_5 \cap k_3$

## Distances

$$d_0 = |FI| = |GH| \text{ (red)} \dots 2$$

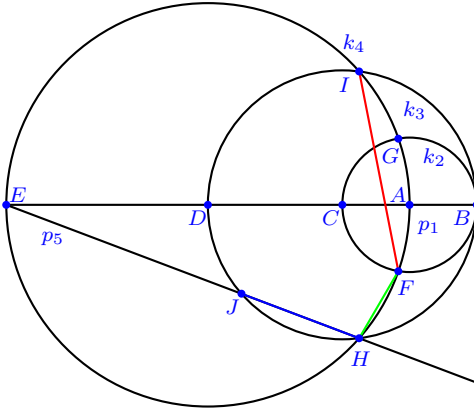
$$d_1 = |IJ| \text{ (blue)} \dots 1$$

$$d_2 = |FH| = |GI| \text{ (green)} \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (4 occurrences)}$$

## 2.30.5 Construction RCC224



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, B)$   
 $D \in p_1 \cap k_3$
  4.  $k_4 = k(D, A)$   
 $E \in p_1 \cap k_4, F \in k_2 \cap k_4, G \in k_2 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  5.  $p_5 = p(E, H)$   
 $J \in p_5 \cap k_3$

## Distances

$$d_0 = |FI| = |GH| \text{ (red) } \dots 2$$

$$d_1 = |HJ| \text{ (blue) } \dots 1$$

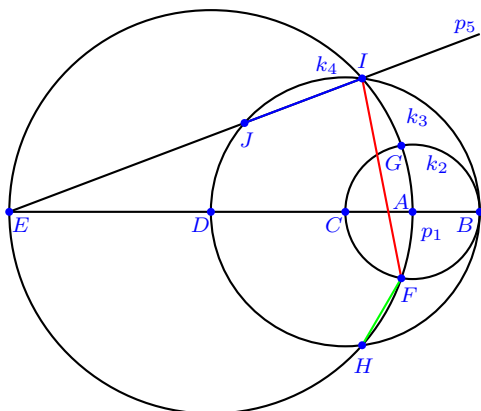
$$d_2 = |FH| = |GI| \text{ (green) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (4 occurrences)}$$



## 2.30.6 Construction RCC231



## Construction Process

$A, B$  given initial points

1.  $p_1 = p(A, B)$
2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
3.  $k_3 = k(C, B)$   
 $D \in p_1 \cap k_3$
4.  $k_4 = k(D, A)$   
 $E \in p_1 \cap k_4, F \in k_2 \cap k_4, G \in k_2 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
5.  $p_5 = p(E, I)$   
 $J \in p_5 \cap k_3$

## Distances

$$d_0 = |FI| = |GH| \text{ (red)} \dots 2$$

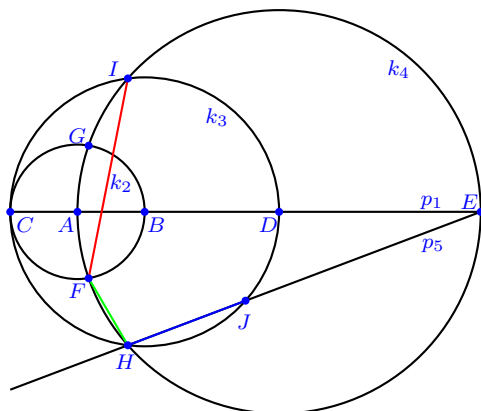
$$d_1 = |IJ| \text{ (blue)} \dots 1$$

$$d_2 = |FH| = |GI| \text{ (green)} \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (4 occurrences)}$$

## 2.30.7 Construction RCC239



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(B, C)$   
 $D \in p_1 \cap k_3$
  4.  $k_4 = k(D, A)$   
 $E \in p_1 \cap k_4, F \in k_2 \cap k_4, G \in k_2 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  5.  $p_5 = p(E, H)$   
 $J \in p_5 \cap k_3$

## Distances

$$d_0 = |FI| = |GH| \text{ (red)} \dots 2$$

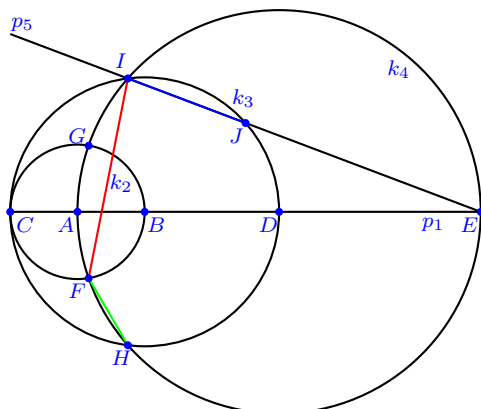
$$d_1 = |HJ| \text{ (blue)} \dots 1$$

$$d_2 = |FH| = |GI| \text{ (green)} \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (4 occurrences)}$$

## 2.30.8 Construction RCC246



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(B, C)$   
 $D \in p_1 \cap k_3$
  4.  $k_4 = k(D, A)$   
 $E \in p_1 \cap k_4, F \in k_2 \cap k_4, G \in k_2 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  5.  $p_5 = p(E, I)$   
 $J \in p_5 \cap k_3$

## Distances

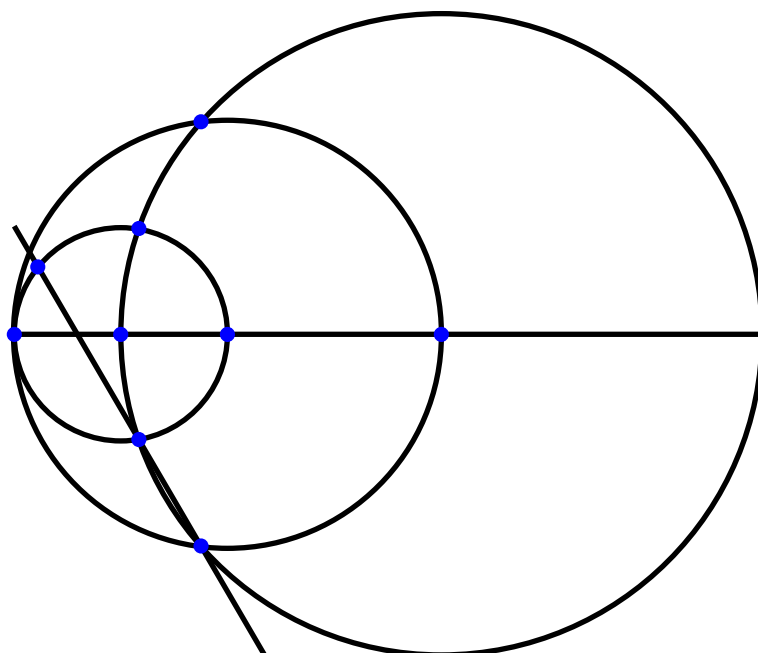
$$d_0 = |FI| = |GH| \text{ (red)} \dots 2$$

$$d_1 = |IJ| \text{ (blue)} \dots 1$$

$$d_2 = |FH| = |GI| \text{ (green)} \dots 2$$

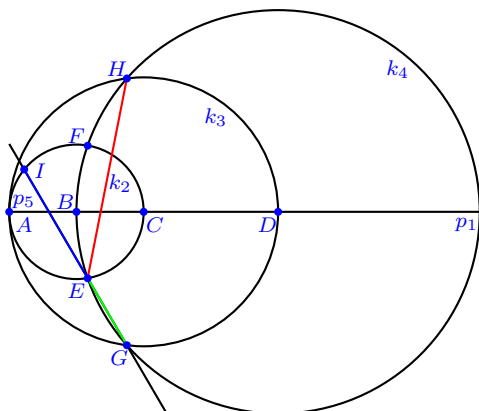
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (4 occurrences)}$$

**2.31 Class of Similar Constructions No. 24****Contained Constructions**

RCC75, RCC84, RCC115, RCC124, RCC225, RCC234, RCC240, RCC249

## 2.31.1 Construction RCC75



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, A)$   
 $D \in p_1 \cap k_3$
  4.  $k_4 = k(D, B)$   
 $E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4$
  5.  $p_5 = p(E, G)$   
 $I \in p_5 \cap k_2$

## Distances

$$d_0 = |EH| = |FG| = |GI| \text{ (red) } \dots 3$$

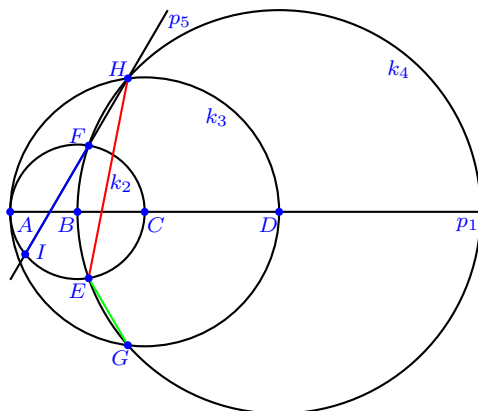
$$d_1 = |EI| \text{ (blue) } \dots 1$$

$$d_2 = |EG| = |FH| \text{ (green) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (5 occurrences)}$$

## 2.31.2 Construction RCC84



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, A)$   
 $D \in p_1 \cap k_3$
  4.  $k_4 = k(D, B)$   
 $E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4$
  5.  $p_5 = p(F, H)$   
 $I \in p_5 \cap k_2$

## Distances

$$d_0 = |EH| = |FG| = |HI| \text{ (red) } \dots 3$$

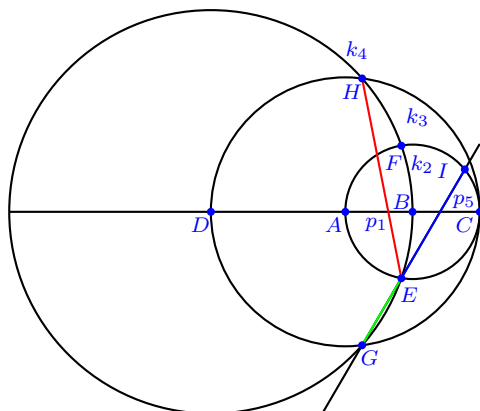
$$d_1 = |FI| \text{ (blue) } \dots 1$$

$$d_2 = |EG| = |FH| \text{ (green) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (5 occurrences)}$$

## 2.31.3 Construction RCC115



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(A, C)$   
 $D \in p_1 \cap k_3$
  4.  $k_4 = k(D, B)$   
 $E \in k_2 \cap k_4, F \in k_2 \cap k_3, G \in k_3 \cap k_4, H \in k_3 \cap k_4$
  5.  $p_5 = p(E, G)$   
 $I \in p_5 \cap k_2$

## Distances

$$d_0 = |EH| = |FG| = |GI| \text{ (red) } \dots 3$$

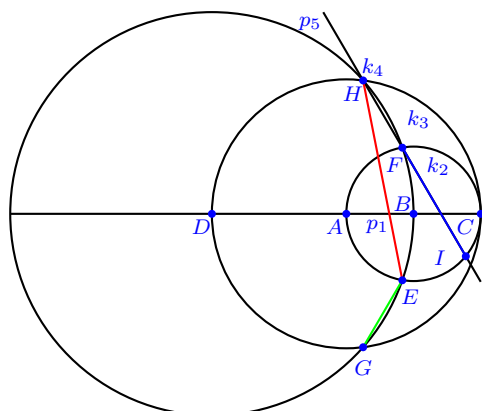
$$d_1 = |EI| \text{ (blue) } \dots 1$$

$$d_2 = |EG| = |FH| \text{ (green) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (5 occurrences)}$$

## 2.31.4 Construction RCC124



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(A, C)$   
 $D \in p_1 \cap k_3$
  4.  $k_4 = k(D, B)$   
 $E \in k_2 \cap k_4, F \in k_2 \cap k_1, G \in k_3 \cap k_4, H \in k_3 \cap k_4$
  5.  $p_5 = p(F, H)$   
 $I \in p_5 \cap k_2$

## Distances

$$d_0 = |EH| = |FG| = |HI| \text{ (red) } \dots 3$$

$$d_1 = |FI| \text{ (blue) } \dots 1$$

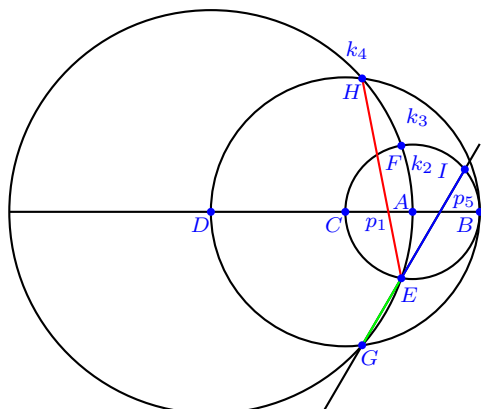
$$d_2 = |EG| = |FH| \text{ (green) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (5 occurrences)}$$



## 2.31.5 Construction RCC225



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, B)$   
 $D \in p_1 \cap k_3$
  4.  $k_4 = k(D, A)$   
 $E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4$
  5.  $p_5 = p(E, G)$   
 $I \in p_5 \cap k_2$

## Distances

$$d_0 = |EH| = |FG| = |GI| \text{ (red)} \dots 3$$

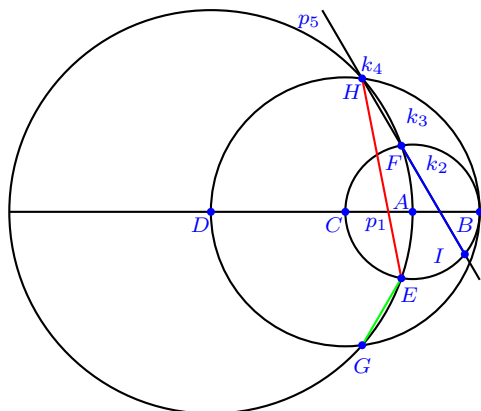
$$d_1 = |EI| \text{ (blue)} \dots 1$$

$$d_2 = |EG| = |FH| \text{ (green)} \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (5 occurrences)}$$

## 2.31.6 Construction RCC234



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, B)$   
 $D \in p_1 \cap k_3$
  4.  $k_4 = k(D, A)$   
 $E \in k_2 \cap k_4$ ,  $F \in k_2 \cap k_3$ ,  $G \in k_3 \cap k_4$ ,  $H \in k_3 \cap k_4$
  5.  $p_5 = p(F, H)$   
 $I \in p_5 \cap k_2$

## Distances

$$d_0 = |EH| = |FG| = |HI| \text{ (red) } \dots 3$$

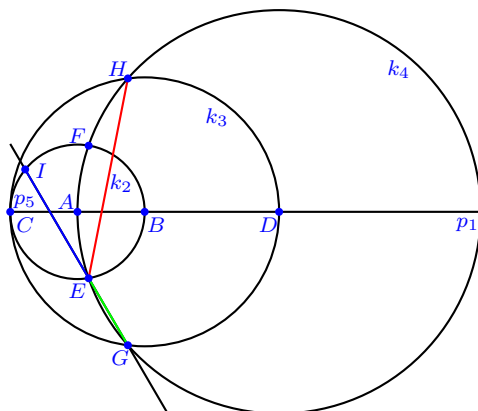
$$d_1 = |FI| \text{ (blue) } \dots 1$$

$$d_2 = |EG| = |FH| \text{ (green) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (5 occurrences)}$$

## 2.31.7 Construction RCC240



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(B, C)$   
 $D \in p_1 \cap k_3$
  4.  $k_4 = k(D, A)$   
 $E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4$
  5.  $p_5 = p(E, G)$   
 $I \in p_5 \cap k_2$

## Distances

$$d_0 = |EH| = |FG| = |GI| \text{ (red) } \dots 3$$

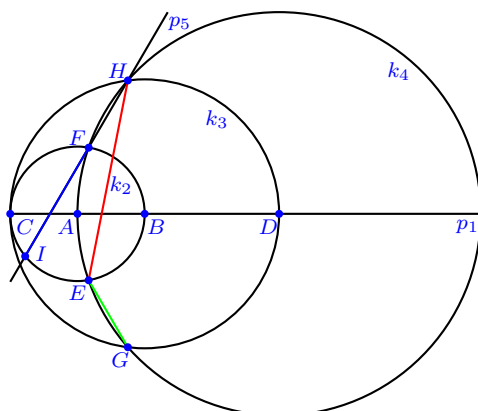
$$d_1 = |EI| \text{ (blue) } \dots 1$$

$$d_2 = |EG| = |FH| \text{ (green) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (5 occurrences)}$$

## 2.31.8 Construction RCC249



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(B, C)$   
 $D \in p_1 \cap k_3$
  4.  $k_4 = k(D, A)$   
 $E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4$
  5.  $p_5 = p(F, H)$   
 $I \in p_5 \cap k_2$

## Distances

$$d_0 = |EH| = |FG| = |HI| \text{ (red) } \dots 3$$

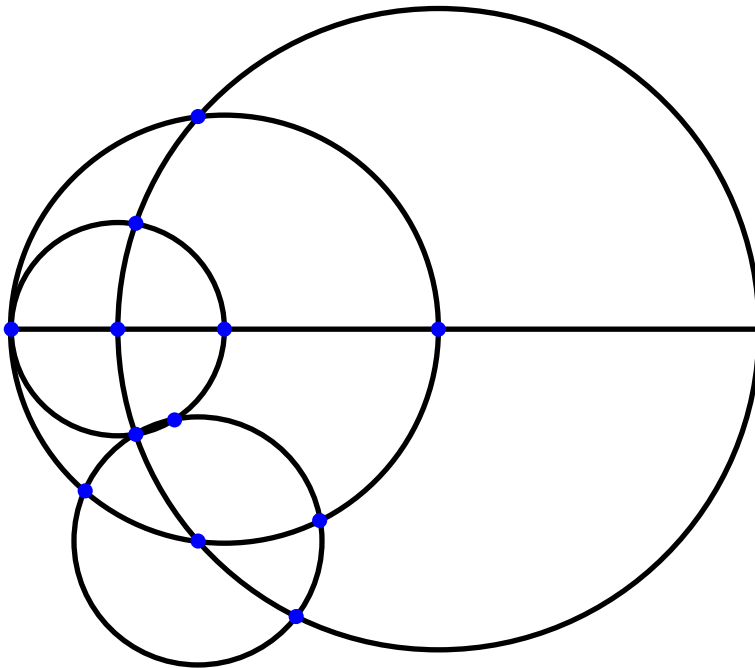
$$d_1 = |FI| \text{ (blue) } \dots 1$$

$$d_2 = |EG| = |FH| \text{ (green) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (5 occurrences)}$$

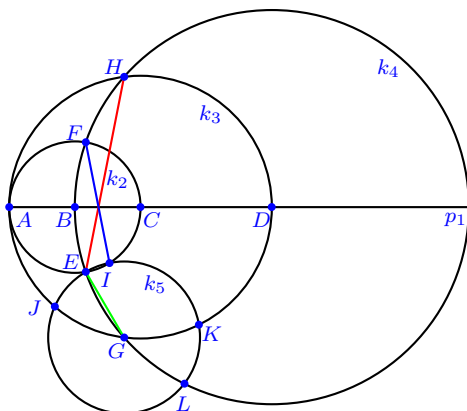
### 2.32 Class of Similar Constructions No. 25



#### Contained Constructions

RCC76, RCC85, RCC116, RCC125, RCC226, RCC235, RCC241, RCC250

## 2.32.1 Construction RCC76



## Construction Process

$A, B$  given initial points

1.  $p_1 = p(A, B)$
2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
3.  $k_3 = k(C, A)$   
 $D \in p_1 \cap k_3$
4.  $k_4 = k(D, B)$   
 $E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4$
5.  $k_5 = k(G, E)$   
 $I \in k_2 \cap k_5, J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5$

## Distances

$$d_0 = |EH| = |FG| \text{ (red) } \dots 2$$

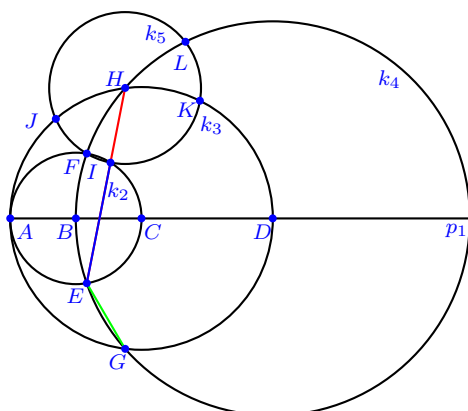
$$d_1 = |FI| \text{ (blue) } \dots 1$$

$$d_2 = |EG| = |FH| = |GI| = |GJ| = |GK| = |GL| \text{ (green) } \dots 6$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)}$$

## 2.32.2 Construction RCC85



## Construction Process

$A, B$  given initial points

1.  $p_1 = p(A, B)$
2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
3.  $k_3 = k(C, A)$   
 $D \in p_1 \cap k_3$
4.  $k_4 = k(D, B)$   
 $E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4$
5.  $k_5 = k(H, F)$   
 $I \in k_2 \cap k_5, J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5$

## Distances

$$d_0 = |EH| = |FG| \text{ (red) } \dots 2$$

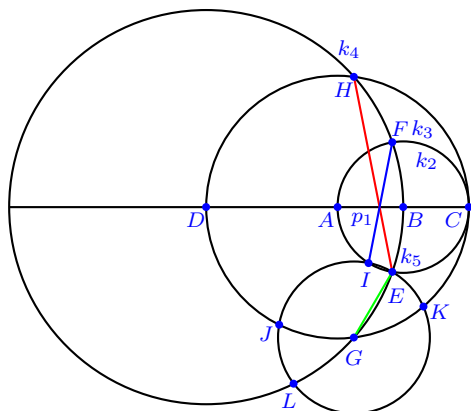
$$d_1 = |EI| \text{ (blue) } \dots 1$$

$$d_2 = |EG| = |FH| = |HI| = |HJ| = |HK| = |HL| \text{ (green) } \dots 6$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)}$$

## 2.32.3 Construction RCC116



## Construction Process

$A, B$  given initial points

1.  $p_1 = p(A, B)$
2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
3.  $k_3 = k(A, C)$   
 $D \in p_1 \cap k_3$
4.  $k_4 = k(D, B)$   
 $E \in k_2 \cap k_4, F \in k_2 \cap k_3, G \in k_3 \cap k_4, H \in k_3 \cap k_4$
5.  $k_5 = k(G, E)$   
 $I \in k_2 \cap k_5, J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5$

## Distances

$$d_0 = |EH| = |FG| \text{ (red) } \dots 2$$

$$d_1 = |FI| \text{ (blue) } \dots 1$$

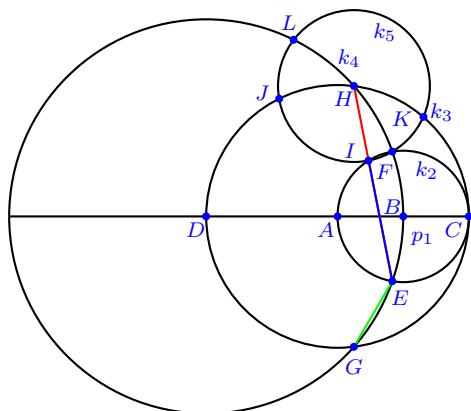
$$d_2 = |EG| = |FH| = |GI| = |GJ| = |GK| = |GL| \text{ (green) } \dots 6$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)}$$



## 2.32.4 Construction RCC125



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(A, C)$   
 $D \in p_1 \cap k_3$
  4.  $k_4 = k(D, B)$   
 $E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(H, F)$   
 $I \in k_2 \cap k_5, J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5$

## Distances

$$d_0 = |EH| = |FG| \text{ (red) } \dots 2$$

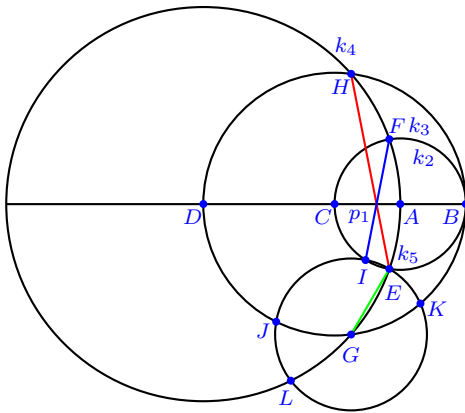
$$d_1 = |EI| \text{ (blue) } \dots 1$$

$$d_2 = |EG| = |FH| = |HI| = |HJ| = |HK| = |HL| \text{ (green) } \dots 6$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)}$$

## 2.32.5 Construction RCC226



## Construction Process

$A, B$  given initial points

1.  $p_1 = p(A, B)$
2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
3.  $k_3 = k(C, B)$   
 $D \in p_1 \cap k_3$
4.  $k_4 = k(D, A)$   
 $E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4$
5.  $k_5 = k(G, E)$   
 $I \in k_2 \cap k_5, J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5$

## Distances

$$d_0 = |EH| = |FG| \text{ (red) } \dots 2$$

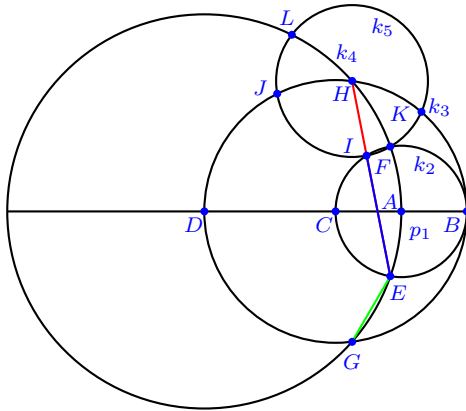
$$d_1 = |FI| \text{ (blue) } \dots 1$$

$$d_2 = |EG| = |FH| = |GI| = |GJ| = |GK| = |GL| \text{ (green) } \dots 6$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)}$$

## 2.32.6 Construction RCC235



## Construction Process

$A, B$  given initial points

1.  $p_1 = p(A, B)$
2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
3.  $k_3 = k(C, B)$   
 $D \in p_1 \cap k_3$
4.  $k_4 = k(D, A)$   
 $E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4$
5.  $k_5 = k(H, F)$   
 $I \in k_2 \cap k_5, J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5$

## Distances

$$d_0 = |EH| = |FG| \text{ (red) } \dots 2$$

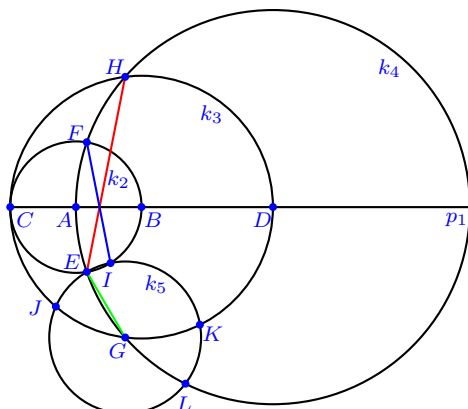
$$d_1 = |EI| \text{ (blue) } \dots 1$$

$$d_2 = |EG| = |FH| = |HI| = |HJ| = |HK| = |HL| \text{ (green) } \dots 6$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)}$$

## 2.32.7 Construction RCC241



## Construction Process

$A, B$  given initial points

1.  $p_1 = p(A, B)$
2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
3.  $k_3 = k(B, C)$   
 $D \in p_1 \cap k_3$
4.  $k_4 = k(D, A)$   
 $E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4$
5.  $k_5 = k(G, E)$   
 $I \in k_2 \cap k_5, J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5$

## Distances

$$d_0 = |EH| = |FG| \text{ (red) } \dots 2$$

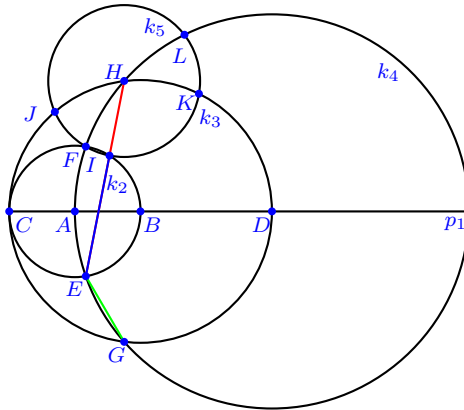
$$d_1 = |FI| \text{ (blue) } \dots 1$$

$$d_2 = |EG| = |FH| = |GI| = |GJ| = |GK| = |GL| \text{ (green) } \dots 6$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)}$$

## 2.32.8 Construction RCC250



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(B, C)$   
 $D \in p_1 \cap k_3$
  4.  $k_4 = k(D, A)$   
 $E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(H, F)$   
 $I \in k_2 \cap k_5, J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5$

## Distances

$$d_0 = |EH| = |FG| \text{ (red)} \dots 2$$

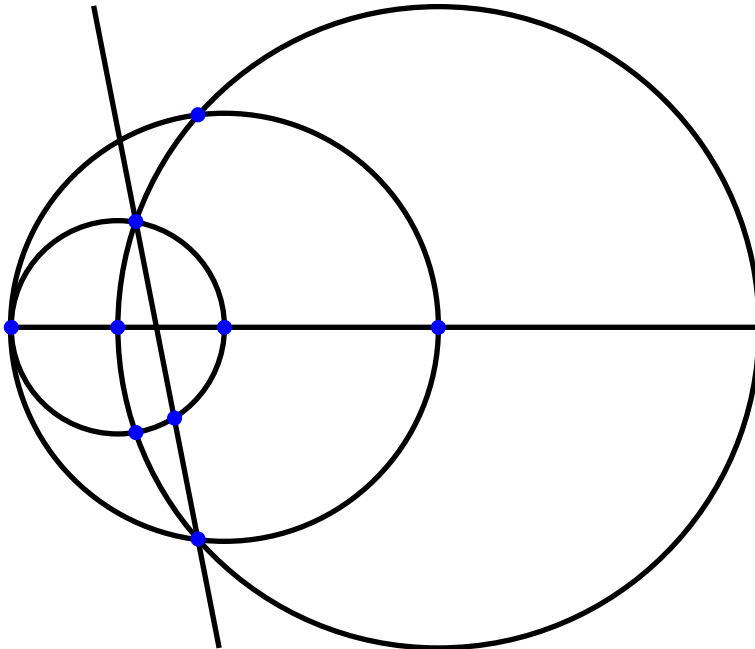
$$d_1 = |EI| \text{ (blue)} \dots 1$$

$$d_2 = |EG| = |FH| = |HI| = |HJ| = |HK| = |HL| \text{ (green)} \dots 6$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)}$$

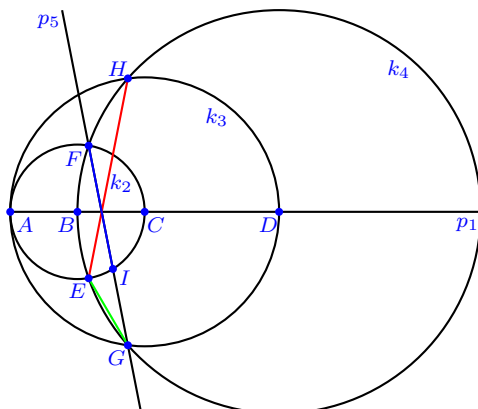
### 2.33 Class of Similar Constructions No. 26



#### Contained Constructions

RCC77, RCC82, RCC117, RCC122, RCC227, RCC232, RCC242, RCC247

## 2.33.1 Construction RCC77



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, A)$   
 $D \in p_1 \cap k_3$
  4.  $k_4 = k(D, B)$   
 $E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4$
  5.  $p_5 = p(F, G)$   
 $I \in p_5 \cap k_2$

## Distances

$$d_0 = |EH| = |FG| \text{ (red) } \dots 2$$

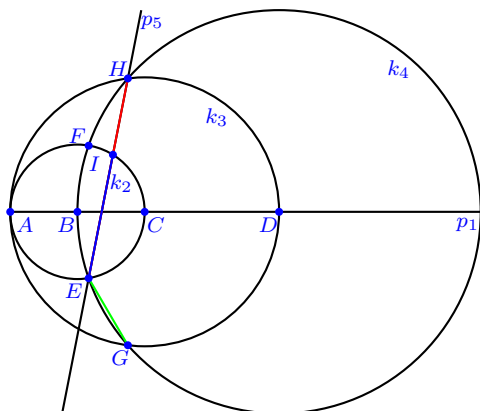
$$d_1 = |FI| \text{ (blue) } \dots 1$$

$$d_2 = |EG| = |FH| = |GI| \text{ (green) } \dots 3$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (5 occurrences)}$$

## 2.33.2 Construction RCC82



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, A)$   
 $D \in p_1 \cap k_3$
  4.  $k_4 = k(D, B)$   
 $E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4$
  5.  $p_5 = p(E, H)$   
 $I \in p_5 \cap k_2$

## Distances

$$d_0 = |EH| = |FG| \text{ (red) } \dots 2$$

$$d_1 = |EI| \text{ (blue) } \dots 1$$

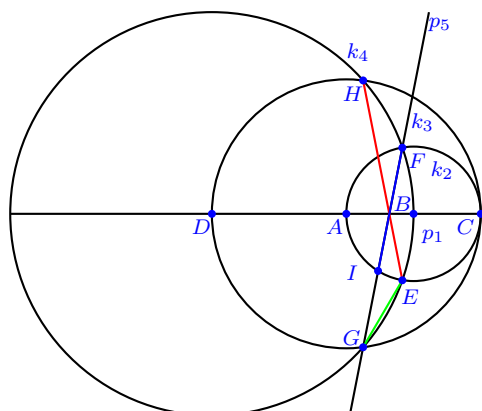
$$d_2 = |EG| = |FH| = |HI| \text{ (green) } \dots 3$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (5 occurrences)}$$



## 2.33.3 Construction RCC117



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(A, C)$   
 $D \in p_1 \cap k_3$
  4.  $k_4 = k(D, B)$   
 $E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4$
  5.  $p_5 = p(F, G)$   
 $I \in p_5 \cap k_2$

## Distances

$$d_0 = |EH| = |FG| \text{ (red) } \dots 2$$

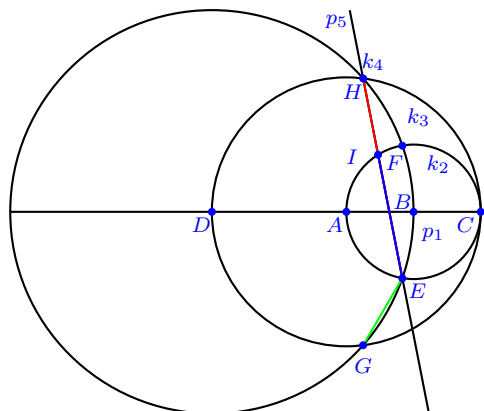
$$d_1 = |FI| \text{ (blue) } \dots 1$$

$$d_2 = |EG| = |FH| = |GI| \text{ (green) } \dots 3$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (5 occurrences)}$$

2.33.4 Construction RCC122



Construction Process

- $A, B$  given initial points
- 1.  $p_1 = p(A, B)$
- 2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
- 3.  $k_3 = k(A, C)$   
 $D \in p_1 \cap k_3$
- 4.  $k_4 = k(D, B)$   
 $E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4$
- 5.  $p_5 = p(E, H)$   
 $I \in p_5 \cap k_2$

Distances

$$d_0 = |EH| = |FG| \text{ (red) } \dots 2$$

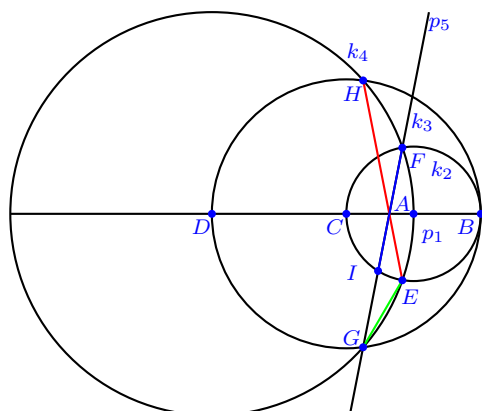
$$d_1 = |EI| \text{ (blue) } \dots 1$$

$$d_2 = |EG| = |FH| = |HI| \text{ (green) } \dots 3$$

Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (5 occurrences)}$$

## 2.33.5 Construction RCC227



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, B)$   
 $D \in p_1 \cap k_3$
  4.  $k_4 = k(D, A)$   
 $E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4$
  5.  $p_5 = p(F, G)$   
 $I \in p_5 \cap k_2$

## Distances

$$d_0 = |EH| = |FG| \text{ (red) } \dots 2$$

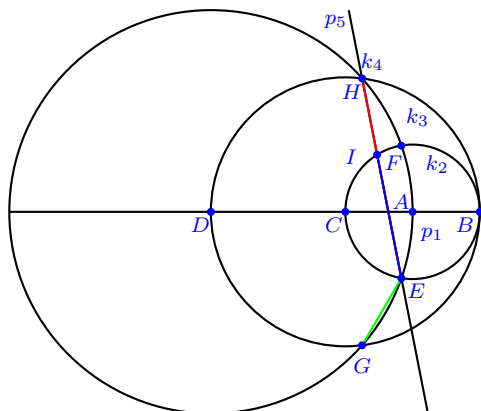
$$d_1 = |FI| \text{ (blue) } \dots 1$$

$$d_2 = |EG| = |FH| = |GI| \text{ (green) } \dots 3$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (5 occurrences)}$$

## 2.33.6 Construction RCC232



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, B)$   
 $D \in p_1 \cap k_3$
  4.  $k_4 = k(D, A)$   
 $E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4$
  5.  $p_5 = p(E, H)$   
 $I \in p_5 \cap k_2$

## Distances

$$d_0 = |EH| = |FG| \text{ (red) } \dots 2$$

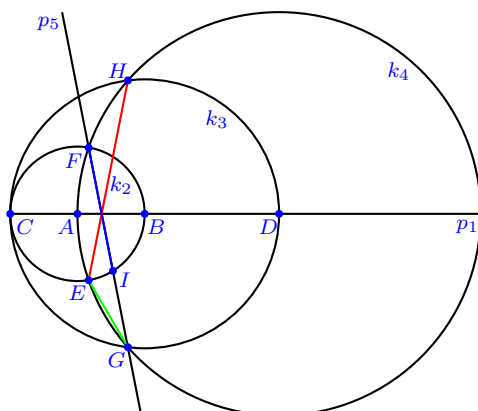
$$d_1 = |EI| \text{ (blue) } \dots 1$$

$$d_2 = |EG| = |FH| = |HI| \text{ (green) } \dots 3$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (5 occurrences)}$$

## 2.33.7 Construction RCC242



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(B, C)$   
 $D \in p_1 \cap k_3$
  4.  $k_4 = k(D, A)$   
 $E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4$
  5.  $p_5 = p(F, G)$   
 $I \in p_5 \cap k_2$

## Distances

$$d_0 = |EH| = |FG| \text{ (red) } \dots 2$$

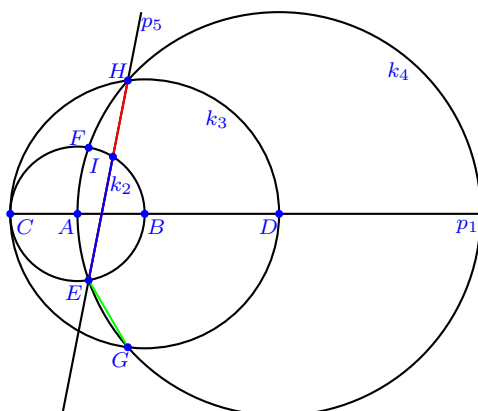
$$d_1 = |FI| \text{ (blue) } \dots 1$$

$$d_2 = |EG| = |FH| = |GI| \text{ (green) } \dots 3$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (5 occurrences)}$$

## 2.33.8 Construction RCC247



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(B, C)$   
 $D \in p_1 \cap k_3$
  4.  $k_4 = k(D, A)$   
 $E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4$
  5.  $p_5 = p(E, H)$   
 $I \in p_5 \cap k_2$

## Distances

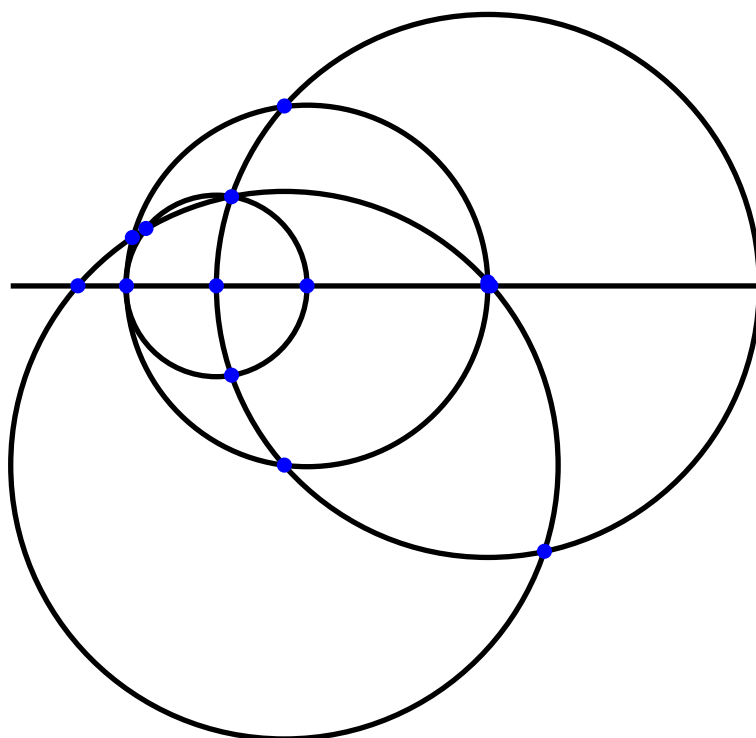
$$d_0 = |EH| = |FG| \text{ (red) } \dots 2$$

$$d_1 = |EI| \text{ (blue) } \dots 1$$

$$d_2 = |EG| = |FH| = |HI| \text{ (green) } \dots 3$$

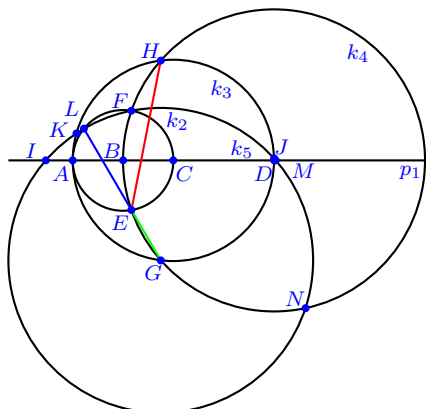
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (5 occurrences)}$$

**2.34 Class of Similar Constructions No. 27****Contained Constructions**

RCC78, RCC83, RCC118, RCC123, RCC228, RCC233, RCC243, RCC248

## 2.34.1 Construction RCC78



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, A)$   
 $D \in p_1 \cap k_3$
  4.  $k_4 = k(D, B)$   
 $E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(G, F)$   
 $I \in p_1 \cap k_5, J \in p_1 \cap k_5, K \in k_2 \cap k_5, L \in k_3 \cap k_5, M \in k_3 \cap k_5,$   
 $N \in k_4 \cap k_5$

## Distances

$$d_0 = |EH| = |FG| = |GI| = |GJ| = |GK| = |GL| = |GM| = |GN| = |HI| = |HJ| \text{ (red)} \dots 10$$

$$d_1 = |EK| \text{ (blue)} \dots 1$$

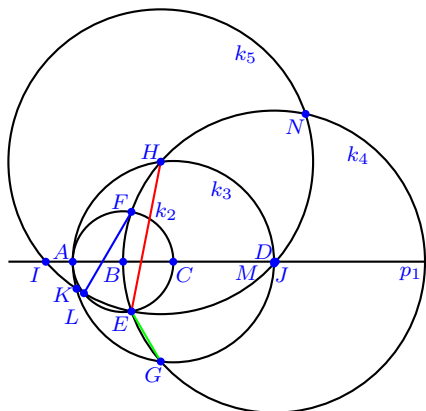
$$d_2 = |EG| = |FH| \text{ (green)} \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (12 occurrences)}$$



## 2.34.2 Construction RCC83



## Construction Process

$A, B$  given initial points

1.  $p_1 = p(A, B)$
2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
3.  $k_3 = k(C, A)$   
 $D \in p_1 \cap k_3$
4.  $k_4 = k(D, B)$   
 $E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4$
5.  $k_5 = k(H, E)$   
 $I \in p_1 \cap k_5, J \in p_1 \cap k_5, K \in k_2 \cap k_5, L \in k_3 \cap k_5, M \in k_3 \cap k_5,$   
 $N \in k_4 \cap k_5$

## Distances

$$d_0 = |EH| = |FG| = |GI| = |GJ| = |HI| = |HJ| = |HK| = |HL| = |HM| = |HN| \text{ (red)} \dots 10$$

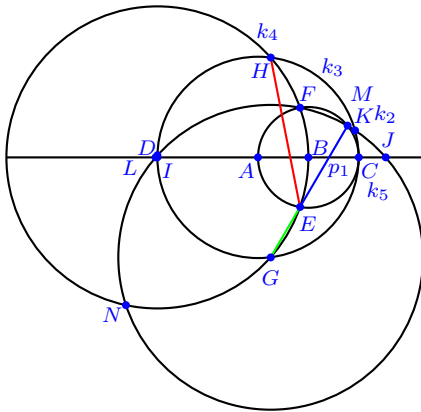
$$d_1 = |FK| \text{ (blue)} \dots 1$$

$$d_2 = |EG| = |FH| \text{ (green)} \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (12 occurrences)}$$

**2.34.3 Construction RCC118**



**Construction Process**

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(A, C)$   
 $D \in p_1 \cap k_3$
  4.  $k_4 = k(D, B)$   
 $E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(G, F)$   
 $I \in p_1 \cap k_5, J \in p_1 \cap k_5, K \in k_2 \cap k_5, L \in k_3 \cap k_5, M \in k_3 \cap k_5,$   
 $N \in k_4 \cap k_5$

**Distances**

$$d_0 = |EH| = |FG| = |GI| = |GJ| = |GK| = |GL| = |GM| = |GN| = |HI| = |HJ| \text{ (red)} \dots 10$$

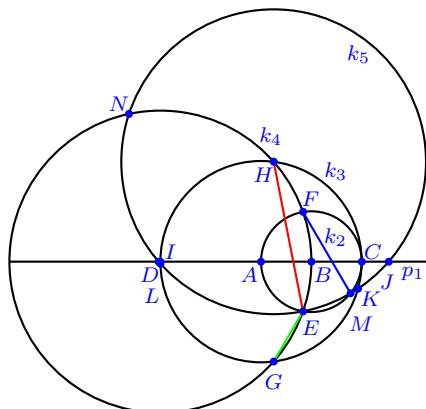
$$d_1 = |EK| \text{ (blue)} \dots 1$$

$$d_2 = |EG| = |FH| \text{ (green)} \dots 2$$

**Occurrences of the Golden Ratio**

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (12 occurrences)}$$

## 2.34.4 Construction RCC123



## Construction Process

$A, B$  given initial points

1.  $p_1 = p(A, B)$
2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
3.  $k_3 = k(A, C)$   
 $D \in p_1 \cap k_3$
4.  $k_4 = k(D, B)$   
 $E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4$
5.  $k_5 = k(H, E)$   
 $I \in p_1 \cap k_5, J \in p_1 \cap k_5, K \in k_2 \cap k_5, L \in k_3 \cap k_5, M \in k_3 \cap k_5,$   
 $N \in k_4 \cap k_5$

## Distances

$$d_0 = |EH| = |FG| = |GI| = |GJ| = |HI| = |HJ| = |HK| = |HL| = |HM| = |HN| \text{ (red) } \dots 10$$

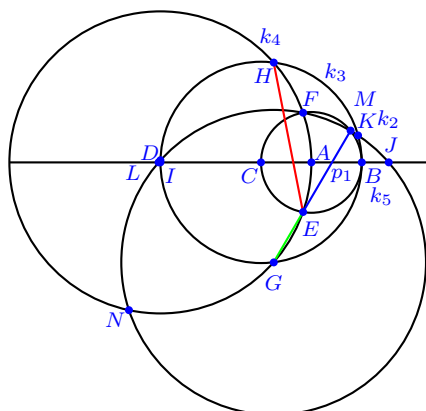
$$d_1 = |FK| \text{ (blue) } \dots 1$$

$$d_2 = |EG| = |FH| \text{ (green) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (12 occurrences)}$$

## 2.34.5 Construction RCC228



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, B)$   
 $D \in p_1 \cap k_3$
  4.  $k_4 = k(D, A)$   
 $E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(G, F)$   
 $I \in p_1 \cap k_5, J \in p_1 \cap k_5, K \in k_2 \cap k_5, L \in k_3 \cap k_5, M \in k_3 \cap k_5,$   
 $N \in k_4 \cap k_5$

## Distances

$$d_0 = |EH| = |FG| = |GI| = |GJ| = |GK| = |GL| = |GM| = |GN| = |HI| = |HJ| \text{ (red)} \dots 10$$

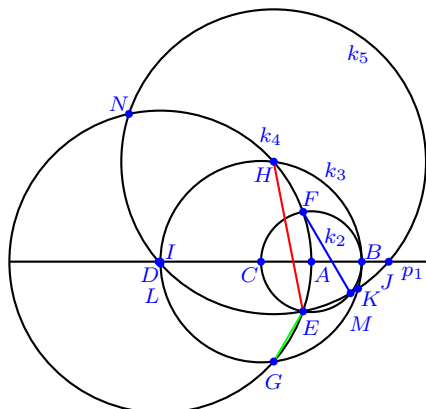
$$d_1 = |EK| \text{ (blue)} \dots 1$$

$$d_2 = |EG| = |FH| \text{ (green)} \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (12 occurrences)}$$

## 2.34.6 Construction RCC233



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, B)$   
 $D \in p_1 \cap k_3$
  4.  $k_4 = k(D, A)$   
 $E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(H, E)$   
 $I \in p_1 \cap k_5, J \in p_1 \cap k_5, K \in k_2 \cap k_5, L \in k_3 \cap k_5, M \in k_3 \cap k_5,$   
 $N \in k_4 \cap k_5$

## Distances

$$d_0 = |EH| = |FG| = |GI| = |GJ| = |HI| = |HJ| = |HK| = |HL| = |HM| = |HN| \text{ (red) } \dots 10$$

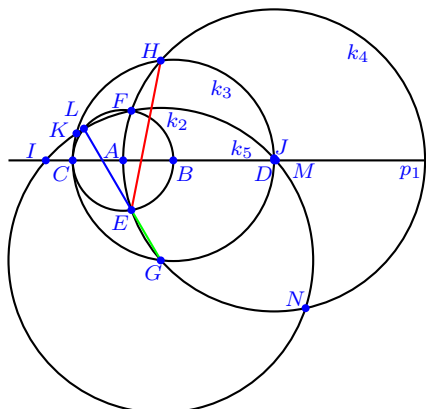
$$d_1 = |FK| \text{ (blue) } \dots 1$$

$$d_2 = |EG| = |FH| \text{ (green) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (12 occurrences)}$$

## 2.34.7 Construction RCC243



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(B, C)$   
 $D \in p_1 \cap k_3$
  4.  $k_4 = k(D, A)$   
 $E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(G, F)$   
 $I \in p_1 \cap k_5, J \in p_1 \cap k_5, K \in k_2 \cap k_5, L \in k_3 \cap k_5, M \in k_3 \cap k_5,$   
 $N \in k_4 \cap k_5$

## Distances

$$d_0 = |EH| = |FG| = |GI| = |GJ| = |GK| = |GL| = |GM| = |GN| = |HI| = |HJ| \text{ (red)} \dots 10$$

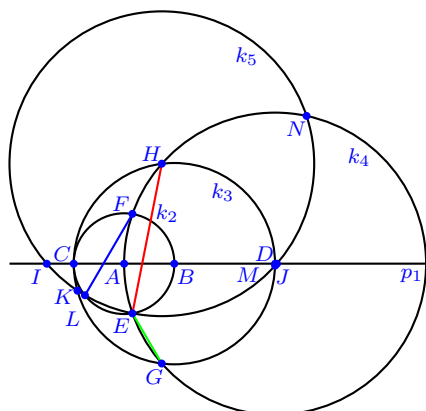
$$d_1 = |EK| \text{ (blue)} \dots 1$$

$$d_2 = |EG| = |FH| \text{ (green)} \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (12 occurrences)}$$

## 2.34.8 Construction RCC248



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(B, C)$   
 $D \in p_1 \cap k_3$
  4.  $k_4 = k(D, A)$   
 $E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(H, E)$   
 $I \in p_1 \cap k_5, J \in p_1 \cap k_5, K \in k_2 \cap k_5, L \in k_3 \cap k_5, M \in k_3 \cap k_5,$   
 $N \in k_4 \cap k_5$

## Distances

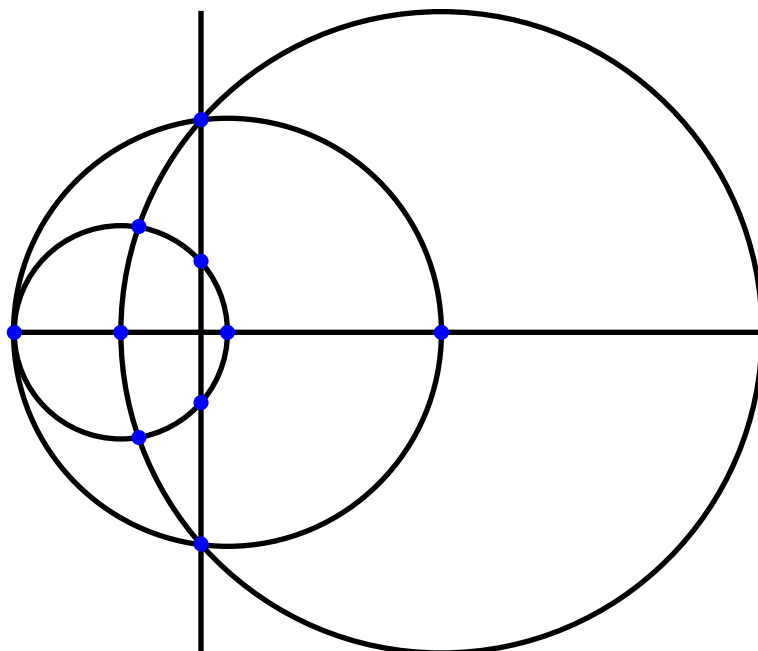
$$d_0 = |EH| = |FG| = |GI| = |GJ| = |HI| = |HJ| = |HK| = |HL| = |HM| = |HN| \text{ (red) } \dots 10$$

$$d_1 = |FK| \text{ (blue) } \dots 1$$

$$d_2 = |EG| = |FH| \text{ (green) } \dots 2$$

## Occurrences of the Golden Ratio

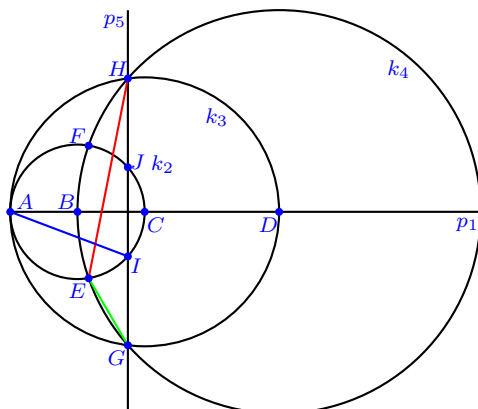
$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (12 occurrences)}$$

**2.35 Class of Similar Constructions No. 28****Contained Constructions**

RCC86, RCC126, RCC236, RCC251



## 2.35.1 Construction RCC86



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, A)$   
 $D \in p_1 \cap k_3$
  4.  $k_4 = k(D, B)$   
 $E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4$
  5.  $p_5 = p(G, H)$   
 $I \in p_5 \cap k_2, J \in p_5 \cap k_2$

## Distances

$$d_0 = |EH| = |FG| \text{ (red) } \dots 2$$

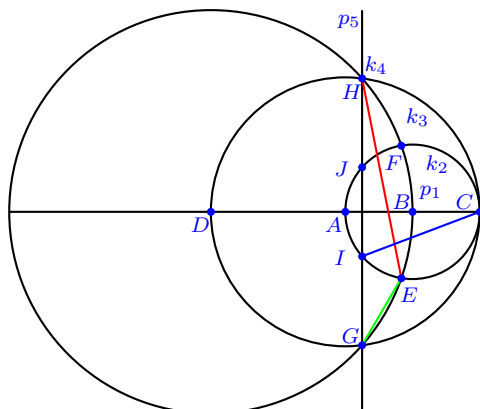
$$d_1 = |AI| = |AJ| \text{ (blue) } \dots 2$$

$$d_2 = |EG| = |FH| \text{ (green) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)}$$

## 2.35.2 Construction RCC126



## Construction Process

$A, B$  given initial points

1.  $p_1 = p(A, B)$
2.  $k_2 = k(B, A)$   
 $C \in p_1 \cap k_2$
3.  $k_3 = k(A, C)$   
 $D \in p_1 \cap k_3$
4.  $k_4 = k(D, B)$   
 $E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4$
5.  $p_5 = p(G, H)$   
 $I \in p_5 \cap k_2, J \in p_5 \cap k_2$

## Distances

$$d_0 = |EH| = |FG| \text{ (red) } \dots 2$$

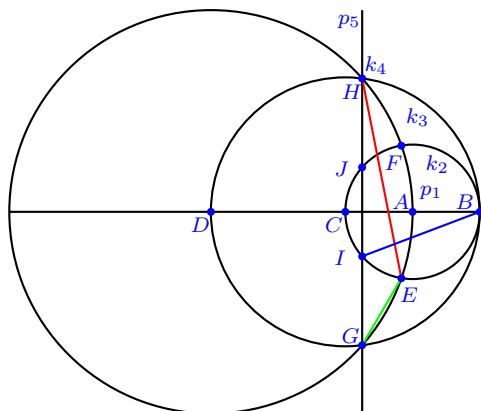
$$d_1 = |CI| = |CJ| \text{ (blue) } \dots 2$$

$$d_2 = |EG| = |FH| \text{ (green) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)}$$

## 2.35.3 Construction RCC236



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(C, B)$   
 $D \in p_1 \cap k_3$
  4.  $k_4 = k(D, A)$   
 $E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4$
  5.  $p_5 = p(G, H)$   
 $I \in p_5 \cap k_2, J \in p_5 \cap k_2$

## Distances

$$d_0 = |EH| = |FG| \text{ (red) } \dots 2$$

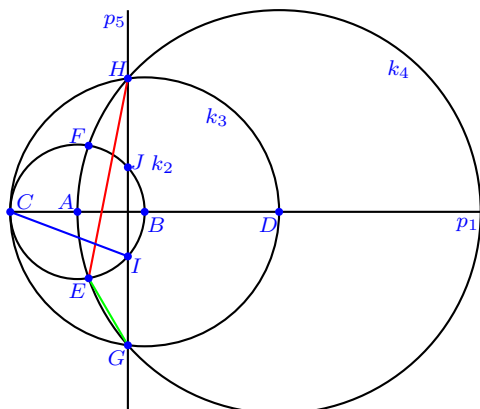
$$d_1 = |BI| = |BJ| \text{ (blue) } \dots 2$$

$$d_2 = |EG| = |FH| \text{ (green) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)}$$

## 2.35.4 Construction RCC251



## Construction Process

- $A, B$  given initial points
1.  $p_1 = p(A, B)$
  2.  $k_2 = k(A, B)$   
 $C \in p_1 \cap k_2$
  3.  $k_3 = k(B, C)$   
 $D \in p_1 \cap k_3$
  4.  $k_4 = k(D, A)$   
 $E \in k_2 \cap k_4, F \in k_2 \cap k_4, G \in k_3 \cap k_4, H \in k_3 \cap k_4$
  5.  $p_5 = p(G, H)$   
 $I \in p_5 \cap k_2, J \in p_5 \cap k_2$

## Distances

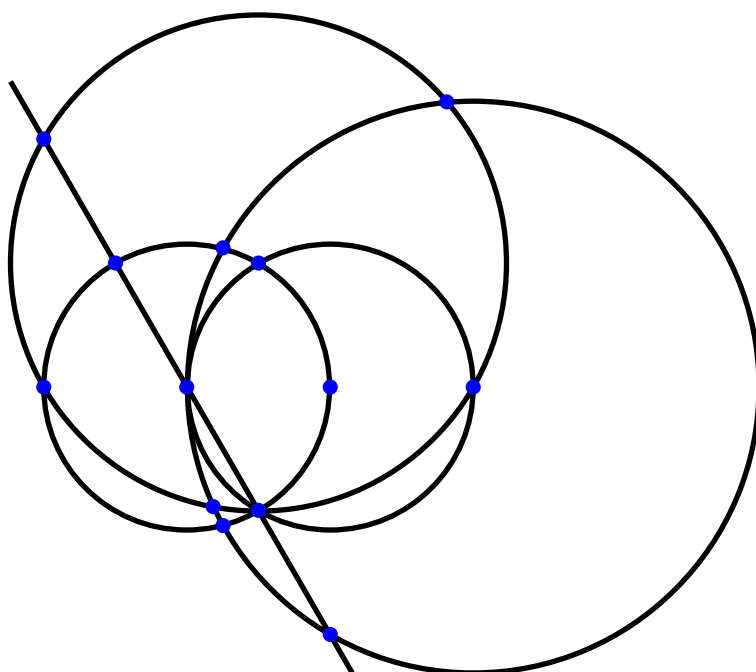
$$d_0 = |EH| = |FG| \text{ (red) } \dots 2$$

$$d_1 = |CI| = |CJ| \text{ (blue) } \dots 2$$

$$d_2 = |EG| = |FH| \text{ (green) } \dots 2$$

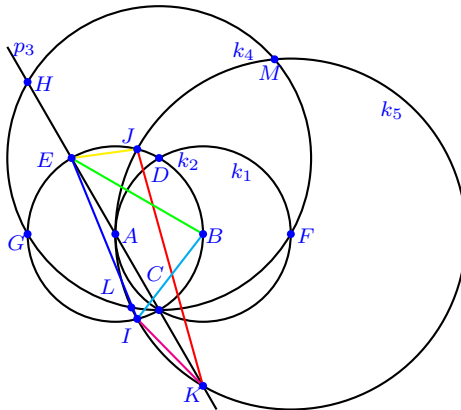
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)}$$

**2.36 Class of Similar Constructions No. 29****Contained Constructions**

RCC254, RCC290, RCC304, RCC336

## 2.36.1 Construction RCC254



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $p_3 = p(A, C)$   
 $E \in p_3 \cap k_2$
  4.  $k_4 = k(D, C)$   
 $F \in k_1 \cap k_4, G \in k_2 \cap k_4, H \in p_3 \cap k_4$
  5.  $k_5 = k(F, A)$   
 $I \in k_2 \cap k_5, J \in k_2 \cap k_5, K \in p_3 \cap k_5, L \in k_4 \cap k_5, M \in k_4 \cap k_5$

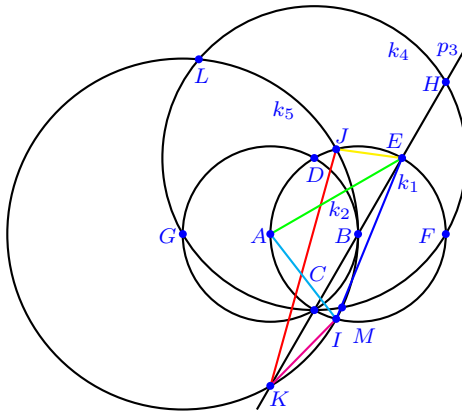
## Distances

$$\begin{aligned}
 d_0 &= |JK| \text{ (red)} \dots 1 \\
 d_1 &= |EI| \text{ (blue)} \dots 1 \\
 d_2 &= |BE| = |BK| = |CD| = |CF| = |CG| = |DF| = |DG| = |DH| = \\
 &= |DL| = |DM| = |GH| \text{ (green)} \dots 11 \\
 d_3 &= |BI| = |BJ| \text{ (cyan)} \dots 2 \\
 d_4 &= |IK| \text{ (magenta)} \dots 1 \\
 d_5 &= |EJ| \text{ (yellow)} \dots 1
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (26 occurrences)}$$

## 2.36.2 Construction RCC290



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $p_3 = p(B, C)$   
 $E \in p_3 \cap k_1$
  4.  $k_4 = k(D, C)$   
 $F \in k_1 \cap k_4, G \in k_2 \cap k_4, H \in p_3 \cap k_4$
  5.  $k_5 = k(G, B)$   
 $I \in k_1 \cap k_5, J \in k_1 \cap k_5, K \in p_3 \cap k_5, L \in k_4 \cap k_5, M \in k_4 \cap k_5$

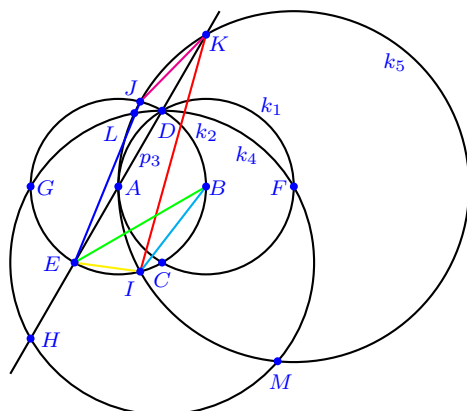
## Distances

$$\begin{aligned}
 d_0 &= |JK| \text{ (red)} \dots 1 \\
 d_1 &= |EI| \text{ (blue)} \dots 1 \\
 d_2 &= |AE| = |AK| = |CD| = |CF| = |CG| = |DF| = |DG| = |DH| = \\
 &= |DL| = |DM| = |FH| \text{ (green)} \dots 11 \\
 d_3 &= |AI| = |AJ| \text{ (cyan)} \dots 2 \\
 d_4 &= |IK| \text{ (magenta)} \dots 1 \\
 d_5 &= |EJ| \text{ (yellow)} \dots 1
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (26 occurrences)}$$

## 2.36.3 Construction RCC304



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $p_3 = p(A, D)$   
 $E \in p_3 \cap k_2$
  4.  $k_4 = k(C, D)$   
 $F \in k_1 \cap k_4, G \in k_2 \cap k_4, H \in p_3 \cap k_4$
  5.  $k_5 = k(F, A)$   
 $I \in k_2 \cap k_5, J \in k_2 \cap k_5, K \in p_3 \cap k_5, L \in k_4 \cap k_5, M \in k_4 \cap k_5$

## Distances

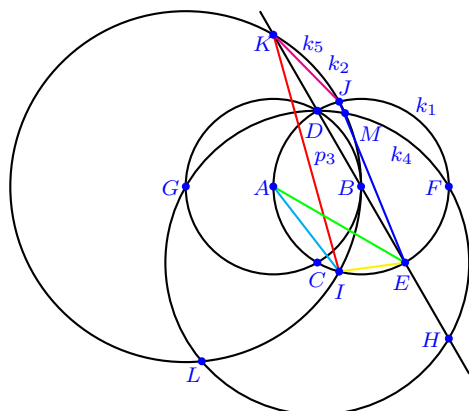
$$\begin{aligned}
 d_0 &= |IK| \text{ (red)} \dots 1 \\
 d_1 &= |EJ| \text{ (blue)} \dots 1 \\
 d_2 &= |BE| = |BK| = |CD| = |CF| = |CG| = |CH| = |CL| = |CM| = \\
 &= |DF| = |DG| = |GH| \text{ (green)} \dots 11 \\
 d_3 &= |BI| = |BJ| \text{ (cyan)} \dots 2 \\
 d_4 &= |JK| \text{ (magenta)} \dots 1 \\
 d_5 &= |EI| \text{ (yellow)} \dots 1
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (26 occurrences)}$$



## 2.36.4 Construction RCC336



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $p_3 = p(B, D)$   
 $E \in p_3 \cap k_1$
  4.  $k_4 = k(C, D)$   
 $F \in k_1 \cap k_4, G \in k_2 \cap k_4, H \in p_3 \cap k_4$
  5.  $k_5 = k(G, B)$   
 $I \in k_1 \cap k_5, J \in k_1 \cap k_5, K \in p_3 \cap k_5, L \in k_4 \cap k_5, M \in k_4 \cap k_5$

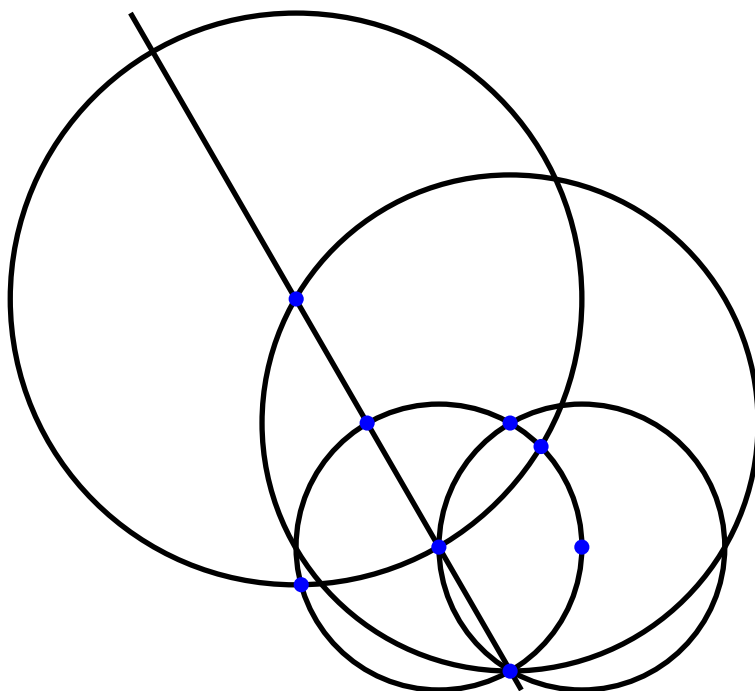
## Distances

$$\begin{aligned}
 d_0 &= |IK| \text{ (red)} \dots 1 \\
 d_1 &= |EJ| \text{ (blue)} \dots 1 \\
 d_2 &= |AE| = |AK| = |CD| = |CF| = |CG| = |CH| = |CL| = |CM| = \\
 &= |DF| = |DG| = |FH| \text{ (green)} \dots 11 \\
 d_3 &= |AI| = |AJ| \text{ (cyan)} \dots 2 \\
 d_4 &= |JK| \text{ (magenta)} \dots 1 \\
 d_5 &= |EI| \text{ (yellow)} \dots 1
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (26 occurrences)}$$

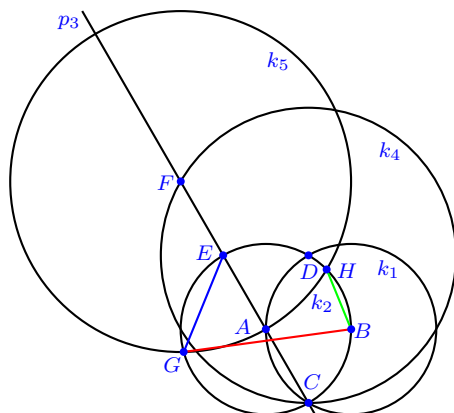
### 2.37 Class of Similar Constructions No. 30



#### Contained Constructions

RCC255, RCC291, RCC305, RCC337

## 2.37.1 Construction RCC255



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $p_3 = p(A, C)$   
 $E \in p_3 \cap k_2$
  4.  $k_4 = k(D, C)$   
 $F \in p_3 \cap k_4$
  5.  $k_5 = k(F, A)$   
 $G \in k_2 \cap k_5, H \in k_2 \cap k_5$

## Distances

$$d_0 = |BG| \text{ (red) } \dots 1$$

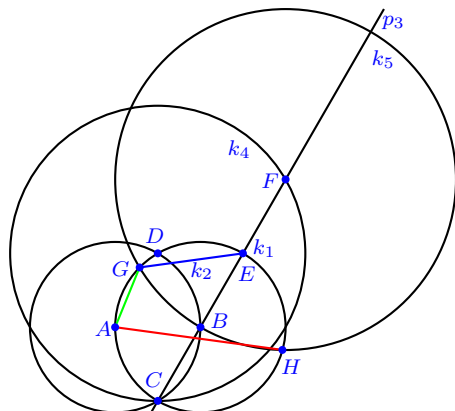
$$d_1 = |EG| = |EH| \text{ (blue) } \dots 2$$

$$d_2 = |BH| \text{ (green) } \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (4 occurrences)}$$

## 2.37.2 Construction RCC291



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $p_3 = p(B, C)$   
 $E \in p_3 \cap k_1$
  4.  $k_4 = k(D, C)$   
 $F \in p_3 \cap k_4$
  5.  $k_5 = k(F, B)$   
 $G \in k_1 \cap k_5, H \in k_1 \cap k_5$

## Distances

$$d_0 = |AH| \text{ (red) } \dots 1$$

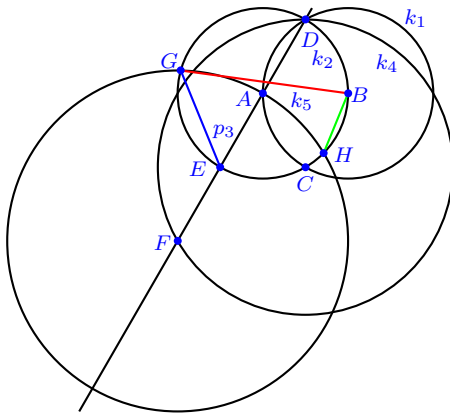
$$d_1 = |EG| = |EH| \text{ (blue) } \dots 2$$

$$d_2 = |AG| \text{ (green) } \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (4 occurrences)}$$

## 2.37.3 Construction RCC305



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $p_3 = p(A, D)$   
 $E \in p_3 \cap k_2$
  4.  $k_4 = k(C, D)$   
 $F \in p_3 \cap k_4$
  5.  $k_5 = k(F, A)$   
 $G \in k_2 \cap k_5, H \in k_2 \cap k_5$

## Distances

$$d_0 = |BG| \text{ (red) } \dots 1$$

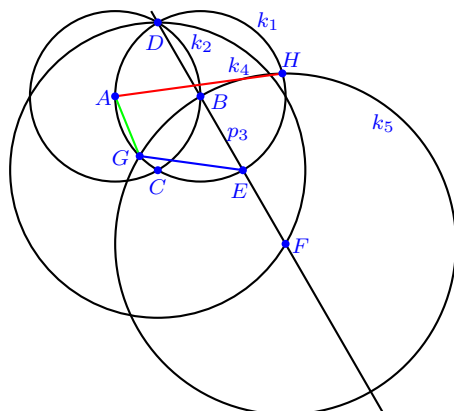
$$d_1 = |EG| = |EH| \text{ (blue) } \dots 2$$

$$d_2 = |BH| \text{ (green) } \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (4 occurrences)}$$

## 2.37.4 Construction RCC337



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $p_3 = p(B, D)$   
 $E \in p_3 \cap k_1$
  4.  $k_4 = k(C, D)$   
 $F \in p_3 \cap k_4$
  5.  $k_5 = k(F, B)$   
 $G \in k_1 \cap k_5, H \in k_1 \cap k_5$

## Distances

$$d_0 = |AH| \text{ (red) } \dots 1$$

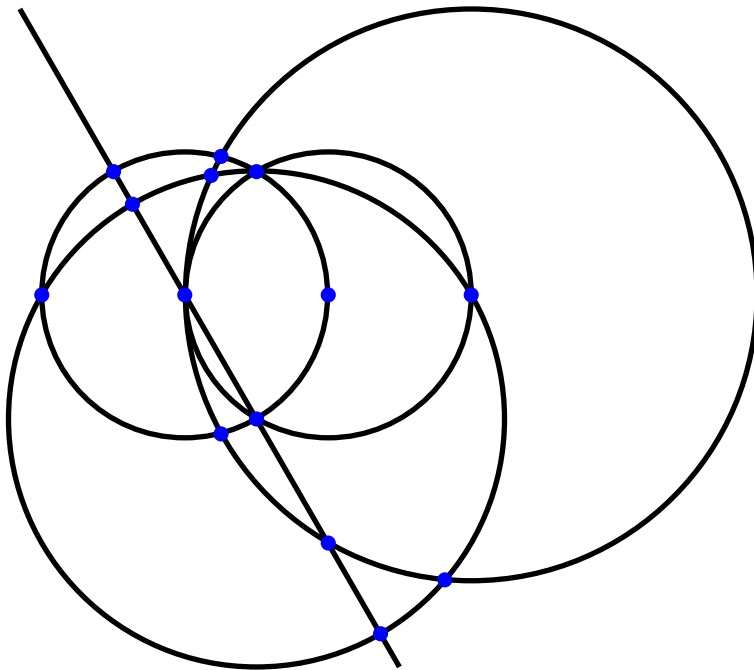
$$d_1 = |EG| = |EH| \text{ (blue) } \dots 2$$

$$d_2 = |AG| \text{ (green) } \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (4 occurrences)}$$

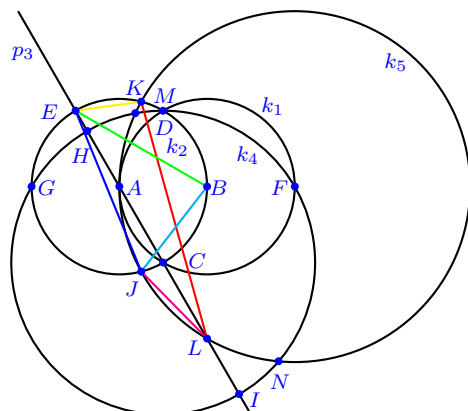
### 2.38 Class of Similar Constructions No. 31



#### Contained Constructions

RCC256, RCC292, RCC302, RCC334

## 2.38.1 Construction RCC256



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $p_3 = p(A, C)$   
 $E \in p_3 \cap k_2$
  4.  $k_4 = k(C, D)$   
 $F \in k_1 \cap k_4, G \in k_2 \cap k_4, H \in p_3 \cap k_4, I \in p_3 \cap k_4$
  5.  $k_5 = k(F, A)$   
 $J \in k_2 \cap k_5, K \in k_2 \cap k_5, L \in p_3 \cap k_5, M \in k_4 \cap k_5, N \in k_4 \cap k_5$

## Distances

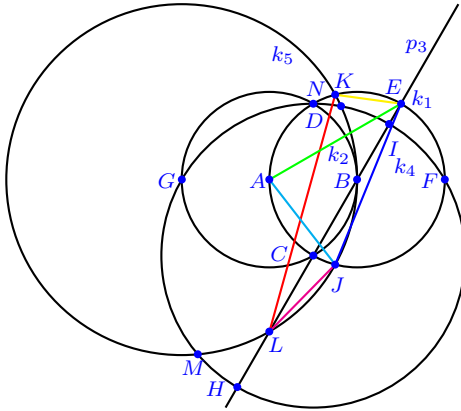
$$\begin{aligned}
 d_0 &= |KL| \text{ (red)} \dots 1 \\
 d_1 &= |EJ| \text{ (blue)} \dots 1 \\
 d_2 &= |BE| = |BL| = |CD| = |CF| = |CG| = |CH| = |CI| = |CM| = \\
 &= |CN| = |DF| = |DG| \text{ (green)} \dots 11 \\
 d_3 &= |BJ| = |BK| \text{ (cyan)} \dots 2 \\
 d_4 &= |JL| \text{ (magenta)} \dots 1 \\
 d_5 &= |EK| \text{ (yellow)} \dots 1
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (26 occurrences)}$$



**2.38.2 Construction RCC292**



**Construction Process**

- $A, B$  given initial points
- 1.  $k_1 = k(B, A)$
- 2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
- 3.  $p_3 = p(B, C)$   
 $E \in p_3 \cap k_1$
- 4.  $k_4 = k(C, D)$   
 $F \in k_1 \cap k_4, G \in k_2 \cap k_4, H \in p_3 \cap k_4, I \in p_3 \cap k_4$
- 5.  $k_5 = k(G, B)$   
 $J \in k_1 \cap k_5, K \in k_1 \cap k_5, L \in p_3 \cap k_5, M \in k_4 \cap k_5, N \in k_4 \cap k_5$

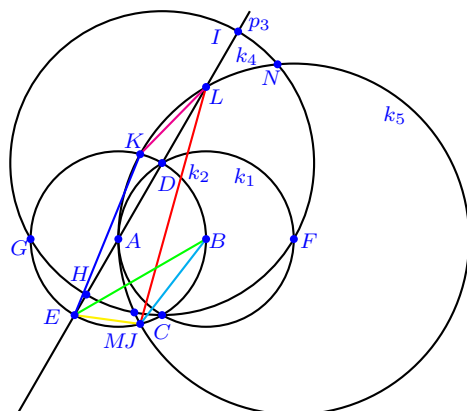
**Distances**

- $d_0 = |KL|$  (red) ... 1
- $d_1 = |EJ|$  (blue) ... 1
- $d_2 = |AE| = |AL| = |CD| = |CF| = |CG| = |CH| = |CI| = |CM| = |CN| = |DF| = |DG|$  (green) ... 11
- $d_3 = |AJ| = |AK|$  (cyan) ... 2
- $d_4 = |JL|$  (magenta) ... 1
- $d_5 = |EK|$  (yellow) ... 1

**Occurrences of the Golden Ratio**

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (26 occurrences)}$$

## 2.38.3 Construction RCC302



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $p_3 = p(A, D)$   
 $E \in p_3 \cap k_2$
  4.  $k_4 = k(D, C)$   
 $F \in k_1 \cap k_4, G \in k_2 \cap k_4, H \in p_3 \cap k_4, I \in p_3 \cap k_4$
  5.  $k_5 = k(F, A)$   
 $J \in k_2 \cap k_5, K \in k_2 \cap k_5, L \in p_3 \cap k_5, M \in k_4 \cap k_5, N \in k_4 \cap k_5$

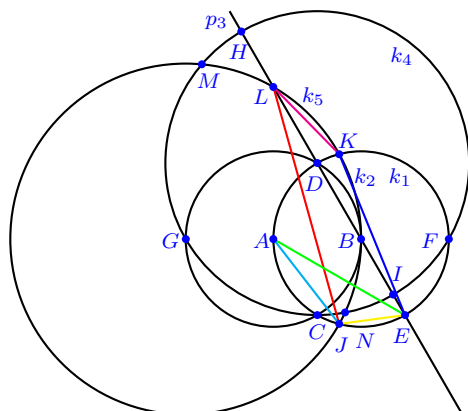
## Distances

$$\begin{aligned}
 d_0 &= |JL| \text{ (red)} \dots 1 \\
 d_1 &= |EK| \text{ (blue)} \dots 1 \\
 d_2 &= |BE| = |BL| = |CD| = |CF| = |CG| = |DF| = |DG| = |DH| = \\
 &= |DI| = |DM| = |DN| \text{ (green)} \dots 11 \\
 d_3 &= |BJ| = |BK| \text{ (cyan)} \dots 2 \\
 d_4 &= |KL| \text{ (magenta)} \dots 1 \\
 d_5 &= |EJ| \text{ (yellow)} \dots 1
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (26 occurrences)}$$

## 2.38.4 Construction RCC334



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $p_3 = p(B, D)$   
 $E \in p_3 \cap k_1$
  4.  $k_4 = k(D, C)$   
 $F \in k_1 \cap k_4, G \in k_2 \cap k_4, H \in p_3 \cap k_4, I \in p_3 \cap k_4$
  5.  $k_5 = k(G, B)$   
 $J \in k_1 \cap k_5, K \in k_1 \cap k_5, L \in p_3 \cap k_5, M \in k_4 \cap k_5, N \in k_4 \cap k_5$

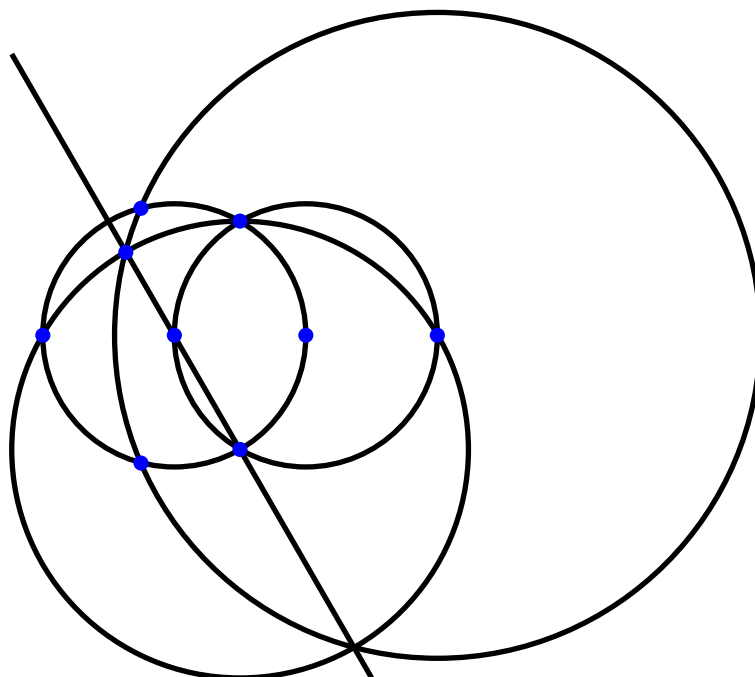
## Distances

$$\begin{aligned}
 d_0 &= |JL| \text{ (red)} \dots 1 \\
 d_1 &= |EK| \text{ (blue)} \dots 1 \\
 d_2 &= |AE| = |AL| = |CD| = |CF| = |CG| = |DF| = |DG| = |DH| = \\
 &= |DI| = |DM| = |DN| \text{ (green)} \dots 11 \\
 d_3 &= |AJ| = |AK| \text{ (cyan)} \dots 2 \\
 d_4 &= |KL| \text{ (magenta)} \dots 1 \\
 d_5 &= |EJ| \text{ (yellow)} \dots 1
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (26 occurrences)}$$

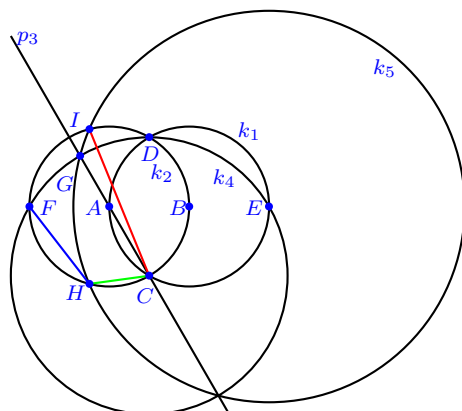
### 2.39 Class of Similar Constructions No. 32



#### Contained Constructions

RCC257, RCC293, RCC303, RCC335

## 2.39.1 Construction RCC257



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $p_3 = p(A, C)$
  4.  $k_4 = k(C, D)$   
 $E \in k_1 \cap k_4, F \in k_2 \cap k_4, G \in p_3 \cap k_4$
  5.  $k_5 = k(E, G)$   
 $H \in k_2 \cap k_5, I \in k_2 \cap k_5$

## Distances

$$d_0 = |CI| = |DH| \text{ (red) } \dots 2$$

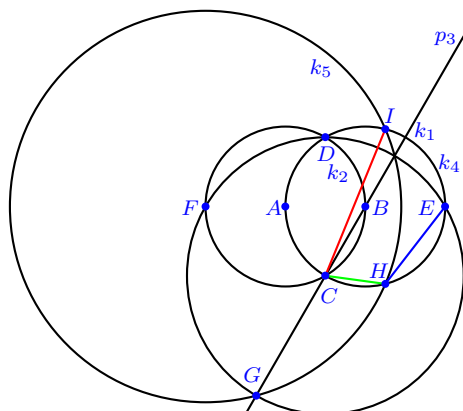
$$d_1 = |FH| = |FI| \text{ (blue) } \dots 2$$

$$d_2 = |CH| = |DI| \text{ (green) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)}$$

## 2.39.2 Construction RCC293



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $p_3 = p(B, C)$
  4.  $k_4 = k(C, D)$   
 $E \in k_1 \cap k_4, F \in k_2 \cap k_4, G \in p_3 \cap k_4$
  5.  $k_5 = k(F, G)$   
 $H \in k_1 \cap k_5, I \in k_1 \cap k_5$

## Distances

$$d_0 = |CI| = |DH| \text{ (red) } \dots 2$$

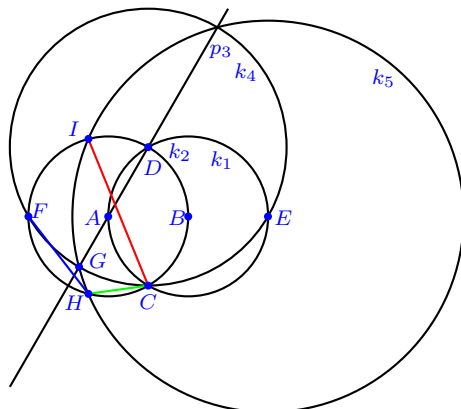
$$d_1 = |EH| = |EI| \text{ (blue) } \dots 2$$

$$d_2 = |CH| = |DI| \text{ (green) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)}$$

## 2.39.3 Construction RCC303



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $p_3 = p(A, D)$
  4.  $k_4 = k(D, C)$   
 $E \in k_1 \cap k_4, F \in k_2 \cap k_4, G \in p_3 \cap k_4$
  5.  $k_5 = k(E, G)$   
 $H \in k_2 \cap k_5, I \in k_2 \cap k_5$

## Distances

$$d_0 = |CI| = |DH| \text{ (red) } \dots 2$$

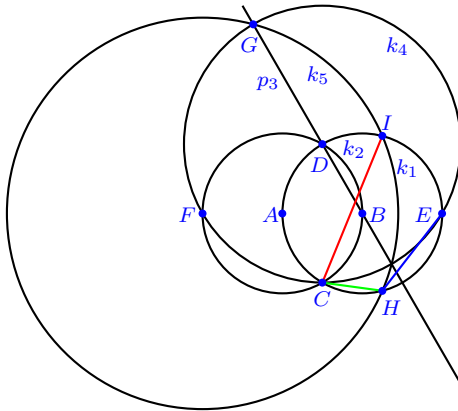
$$d_1 = |FH| = |FI| \text{ (blue) } \dots 2$$

$$d_2 = |CH| = |DI| \text{ (green) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)}$$

### 2.39.4 Construction RCC335



#### Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $p_3 = p(B, D)$
  4.  $k_4 = k(D, C)$   
 $E \in k_1 \cap k_4, F \in k_2 \cap k_4, G \in p_3 \cap k_4$
  5.  $k_5 = k(F, G)$   
 $H \in k_1 \cap k_5, I \in k_1 \cap k_5$

#### Distances

$$d_0 = |CI| = |DH| \text{ (red) } \dots 2$$

$$d_1 = |EH| = |EI| \text{ (blue) } \dots 2$$

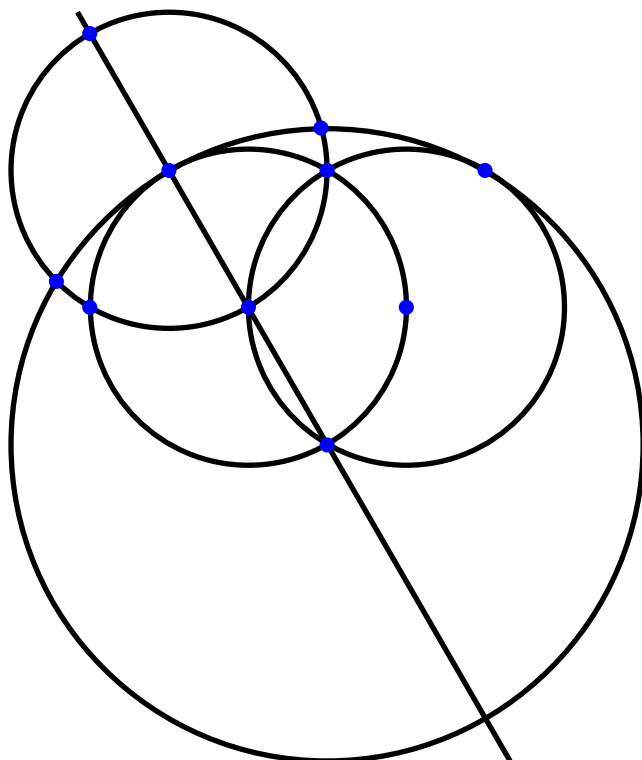
$$d_2 = |CH| = |DI| \text{ (green) } \dots 2$$

#### Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)}$$



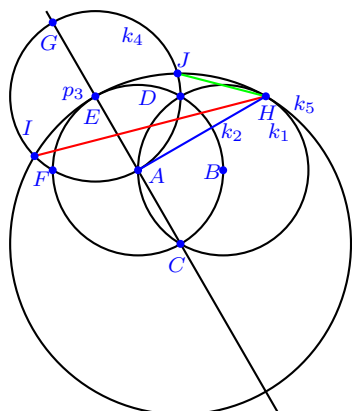
## 2.40 Class of Similar Constructions No. 33



### Contained Constructions

RCC258, RCC295, RCC306, RCC339

## 2.40.1 Construction RCC258



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $p_3 = p(A, C)$   
 $E \in p_3 \cap k_2$
  4.  $k_4 = k(E, A)$   
 $F \in k_2 \cap k_4, G \in p_3 \cap k_4$
  5.  $k_5 = k(C, E)$   
 $H \in k_1 \cap k_5, I \in k_4 \cap k_5, J \in k_4 \cap k_5$

## Distances

$$d_0 = |HI| \text{ (red) } \dots 1$$

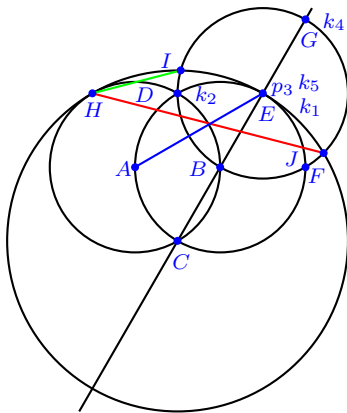
$$d_1 = |AH| = |BE| = |CD| = |CF| = |DF| = |DG| = |FG| \text{ (blue) } \dots 7$$

$$d_2 = |HJ| \text{ (green) } \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (14 occurrences)}$$

2.40.2 Construction RCC295



Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $p_3 = p(B, C)$   
 $E \in p_3 \cap k_1$
  4.  $k_4 = k(E, B)$   
 $F \in k_1 \cap k_4, G \in p_3 \cap k_4$
  5.  $k_5 = k(C, E)$   
 $H \in k_2 \cap k_5, I \in k_4 \cap k_5, J \in k_4 \cap k_5$

Distances

$$d_0 = |HJ| \text{ (red) } \dots 1$$

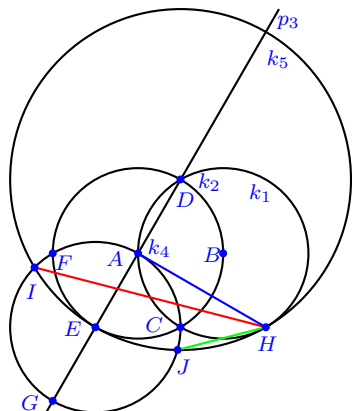
$$d_1 = |AE| = |BH| = |CD| = |CF| = |DF| = |DG| = |FG| \text{ (blue) } \dots 7$$

$$d_2 = |HI| \text{ (green) } \dots 1$$

Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (14 occurrences)}$$

## 2.40.3 Construction RCC306



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $p_3 = p(A, D)$   
 $E \in p_3 \cap k_2$
  4.  $k_4 = k(E, A)$   
 $F \in k_2 \cap k_4, G \in p_3 \cap k_4$
  5.  $k_5 = k(D, E)$   
 $H \in k_1 \cap k_5, I \in k_4 \cap k_5, J \in k_4 \cap k_5$

## Distances

$$d_0 = |HI| \text{ (red) } \dots 1$$

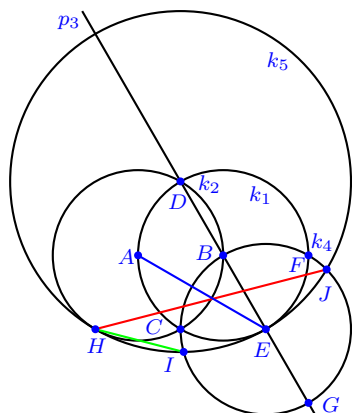
$$d_1 = |AH| = |BE| = |CD| = |CF| = |CG| = |DF| = |FG| \text{ (blue) } \dots 7$$

$$d_2 = |HJ| \text{ (green) } \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (14 occurrences)}$$

2.40.4 Construction RCC339



Construction Process

- $A, B$  given initial points
- 1.  $k_1 = k(B, A)$
- 2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
- 3.  $p_3 = p(B, D)$   
 $E \in p_3 \cap k_1$
- 4.  $k_4 = k(E, B)$   
 $F \in k_1 \cap k_4, G \in p_3 \cap k_4$
- 5.  $k_5 = k(D, E)$   
 $H \in k_2 \cap k_5, I \in k_4 \cap k_5, J \in k_4 \cap k_5$

Distances

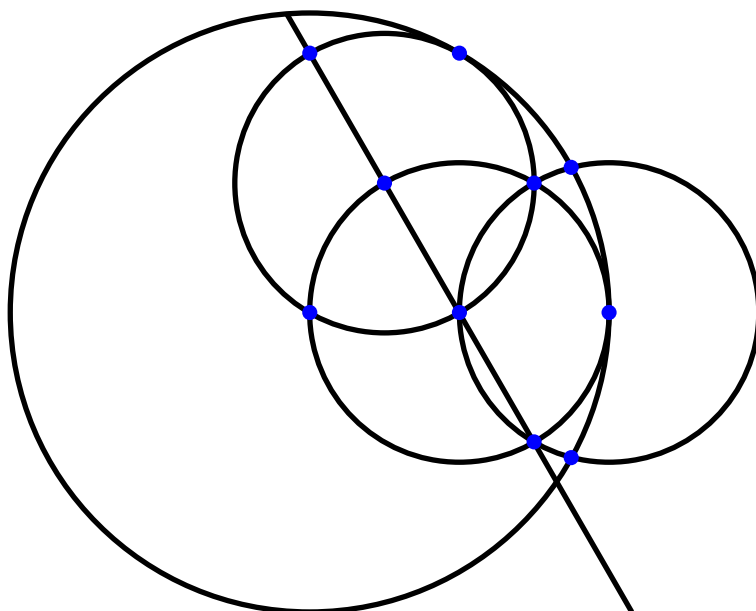
$$d_0 = |HJ| \text{ (red) } \dots 1$$

$$d_1 = |AE| = |BH| = |CD| = |CF| = |CG| = |DF| = |FG| \text{ (blue) } \dots 7$$

$$d_2 = |HI| \text{ (green) } \dots 1$$

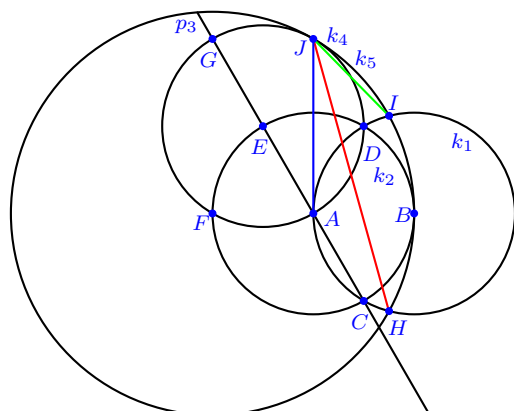
Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (14 occurrences)}$$

**2.41 Class of Similar Constructions No. 34****Contained Constructions**

RCC259, RCC296, RCC307, RCC340

## 2.41.1 Construction RCC259



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $p_3 = p(A, C)$   
 $E \in p_3 \cap k_2$
  4.  $k_4 = k(E, A)$   
 $F \in k_2 \cap k_4, G \in p_3 \cap k_4$
  5.  $k_5 = k(F, B)$   
 $H \in k_1 \cap k_5, I \in k_1 \cap k_5, J \in k_4 \cap k_5$

## Distances

$$d_0 = |HJ| \text{ (red)} \dots 1$$

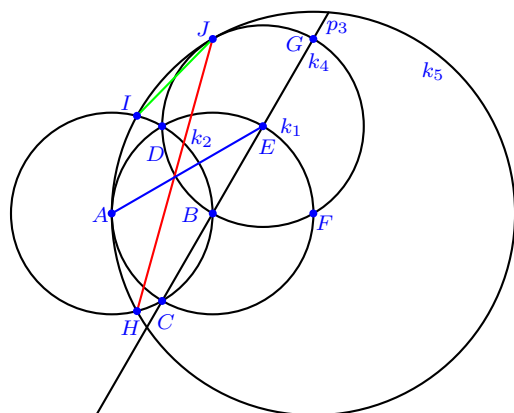
$$d_1 = |AJ| = |BE| = |CD| = |CF| = |DF| = |DG| = |FG| \text{ (blue)} \dots 7$$

$$d_2 = |IJ| \text{ (green)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (14 occurrences)}$$

## 2.41.2 Construction RCC296



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $p_3 = p(B, C)$   
 $E \in p_3 \cap k_1$
  4.  $k_4 = k(E, B)$   
 $F \in k_1 \cap k_4, G \in p_3 \cap k_4$
  5.  $k_5 = k(F, A)$   
 $H \in k_2 \cap k_5, I \in k_2 \cap k_5, J \in k_4 \cap k_5$

## Distances

$$d_0 = |HJ| \text{ (red) } \dots 1$$

$$d_1 = |AE| = |BJ| = |CD| = |CF| = |DF| = |DG| = |FG| \text{ (blue) } \dots 7$$

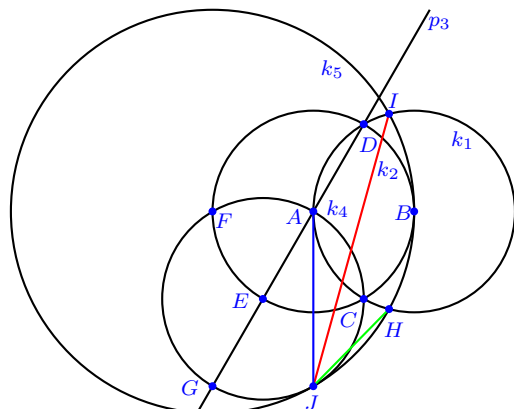
$$d_2 = |IJ| \text{ (green) } \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (14 occurrences)}$$



## 2.41.3 Construction RCC307



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $p_3 = p(A, D)$   
 $E \in p_3 \cap k_2$
  4.  $k_4 = k(E, A)$   
 $F \in k_2 \cap k_4, G \in p_3 \cap k_4$
  5.  $k_5 = k(F, B)$   
 $H \in k_1 \cap k_5, I \in k_1 \cap k_5, J \in k_4 \cap k_5$

## Distances

$$d_0 = |IJ| \text{ (red) } \dots 1$$

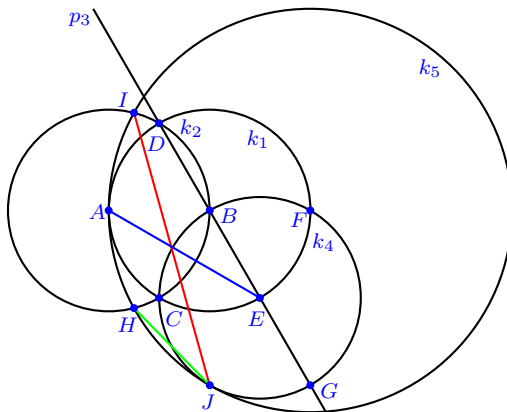
$$d_1 = |AJ| = |BE| = |CD| = |CF| = |CG| = |DF| = |FG| \text{ (blue) } \dots 7$$

$$d_2 = |HJ| \text{ (green) } \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (14 occurrences)}$$

## 2.41.4 Construction RCC340



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $p_3 = p(B, D)$   
 $E \in p_3 \cap k_1$
  4.  $k_4 = k(E, B)$   
 $F \in k_1 \cap k_4, G \in p_3 \cap k_4$
  5.  $k_5 = k(F, A)$   
 $H \in k_2 \cap k_5, I \in k_2 \cap k_5, J \in k_4 \cap k_5$

## Distances

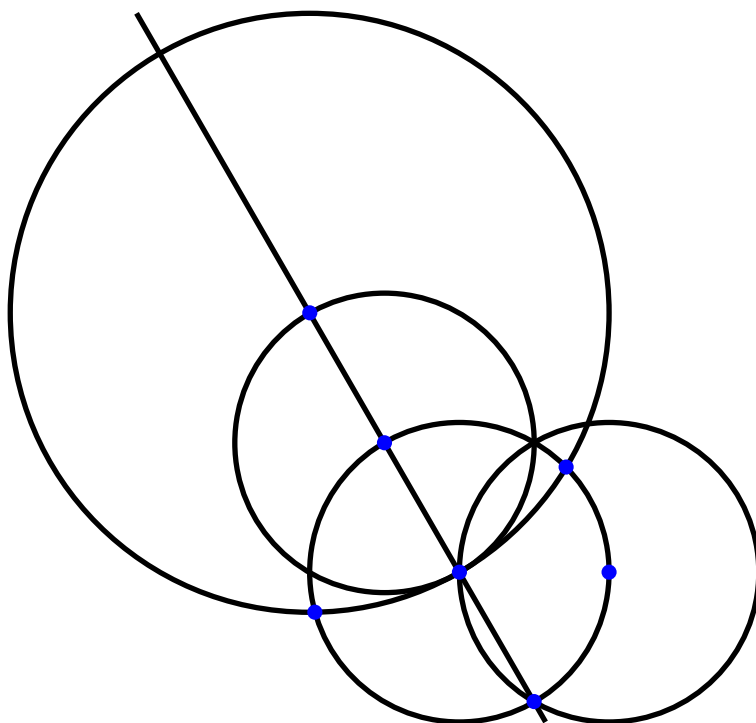
$$d_0 = |IJ| \text{ (red)} \dots 1$$

$$d_1 = |AE| = |BJ| = |CD| = |CF| = |CG| = |DF| = |FG| \text{ (blue)} \dots 7$$

$$d_2 = |HJ| \text{ (green)} \dots 1$$

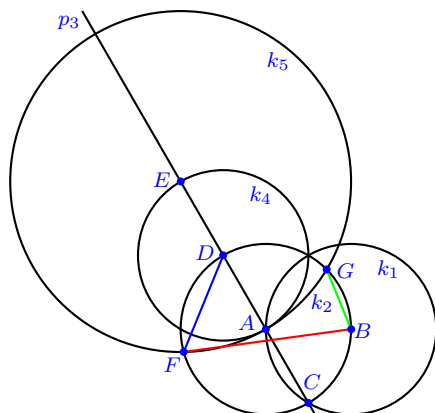
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (14 occurrences)}$$

**2.42 Class of Similar Constructions No. 35****Contained Constructions**

RCC260, RCC297, RCC308, RCC341

## 2.42.1 Construction RCC260



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2$
  3.  $p_3 = p(A, C)$   
 $D \in p_3 \cap k_2$
  4.  $k_4 = k(D, A)$   
 $E \in p_3 \cap k_4$
  5.  $k_5 = k(E, A)$   
 $F \in k_2 \cap k_5, G \in k_2 \cap k_5$

## Distances

$$d_0 = |BF| \text{ (red) } \dots 1$$

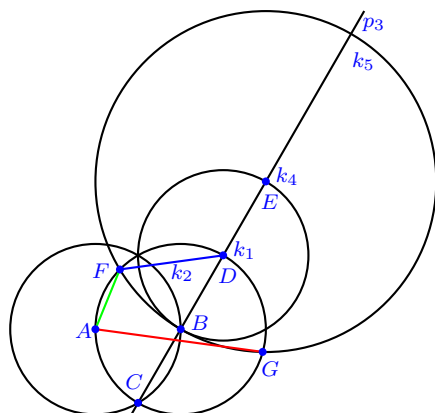
$$d_1 = |DF| = |DG| \text{ (blue) } \dots 2$$

$$d_2 = |BG| \text{ (green) } \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (4 occurrences)}$$

## 2.42.2 Construction RCC297



## Construction Process

$A, B$  given initial points

1.  $k_1 = k(B, A)$
2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2$
3.  $p_3 = p(B, C)$   
 $D \in p_3 \cap k_1$
4.  $k_4 = k(D, B)$   
 $E \in p_3 \cap k_4$
5.  $k_5 = k(E, B)$   
 $F \in k_1 \cap k_5, G \in k_1 \cap k_5$

## Distances

$$d_0 = |AG| \text{ (red) } \dots 1$$

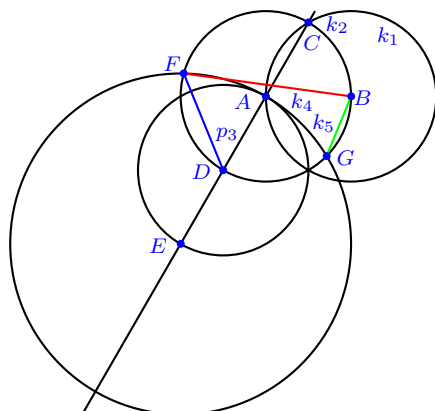
$$d_1 = |DF| = |DG| \text{ (blue) } \dots 2$$

$$d_2 = |AF| \text{ (green) } \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (4 occurrences)}$$

## 2.42.3 Construction RCC308



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2$
  3.  $p_3 = p(A, C)$   
 $D \in p_3 \cap k_2$
  4.  $k_4 = k(D, A)$   
 $E \in p_3 \cap k_4$
  5.  $k_5 = k(E, A)$   
 $F \in k_2 \cap k_5, G \in k_2 \cap k_5$

## Distances

$$d_0 = |BF| \text{ (red)} \dots 1$$

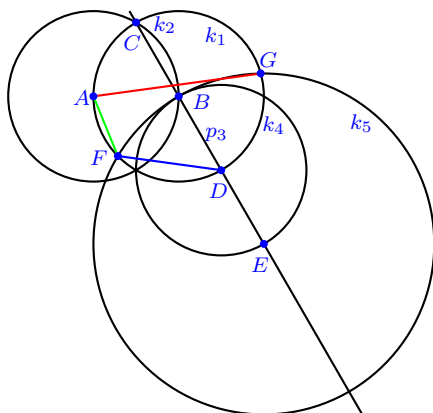
$$d_1 = |DF| = |DG| \text{ (blue)} \dots 2$$

$$d_2 = |BG| \text{ (green)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (4 occurrences)}$$

## 2.42.4 Construction RCC341



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2$
  3.  $p_3 = p(B, C)$   
 $D \in p_3 \cap k_1$
  4.  $k_4 = k(D, B)$   
 $E \in p_3 \cap k_4$
  5.  $k_5 = k(E, B)$   
 $F \in k_1 \cap k_5, G \in k_1 \cap k_5$

## Distances

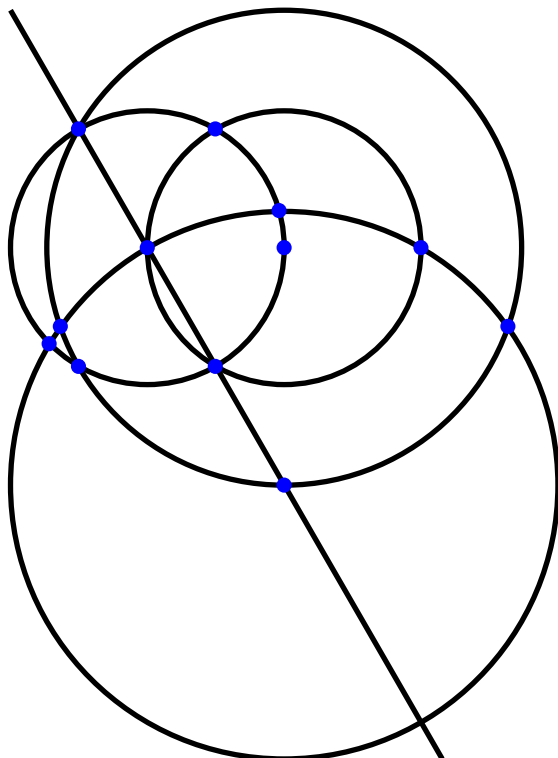
$$d_0 = |AG| \text{ (red)} \dots 1$$

$$d_1 = |DF| = |DG| \text{ (blue)} \dots 2$$

$$d_2 = |AF| \text{ (green)} \dots 1$$

## Occurrences of the Golden Ratio

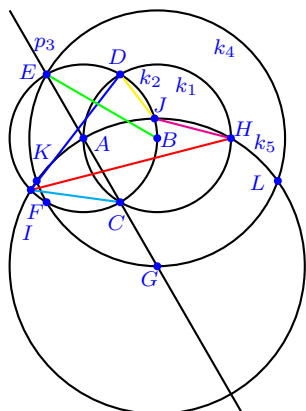
$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (4 occurrences)}$$

**2.43 Class of Similar Constructions No. 36****Contained Constructions**

RCC261, RCC294, RCC309, RCC338



## 2.43.1 Construction RCC261



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $p_3 = p(A, C)$   
 $E \in p_3 \cap k_2$
  4.  $k_4 = k(B, E)$   
 $F \in k_2 \cap k_4, G \in p_3 \cap k_4$
  5.  $k_5 = k(G, A)$   
 $H \in k_1 \cap k_5, I \in k_2 \cap k_5, J \in k_2 \cap k_5, K \in k_4 \cap k_5, L \in k_4 \cap k_5$

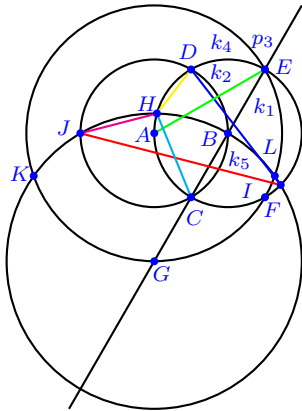
## Distances

$$\begin{aligned}
 d_0 &= |HI| \text{ (red)} \dots 1 \\
 d_1 &= |DI| \text{ (blue)} \dots 1 \\
 d_2 &= |BE| = |BF| = |BG| = |BK| = |BL| = |CD| = |CH| = |DH| = \\
 &|EF| = |FG| \text{ (green)} \dots 10 \\
 d_3 &= |CI| = |CJ| \text{ (cyan)} \dots 2 \\
 d_4 &= |HJ| \text{ (magenta)} \dots 1 \\
 d_5 &= |DJ| \text{ (yellow)} \dots 1
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (24 occurrences)}$$

**2.43.2 Construction RCC294**



**Construction Process**

- $A, B$  given initial points
- 1.  $k_1 = k(B, A)$
- 2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
- 3.  $p_3 = p(B, C)$   
 $E \in p_3 \cap k_1$
- 4.  $k_4 = k(A, E)$   
 $F \in k_1 \cap k_4, G \in p_3 \cap k_4$
- 5.  $k_5 = k(G, B)$   
 $H \in k_1 \cap k_5, I \in k_1 \cap k_5, J \in k_2 \cap k_5, K \in k_4 \cap k_5, L \in k_4 \cap k_5$

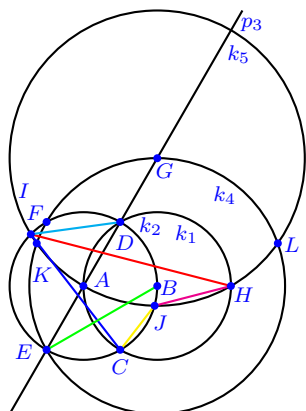
**Distances**

- $d_0 = |IJ|$  (red) ... 1
- $d_1 = |DI|$  (blue) ... 1
- $d_2 = |AE| = |AF| = |AG| = |AK| = |AL| = |CD| = |CJ| = |DJ| = |EF| = |FG|$  (green) ... 10
- $d_3 = |CH| = |CI|$  (cyan) ... 2
- $d_4 = |HJ|$  (magenta) ... 1
- $d_5 = |DH|$  (yellow) ... 1

**Occurrences of the Golden Ratio**

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (24 occurrences)}$$

## 2.43.3 Construction RCC309



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $p_3 = p(A, D)$   
 $E \in p_3 \cap k_2$
  4.  $k_4 = k(B, E)$   
 $F \in k_2 \cap k_4, G \in p_3 \cap k_4$
  5.  $k_5 = k(G, A)$   
 $H \in k_1 \cap k_5, I \in k_2 \cap k_5, J \in k_2 \cap k_5, K \in k_4 \cap k_5, L \in k_4 \cap k_5$

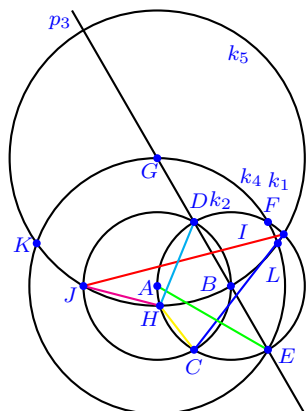
## Distances

$$\begin{aligned}
 d_0 &= |HI| \text{ (red)} \dots 1 \\
 d_1 &= |CI| \text{ (blue)} \dots 1 \\
 d_2 &= |BE| = |BF| = |BG| = |BK| = |BL| = |CD| = |CH| = |DH| = \\
 &= |EF| = |FG| \text{ (green)} \dots 10 \\
 d_3 &= |DI| = |DJ| \text{ (cyan)} \dots 2 \\
 d_4 &= |HJ| \text{ (magenta)} \dots 1 \\
 d_5 &= |CJ| \text{ (yellow)} \dots 1
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (24 occurrences)}$$

## 2.43.4 Construction RCC338



## Construction Process

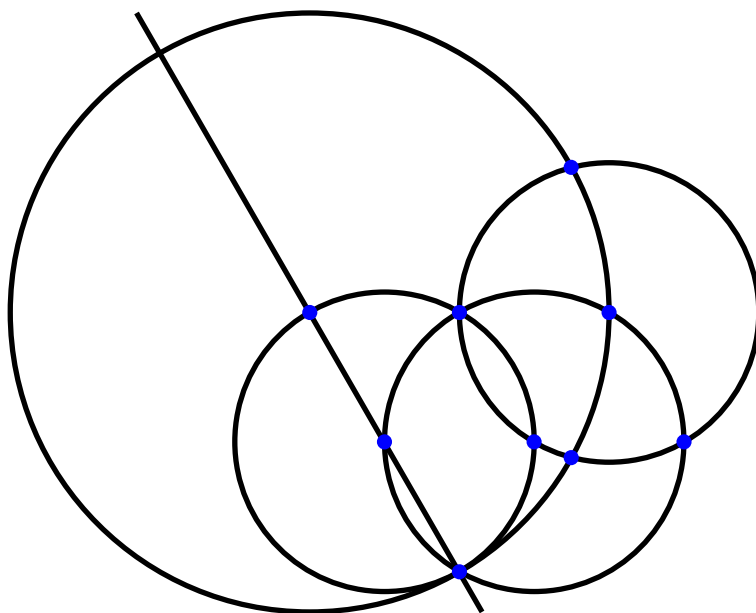
- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $p_3 = p(B, D)$   
 $E \in p_3 \cap k_1$
  4.  $k_4 = k(A, E)$   
 $F \in k_1 \cap k_4, G \in p_3 \cap k_4$
  5.  $k_5 = k(G, B)$   
 $H \in k_1 \cap k_5, I \in k_1 \cap k_5, J \in k_2 \cap k_5, K \in k_4 \cap k_5, L \in k_4 \cap k_5$

## Distances

$$\begin{aligned}
 d_0 &= |IJ| \text{ (red)} \dots 1 \\
 d_1 &= |CI| \text{ (blue)} \dots 1 \\
 d_2 &= |AE| = |AF| = |AG| = |AK| = |AL| = |CD| = |CJ| = |DJ| = \\
 &|EF| = |FG| \text{ (green)} \dots 10 \\
 d_3 &= |DH| = |DI| \text{ (cyan)} \dots 2 \\
 d_4 &= |HJ| \text{ (magenta)} \dots 1 \\
 d_5 &= |CH| \text{ (yellow)} \dots 1
 \end{aligned}$$

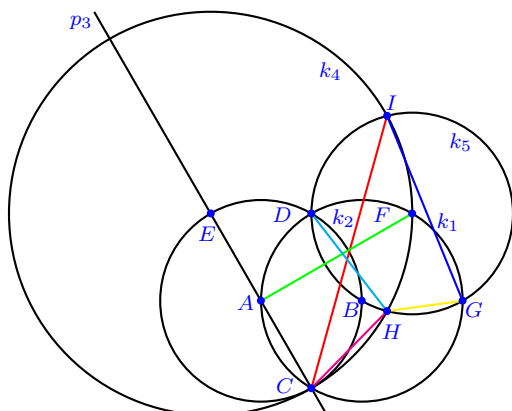
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (24 occurrences)}$$

**2.44 Class of Similar Constructions No. 37****Contained Constructions**

RCC262, RCC298, RCC310, RCC342

## 2.44.1 Construction RCC262



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $p_3 = p(A, C)$   
 $E \in p_3 \cap k_2$
  4.  $k_4 = k(E, C)$   
 $F \in k_1 \cap k_4$
  5.  $k_5 = k(F, B)$   
 $G \in k_1 \cap k_5, H \in k_4 \cap k_5, I \in k_4 \cap k_5$

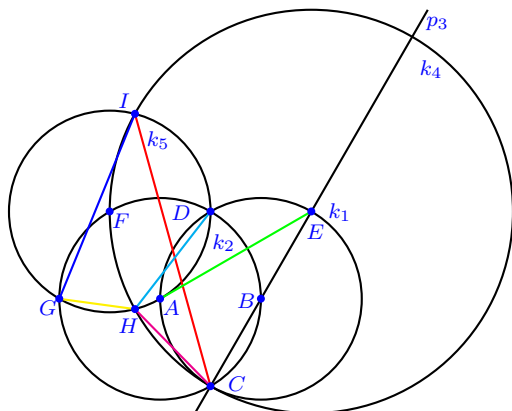
## Distances

- $d_0 = |CI|$  (red) ... 1  
 $d_1 = |GI|$  (blue) ... 1  
 $d_2 = |AF| = |BE| = |CD| = |CG| = |DG|$  (green) ... 5  
 $d_3 = |DH| = |DI|$  (cyan) ... 2  
 $d_4 = |CH|$  (magenta) ... 1  
 $d_5 = |GH|$  (yellow) ... 1

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (14 occurrences)}$$

## 2.44.2 Construction RCC298



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $p_3 = p(B, C)$   
 $E \in p_3 \cap k_1$
  4.  $k_4 = k(E, C)$   
 $F \in k_2 \cap k_4$
  5.  $k_5 = k(F, A)$   
 $G \in k_2 \cap k_5, H \in k_4 \cap k_5, I \in k_4 \cap k_5$

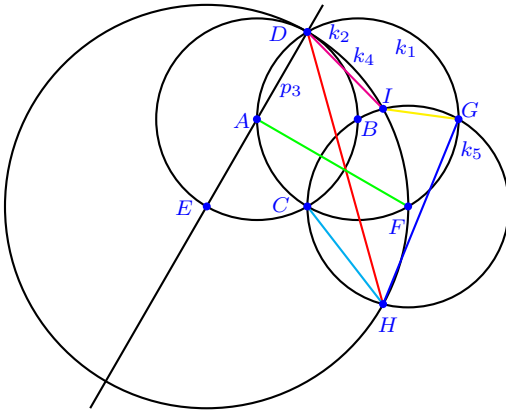
## Distances

- $d_0 = |CI|$  (red) ... 1  
 $d_1 = |GI|$  (blue) ... 1  
 $d_2 = |AE| = |BF| = |CD| = |CG| = |DG|$  (green) ... 5  
 $d_3 = |DH| = |DI|$  (cyan) ... 2  
 $d_4 = |CH|$  (magenta) ... 1  
 $d_5 = |GH|$  (yellow) ... 1

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (14 occurrences)}$$

**2.44.3 Construction RCC310**



**Construction Process**

- $A, B$  given initial points
- 1.  $k_1 = k(B, A)$
- 2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
- 3.  $p_3 = p(A, D)$   
 $E \in p_3 \cap k_2$
- 4.  $k_4 = k(E, D)$   
 $F \in k_1 \cap k_4$
- 5.  $k_5 = k(F, B)$   
 $G \in k_1 \cap k_5, H \in k_4 \cap k_5, I \in k_4 \cap k_5$

**Distances**

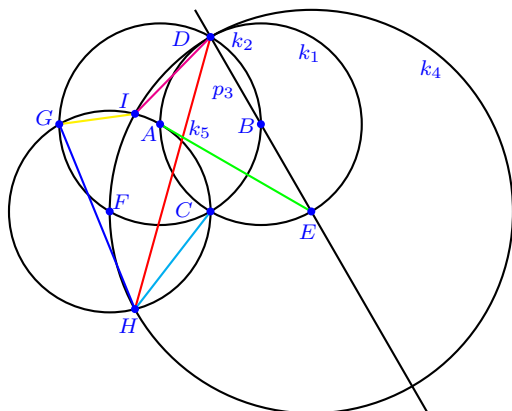
- $d_0 = |DH|$  (red) ... 1
- $d_1 = |GH|$  (blue) ... 1
- $d_2 = |AF| = |BE| = |CD| = |CG| = |DG|$  (green) ... 5
- $d_3 = |CH| = |CI|$  (cyan) ... 2
- $d_4 = |DI|$  (magenta) ... 1
- $d_5 = |GI|$  (yellow) ... 1

**Occurrences of the Golden Ratio**

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (14 occurrences)}$$



## 2.44.4 Construction RCC342



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $p_3 = p(B, D)$   
 $E \in p_3 \cap k_1$
  4.  $k_4 = k(E, D)$   
 $F \in k_2 \cap k_4$
  5.  $k_5 = k(F, A)$   
 $G \in k_2 \cap k_5, H \in k_4 \cap k_5, I \in k_4 \cap k_5$

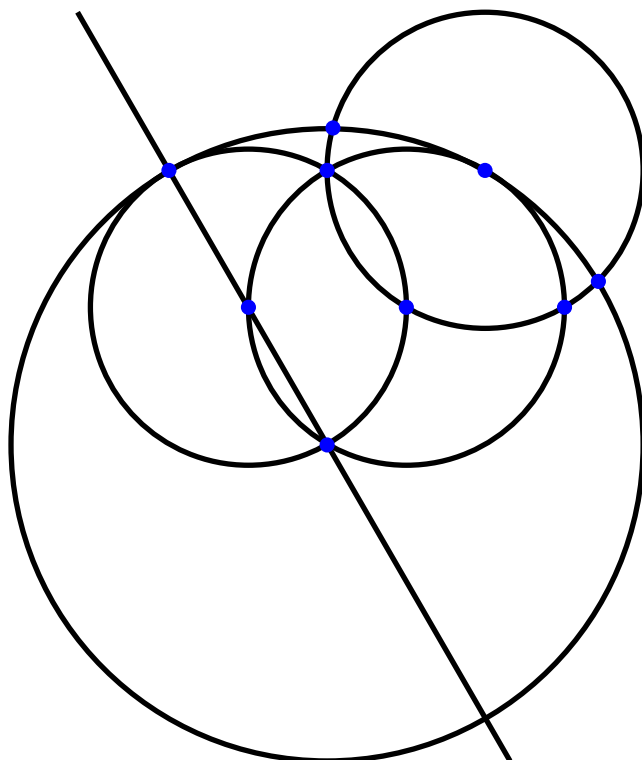
## Distances

$$\begin{aligned}
 d_0 &= |DH| \text{ (red)} \dots 1 \\
 d_1 &= |GH| \text{ (blue)} \dots 1 \\
 d_2 &= |AE| = |BF| = |CD| = |CG| = |DG| \text{ (green)} \dots 5 \\
 d_3 &= |CH| = |CI| \text{ (cyan)} \dots 2 \\
 d_4 &= |DI| \text{ (magenta)} \dots 1 \\
 d_5 &= |GI| \text{ (yellow)} \dots 1
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (14 occurrences)}$$

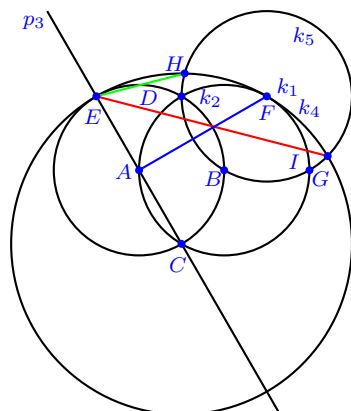
## 2.45 Class of Similar Constructions No. 38



### Contained Constructions

RCC263, RCC299, RCC311, RCC343

## 2.45.1 Construction RCC263



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $p_3 = p(A, C)$   
 $E \in p_3 \cap k_2$
  4.  $k_4 = k(C, E)$   
 $F \in k_1 \cap k_4$
  5.  $k_5 = k(F, B)$   
 $G \in k_1 \cap k_5, H \in k_4 \cap k_5, I \in k_4 \cap k_5$

## Distances

$$d_0 = |EI| \text{ (red)} \dots 1$$

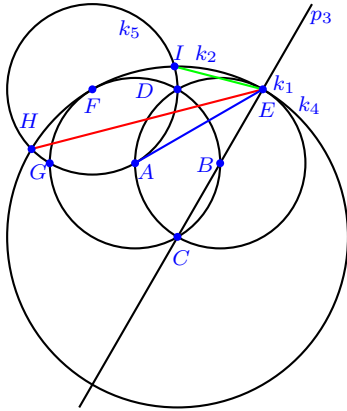
$$d_1 = |AF| = |BE| = |CD| = |CG| = |DG| \text{ (blue)} \dots 5$$

$$d_2 = |EH| \text{ (green)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (10 occurrences)}$$

**2.45.2 Construction RCC299**



**Construction Process**

- $A, B$  given initial points
- 1.  $k_1 = k(B, A)$
- 2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
- 3.  $p_3 = p(B, C)$   
 $E \in p_3 \cap k_1$
- 4.  $k_4 = k(C, E)$   
 $F \in k_2 \cap k_4$
- 5.  $k_5 = k(F, A)$   
 $G \in k_2 \cap k_5, H \in k_4 \cap k_5, I \in k_4 \cap k_5$

**Distances**

$$d_0 = |EH| \text{ (red)} \dots 1$$

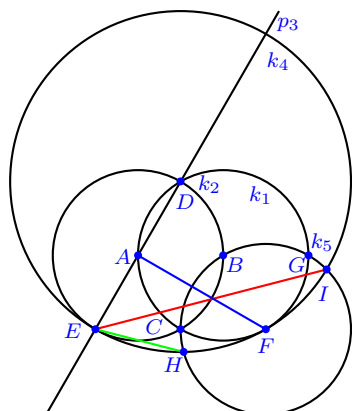
$$d_1 = |AE| = |BF| = |CD| = |CG| = |DG| \text{ (blue)} \dots 5$$

$$d_2 = |EI| \text{ (green)} \dots 1$$

**Occurrences of the Golden Ratio**

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (10 occurrences)}$$

## 2.45.3 Construction RCC311



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2$ ,  $D \in k_1 \cap k_2$
  3.  $p_3 = p(A, D)$   
 $E \in p_3 \cap k_2$
  4.  $k_4 = k(D, E)$   
 $F \in k_1 \cap k_4$
  5.  $k_5 = k(F, B)$   
 $G \in k_1 \cap k_5$ ,  $H \in k_4 \cap k_5$ ,  $I \in k_4 \cap k_5$

## Distances

$$d_0 = |EI| \text{ (red)} \dots 1$$

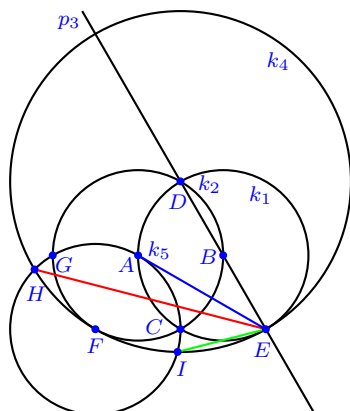
$$d_1 = |AF| = |BE| = |CD| = |CG| = |DG| \text{ (blue)} \dots 5$$

$$d_2 = |EH| \text{ (green)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (10 occurrences)}$$

## 2.45.4 Construction RCC343



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $p_3 = p(B, D)$   
 $E \in p_3 \cap k_1$
  4.  $k_4 = k(D, E)$   
 $F \in k_2 \cap k_4$
  5.  $k_5 = k(F, A)$   
 $G \in k_2 \cap k_5, H \in k_4 \cap k_5, I \in k_4 \cap k_5$

## Distances

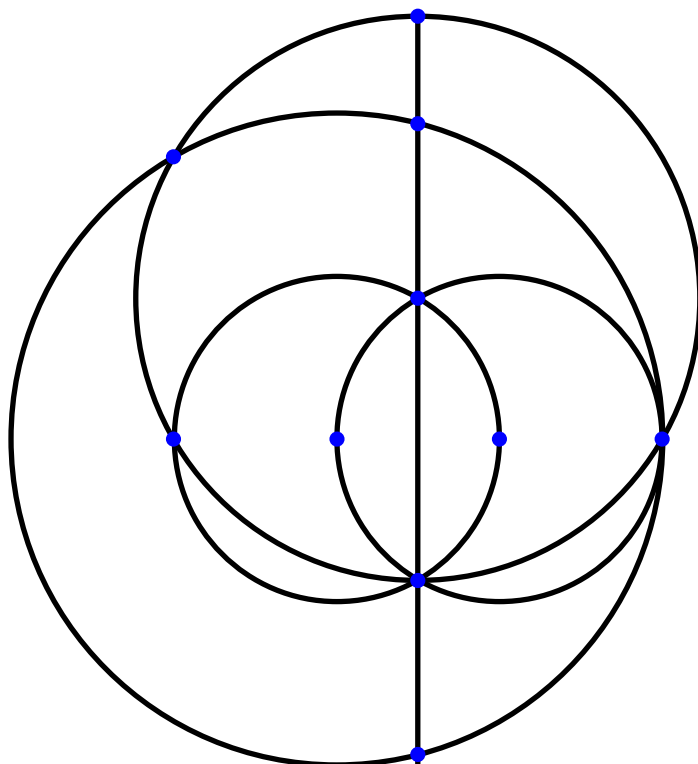
$$d_0 = |EH| \text{ (red)} \dots 1$$

$$d_1 = |AE| = |BF| = |CD| = |CG| = |DG| \text{ (blue)} \dots 5$$

$$d_2 = |EI| \text{ (green)} \dots 1$$

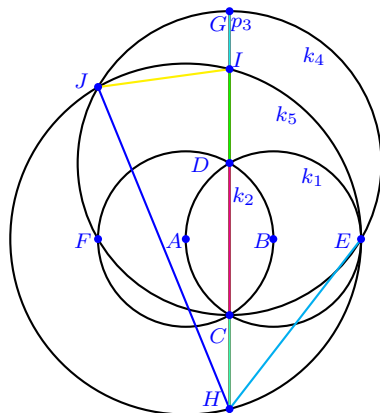
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (10 occurrences)}$$

**2.46 Class of Similar Constructions No. 39****Contained Constructions**

RCC344, RCC345, RCC346, RCC347

## 2.46.1 Construction RCC344



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $p_3 = p(C, D)$
  4.  $k_4 = k(D, C)$   
 $E \in k_1 \cap k_4, F \in k_2 \cap k_4, G \in p_3 \cap k_4$
  5.  $k_5 = k(A, E)$   
 $H \in p_3 \cap k_5, I \in p_3 \cap k_5, J \in k_4 \cap k_5$

## Distances

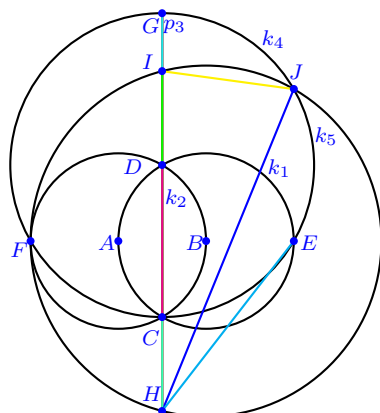
- $d_0 = |GH|$  (red) ... 1  
 $d_1 = |HJ|$  (blue) ... 1  
 $d_2 = |CI| = |DH|$  (green) ... 2  
 $d_3 = |EH| = |EI| = |FH| = |FI|$  (cyan) ... 4  
 $d_4 = |CD| = |CE| = |CF| = |DE| = |DF| = |DG| = |DJ| = |FJ| = |GJ|$  (magenta) ... 9  
 $d_5 = |IJ|$  (yellow) ... 1  
 $d_6 = |CH| = |DI|$  (grass) ... 2  
 $d_7 = |GI|$  (sea) ... 1

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} = \frac{d_4}{d_6} = \frac{d_6}{d_7} \quad (48 \text{ occurrences})$$



## 2.46.2 Construction RCC345



## Construction Process

- $A, B$  given initial points
- $k_1 = k(B, A)$
  - $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  - $p_3 = p(C, D)$
  - $k_4 = k(D, C)$   
 $E \in k_1 \cap k_4, F \in k_2 \cap k_4, G \in p_3 \cap k_4$
  - $k_5 = k(B, F)$   
 $H \in p_3 \cap k_5, I \in p_3 \cap k_5, J \in k_4 \cap k_5$

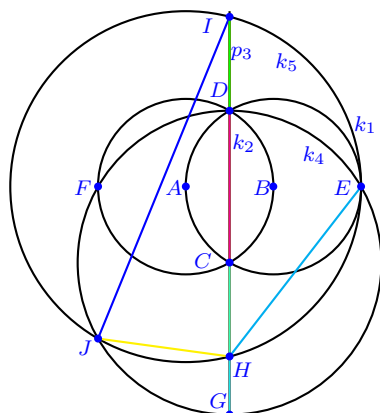
## Distances

- $d_0 = |GH|$  (red) ... 1  
 $d_1 = |HJ|$  (blue) ... 1  
 $d_2 = |CI| = |DH|$  (green) ... 2  
 $d_3 = |EH| = |EI| = |FH| = |FI|$  (cyan) ... 4  
 $d_4 = |CD| = |CE| = |CF| = |DE| = |DF| = |DG| = |DJ| = |EJ| = |GJ|$  (magenta) ... 9  
 $d_5 = |IJ|$  (yellow) ... 1  
 $d_6 = |CH| = |DI|$  (grass) ... 2  
 $d_7 = |GI|$  (sea) ... 1

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} = \frac{d_4}{d_6} = \frac{d_6}{d_7} \quad (48 \text{ occurrences})$$

## 2.46.3 Construction RCC346



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $p_3 = p(C, D)$
  4.  $k_4 = k(C, D)$   
 $E \in k_1 \cap k_4, F \in k_2 \cap k_4, G \in p_3 \cap k_4$
  5.  $k_5 = k(A, E)$   
 $H \in p_3 \cap k_5, I \in p_3 \cap k_5, J \in k_4 \cap k_5$

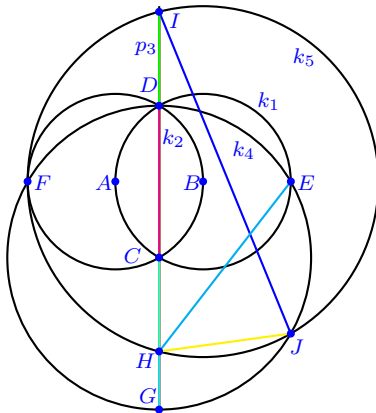
## Distances

- $d_0 = |GI|$  (red) ... 1  
 $d_1 = |IJ|$  (blue) ... 1  
 $d_2 = |CI| = |DH|$  (green) ... 2  
 $d_3 = |EH| = |EI| = |FH| = |FI|$  (cyan) ... 4  
 $d_4 = |CD| = |CE| = |CF| = |CG| = |CJ| = |DE| = |DF| = |FJ| = |GJ|$  (magenta) ... 9  
 $d_5 = |HJ|$  (yellow) ... 1  
 $d_6 = |CH| = |DI|$  (grass) ... 2  
 $d_7 = |GH|$  (sea) ... 1

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} = \frac{d_4}{d_6} = \frac{d_6}{d_7} \quad (48 \text{ occurrences})$$

## 2.46.4 Construction RCC347



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $p_3 = p(C, D)$
  4.  $k_4 = k(C, D)$   
 $E \in k_1 \cap k_4, F \in k_2 \cap k_4, G \in p_3 \cap k_4$
  5.  $k_5 = k(B, F)$   
 $H \in p_3 \cap k_5, I \in p_3 \cap k_5, J \in k_4 \cap k_5$

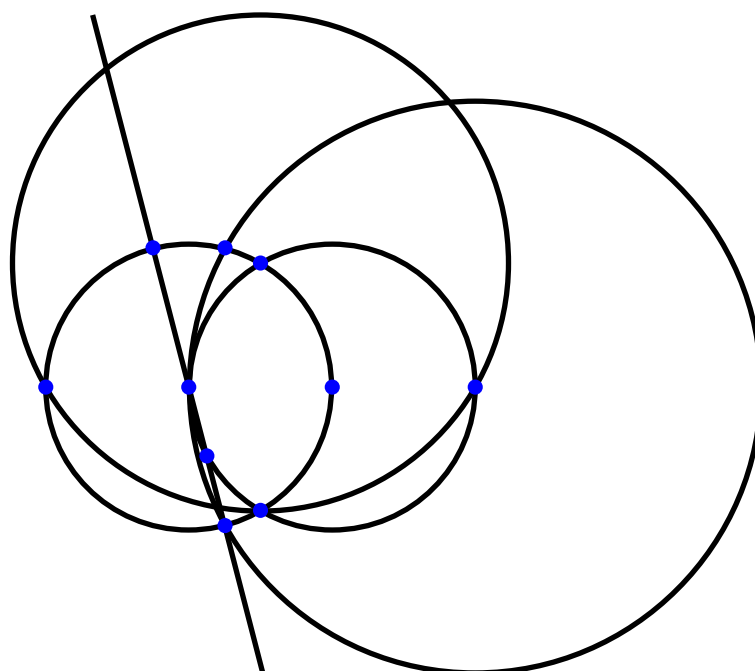
## Distances

- $d_0 = |GI|$  (red) ... 1  
 $d_1 = |IJ|$  (blue) ... 1  
 $d_2 = |CI| = |DH|$  (green) ... 2  
 $d_3 = |EH| = |EI| = |FH| = |FI|$  (cyan) ... 4  
 $d_4 = |CD| = |CE| = |CF| = |CG| = |CJ| = |DE| = |DF| = |EJ| = |GJ|$  (magenta) ... 9  
 $d_5 = |HJ|$  (yellow) ... 1  
 $d_6 = |CH| = |DI|$  (grass) ... 2  
 $d_7 = |GH|$  (sea) ... 1

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} = \frac{d_4}{d_6} = \frac{d_6}{d_7} \quad (48 \text{ occurrences})$$

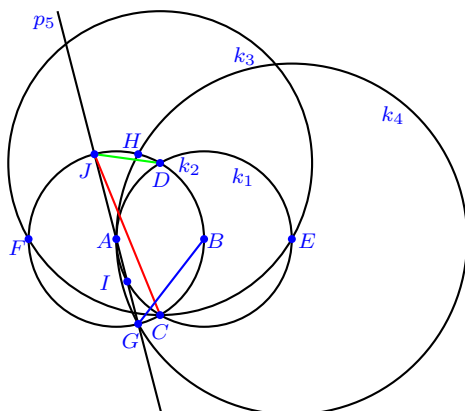
## 2.47 Class of Similar Constructions No. 40



### Contained Constructions

RCC350, RCC380, RCC416, RCC446

## 2.47.1 Construction RCC350



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, C)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, A)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
  5.  $p_5 = p(A, G)$   
 $I \in p_5 \cap k_1, J \in p_5 \cap k_2$

## Distances

$$d_0 = |CJ| \text{ (red)} \dots 1$$

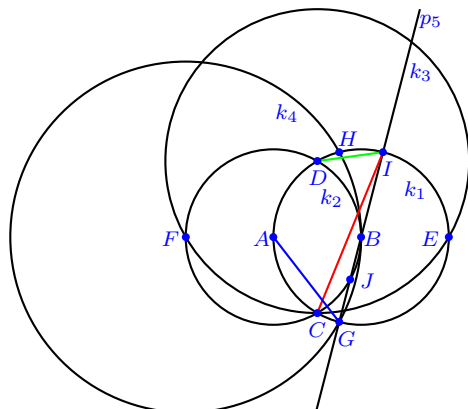
$$d_1 = |BG| = |BH| = |FI| = |FJ| \text{ (blue)} \dots 4$$

$$d_2 = |DJ| \text{ (green)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)}$$

## 2.47.2 Construction RCC380



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, C)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, B)$   
 $G \in k_1 \cap k_4, H \in k_1 \cap k_4$
  5.  $p_5 = p(B, G)$   
 $I \in p_5 \cap k_1, J \in p_5 \cap k_2$

## Distances

$$d_0 = |CI| \text{ (red)} \dots 1$$

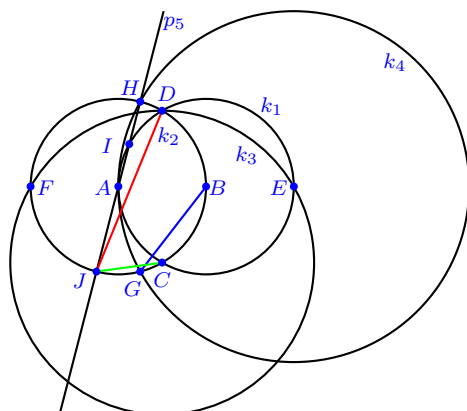
$$d_1 = |AG| = |AH| = |EI| = |EJ| \text{ (blue)} \dots 4$$

$$d_2 = |DI| \text{ (green)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)}$$

## 2.47.3 Construction RCC416



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, D)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, A)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
  5.  $p_5 = p(A, H)$   
 $I \in p_5 \cap k_1, J \in p_5 \cap k_2$

## Distances

$$d_0 = |DJ| \text{ (red)} \dots 1$$

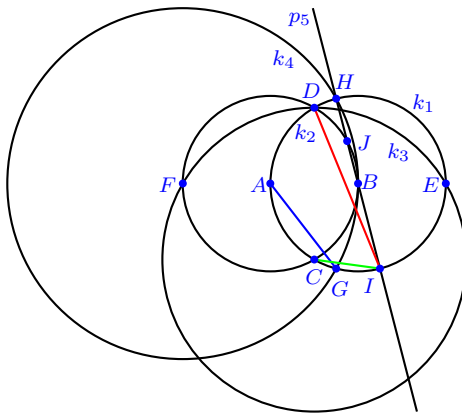
$$d_1 = |BG| = |BH| = |FI| = |FJ| \text{ (blue)} \dots 4$$

$$d_2 = |CJ| \text{ (green)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)}$$

## 2.47.4 Construction RCC446



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, D)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, B)$   
 $G \in k_1 \cap k_4, H \in k_1 \cap k_4$
  5.  $p_5 = p(B, H)$   
 $I \in p_5 \cap k_1, J \in p_5 \cap k_2$

## Distances

$$d_0 = |DI| \text{ (red)} \dots 1$$

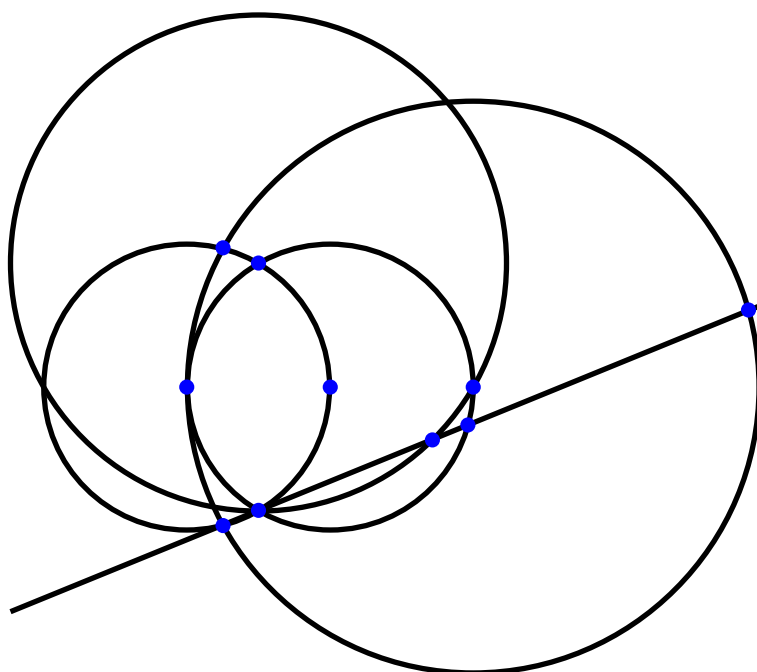
$$d_1 = |AG| = |AH| = |EI| = |EJ| \text{ (blue)} \dots 4$$

$$d_2 = |CI| \text{ (green)} \dots 1$$

## Occurrences of the Golden Ratio

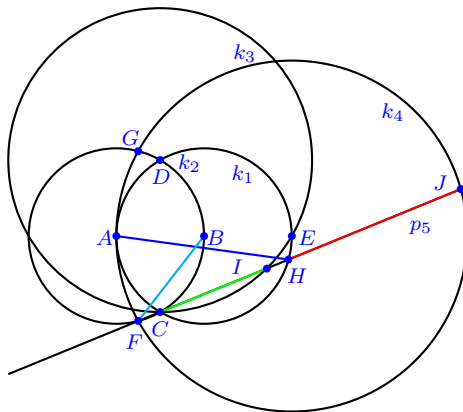
$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)}$$



**2.48 Class of Similar Constructions No. 41****Contained Constructions**

RCC352, RCC382, RCC421, RCC451

## 2.48.1 Construction RCC352



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, C)$   
 $E \in k_1 \cap k_3$
  4.  $k_4 = k(E, A)$   
 $F \in k_2 \cap k_4, G \in k_2 \cap k_4$
  5.  $p_5 = p(C, F)$   
 $H \in p_5 \cap k_1, I \in p_5 \cap k_3, J \in p_5 \cap k_4$

## Distances

$$d_0 = |HJ| \text{ (red)} \dots 1$$

$$d_1 = |AH| \text{ (blue)} \dots 1$$

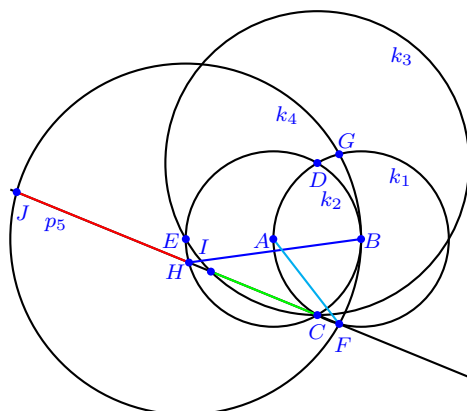
$$d_2 = |CI| \text{ (green)} \dots 1$$

$$d_3 = |BF| = |BG| \text{ (cyan)} \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} \text{ (3 occurrences)}$$

## 2.48.2 Construction RCC382



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, C)$   
 $E \in k_2 \cap k_3$
  4.  $k_4 = k(E, B)$   
 $F \in k_1 \cap k_4, G \in k_1 \cap k_4$
  5.  $p_5 = p(C, F)$   
 $H \in p_5 \cap k_2, I \in p_5 \cap k_3, J \in p_5 \cap k_4$

## Distances

$$d_0 = |HJ| \text{ (red)} \dots 1$$

$$d_1 = |BH| \text{ (blue)} \dots 1$$

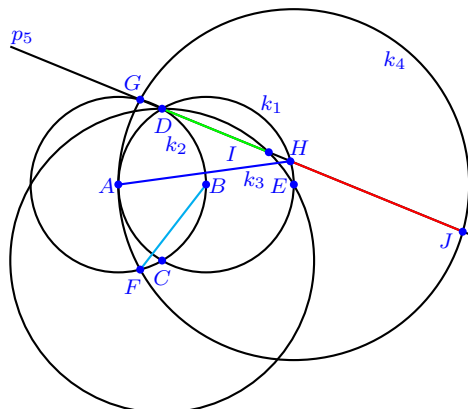
$$d_2 = |CI| \text{ (green)} \dots 1$$

$$d_3 = |AF| = |AG| \text{ (cyan)} \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} \text{ (3 occurrences)}$$

## 2.48.3 Construction RCC421



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, D)$   
 $E \in k_1 \cap k_3$
  4.  $k_4 = k(E, A)$   
 $F \in k_2 \cap k_4, G \in k_2 \cap k_4$
  5.  $p_5 = p(D, G)$   
 $H \in p_5 \cap k_1, I \in p_5 \cap k_3, J \in p_5 \cap k_4$

## Distances

$$d_0 = |HJ| \text{ (red) } \dots 1$$

$$d_1 = |AH| \text{ (blue) } \dots 1$$

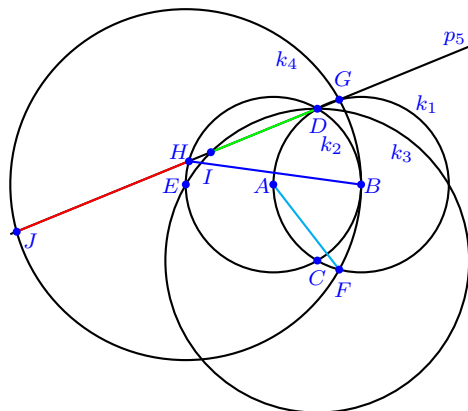
$$d_2 = |DI| \text{ (green) } \dots 1$$

$$d_3 = |BF| = |BG| \text{ (cyan) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} \text{ (3 occurrences)}$$

## 2.48.4 Construction RCC451



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, D)$   
 $E \in k_2 \cap k_3$
  4.  $k_4 = k(E, B)$   
 $F \in k_1 \cap k_4, G \in k_1 \cap k_4$
  5.  $p_5 = p(D, G)$   
 $H \in p_5 \cap k_2, I \in p_5 \cap k_3, J \in p_5 \cap k_4$

## Distances

$$d_0 = |HJ| \text{ (red)} \dots 1$$

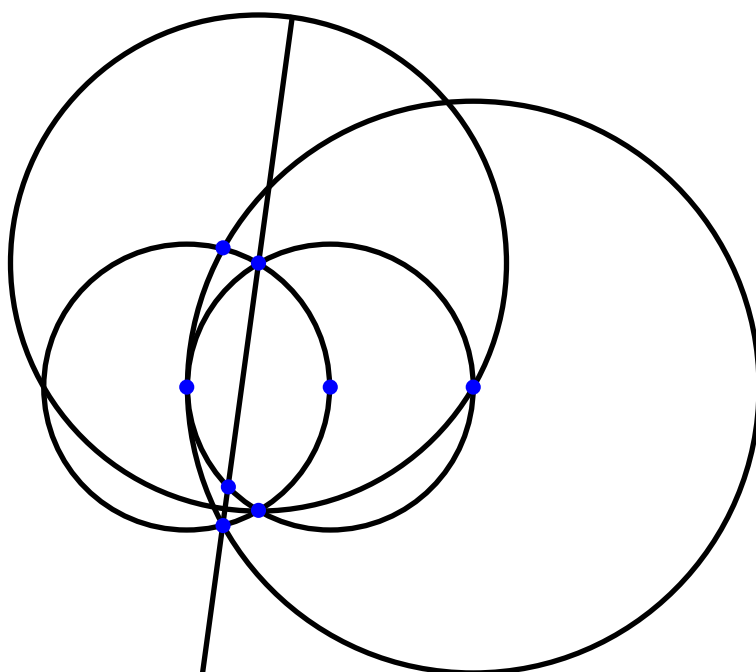
$$d_1 = |BH| \text{ (blue)} \dots 1$$

$$d_2 = |DI| \text{ (green)} \dots 1$$

$$d_3 = |AF| = |AG| \text{ (cyan)} \dots 2$$

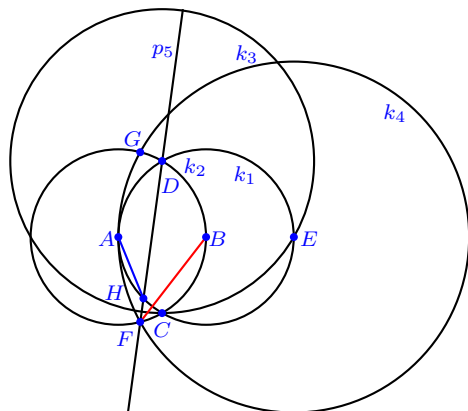
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} \text{ (3 occurrences)}$$

**2.49 Class of Similar Constructions No. 42****Contained Constructions**

RCC355, RCC385, RCC418, RCC448

## 2.49.1 Construction RCC355



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, C)$   
 $E \in k_1 \cap k_3$
  4.  $k_4 = k(E, A)$   
 $F \in k_2 \cap k_4, G \in k_2 \cap k_4$
  5.  $p_5 = p(D, F)$   
 $H \in p_5 \cap k_1$

## Distances

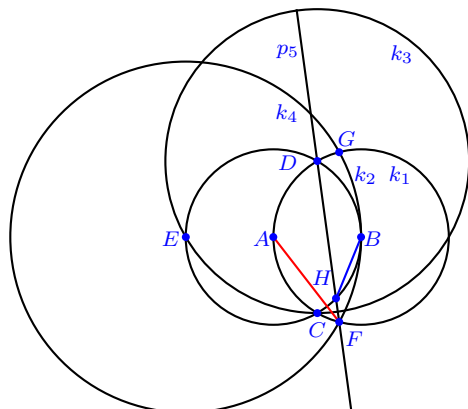
$$d_0 = |BF| = |BG| \text{ (red)} \dots 2$$

$$d_1 = |AH| \text{ (blue)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (2 occurrences)}$$

## 2.49.2 Construction RCC385



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, C)$   
 $E \in k_2 \cap k_3$
  4.  $k_4 = k(E, B)$   
 $F \in k_1 \cap k_4, G \in k_1 \cap k_4$
  5.  $p_5 = p(D, F)$   
 $H \in p_5 \cap k_2$

## Distances

$$d_0 = |AF| = |AG| \text{ (red) } \dots 2$$

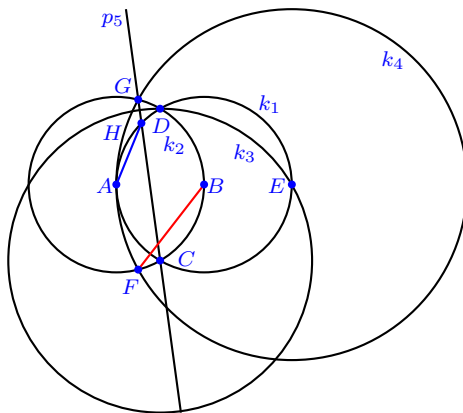
$$d_1 = |BH| \text{ (blue) } \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (2 occurrences)}$$



## 2.49.3 Construction RCC418



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, D)$   
 $E \in k_1 \cap k_3$
  4.  $k_4 = k(E, A)$   
 $F \in k_2 \cap k_4, G \in k_2 \cap k_4$
  5.  $p_5 = p(C, G)$   
 $H \in p_5 \cap k_1$

## Distances

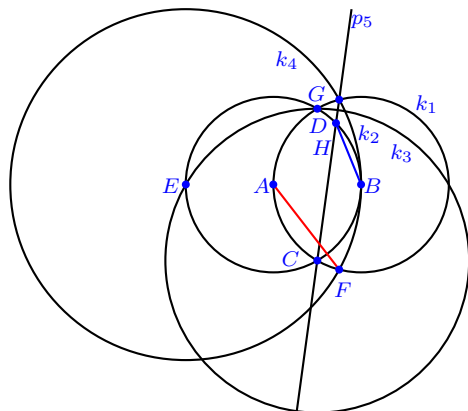
$$d_0 = |BF| = |BG| \text{ (red)} \dots 2$$

$$d_1 = |AH| \text{ (blue)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (2 occurrences)}$$

## 2.49.4 Construction RCC448



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, D)$   
 $E \in k_2 \cap k_3$
  4.  $k_4 = k(E, B)$   
 $F \in k_1 \cap k_4, G \in k_1 \cap k_4$
  5.  $p_5 = p(C, G)$   
 $H \in p_5 \cap k_2$

## Distances

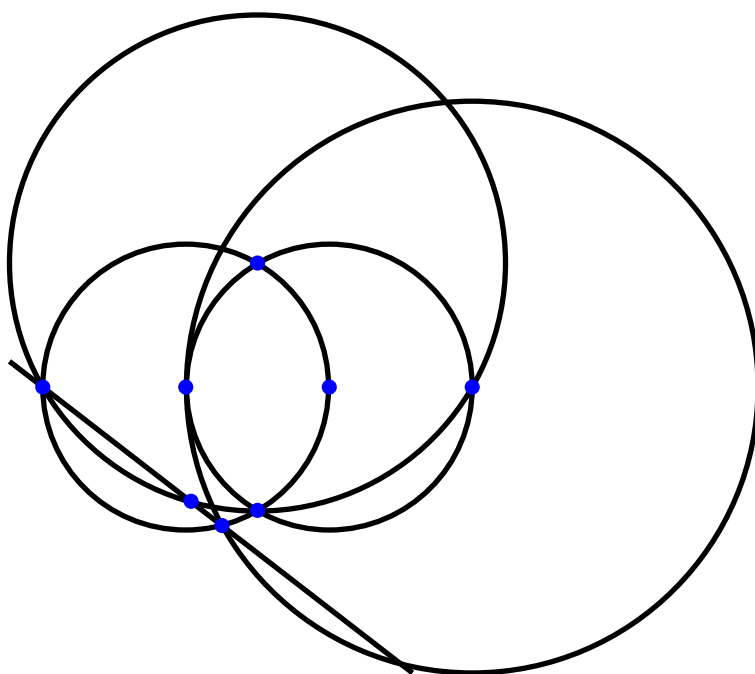
$$d_0 = |AF| = |AG| \text{ (red) } \dots 2$$

$$d_1 = |BH| \text{ (blue) } \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (2 occurrences)}$$

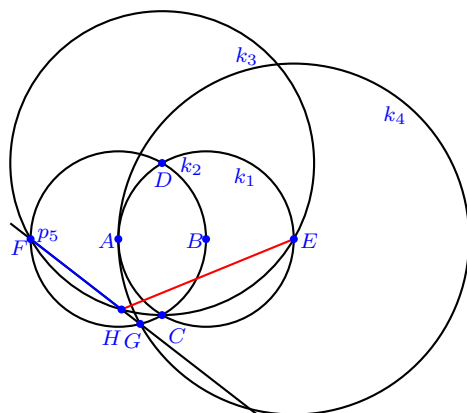
## 2.50 Class of Similar Constructions No. 43



### Contained Constructions

RCC359, RCC388, RCC425, RCC454

## 2.50.1 Construction RCC359



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, C)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, A)$   
 $G \in k_2 \cap k_4$
  5.  $p_5 = p(F, G)$   
 $H \in p_5 \cap k_3$

## Distances

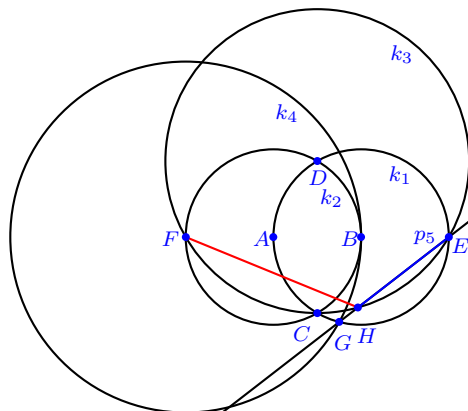
$$d_0 = |EH| \text{ (red)} \dots 1$$

$$d_1 = |FH| \text{ (blue)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (1 occurrence)}$$

## 2.50.2 Construction RCC388



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, C)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, B)$   
 $G \in k_1 \cap k_4$
  5.  $p_5 = p(E, G)$   
 $H \in p_5 \cap k_3$

## Distances

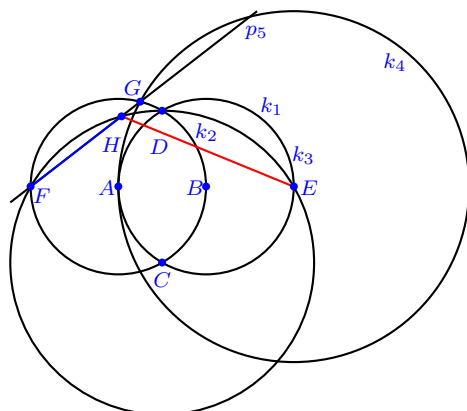
$$d_0 = |FH| \text{ (red) } \dots 1$$

$$d_1 = |EH| \text{ (blue) } \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (1 occurrence)}$$

## 2.50.3 Construction RCC425



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, D)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, A)$   
 $G \in k_2 \cap k_4$
  5.  $p_5 = p(F, G)$   
 $H \in p_5 \cap k_3$

## Distances

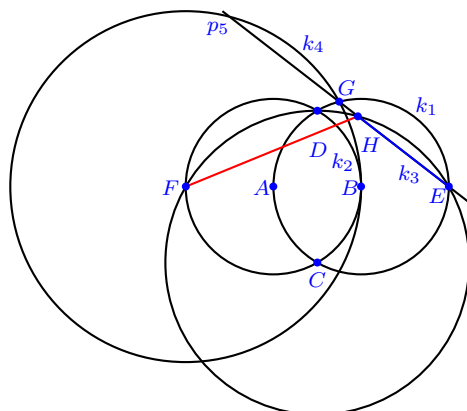
$$d_0 = |EH| \text{ (red) } \dots 1$$

$$d_1 = |FH| \text{ (blue) } \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (1 occurrence)}$$

## 2.50.4 Construction RCC454



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, D)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, B)$   
 $G \in k_1 \cap k_4$
  5.  $p_5 = p(E, G)$   
 $H \in p_5 \cap k_3$

## Distances

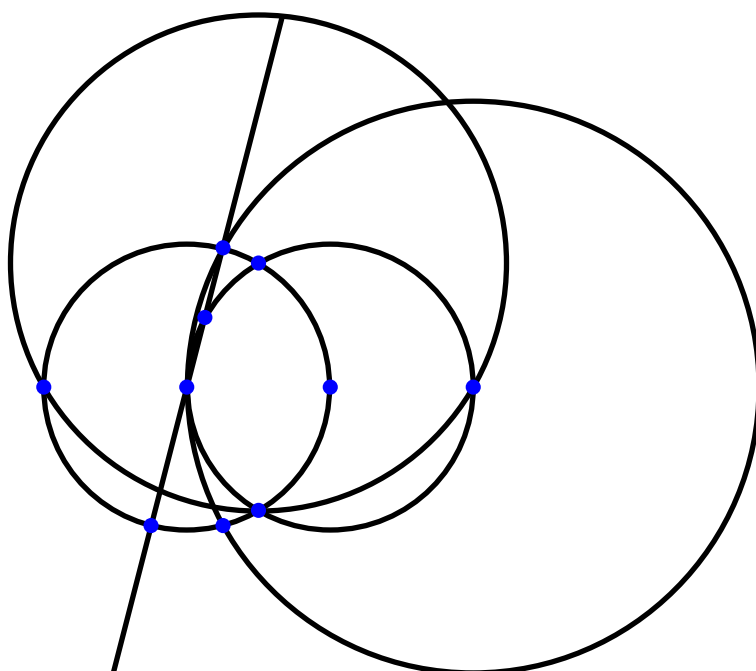
$$d_0 = |FH| \text{ (red) } \dots 1$$

$$d_1 = |EH| \text{ (blue) } \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (1 occurrence)}$$

## 2.51 Class of Similar Constructions No. 44

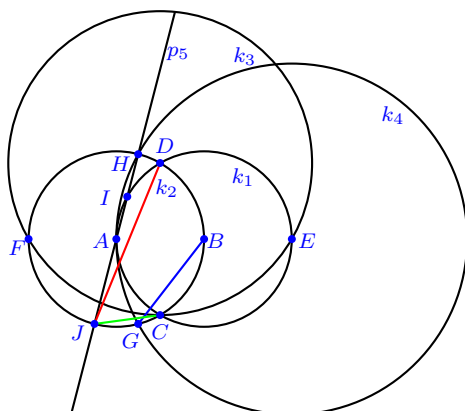


### Contained Constructions

RCC361, RCC391, RCC405, RCC435



## 2.51.1 Construction RCC361



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, C)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, A)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
  5.  $p_5 = p(A, H)$   
 $I \in p_5 \cap k_1, J \in p_5 \cap k_2$

## Distances

$$d_0 = |DJ| \text{ (red)} \dots 1$$

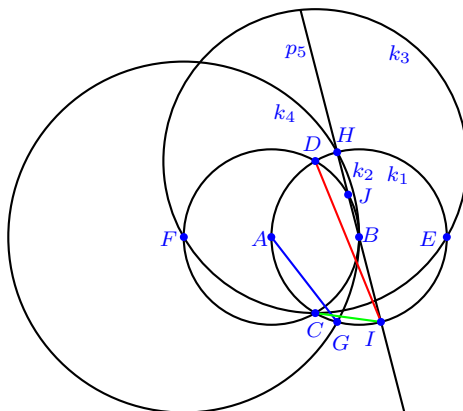
$$d_1 = |BG| = |BH| = |FI| = |FJ| \text{ (blue)} \dots 4$$

$$d_2 = |CJ| \text{ (green)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)}$$

**2.51.2 Construction RCC391**



**Construction Process**

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, C)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, B)$   
 $G \in k_1 \cap k_4, H \in k_1 \cap k_4$
  5.  $p_5 = p(B, H)$   
 $I \in p_5 \cap k_1, J \in p_5 \cap k_2$

**Distances**

$$d_0 = |DI| \text{ (red)} \dots 1$$

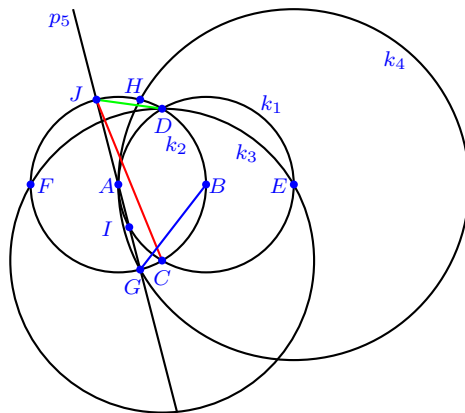
$$d_1 = |AG| = |AH| = |EI| = |EJ| \text{ (blue)} \dots 4$$

$$d_2 = |CI| \text{ (green)} \dots 1$$

**Occurrences of the Golden Ratio**

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)}$$

## 2.51.3 Construction RCC405



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, D)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, A)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
  5.  $p_5 = p(A, G)$   
 $I \in p_5 \cap k_1, J \in p_5 \cap k_2$

## Distances

$$d_0 = |CJ| \text{ (red)} \dots 1$$

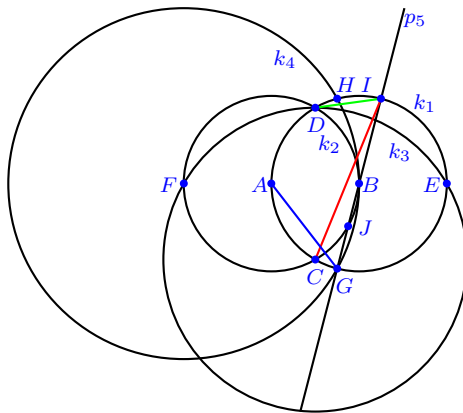
$$d_1 = |BG| = |BH| = |FI| = |FJ| \text{ (blue)} \dots 4$$

$$d_2 = |DJ| \text{ (green)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)}$$

## 2.51.4 Construction RCC435



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, D)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, B)$   
 $G \in k_1 \cap k_4, H \in k_1 \cap k_4$
  5.  $p_5 = p(B, G)$   
 $I \in p_5 \cap k_1, J \in p_5 \cap k_2$

## Distances

$$d_0 = |CI| \text{ (red)} \dots 1$$

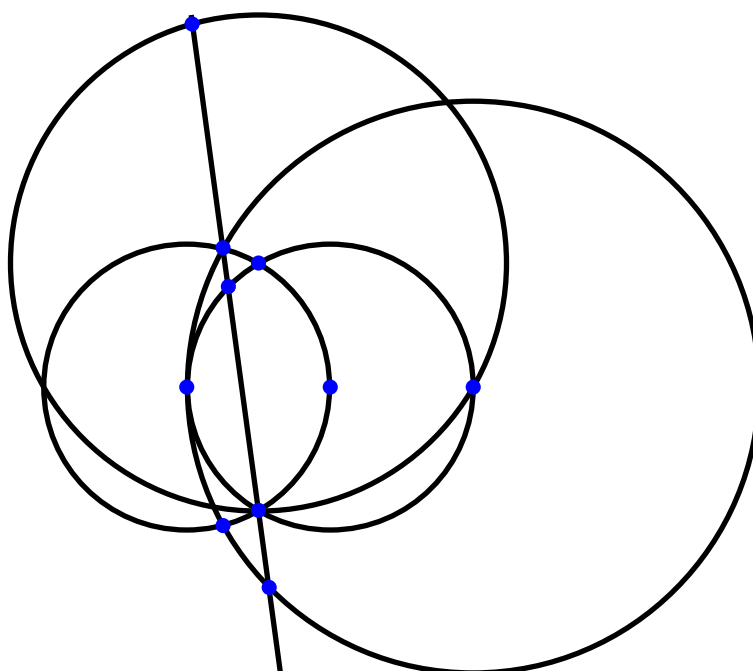
$$d_1 = |AG| = |AH| = |EI| = |EJ| \text{ (blue)} \dots 4$$

$$d_2 = |DI| \text{ (green)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)}$$

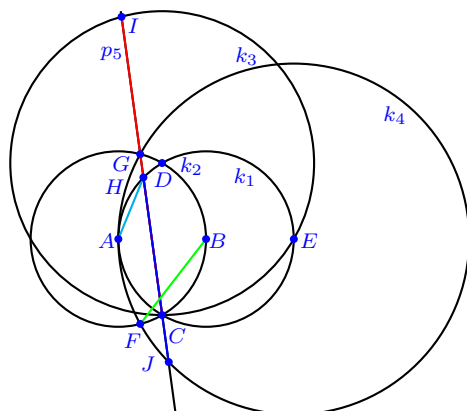
## 2.52 Class of Similar Constructions No. 45



### Contained Constructions

RCC363, RCC393, RCC410, RCC440

## 2.52.1 Construction RCC363



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, C)$   
 $E \in k_1 \cap k_3$
  4.  $k_4 = k(E, A)$   
 $F \in k_2 \cap k_4, G \in k_2 \cap k_4$
  5.  $p_5 = p(C, G)$   
 $H \in p_5 \cap k_1, I \in p_5 \cap k_3, J \in p_5 \cap k_4$

## Distances

$$d_0 = |CI| \text{ (red)} \dots 1$$

$$d_1 = |HJ| \text{ (blue)} \dots 1$$

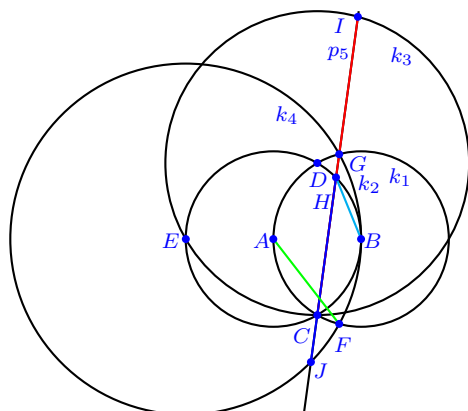
$$d_2 = |BF| = |BG| \text{ (green)} \dots 2$$

$$d_3 = |AH| \text{ (cyan)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_2}{d_3} \text{ (3 occurrences)}$$

## 2.52.2 Construction RCC393



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, C)$   
 $E \in k_2 \cap k_3$
  4.  $k_4 = k(E, B)$   
 $F \in k_1 \cap k_4, G \in k_1 \cap k_4$
  5.  $p_5 = p(C, G)$   
 $H \in p_5 \cap k_2, I \in p_5 \cap k_3, J \in p_5 \cap k_4$

## Distances

$$d_0 = |CI| \text{ (red)} \dots 1$$

$$d_1 = |HJ| \text{ (blue)} \dots 1$$

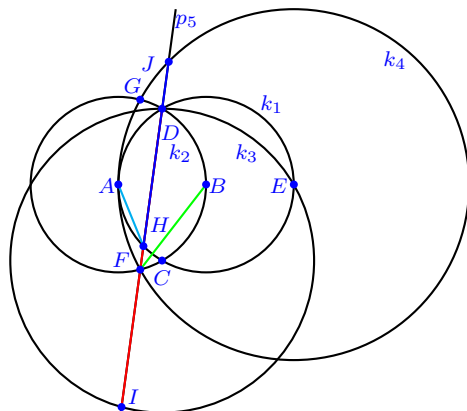
$$d_2 = |AF| = |AG| \text{ (green)} \dots 2$$

$$d_3 = |BH| \text{ (cyan)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_2}{d_3} \text{ (3 occurrences)}$$

## 2.52.3 Construction RCC410



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, D)$   
 $E \in k_1 \cap k_3$
  4.  $k_4 = k(E, A)$   
 $F \in k_2 \cap k_4, G \in k_2 \cap k_4$
  5.  $p_5 = p(D, F)$   
 $H \in p_5 \cap k_1, I \in p_5 \cap k_3, J \in p_5 \cap k_4$

## Distances

$$d_0 = |DI| \text{ (red)} \dots 1$$

$$d_1 = |HJ| \text{ (blue)} \dots 1$$

$$d_2 = |BF| = |BG| \text{ (green)} \dots 2$$

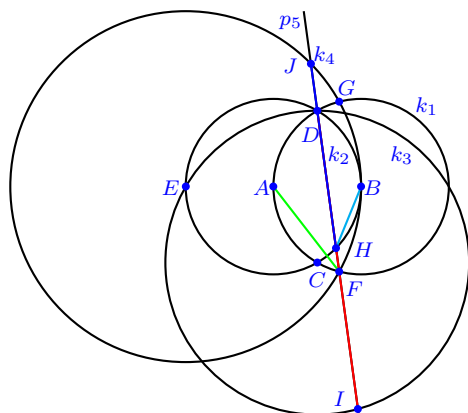
$$d_3 = |AH| \text{ (cyan)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_2}{d_3} \text{ (3 occurrences)}$$



## 2.52.4 Construction RCC440



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, D)$   
 $E \in k_2 \cap k_3$
  4.  $k_4 = k(E, B)$   
 $F \in k_1 \cap k_4, G \in k_1 \cap k_4$
  5.  $p_5 = p(D, F)$   
 $H \in p_5 \cap k_2, I \in p_5 \cap k_3, J \in p_5 \cap k_4$

## Distances

$$d_0 = |DI| \text{ (red)} \dots 1$$

$$d_1 = |HJ| \text{ (blue)} \dots 1$$

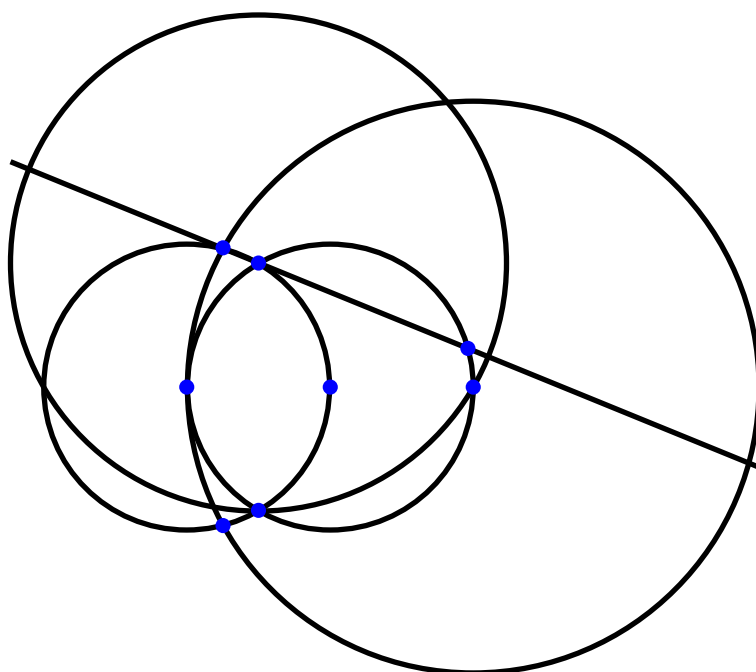
$$d_2 = |AF| = |AG| \text{ (green)} \dots 2$$

$$d_3 = |BH| \text{ (cyan)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_2}{d_3} \text{ (3 occurrences)}$$

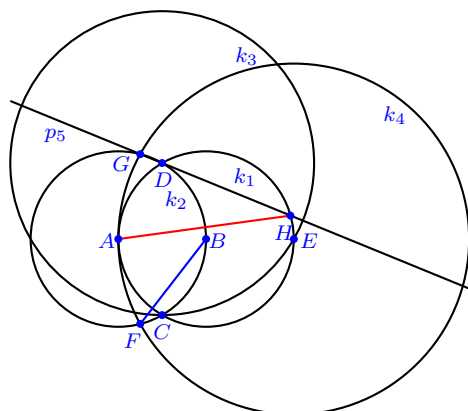
## 2.53 Class of Similar Constructions No. 46



### Contained Constructions

RCC366, RCC396, RCC407, RCC437

## 2.53.1 Construction RCC366



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, C)$   
 $E \in k_1 \cap k_3$
  4.  $k_4 = k(E, A)$   
 $F \in k_2 \cap k_4, G \in k_2 \cap k_4$
  5.  $p_5 = p(D, G)$   
 $H \in p_5 \cap k_1$

## Distances

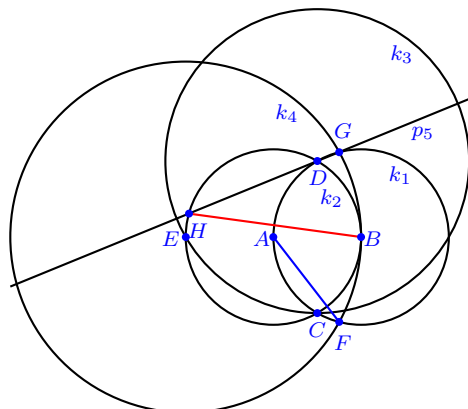
$$d_0 = |AH| \text{ (red) } \dots 1$$

$$d_1 = |BF| = |BG| \text{ (blue) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (2 occurrences)}$$

## 2.53.2 Construction RCC396



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, C)$   
 $E \in k_2 \cap k_3$
  4.  $k_4 = k(E, B)$   
 $F \in k_1 \cap k_4, G \in k_1 \cap k_4$
  5.  $p_5 = p(D, G)$   
 $H \in p_5 \cap k_2$

## Distances

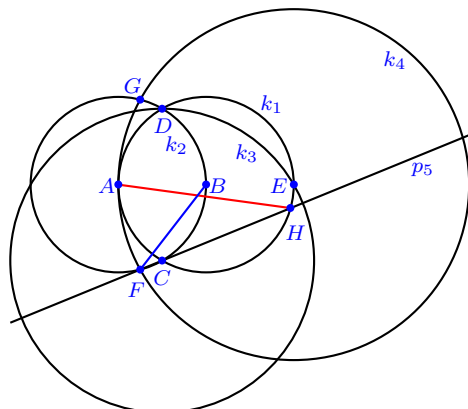
$$d_0 = |BH| \text{ (red) } \dots 1$$

$$d_1 = |AF| = |AG| \text{ (blue) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (2 occurrences)}$$

## 2.53.3 Construction RCC407



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, D)$   
 $E \in k_1 \cap k_3$
  4.  $k_4 = k(E, A)$   
 $F \in k_2 \cap k_4, G \in k_2 \cap k_4$
  5.  $p_5 = p(C, F)$   
 $H \in p_5 \cap k_1$

## Distances

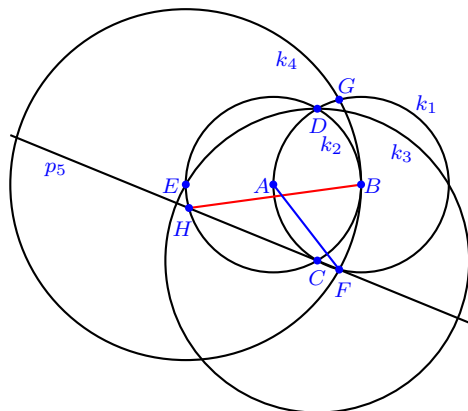
$$d_0 = |AH| \text{ (red)} \dots 1$$

$$d_1 = |BF| = |BG| \text{ (blue)} \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (2 occurrences)}$$

## 2.53.4 Construction RCC437



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, D)$   
 $E \in k_2 \cap k_3$
  4.  $k_4 = k(E, B)$   
 $F \in k_1 \cap k_4, G \in k_1 \cap k_4$
  5.  $p_5 = p(C, F)$   
 $H \in p_5 \cap k_2$

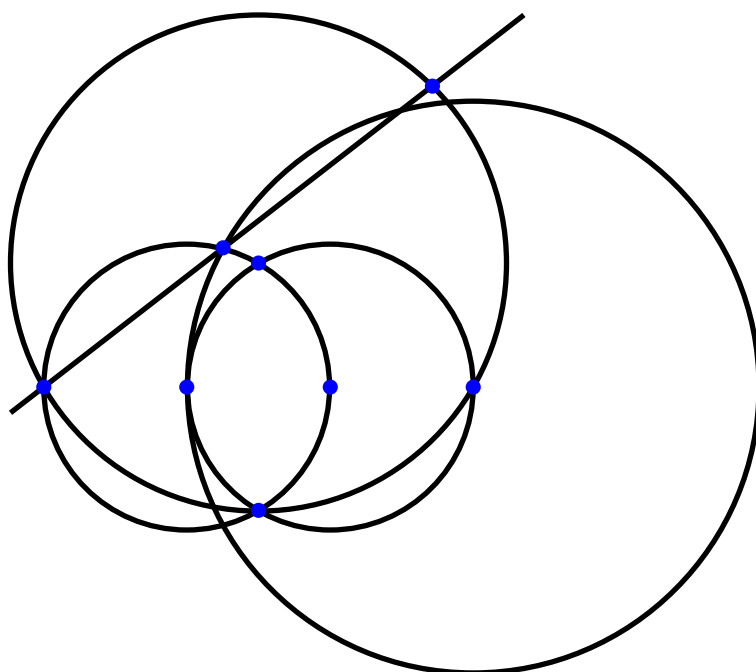
## Distances

$$d_0 = |BH| \text{ (red)} \dots 1$$

$$d_1 = |AF| = |AG| \text{ (blue)} \dots 2$$

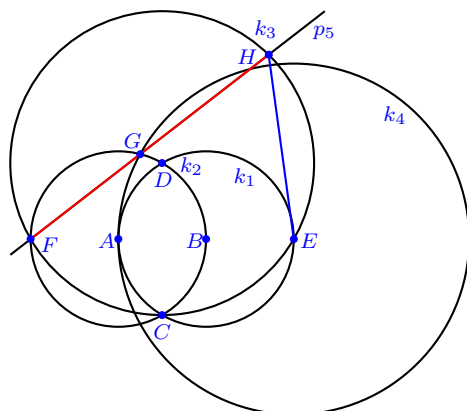
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (2 occurrences)}$$

**2.54 Class of Similar Constructions No. 47****Contained Constructions**

RCC370, RCC399, RCC414, RCC443

## 2.54.1 Construction RCC370



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, C)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, A)$   
 $G \in k_2 \cap k_4$
  5.  $p_5 = p(F, G)$   
 $H \in p_5 \cap k_3$

## Distances

$$d_0 = |FH| \text{ (red) } \dots 1$$

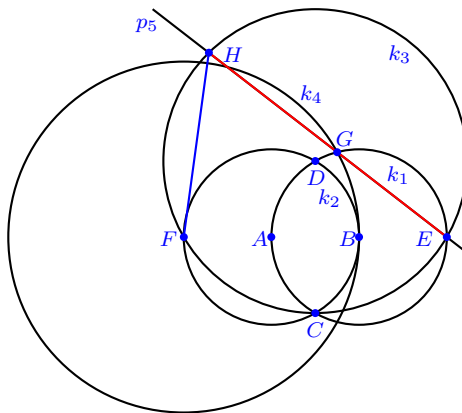
$$d_1 = |EH| \text{ (blue) } \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (1 occurrence)}$$



## 2.54.2 Construction RCC399



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, C)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, B)$   
 $G \in k_1 \cap k_4$
  5.  $p_5 = p(E, G)$   
 $H \in p_5 \cap k_3$

## Distances

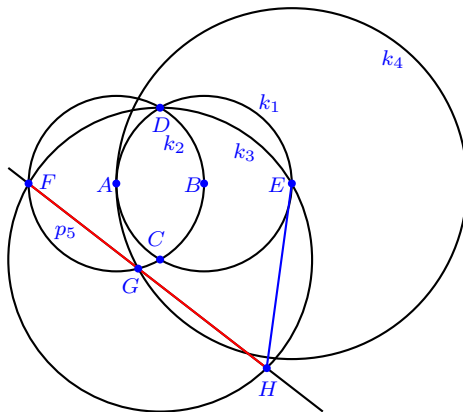
$$d_0 = |EH| \text{ (red) } \dots 1$$

$$d_1 = |FH| \text{ (blue) } \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (1 occurrence)}$$

## 2.54.3 Construction RCC414



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, D)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, A)$   
 $G \in k_2 \cap k_4$
  5.  $p_5 = p(F, G)$   
 $H \in p_5 \cap k_3$

## Distances

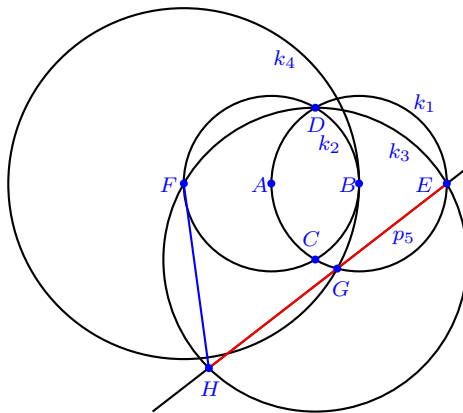
$$d_0 = |FH| \text{ (red) } \dots 1$$

$$d_1 = |EH| \text{ (blue) } \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (1 occurrence)}$$

## 2.54.4 Construction RCC443



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, D)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, B)$   
 $G \in k_1 \cap k_4$
  5.  $p_5 = p(E, G)$   
 $H \in p_5 \cap k_3$

## Distances

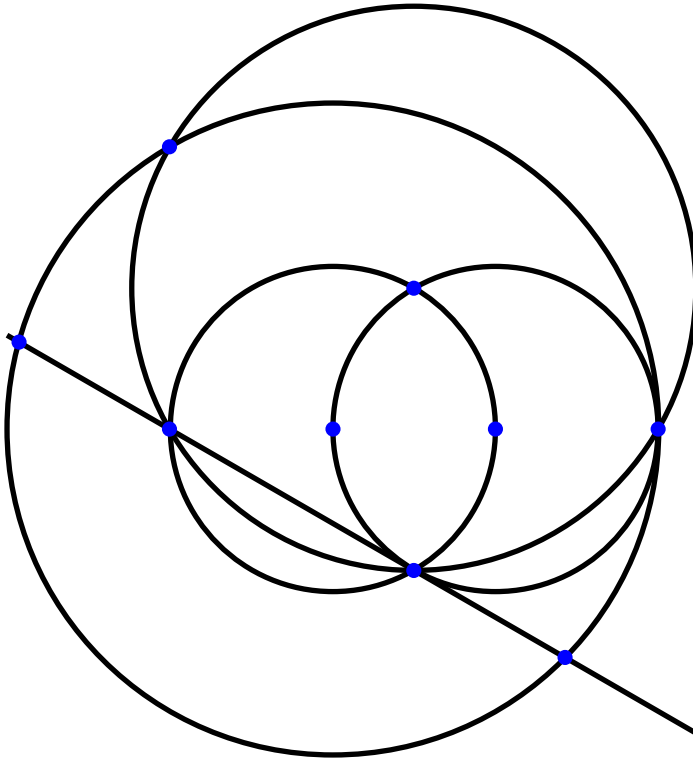
$$d_0 = |EH| \text{ (red) } \dots 1$$

$$d_1 = |FH| \text{ (blue) } \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (1 occurrence)}$$

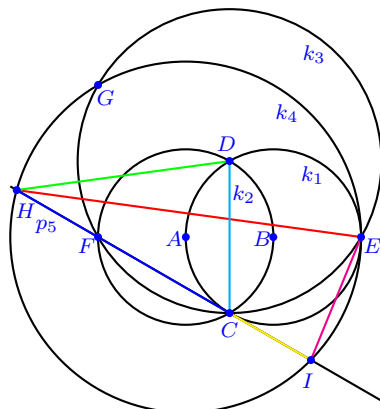
## 2.55 Class of Similar Constructions No. 48



### Contained Constructions

RCC373, RCC377, RCC429, RCC433

## 2.55.1 Construction RCC373



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, C)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(A, E)$   
 $G \in k_3 \cap k_4$
  5.  $p_5 = p(C, F)$   
 $H \in p_5 \cap k_4, I \in p_5 \cap k_4$

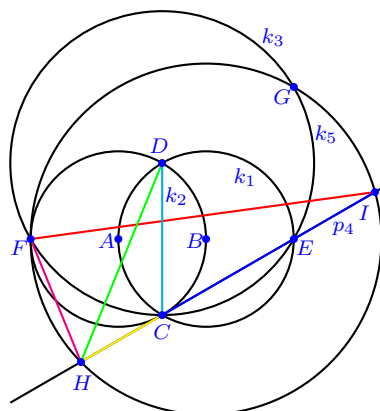
## Distances

$$\begin{aligned}
 d_0 &= |EH| = |GI| \text{ (red)} \dots 2 \\
 d_1 &= |CH| = |FI| \text{ (blue)} \dots 2 \\
 d_2 &= |DH| = |DI| \text{ (green)} \dots 2 \\
 d_3 &= |CD| = |CE| = |CF| = |DE| = |DF| = |DG| = |FG| \text{ (cyan)} \dots 7 \\
 d_4 &= |EI| = |GH| \text{ (magenta)} \dots 2 \\
 d_5 &= |CI| = |FH| \text{ (yellow)} \dots 2
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (36 occurrences)}$$

## 2.55.2 Construction RCC377



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, C)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $p_4 = p(C, E)$
  5.  $k_5 = k(B, F)$   
 $G \in k_3 \cap k_5, H \in p_4 \cap k_5, I \in p_4 \cap k_5$

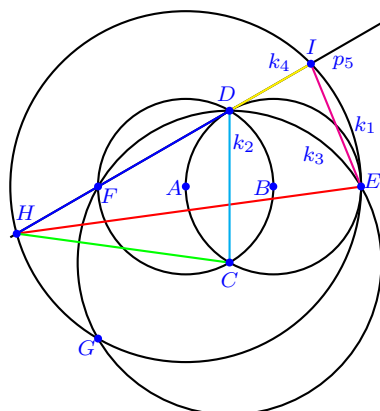
## Distances

$$\begin{aligned}
 d_0 &= |FI| = |GH| \text{ (red)} \dots 2 \\
 d_1 &= |CI| = |EH| \text{ (blue)} \dots 2 \\
 d_2 &= |DH| = |DI| \text{ (green)} \dots 2 \\
 d_3 &= |CD| = |CE| = |CF| = |DE| = |DF| = |DG| = |EG| \text{ (cyan)} \dots 7 \\
 d_4 &= |FH| = |GI| \text{ (magenta)} \dots 2 \\
 d_5 &= |CH| = |EI| \text{ (yellow)} \dots 2
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (36 occurrences)}$$

## 2.55.3 Construction RCC429



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, D)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(A, E)$   
 $G \in k_3 \cap k_4$
  5.  $p_5 = p(D, F)$   
 $H \in p_5 \cap k_4, I \in p_5 \cap k_4$

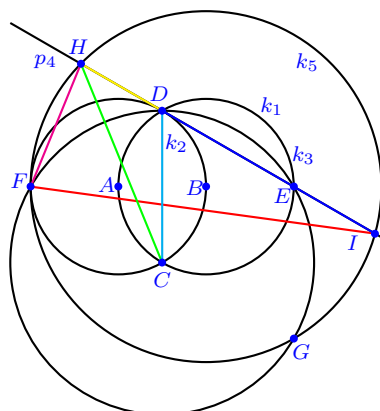
## Distances

$$\begin{aligned}
 d_0 &= |EH| = |GI| \text{ (red)} \dots 2 \\
 d_1 &= |DH| = |FI| \text{ (blue)} \dots 2 \\
 d_2 &= |CH| = |CI| \text{ (green)} \dots 2 \\
 d_3 &= |CD| = |CE| = |CF| = |CG| = |DE| = |DF| = |FG| \text{ (cyan)} \dots 7 \\
 d_4 &= |EI| = |GH| \text{ (magenta)} \dots 2 \\
 d_5 &= |DI| = |FH| \text{ (yellow)} \dots 2
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (36 occurrences)}$$

## 2.55.4 Construction RCC433



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, D)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $p_4 = p(D, E)$
  5.  $k_5 = k(B, F)$   
 $G \in k_3 \cap k_5, H \in p_4 \cap k_5, I \in p_4 \cap k_5$

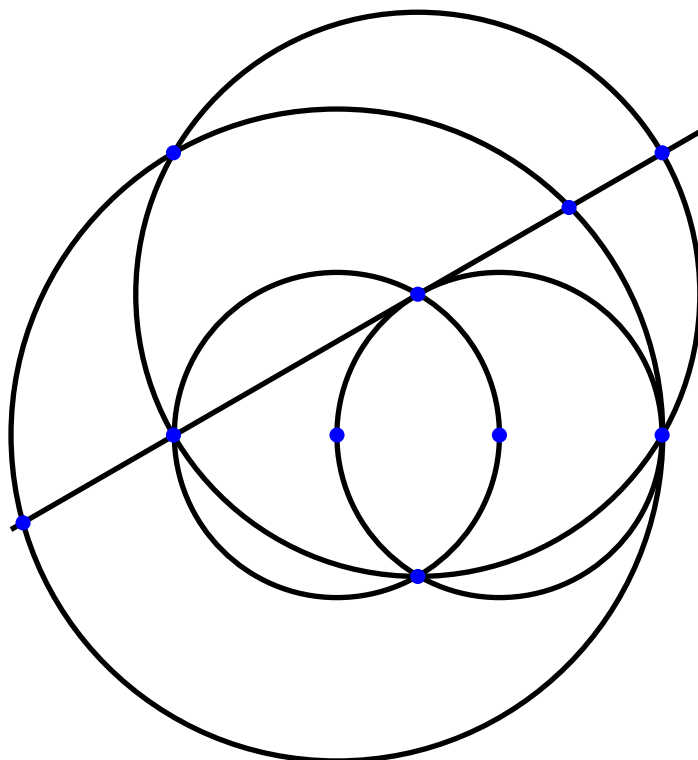
## Distances

$$\begin{aligned}
 d_0 &= |FI| = |GH| \text{ (red)} \dots 2 \\
 d_1 &= |DI| = |EH| \text{ (blue)} \dots 2 \\
 d_2 &= |CH| = |CI| \text{ (green)} \dots 2 \\
 d_3 &= |CD| = |CE| = |CF| = |CG| = |DE| = |DF| = |EG| \text{ (cyan)} \dots 7 \\
 d_4 &= |FH| = |GI| \text{ (magenta)} \dots 2 \\
 d_5 &= |DH| = |EI| \text{ (yellow)} \dots 2
 \end{aligned}$$

## Occurrences of the Golden Ratio

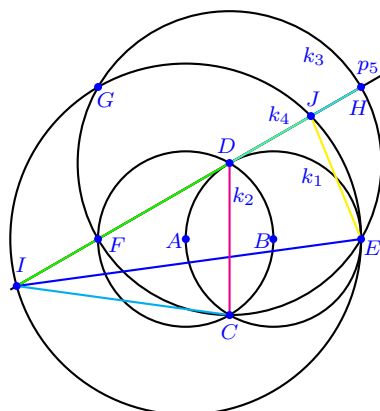
$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (36 occurrences)}$$



**2.56 Class of Similar Constructions No. 49****Contained Constructions**

RCC374, RCC378, RCC428, RCC432

## 2.56.1 Construction RCC374



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, C)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(A, E)$   
 $G \in k_3 \cap k_4$
  5.  $p_5 = p(D, F)$   
 $H \in p_5 \cap k_3, I \in p_5 \cap k_4, J \in p_5 \cap k_4$

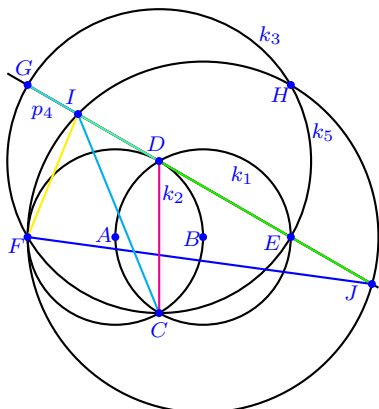
## Distances

$$\begin{aligned}
 d_0 &= |HI| \text{ (red)} \dots 1 \\
 d_1 &= |EI| \text{ (blue)} \dots 1 \\
 d_2 &= |DI| = |FJ| \text{ (green)} \dots 2 \\
 d_3 &= |CI| = |CJ| = |GI| = |GJ| \text{ (cyan)} \dots 4 \\
 d_4 &= |CD| = |CE| = |CF| = |DE| = |DF| = |DG| = |DH| = |EH| = |FG| \text{ (magenta)} \dots 9 \\
 d_5 &= |EJ| \text{ (yellow)} \dots 1 \\
 d_6 &= |DJ| = |FI| \text{ (grass)} \dots 2 \\
 d_7 &= |HJ| \text{ (sea)} \dots 1
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} = \frac{d_4}{d_6} = \frac{d_6}{d_7} \text{ (48 occurrences)}$$

## 2.56.2 Construction RCC378



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, C)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $p_4 = p(D, E)$   
 $G \in p_4 \cap k_3$
  5.  $k_5 = k(B, F)$   
 $H \in k_3 \cap k_5, I \in p_4 \cap k_5, J \in p_4 \cap k_5$

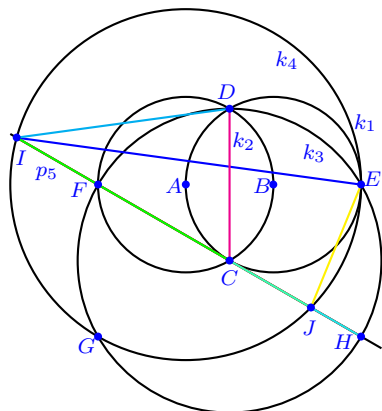
## Distances

- $d_0 = |GJ|$  (red) ... 1  
 $d_1 = |FJ|$  (blue) ... 1  
 $d_2 = |DJ| = |EI|$  (green) ... 2  
 $d_3 = |CI| = |CJ| = |HI| = |HJ|$  (cyan) ... 4  
 $d_4 = |CD| = |CE| = |CF| = |DE| = |DF| = |DG| = |DH| = |EH| = |FG|$  (magenta) ... 9  
 $d_5 = |FI|$  (yellow) ... 1  
 $d_6 = |DI| = |EJ|$  (grass) ... 2  
 $d_7 = |GI|$  (sea) ... 1

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} = \frac{d_4}{d_6} = \frac{d_6}{d_7} \quad (48 \text{ occurrences})$$

## 2.56.3 Construction RCC428



## Construction Process

- $A, B$  given initial points
- $k_1 = k(B, A)$
  - $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  - $k_3 = k(C, D)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  - $k_4 = k(A, E)$   
 $G \in k_3 \cap k_4$
  - $p_5 = p(C, F)$   
 $H \in p_5 \cap k_3, I \in p_5 \cap k_4, J \in p_5 \cap k_4$

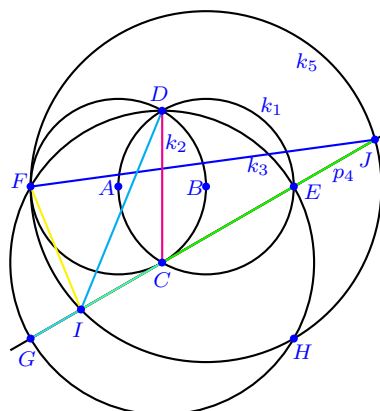
## Distances

- $d_0 = |HI|$  (red) ... 1  
 $d_1 = |EI|$  (blue) ... 1  
 $d_2 = |CI| = |FJ|$  (green) ... 2  
 $d_3 = |DI| = |DJ| = |GI| = |GJ|$  (cyan) ... 4  
 $d_4 = |CD| = |CE| = |CF| = |CG| = |CH| = |DE| = |DF| = |EH| = |FG|$  (magenta) ... 9  
 $d_5 = |EJ|$  (yellow) ... 1  
 $d_6 = |CJ| = |FI|$  (grass) ... 2  
 $d_7 = |HJ|$  (sea) ... 1

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} = \frac{d_4}{d_6} = \frac{d_6}{d_7} \quad (48 \text{ occurrences})$$

## 2.56.4 Construction RCC432



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, D)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $p_4 = p(C, E)$   
 $G \in p_4 \cap k_3$
  5.  $k_5 = k(B, F)$   
 $H \in k_3 \cap k_5, I \in p_4 \cap k_5, J \in p_4 \cap k_5$

## Distances

- $d_0 = |GJ|$  (red) ... 1  
 $d_1 = |FJ|$  (blue) ... 1  
 $d_2 = |CJ| = |EI|$  (green) ... 2  
 $d_3 = |DI| = |DJ| = |HI| = |HJ|$  (cyan) ... 4  
 $d_4 = |CD| = |CE| = |CF| = |CG| = |CH| = |DE| = |DF| = |EH| = |FG|$  (magenta) ... 9  
 $d_5 = |FI|$  (yellow) ... 1  
 $d_6 = |CI| = |EJ|$  (grass) ... 2  
 $d_7 = |GI|$  (sea) ... 1

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} = \frac{d_4}{d_6} = \frac{d_6}{d_7} \quad (48 \text{ occurrences})$$

## 2.57 MM

The constructions described in this section consist only of circles; the processes use a compass to measure the distance between two points for the radius (a circle with a given center and radius).

These constructions are sometimes called Mascheroni's or Mohr-Mascheroni's constructions. Lorenzo Mascheroni described the constructions in [2] in 1797, and Georg Mohr described the identical constructions one hundred years prior in [3]. For the names of these constructions, we selected the prefix MM.

The shortest Mohr-Mascheroni construction of the golden ratio requires 5 steps. In total, 368 of these constructions resulting from 988 different processes have been noted. These constructions form 80 classes of mutually similar constructions.

The constructions are labeled MM1 to MM368.

From this list (the numbering remains unaltered), we omit 114 constructions that contain only circles with a radius equal to the distance from the center to the other point; these constructions are mentioned in the list of CCO constructions.

### Remarks

Several MM constructions are analogous to RCC constructions, in which a straight line is replaced by a circle.

When construction RCC361 is compared with construction MM331, straight line  $p_5$  is replaced by circle  $k_5$ . The resulting points of intersection are identical. In MM331, more usable points of intersection are noted; therefore, the golden ratio can be found more frequently.

In the analysis of RCC constructions, we compared RC3 and RCC35. Here we can look for comparison with those constructions, where we would replace the straight line  $p_1$  by a circle. Such construction can really be found. It contains five lines, but measuring or transferring the distance to the radius of the circle is not required; therefore, we will find it between the constructions of CCO.

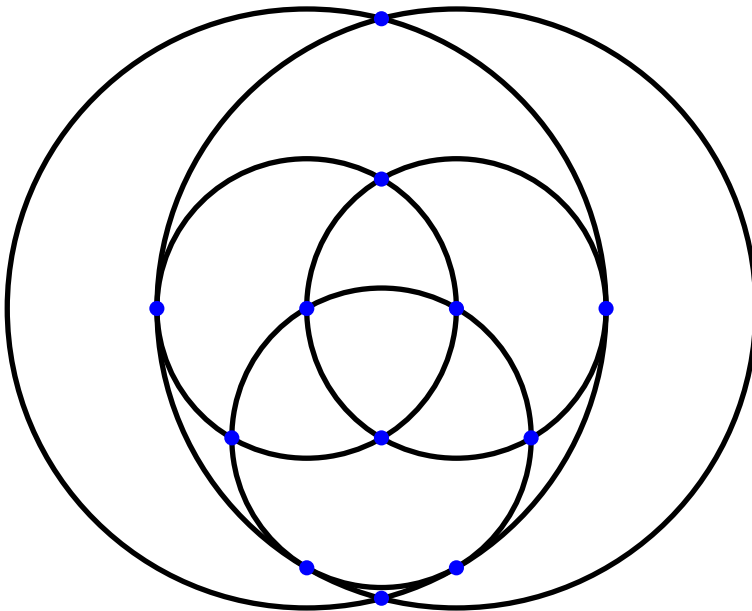
## 2.58 Constructions Omitted Due to Identity

MM1 ( RCC267 CCO2), MM2 ( RCC266 CCO1), MM3 ( RCC269 CCO4),  
 MM4 ( RCC268 CCO3), MM5 ( RCC276 CCO11), MM6 ( RCC277 CCO12),  
 MM7 ( RCC288 CCO23), MM8 ( RCC289 CCO24), MM9 ( RCC273 CCO8),  
 MM10 ( RCC272 CCO7), MM11 ( RCC271 CCO6), MM12 ( RCC270 CCO5),  
 MM13 ( RCC283 CCO18), MM14 ( RCC282 CCO17), MM15 ( RCC284 CCO19),  
 MM16 ( RCC285 CCO20), MM17 ( RCC286 CCO21), MM18 ( RCC287 CCO22),  
 MM19 ( RCC278 CCO13), MM20 ( RCC279 CCO14), MM21 ( RCC280 CCO15),  
 MM22 ( RCC281 CCO16), MM23 ( RCC274 CCO9), MM24 ( RCC275 CCO10),  
 MM46 ( RCC320 CCO31), MM47 ( RCC321 CCO32), MM48 ( RCC332 CCO43),  
 MM49 ( RCC333 CCO44), MM50 ( RCC317 CCO28), MM51 ( RCC316 CCO27),  
 MM52 ( RCC315 CCO26), MM53 ( RCC314 CCO25), MM54 ( RCC327 CCO38),  
 MM55 ( RCC326 CCO37), MM56 ( RCC328 CCO39), MM57 ( RCC329 CCO40),  
 MM58 ( RCC330 CCO41), MM59 ( RCC331 CCO42), MM60 ( RCC322 CCO33),  
 MM61 ( RCC323 CCO34), MM62 ( RCC324 CCO35), MM63 ( RCC325 CCO36),  
 MM64 ( RCC318 CCO29), MM65 ( RCC319 CCO30), MM147 ( RCC404 CCO81),  
 MM148 ( RCC431 CCO98), MM149 ( RCC427 CCO96), MM152 ( RCC403 CCO80),  
 MM153 ( RCC430 CCO97), MM156 ( RCC434 CCO99), MM157 ( RCC457 CCO114),  
 MM166 ( RCC436 CCO100), MM168 ( RCC445 CCO106), MM169 ( RCC447 CCO107),  
 MM171 ( RCC456 CCO113), MM178 ( RCC439 CCO102), MM179 ( RCC453 CCO111),  
 MM182 ( RCC438 CCO101), MM183 ( RCC452 CCO110), MM186 ( RCC450 CCO109),  
 MM187 ( RCC442 CCO104), MM190 ( RCC441 CCO103), MM191 ( RCC449 CCO108),  
 MM196 ( RCC444 CCO105), MM197 ( RCC455 CCO112), MM212 ( RCC406 CCO82),  
 MM214 ( RCC413 CCO87), MM215 ( RCC417 CCO89), MM217 ( RCC424 CCO94),  
 MM224 ( RCC409 CCO84), MM225 ( RCC423 CCO93), MM228 ( RCC408 CCO83),  
 MM229 ( RCC422 CCO92), MM232 ( RCC420 CCO91), MM233 ( RCC412 CCO86),  
 MM236 ( RCC411 CCO85), MM237 ( RCC419 CCO90), MM242 ( RCC415 CCO88),  
 MM243 ( RCC426 CCO95), MM258 ( RCC349 CCO46), MM259 ( RCC376 CCO63),  
 MM260 ( RCC372 CCO61), MM263 ( RCC348 CCO45), MM264 ( RCC375 CCO62),  
 MM267 ( RCC379 CCO64), MM268 ( RCC402 CCO79), MM277 ( RCC381 CCO65),  
 MM279 ( RCC390 CCO71), MM280 ( RCC392 CCO72), MM282 ( RCC401 CCO78),  
 MM289 ( RCC384 CCO67), MM290 ( RCC398 CCO76), MM293 ( RCC383 CCO66),  
 MM294 ( RCC397 CCO75), MM297 ( RCC395 CCO74), MM298 ( RCC387 CCO69),  
 MM301 ( RCC386 CCO68), MM302 ( RCC394 CCO73), MM307 ( RCC389 CCO70),  
 MM308 ( RCC400 CCO77), MM323 ( RCC351 CCO47), MM325 ( RCC358 CCO52),  
 MM326 ( RCC362 CCO54), MM328 ( RCC369 CCO59), MM335 ( RCC354 CCO49),  
 MM336 ( RCC368 CCO58), MM339 ( RCC353 CCO48), MM340 ( RCC367 CCO57),  
 MM343 ( RCC365 CCO56), MM344 ( RCC357 CCO51), MM347 ( RCC356 CCO50),  
 MM348 ( RCC364 CCO55), MM353 ( RCC360 CCO53), MM354 ( RCC371 CCO60),

Omitted 114 constructions.

Contains 254 constructions.

## 2.59 Class of Similar Constructions No. 50

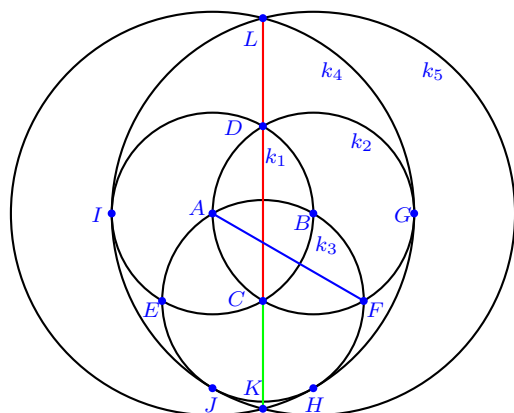


### Contained Constructions

MM25, MM26, MM33, MM66, MM68, MM75



## 2.59.1 Construction MM25



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |AB|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(A, |DE|)$   
 $G \in k_2 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(B, |DE|)$   
 $I \in k_1 \cap k_5, J \in k_3 \cap k_5, K \in k_4 \cap k_5, L \in k_4 \cap k_5$

## Distances

$$d_0 = |CL| = |DK| \text{ (red) } \dots 2$$

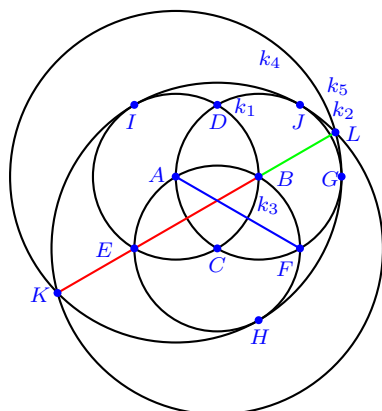
$$d_1 = |AF| = |AJ| = |BE| = |BH| = |CD| = |CG| = |CI| = |DG| = |DI| = |EH| = |FJ| \text{ (blue) } \dots 11$$

$$d_2 = |CK| = |DL| \text{ (green) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (44 occurrences)}$$

## 2.59.2 Construction MM26



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |AB|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(A, |DE|)$   
 $G \in k_2 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(C, |DE|)$   
 $I \in k_1 \cap k_5, J \in k_2 \cap k_5, K \in k_4 \cap k_5, L \in k_4 \cap k_5$

## Distances

$$d_0 = |BK| = |EL| \text{ (red) } \dots 2$$

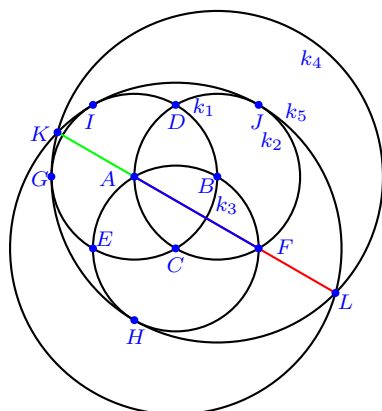
$$d_1 = |AF| = |AJ| = |BE| = |BH| = |BI| = |CD| = |CG| = |DG| = |EH| = |EI| = |FJ| \text{ (blue) } \dots 11$$

$$d_2 = |BL| = |EK| \text{ (green) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (44 occurrences)}$$

## 2.59.3 Construction MM33



## Construction Process

$A, B$  given initial points

1.  $k_1 = k(A, |AB|)$
2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
3.  $k_3 = k(C, |AB|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
4.  $k_4 = k(B, |DE|)$   
 $G \in k_1 \cap k_4, H \in k_3 \cap k_4$
5.  $k_5 = k(C, |DE|)$   
 $I \in k_1 \cap k_5, J \in k_2 \cap k_5, K \in k_4 \cap k_5, L \in k_4 \cap k_5$

## Distances

$$d_0 = |AL| = |FK| \text{ (red) } \dots 2$$

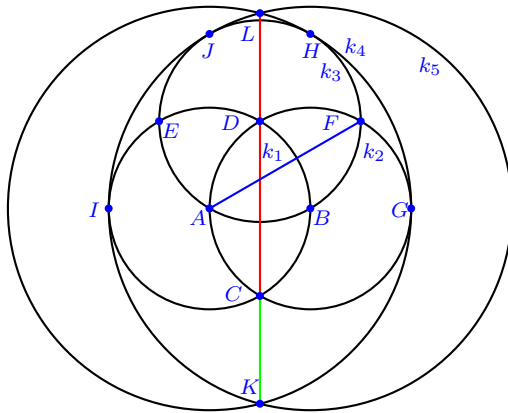
$$d_1 = |AF| = |AH| = |AJ| = |BE| = |BI| = |CD| = |CG| = |DG| = |EI| = |FH| = |FJ| \text{ (blue) } \dots 11$$

$$d_2 = |AK| = |FL| \text{ (green) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (44 occurrences)}$$

## 2.59.4 Construction MM66



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |AB|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(A, |CE|)$   
 $G \in k_2 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(B, |CE|)$   
 $I \in k_1 \cap k_5, J \in k_3 \cap k_5, K \in k_4 \cap k_5, L \in k_4 \cap k_5$

## Distances

$$d_0 = |CL| = |DK| \text{ (red)} \dots 2$$

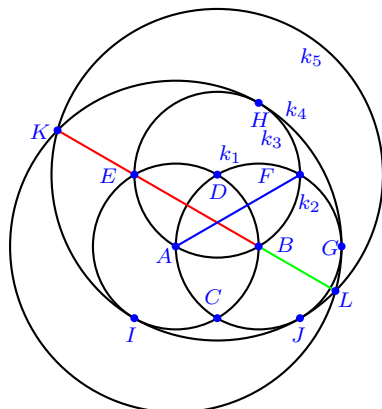
$$d_1 = |AF| = |AJ| = |BE| = |BH| = |CD| = |CG| = |CI| = |DG| = |DI| = |EH| = |FJ| \text{ (blue)} \dots 11$$

$$d_2 = |CK| = |DL| \text{ (green)} \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (44 occurrences)}$$

## 2.59.5 Construction MM68



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |AB|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(A, |CE|)$   
 $G \in k_2 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(D, |CE|)$   
 $I \in k_1 \cap k_5, J \in k_2 \cap k_5, K \in k_4 \cap k_5, L \in k_4 \cap k_5$

## Distances

$$d_0 = |BK| = |EL| \text{ (red) } \dots 2$$

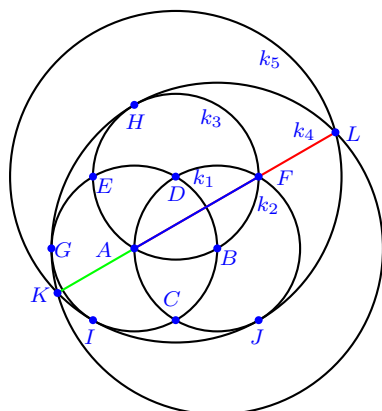
$$d_1 = |AF| = |AJ| = |BE| = |BH| = |BI| = |CD| = |CG| = |DG| = |EH| = |EI| = |FJ| \text{ (blue) } \dots 11$$

$$d_2 = |BL| = |EK| \text{ (green) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (44 occurrences)}$$

## 2.59.6 Construction MM75



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |AB|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(B, |CE|)$   
 $G \in k_1 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(D, |CE|)$   
 $I \in k_1 \cap k_5, J \in k_2 \cap k_5, K \in k_4 \cap k_5, L \in k_4 \cap k_5$

## Distances

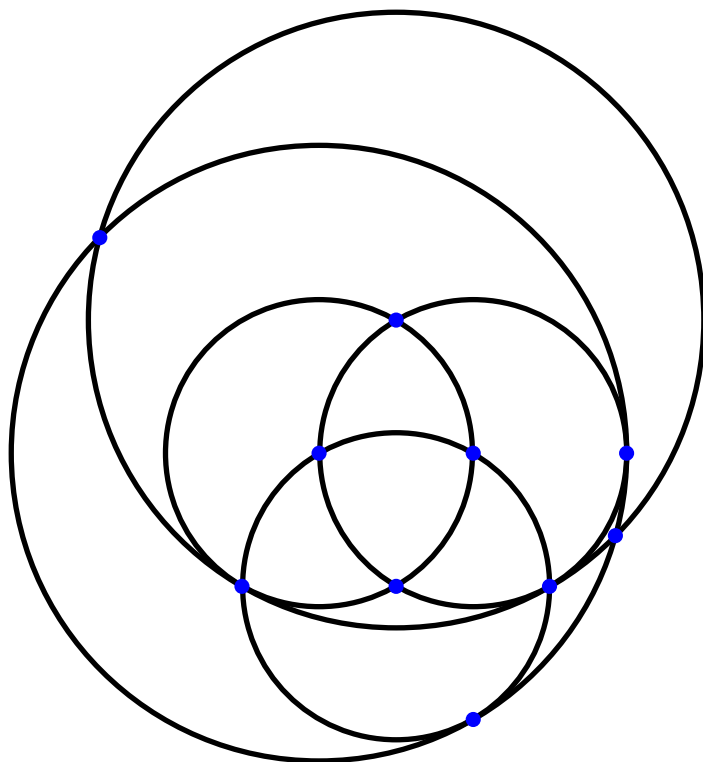
$$d_0 = |AL| = |FK| \text{ (red) } \dots 2$$

$$d_1 = |AF| = |AH| = |AJ| = |BE| = |BI| = |CD| = |CG| = |DG| = |EI| = |FH| = |FJ| \text{ (blue) } \dots 11$$

$$d_2 = |AK| = |FL| \text{ (green) } \dots 2$$

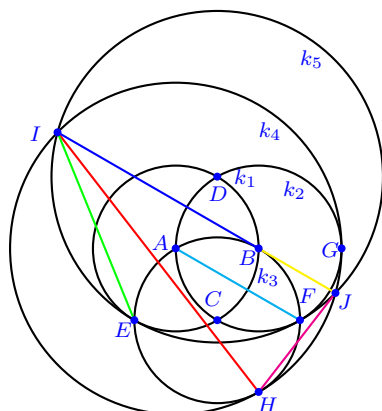
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (44 occurrences)}$$

**2.60 Class of Similar Constructions No. 51****Contained Constructions**

MM27, MM28, MM34, MM35, MM40, MM41, MM67, MM69, MM74, MM76,  
MM81, MM82

## 2.60.1 Construction MM27



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |AB|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(A, |DE|)$   
 $G \in k_2 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(D, |DE|)$   
 $I \in k_4 \cap k_5, J \in k_4 \cap k_5$

## Distances

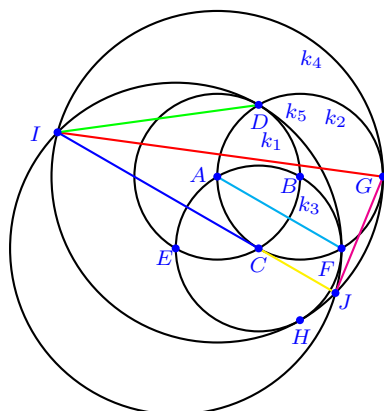
- $d_0 = |HI|$  (red) ... 1  
 $d_1 = |BI|$  (blue) ... 1  
 $d_2 = |EI| = |EJ|$  (green) ... 2  
 $d_3 = |AF| = |BE| = |BH| = |CD| = |CG| = |DG| = |EH|$  (cyan) ... 7  
 $d_4 = |HJ|$  (magenta) ... 1  
 $d_5 = |BJ|$  (yellow) ... 1

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \quad (18 \text{ occurrences})$$



## 2.60.2 Construction MM28



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |AB|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(A, |DE|)$   
 $G \in k_2 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(E, |DE|)$   
 $I \in k_4 \cap k_5, J \in k_4 \cap k_5$

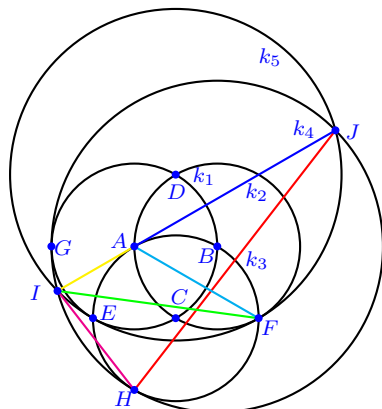
## Distances

- $d_0 = |GI|$  (red) ... 1  
 $d_1 = |CI|$  (blue) ... 1  
 $d_2 = |DI| = |DJ|$  (green) ... 2  
 $d_3 = |AF| = |BE| = |BH| = |CD| = |CG| = |DG| = |EH|$  (cyan) ... 7  
 $d_4 = |GJ|$  (magenta) ... 1  
 $d_5 = |CJ|$  (yellow) ... 1

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \quad (18 \text{ occurrences})$$

## 2.60.3 Construction MM34



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |AB|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(B, |DE|)$   
 $G \in k_1 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(D, |DE|)$   
 $I \in k_4 \cap k_5, J \in k_4 \cap k_5$

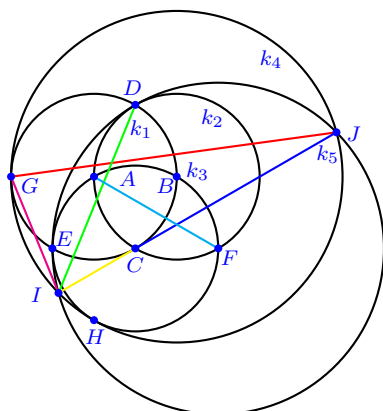
## Distances

- $d_0 = |HJ|$  (red) ... 1  
 $d_1 = |AJ|$  (blue) ... 1  
 $d_2 = |FI| = |FJ|$  (green) ... 2  
 $d_3 = |AF| = |AH| = |BE| = |CD| = |CG| = |DG| = |FH|$  (cyan) ... 7  
 $d_4 = |HI|$  (magenta) ... 1  
 $d_5 = |AI|$  (yellow) ... 1

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \quad (18 \text{ occurrences})$$

## 2.60.4 Construction MM35



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |AB|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(B, |DE|)$   
 $G \in k_1 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(F, |DE|)$   
 $I \in k_4 \cap k_5, J \in k_4 \cap k_5$

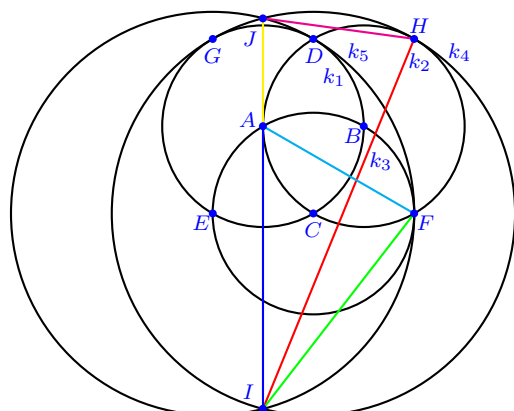
## Distances

$$\begin{aligned}
 d_0 &= |GJ| \text{ (red)} \dots 1 \\
 d_1 &= |CJ| \text{ (blue)} \dots 1 \\
 d_2 &= |DI| = |DJ| \text{ (green)} \dots 2 \\
 d_3 &= |AF| = |AH| = |BE| = |CD| = |CG| = |DG| = |FH| \text{ (cyan)} \dots 7 \\
 d_4 &= |GI| \text{ (magenta)} \dots 1 \\
 d_5 &= |CI| \text{ (yellow)} \dots 1
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (18 occurrences)}$$

## 2.60.5 Construction MM40



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |AB|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(C, |DE|)$   
 $G \in k_1 \cap k_4, H \in k_2 \cap k_4$
  5.  $k_5 = k(E, |DE|)$   
 $I \in k_4 \cap k_5, J \in k_4 \cap k_5$

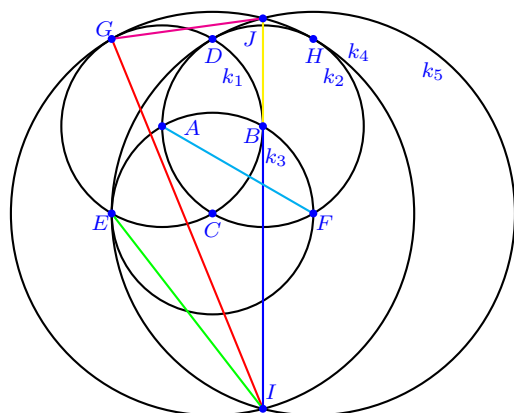
## Distances

- $d_0 = |HI|$  (red) ... 1  
 $d_1 = |AI|$  (blue) ... 1  
 $d_2 = |FI| = |FJ|$  (green) ... 2  
 $d_3 = |AF| = |AH| = |BE| = |BG| = |CD| = |EG| = |FH|$  (cyan) ... 7  
 $d_4 = |HJ|$  (magenta) ... 1  
 $d_5 = |AJ|$  (yellow) ... 1

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (18 occurrences)}$$

## 2.60.6 Construction MM41



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |AB|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(C, |DE|)$   
 $G \in k_1 \cap k_4, H \in k_2 \cap k_4$
  5.  $k_5 = k(F, |DE|)$   
 $I \in k_4 \cap k_5, J \in k_4 \cap k_5$

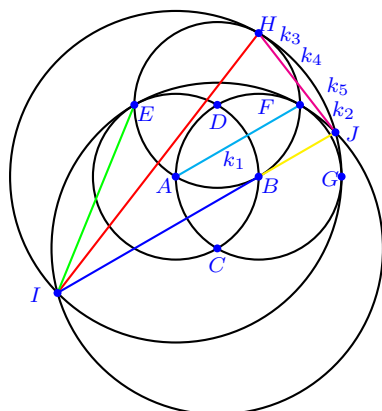
## Distances

- $d_0 = |GI|$  (red) ... 1  
 $d_1 = |BI|$  (blue) ... 1  
 $d_2 = |EI| = |EJ|$  (green) ... 2  
 $d_3 = |AF| = |AH| = |BE| = |BG| = |CD| = |EG| = |FH|$  (cyan) ... 7  
 $d_4 = |GJ|$  (magenta) ... 1  
 $d_5 = |BJ|$  (yellow) ... 1

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (18 occurrences)}$$

## 2.60.7 Construction MM67



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |AB|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(A, |CE|)$   
 $G \in k_2 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(C, |CE|)$   
 $I \in k_4 \cap k_5, J \in k_4 \cap k_5$

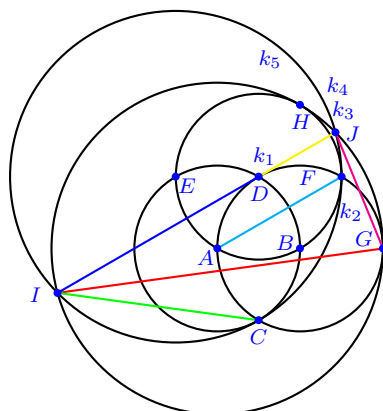
## Distances

- $d_0 = |HI|$  (red) ... 1  
 $d_1 = |BI|$  (blue) ... 1  
 $d_2 = |EI| = |EJ|$  (green) ... 2  
 $d_3 = |AF| = |BE| = |BH| = |CD| = |CG| = |DG| = |EH|$  (cyan) ... 7  
 $d_4 = |HJ|$  (magenta) ... 1  
 $d_5 = |BJ|$  (yellow) ... 1

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \quad (18 \text{ occurrences})$$

## 2.60.8 Construction MM69



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |AB|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(A, |CE|)$   
 $G \in k_2 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(E, |CE|)$   
 $I \in k_4 \cap k_5, J \in k_4 \cap k_5$

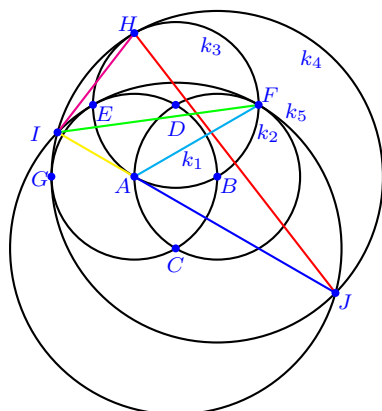
## Distances

- $d_0 = |GI|$  (red) ... 1  
 $d_1 = |DI|$  (blue) ... 1  
 $d_2 = |CI| = |CJ|$  (green) ... 2  
 $d_3 = |AF| = |BE| = |BH| = |CD| = |CG| = |DG| = |EH|$  (cyan) ... 7  
 $d_4 = |GJ|$  (magenta) ... 1  
 $d_5 = |DJ|$  (yellow) ... 1

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \quad (18 \text{ occurrences})$$

## 2.60.9 Construction MM74



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |AB|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(B, |CE|)$   
 $G \in k_1 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(C, |CE|)$   
 $I \in k_4 \cap k_5, J \in k_4 \cap k_5$

## Distances

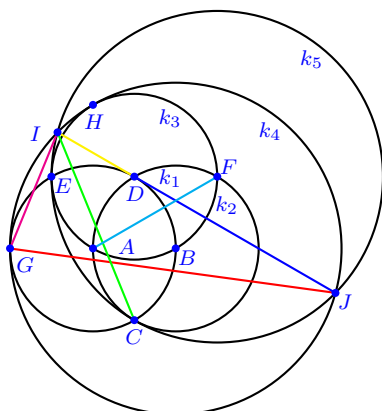
- $d_0 = |HJ|$  (red) ... 1  
 $d_1 = |AJ|$  (blue) ... 1  
 $d_2 = |FI| = |FJ|$  (green) ... 2  
 $d_3 = |AF| = |AH| = |BE| = |CD| = |CG| = |DG| = |FH|$  (cyan) ... 7  
 $d_4 = |HI|$  (magenta) ... 1  
 $d_5 = |AI|$  (yellow) ... 1

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \quad (18 \text{ occurrences})$$



## 2.60.10 Construction MM76



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |AB|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(B, |CE|)$   
 $G \in k_1 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(F, |CE|)$   
 $I \in k_4 \cap k_5, J \in k_4 \cap k_5$

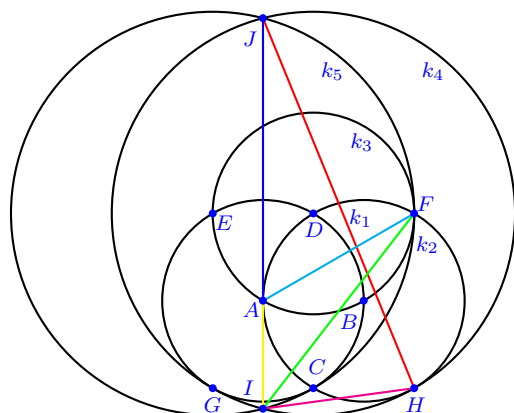
## Distances

- $d_0 = |GJ|$  (red) ... 1  
 $d_1 = |DJ|$  (blue) ... 1  
 $d_2 = |CI| = |CJ|$  (green) ... 2  
 $d_3 = |AF| = |AH| = |BE| = |CD| = |CG| = |DG| = |FH|$  (cyan) ... 7  
 $d_4 = |GI|$  (magenta) ... 1  
 $d_5 = |DI|$  (yellow) ... 1

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \quad (18 \text{ occurrences})$$

## 2.60.11 Construction MM81



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |AB|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(D, |CE|)$   
 $G \in k_1 \cap k_4, H \in k_2 \cap k_4$
  5.  $k_5 = k(E, |CE|)$   
 $I \in k_4 \cap k_5, J \in k_4 \cap k_5$

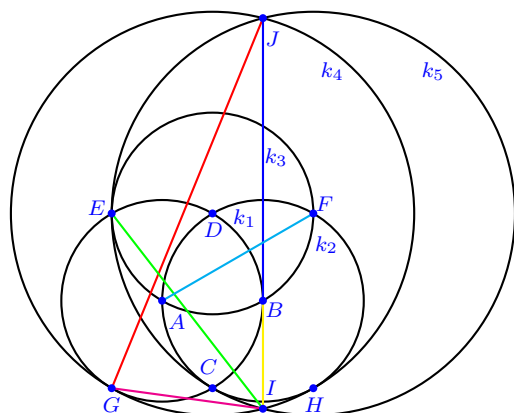
## Distances

- $d_0 = |HJ|$  (red) ... 1  
 $d_1 = |AJ|$  (blue) ... 1  
 $d_2 = |FI| = |FJ|$  (green) ... 2  
 $d_3 = |AF| = |AH| = |BE| = |BG| = |CD| = |EG| = |FH|$  (cyan) ... 7  
 $d_4 = |HI|$  (magenta) ... 1  
 $d_5 = |AI|$  (yellow) ... 1

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (18 occurrences)}$$

## 2.60.12 Construction MM82



## Construction Process

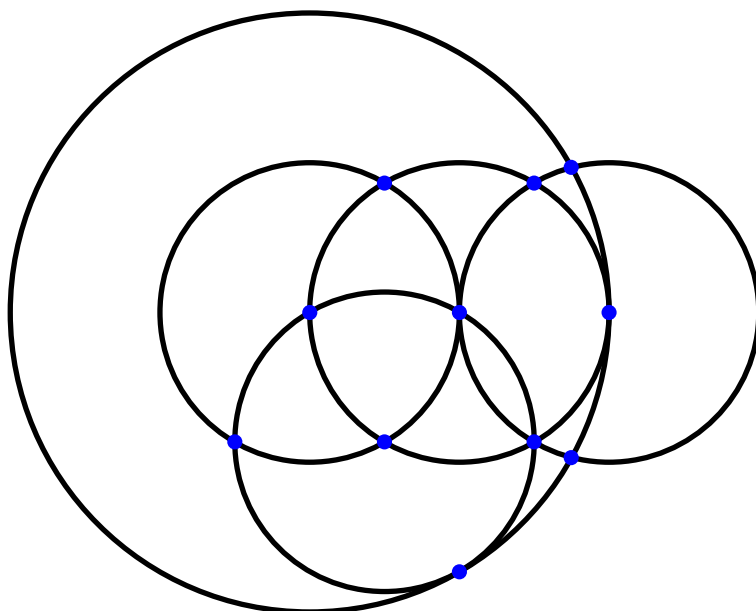
- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |AB|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(D, |CE|)$   
 $G \in k_1 \cap k_4, H \in k_2 \cap k_4$
  5.  $k_5 = k(F, |CE|)$   
 $I \in k_4 \cap k_5, J \in k_4 \cap k_5$

## Distances

- $d_0 = |GJ|$  (red) ... 1  
 $d_1 = |BJ|$  (blue) ... 1  
 $d_2 = |EI| = |EJ|$  (green) ... 2  
 $d_3 = |AF| = |AH| = |BE| = |BG| = |CD| = |EG| = |FH|$  (cyan) ... 7  
 $d_4 = |GI|$  (magenta) ... 1  
 $d_5 = |BI|$  (yellow) ... 1

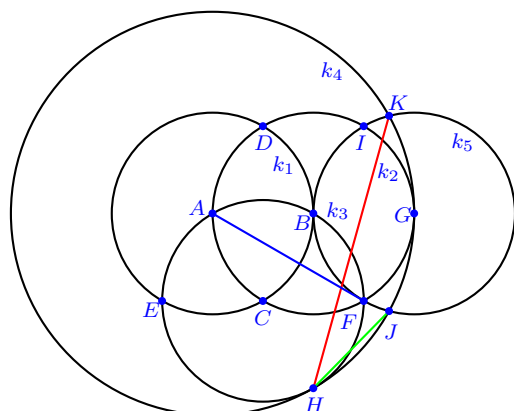
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (18 occurrences)}$$

**2.61 Class of Similar Constructions No. 52****Contained Constructions**

MM29, MM31, MM36, MM38, MM42, MM44, MM70, MM72, MM77, MM79,  
MM83, MM85

## 2.61.1 Construction MM29



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |AB|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(A, |DE|)$   
 $G \in k_2 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(G, |AB|)$   
 $I \in k_2 \cap k_5, J \in k_4 \cap k_5, K \in k_4 \cap k_5$

## Distances

$$d_0 = |HK| \text{ (red)} \dots 1$$

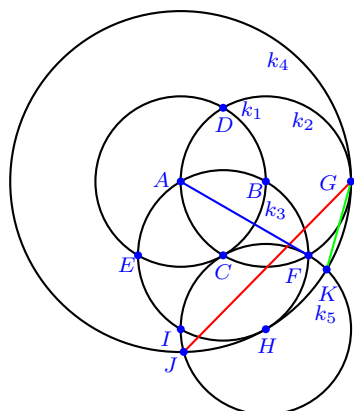
$$d_1 = |AF| = |AI| = |BE| = |BH| = |CD| = |CG| = |DG| = |EH| = |FI| \text{ (blue)} \dots 9$$

$$d_2 = |HJ| \text{ (green)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (18 occurrences)}$$

## 2.61.2 Construction MM31



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |AB|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(A, |DE|)$   
 $G \in k_2 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(H, |AB|)$   
 $I \in k_3 \cap k_5, J \in k_4 \cap k_5, K \in k_4 \cap k_5$

## Distances

$$d_0 = |GJ| \text{ (red)} \dots 1$$

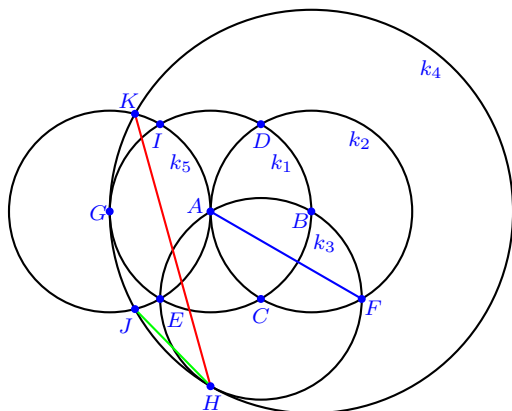
$$d_1 = |AF| = |AI| = |BE| = |BH| = |CD| = |CG| = |DG| = |EH| = |FI| \text{ (blue)} \dots 9$$

$$d_2 = |GK| \text{ (green)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (18 occurrences)}$$

## 2.61.3 Construction MM36



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |AB|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(B, |DE|)$   
 $G \in k_1 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(G, |AB|)$   
 $I \in k_1 \cap k_5, J \in k_4 \cap k_5, K \in k_4 \cap k_5$

## Distances

$$d_0 = |HK| \text{ (red) } \dots 1$$

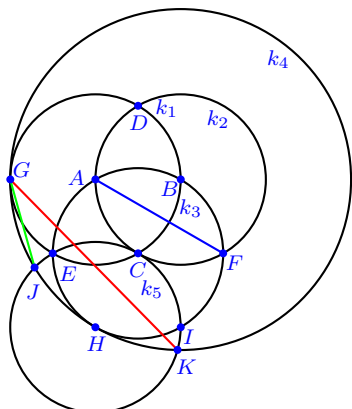
$$d_1 = |AF| = |AH| = |BE| = |BI| = |CD| = |CG| = |DG| = |EI| = |FH| \text{ (blue) } \dots 9$$

$$d_2 = |HJ| \text{ (green) } \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (18 occurrences)}$$

## 2.61.4 Construction MM38



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |AB|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(B, |DE|)$   
 $G \in k_1 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(H, |AB|)$   
 $I \in k_3 \cap k_5, J \in k_4 \cap k_5, K \in k_4 \cap k_5$

## Distances

$$d_0 = |GK| \text{ (red) } \dots 1$$

$$d_1 = |AF| = |AH| = |BE| = |BI| = |CD| = |CG| = |DG| = |EI| = |FH| \text{ (blue) } \dots 9$$

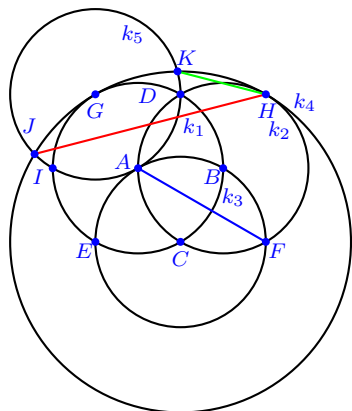
$$d_2 = |GJ| \text{ (green) } \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (18 occurrences)}$$



## 2.61.5 Construction MM42



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |AB|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(C, |DE|)$   
 $G \in k_1 \cap k_4, H \in k_2 \cap k_4$
  5.  $k_5 = k(G, |AB|)$   
 $I \in k_1 \cap k_5, J \in k_4 \cap k_5, K \in k_4 \cap k_5$

## Distances

$$d_0 = |HJ| \text{ (red) } \dots 1$$

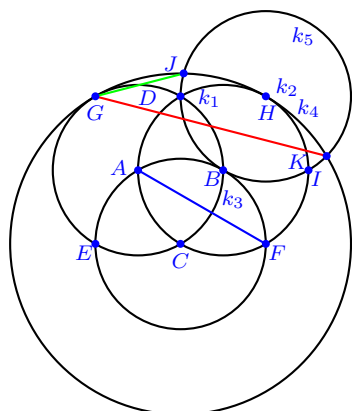
$$d_1 = |AF| = |AH| = |BE| = |BG| = |CD| = |CI| = |DI| = |EG| = |FH| \text{ (blue) } \dots 9$$

$$d_2 = |HK| \text{ (green) } \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (18 occurrences)}$$

## 2.61.6 Construction MM44



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |AB|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(C, |DE|)$   
 $G \in k_1 \cap k_4, H \in k_2 \cap k_4$
  5.  $k_5 = k(H, |AB|)$   
 $I \in k_2 \cap k_5, J \in k_4 \cap k_5, K \in k_4 \cap k_5$

## Distances

$$d_0 = |GK| \text{ (red)} \dots 1$$

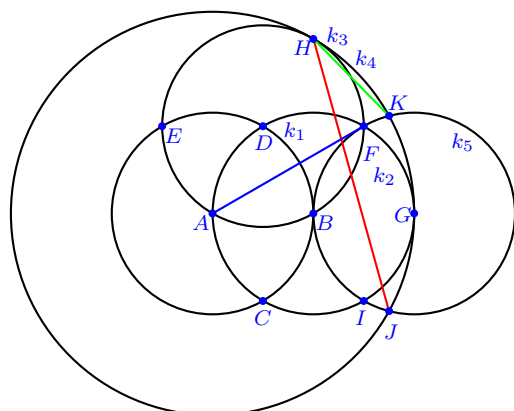
$$d_1 = |AF| = |AH| = |BE| = |BG| = |CD| = |CI| = |DI| = |EG| = |FH| \text{ (blue)} \dots 9$$

$$d_2 = |GJ| \text{ (green)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (18 occurrences)}$$

## 2.61.7 Construction MM70



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |AB|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(A, |CE|)$   
 $G \in k_2 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(G, |AB|)$   
 $I \in k_2 \cap k_5, J \in k_4 \cap k_5, K \in k_4 \cap k_5$

## Distances

$$d_0 = |HJ| \text{ (red) } \dots 1$$

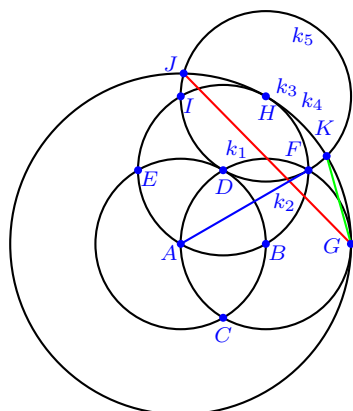
$$d_1 = |AF| = |AI| = |BE| = |BH| = |CD| = |CG| = |DG| = |EH| = |FI| \text{ (blue) } \dots 9$$

$$d_2 = |HK| \text{ (green) } \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (18 occurrences)}$$

## 2.61.8 Construction MM72



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |AB|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(A, |CE|)$   
 $G \in k_2 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(H, |AB|)$   
 $I \in k_3 \cap k_5, J \in k_4 \cap k_5, K \in k_4 \cap k_5$

## Distances

$$d_0 = |GJ| \text{ (red)} \dots 1$$

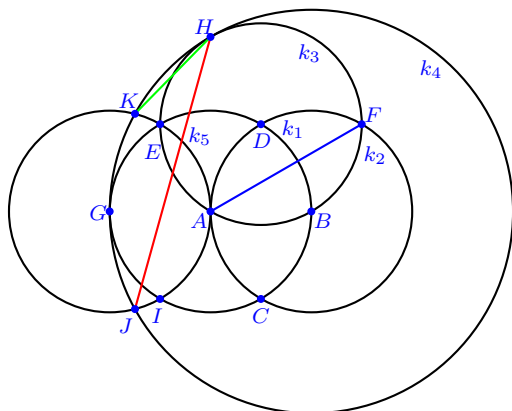
$$d_1 = |AF| = |AI| = |BE| = |BH| = |CD| = |CG| = |DG| = |EH| = |FI| \text{ (blue)} \dots 9$$

$$d_2 = |GK| \text{ (green)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (18 occurrences)}$$

## 2.61.9 Construction MM77



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |AB|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(B, |CE|)$   
 $G \in k_1 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(G, |AB|)$   
 $I \in k_1 \cap k_5, J \in k_4 \cap k_5, K \in k_4 \cap k_5$

## Distances

$$d_0 = |HJ| \text{ (red)} \dots 1$$

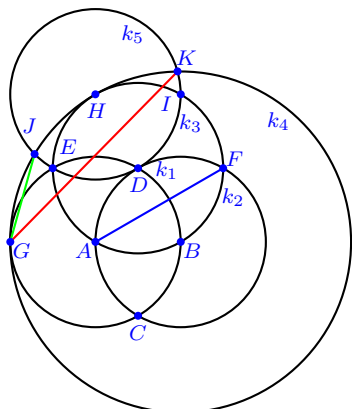
$$d_1 = |AF| = |AH| = |BE| = |BI| = |CD| = |CG| = |DG| = |EI| = |FH| \text{ (blue)} \dots 9$$

$$d_2 = |HK| \text{ (green)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (18 occurrences)}$$

## 2.61.10 Construction MM79



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |AB|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(B, |CE|)$   
 $G \in k_1 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(H, |AB|)$   
 $I \in k_3 \cap k_5, J \in k_4 \cap k_5, K \in k_4 \cap k_5$

## Distances

$$d_0 = |GK| \text{ (red)} \dots 1$$

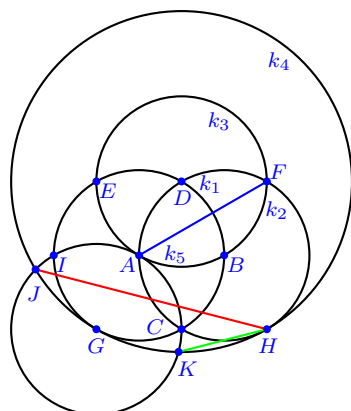
$$d_1 = |AF| = |AH| = |BE| = |BI| = |CD| = |CG| = |DG| = |EI| = |FH| \text{ (blue)} \dots 9$$

$$d_2 = |GJ| \text{ (green)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (18 occurrences)}$$

## 2.61.11 Construction MM83



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |AB|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(D, |CE|)$   
 $G \in k_1 \cap k_4, H \in k_2 \cap k_4$
  5.  $k_5 = k(G, |AB|)$   
 $I \in k_1 \cap k_5, J \in k_4 \cap k_5, K \in k_4 \cap k_5$

## Distances

$$d_0 = |HJ| \text{ (red)} \dots 1$$

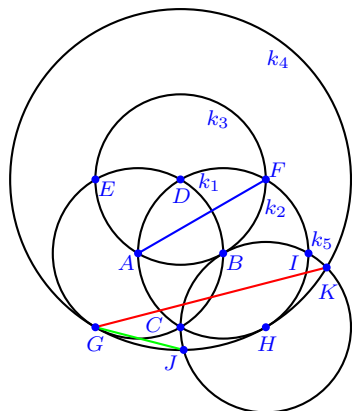
$$d_1 = |AF| = |AH| = |BE| = |BG| = |CD| = |CI| = |DI| = |EG| = |FH| \text{ (blue)} \dots 9$$

$$d_2 = |HK| \text{ (green)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (18 occurrences)}$$

## 2.61.12 Construction MM85



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |AB|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(D, |CE|)$   
 $G \in k_1 \cap k_4, H \in k_2 \cap k_4$
  5.  $k_5 = k(H, |AB|)$   
 $I \in k_2 \cap k_5, J \in k_4 \cap k_5, K \in k_4 \cap k_5$

## Distances

$$d_0 = |GK| \text{ (red)} \dots 1$$

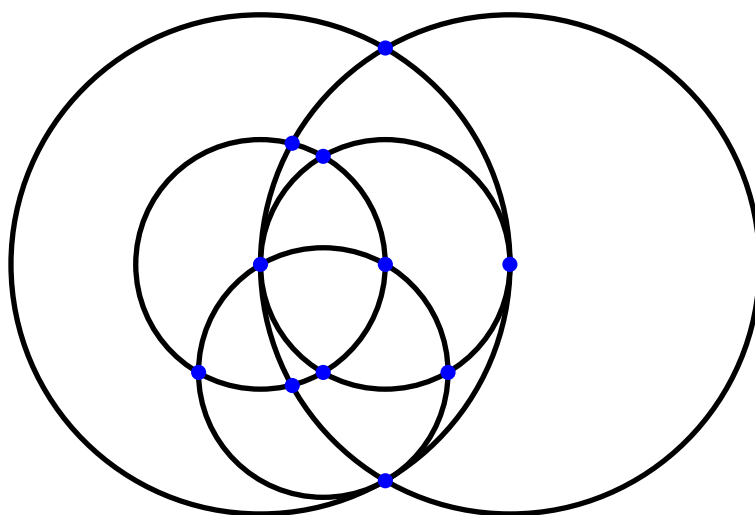
$$d_1 = |AF| = |AH| = |BE| = |BG| = |CD| = |CI| = |DI| = |EG| = |FH| \text{ (blue)} \dots 9$$

$$d_2 = |GJ| \text{ (green)} \dots 1$$

## Occurrences of the Golden Ratio

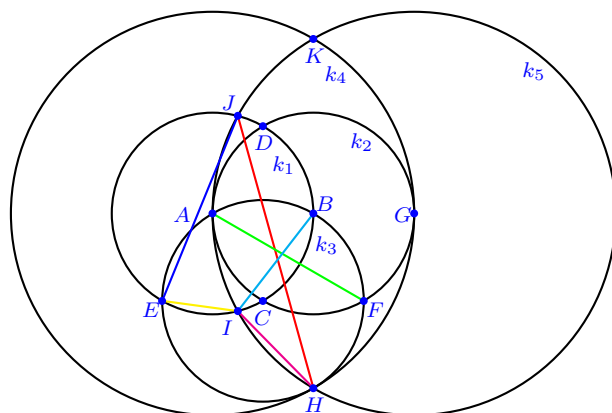
$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (18 occurrences)}$$



**2.62 Class of Similar Constructions No. 53****Contained Constructions**

MM30, MM32, MM37, MM39, MM43, MM45, MM71, MM73, MM78, MM80,  
MM84, MM86

## 2.62.1 Construction MM30



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |AB|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(A, |DE|)$   
 $G \in k_2 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(G, |DE|)$   
 $I \in k_1 \cap k_5, J \in k_1 \cap k_5, K \in k_4 \cap k_5$

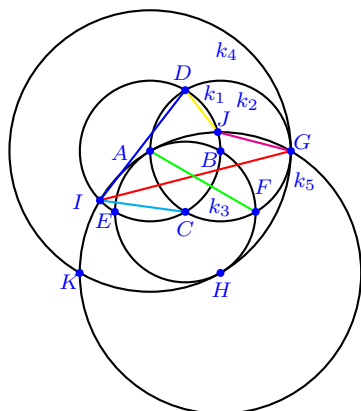
## Distances

$$\begin{aligned}
 d_0 &= |HJ| = |IK| \text{ (red)} \dots 2 \\
 d_1 &= |EJ| \text{ (blue)} \dots 1 \\
 d_2 &= |AF| = |BE| = |BH| = |BK| = |CD| = |CG| = |DG| = |EH| \\
 &\text{ (green)} \dots 8 \\
 d_3 &= |BI| = |BJ| \text{ (cyan)} \dots 2 \\
 d_4 &= |HI| = |JK| \text{ (magenta)} \dots 2 \\
 d_5 &= |EI| \text{ (yellow)} \dots 1
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (36 occurrences)}$$

## 2.62.2 Construction MM32



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |AB|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(A, |DE|)$   
 $G \in k_2 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(H, |DE|)$   
 $I \in k_1 \cap k_5, J \in k_1 \cap k_5, K \in k_4 \cap k_5$

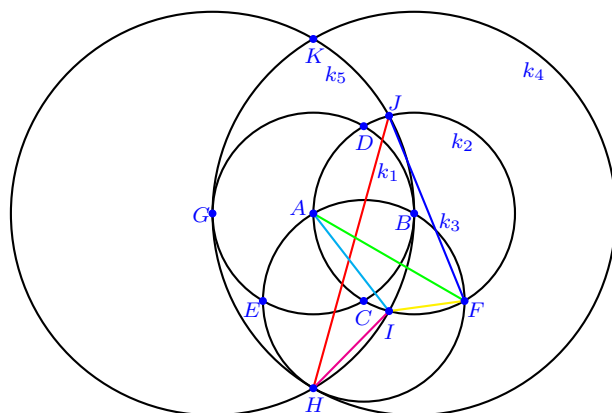
## Distances

$$\begin{aligned}
 d_0 &= |GI| = |JK| \text{ (red)} \dots 2 \\
 d_1 &= |DI| \text{ (blue)} \dots 1 \\
 d_2 &= |AF| = |BE| = |BH| = |CD| = |CG| = |CK| = |DG| = |EH| \\
 &\text{ (green)} \dots 8 \\
 d_3 &= |CI| = |CJ| \text{ (cyan)} \dots 2 \\
 d_4 &= |GJ| = |IK| \text{ (magenta)} \dots 2 \\
 d_5 &= |DJ| \text{ (yellow)} \dots 1
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (36 occurrences)}$$

## 2.62.3 Construction MM37



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |AB|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(B, |DE|)$   
 $G \in k_1 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(G, |DE|)$   
 $I \in k_2 \cap k_5, J \in k_2 \cap k_5, K \in k_4 \cap k_5$

## Distances

$$d_0 = |HJ| = |IK| \text{ (red)} \dots 2$$

$$d_1 = |FJ| \text{ (blue)} \dots 1$$

$$d_2 = |AF| = |AH| = |AK| = |BE| = |CD| = |CG| = |DG| = |FH| \text{ (green)} \dots 8$$

$$d_3 = |AI| = |AJ| \text{ (cyan)} \dots 2$$

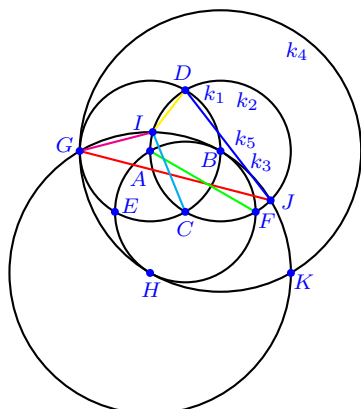
$$d_4 = |HI| = |JK| \text{ (magenta)} \dots 2$$

$$d_5 = |FI| \text{ (yellow)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (36 occurrences)}$$

## 2.62.4 Construction MM39



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |AB|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(B, |DE|)$   
 $G \in k_1 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(H, |DE|)$   
 $I \in k_2 \cap k_5, J \in k_2 \cap k_5, K \in k_4 \cap k_5$

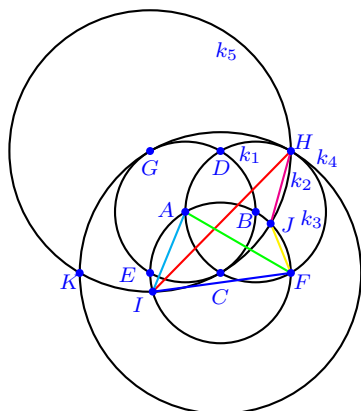
## Distances

$$\begin{aligned}
 d_0 &= |GJ| = |IK| \text{ (red)} \dots 2 \\
 d_1 &= |DJ| \text{ (blue)} \dots 1 \\
 d_2 &= |AF| = |AH| = |BE| = |CD| = |CG| = |CK| = |DG| = |FH| \\
 &\text{(green)} \dots 8 \\
 d_3 &= |CI| = |CJ| \text{ (cyan)} \dots 2 \\
 d_4 &= |GI| = |JK| \text{ (magenta)} \dots 2 \\
 d_5 &= |DI| \text{ (yellow)} \dots 1
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (36 occurrences)}$$

## 2.62.5 Construction MM43



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |AB|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(C, |DE|)$   
 $G \in k_1 \cap k_4, H \in k_2 \cap k_4$
  5.  $k_5 = k(G, |DE|)$   
 $I \in k_3 \cap k_5, J \in k_3 \cap k_5, K \in k_4 \cap k_5$

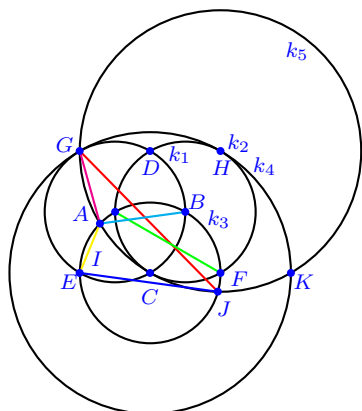
## Distances

$$\begin{aligned}
 d_0 &= |HI| = |JK| \text{ (red)} \dots 2 \\
 d_1 &= |FI| \text{ (blue)} \dots 1 \\
 d_2 &= |AF| = |AH| = |AK| = |BE| = |BG| = |CD| = |EG| = |FH| \\
 &\text{ (green)} \dots 8 \\
 d_3 &= |AI| = |AJ| \text{ (cyan)} \dots 2 \\
 d_4 &= |HJ| = |IK| \text{ (magenta)} \dots 2 \\
 d_5 &= |FJ| \text{ (yellow)} \dots 1
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (36 occurrences)}$$

## 2.62.6 Construction MM45



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |AB|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(C, |DE|)$   
 $G \in k_1 \cap k_4, H \in k_2 \cap k_4$
  5.  $k_5 = k(H, |DE|)$   
 $I \in k_3 \cap k_5, J \in k_3 \cap k_5, K \in k_4 \cap k_5$

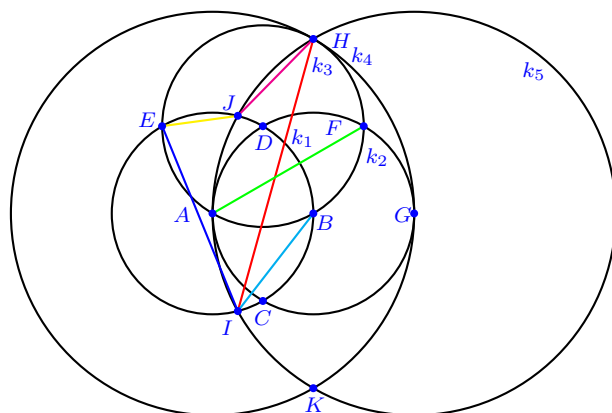
## Distances

$$\begin{aligned}
 d_0 &= |GJ| = |IK| \text{ (red)} \dots 2 \\
 d_1 &= |EJ| \text{ (blue)} \dots 1 \\
 d_2 &= |AF| = |AH| = |BE| = |BG| = |BK| = |CD| = |EG| = |FH| \\
 &\text{(green)} \dots 8 \\
 d_3 &= |BI| = |BJ| \text{ (cyan)} \dots 2 \\
 d_4 &= |GI| = |JK| \text{ (magenta)} \dots 2 \\
 d_5 &= |EI| \text{ (yellow)} \dots 1
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (36 occurrences)}$$

## 2.62.7 Construction MM71



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |AB|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(A, |CE|)$   
 $G \in k_2 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(G, |CE|)$   
 $I \in k_1 \cap k_5, J \in k_1 \cap k_5, K \in k_4 \cap k_5$

## Distances

$$d_0 = |HI| = |JK| \text{ (red)} \dots 2$$

$$d_1 = |EI| \text{ (blue)} \dots 1$$

$$d_2 = |AF| = |BE| = |BH| = |BK| = |CD| = |CG| = |DG| = |EH| \text{ (green)} \dots 8$$

$$d_3 = |BI| = |BJ| \text{ (cyan)} \dots 2$$

$$d_4 = |HJ| = |IK| \text{ (magenta)} \dots 2$$

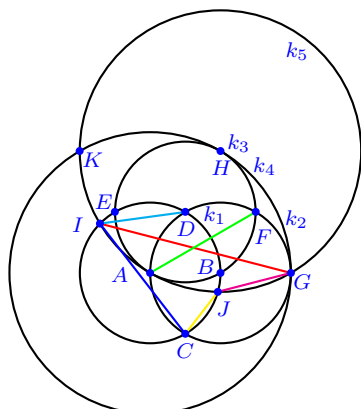
$$d_5 = |EJ| \text{ (yellow)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (36 occurrences)}$$



## 2.62.8 Construction MM73



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |AB|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(A, |CE|)$   
 $G \in k_2 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(H, |CE|)$   
 $I \in k_1 \cap k_5, J \in k_1 \cap k_5, K \in k_4 \cap k_5$

## Distances

$$d_0 = |GI| = |JK| \text{ (red)} \dots 2$$

$$d_1 = |CI| \text{ (blue)} \dots 1$$

$$d_2 = |AF| = |BE| = |BH| = |CD| = |CG| = |DG| = |DK| = |EH| \text{ (green)} \dots 8$$

$$d_3 = |DI| = |DJ| \text{ (cyan)} \dots 2$$

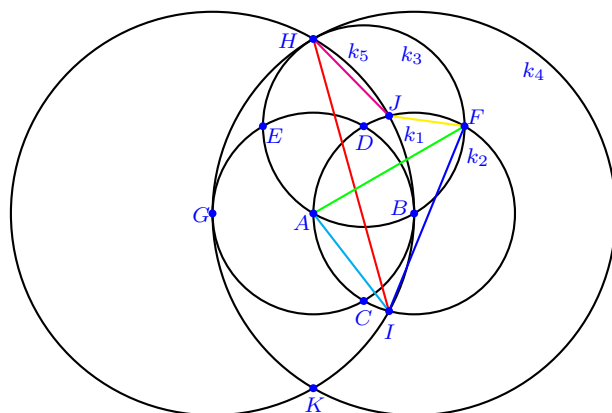
$$d_4 = |GJ| = |IK| \text{ (magenta)} \dots 2$$

$$d_5 = |CJ| \text{ (yellow)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (36 occurrences)}$$

## 2.62.9 Construction MM78



## Construction Process

- $A, B$  given initial points
- $k_1 = k(A, |AB|)$
  - $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  - $k_3 = k(D, |AB|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  - $k_4 = k(B, |CE|)$   
 $G \in k_1 \cap k_4, H \in k_3 \cap k_4$
  - $k_5 = k(G, |CE|)$   
 $I \in k_2 \cap k_5, J \in k_2 \cap k_5, K \in k_4 \cap k_5$

## Distances

$$d_0 = |HI| = |JK| \text{ (red)} \dots 2$$

$$d_1 = |FI| \text{ (blue)} \dots 1$$

$$d_2 = |AF| = |AH| = |AK| = |BE| = |CD| = |CG| = |DG| = |FH| \text{ (green)} \dots 8$$

$$d_3 = |AI| = |AJ| \text{ (cyan)} \dots 2$$

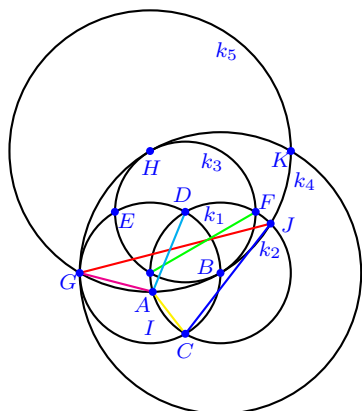
$$d_4 = |HJ| = |IK| \text{ (magenta)} \dots 2$$

$$d_5 = |FJ| \text{ (yellow)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (36 occurrences)}$$

## 2.62.10 Construction MM80



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |AB|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(B, |CE|)$   
 $G \in k_1 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(H, |CE|)$   
 $I \in k_2 \cap k_5, J \in k_2 \cap k_5, K \in k_4 \cap k_5$

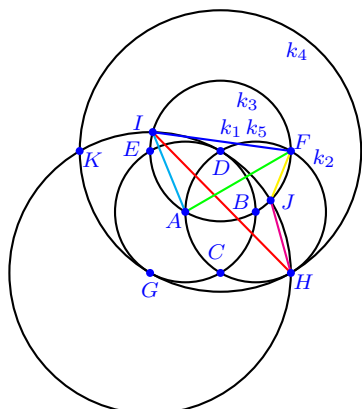
## Distances

$$\begin{aligned}
 d_0 &= |GJ| = |IK| \text{ (red)} \dots 2 \\
 d_1 &= |CJ| \text{ (blue)} \dots 1 \\
 d_2 &= |AF| = |AH| = |BE| = |CD| = |CG| = |DG| = |DK| = |FH| \\
 &\text{(green)} \dots 8 \\
 d_3 &= |DI| = |DJ| \text{ (cyan)} \dots 2 \\
 d_4 &= |GI| = |JK| \text{ (magenta)} \dots 2 \\
 d_5 &= |CI| \text{ (yellow)} \dots 1
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (36 occurrences)}$$

## 2.62.11 Construction MM84



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |AB|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(D, |CE|)$   
 $G \in k_1 \cap k_4, H \in k_2 \cap k_4$
  5.  $k_5 = k(G, |CE|)$   
 $I \in k_3 \cap k_5, J \in k_3 \cap k_5, K \in k_4 \cap k_5$

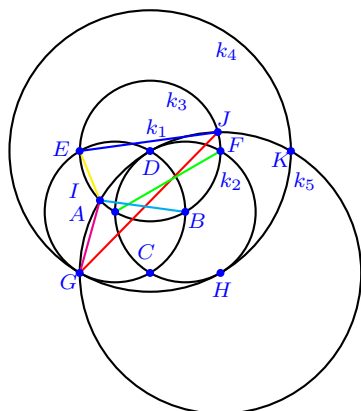
## Distances

$$\begin{aligned}
 d_0 &= |HI| = |JK| \text{ (red)} \dots 2 \\
 d_1 &= |FI| \text{ (blue)} \dots 1 \\
 d_2 &= |AF| = |AH| = |AK| = |BE| = |BG| = |CD| = |EG| = |FH| \\
 &\text{(green)} \dots 8 \\
 d_3 &= |AI| = |AJ| \text{ (cyan)} \dots 2 \\
 d_4 &= |HJ| = |IK| \text{ (magenta)} \dots 2 \\
 d_5 &= |FJ| \text{ (yellow)} \dots 1
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (36 occurrences)}$$

## 2.62.12 Construction MM86



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |AB|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(D, |CE|)$   
 $G \in k_1 \cap k_4, H \in k_2 \cap k_4$
  5.  $k_5 = k(H, |CE|)$   
 $I \in k_3 \cap k_5, J \in k_3 \cap k_5, K \in k_4 \cap k_5$

## Distances

$$d_0 = |GJ| = |IK| \text{ (red)} \dots 2$$

$$d_1 = |EJ| \text{ (blue)} \dots 1$$

$$d_2 = |AF| = |AH| = |BE| = |BG| = |BK| = |CD| = |EG| = |FH| \text{ (green)} \dots 8$$

$$d_3 = |BI| = |BJ| \text{ (cyan)} \dots 2$$

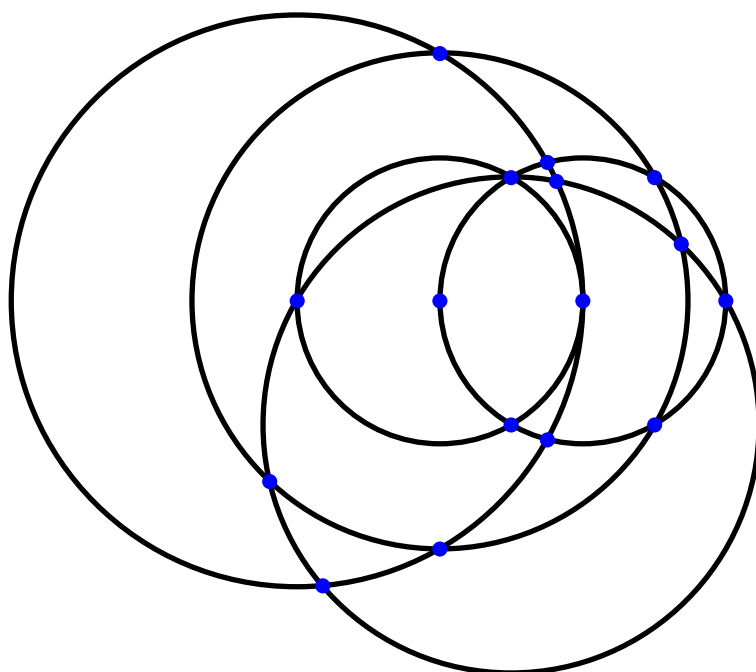
$$d_4 = |GI| = |JK| \text{ (magenta)} \dots 2$$

$$d_5 = |EI| \text{ (yellow)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (36 occurrences)}$$

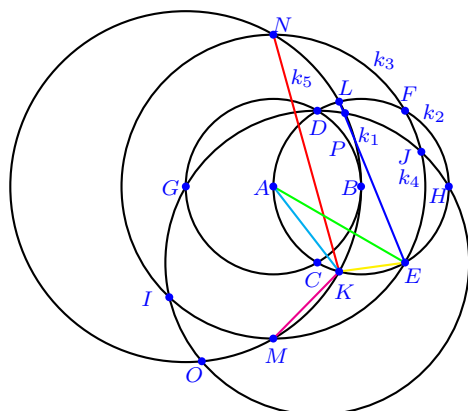
## 2.63 Class of Similar Constructions No. 54



### Contained Constructions

MM87, MM88, MM117, MM118

## 2.63.1 Construction MM87



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(A, |CD|)$   
 $E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(C, |CD|)$   
 $G \in k_1 \cap k_4, H \in k_2 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
  5.  $k_5 = k(G, |DE|)$   
 $K \in k_2 \cap k_5, L \in k_2 \cap k_5, M \in k_3 \cap k_5, N \in k_3 \cap k_5, O \in k_4 \cap k_5,$   
 $P \in k_4 \cap k_5$

## Distances

$$d_0 = |KN| = |LM| \text{ (red)} \dots 2$$

$$d_1 = |EL| = |FK| \text{ (blue)} \dots 2$$

$$d_2 = |AE| = |AF| = |AI| = |AJ| = |AM| = |AN| = |CD| = |CG| = |CH| = |CI| = |CJ| = |CO| = |CP| = |DG| = |DH| = |EF| = |EM| = |FN| \text{ (green)} \dots 18$$

$$d_3 = |AK| = |AL| \text{ (cyan)} \dots 2$$

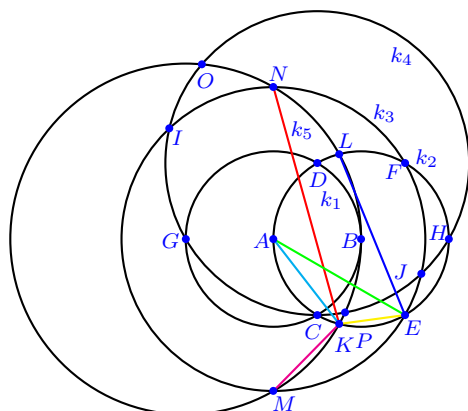
$$d_4 = |KM| = |LN| \text{ (magenta)} \dots 2$$

$$d_5 = |EK| = |FL| \text{ (yellow)} \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (80 occurrences)}$$

## 2.63.2 Construction MM88



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(A, |CD|)$   
 $E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(D, |CD|)$   
 $G \in k_1 \cap k_4, H \in k_2 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
  5.  $k_5 = k(G, |DE|)$   
 $K \in k_2 \cap k_5, L \in k_2 \cap k_5, M \in k_3 \cap k_5, N \in k_3 \cap k_5, O \in k_4 \cap k_5,$   
 $P \in k_4 \cap k_5$

## Distances

$$d_0 = |KN| = |LM| \text{ (red)} \dots 2$$

$$d_1 = |EL| = |FK| \text{ (blue)} \dots 2$$

$$d_2 = |AE| = |AF| = |AI| = |AJ| = |AM| = |AN| = |CD| = |CG| = |CH| = |DG| = |DH| = |DI| = |DJ| = |DO| = |DP| = |EF| = |EM| = |FN| \text{ (green)} \dots 18$$

$$d_3 = |AK| = |AL| \text{ (cyan)} \dots 2$$

$$d_4 = |KM| = |LN| \text{ (magenta)} \dots 2$$

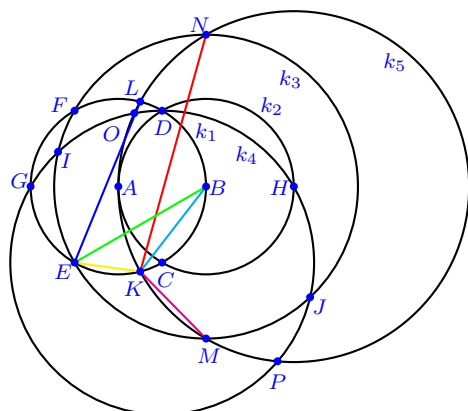
$$d_5 = |EK| = |FL| \text{ (yellow)} \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (80 occurrences)}$$



## 2.63.3 Construction MM117



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(B, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_1 \cap k_3$
  4.  $k_4 = k(C, |CD|)$   
 $G \in k_1 \cap k_4, H \in k_2 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
  5.  $k_5 = k(H, |DE|)$   
 $K \in k_1 \cap k_5, L \in k_1 \cap k_5, M \in k_3 \cap k_5, N \in k_3 \cap k_5, O \in k_4 \cap k_5,$   
 $P \in k_4 \cap k_5$

## Distances

$$d_0 = |KN| = |LM| \text{ (red)} \dots 2$$

$$d_1 = |EL| = |FK| \text{ (blue)} \dots 2$$

$$d_2 = |BE| = |BF| = |BI| = |BJ| = |BM| = |BN| = |CD| = |CG| = |CH| = |CI| = |CJ| = |CO| = |CP| = |DG| = |DH| = |EF| = |EM| = |FN| \text{ (green)} \dots 18$$

$$d_3 = |BK| = |BL| \text{ (cyan)} \dots 2$$

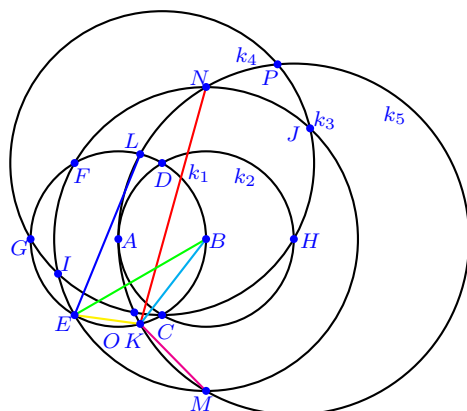
$$d_4 = |KM| = |LN| \text{ (magenta)} \dots 2$$

$$d_5 = |EK| = |FL| \text{ (yellow)} \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (80 occurrences)}$$

## 2.63.4 Construction MM18



## Construction Process

- $A, B$  given initial points
- $k_1 = k(A, |AB|)$
  - $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  - $k_3 = k(B, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_1 \cap k_3$
  - $k_4 = k(D, |CD|)$   
 $G \in k_1 \cap k_4, H \in k_2 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
  - $k_5 = k(H, |DE|)$   
 $K \in k_1 \cap k_5, L \in k_1 \cap k_5, M \in k_3 \cap k_5, N \in k_3 \cap k_5, O \in k_4 \cap k_5,$   
 $P \in k_4 \cap k_5$

## Distances

$$d_0 = |KN| = |LM| \text{ (red)} \dots 2$$

$$d_1 = |EL| = |FK| \text{ (blue)} \dots 2$$

$$d_2 = |BE| = |BF| = |BI| = |BJ| = |BM| = |BN| = |CD| = |CG| = |CH| = |DG| = |DH| = |DI| = |DJ| = |DO| = |DP| = |EF| = |EM| = |FN| \text{ (green)} \dots 18$$

$$d_3 = |BK| = |BL| \text{ (cyan)} \dots 2$$

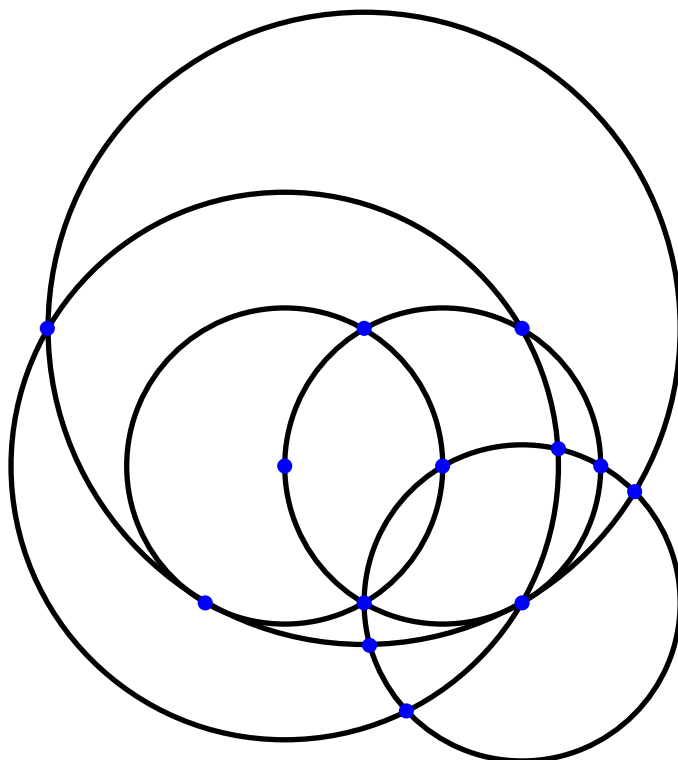
$$d_4 = |KM| = |LN| \text{ (magenta)} \dots 2$$

$$d_5 = |EK| = |FL| \text{ (yellow)} \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (80 occurrences)}$$

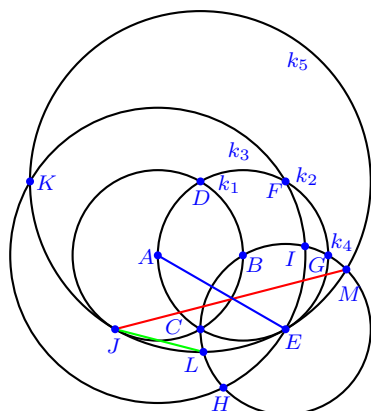
## 2.64 Class of Similar Constructions No. 55



### Contained Constructions

MM89, MM92, MM119, MM122

## 2.64.1 Construction MM89



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(A, |CD|)$   
 $E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, |AB|)$   
 $G \in k_2 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  5.  $k_5 = k(D, |DE|)$   
 $J \in k_1 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5, M \in k_4 \cap k_5$

## Distances

$$d_0 = |JM| \text{ (red)} \dots 1$$

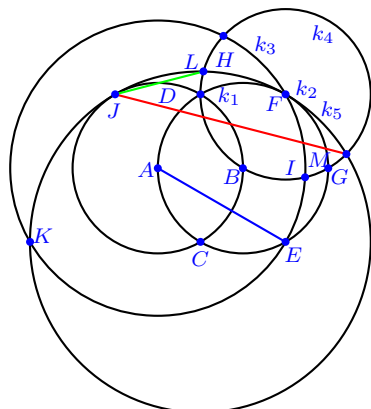
$$d_1 = |AE| = |AF| = |AH| = |AI| = |AK| = |BJ| = |CD| = |CG| = |DG| = |EF| \text{ (blue)} \dots 10$$

$$d_2 = |JL| \text{ (green)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (20 occurrences)}$$

## 2.64.2 Construction MM92



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(A, |CD|)$   
 $E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, |AB|)$   
 $G \in k_2 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  5.  $k_5 = k(C, |DE|)$   
 $J \in k_1 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5, M \in k_4 \cap k_5$

## Distances

$$d_0 = |JM| \text{ (red)} \dots 1$$

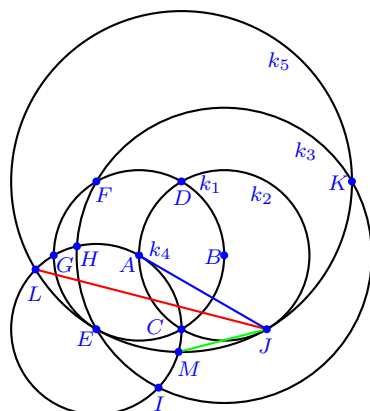
$$d_1 = |AE| = |AF| = |AH| = |AI| = |AK| = |BJ| = |CD| = |CG| = |DG| = |EF| \text{ (blue)} \dots 10$$

$$d_2 = |JL| \text{ (green)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (20 occurrences)}$$

## 2.64.3 Construction MM119



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(B, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_1 \cap k_3$
  4.  $k_4 = k(E, |AB|)$   
 $G \in k_1 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  5.  $k_5 = k(D, |DE|)$   
 $J \in k_2 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5, M \in k_4 \cap k_5$

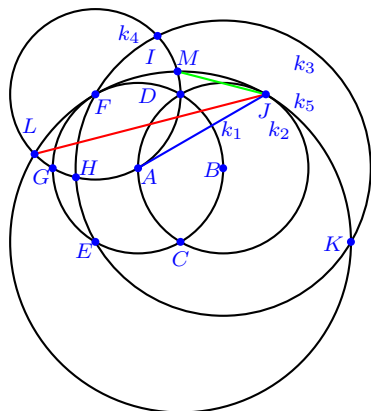
## Distances

$$\begin{aligned}
 d_0 &= |JL| \text{ (red)} \dots 1 \\
 d_1 &= |AJ| = |BE| = |BF| = |BH| = |BI| = |BK| = |CD| = |CG| = \\
 &|DG| = |EF| \text{ (blue)} \dots 10 \\
 d_2 &= |JM| \text{ (green)} \dots 1
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (20 occurrences)}$$

## 2.64.4 Construction MM122



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(B, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_1 \cap k_3$
  4.  $k_4 = k(F, |AB|)$   
 $G \in k_1 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  5.  $k_5 = k(C, |DE|)$   
 $J \in k_2 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5, M \in k_4 \cap k_5$

## Distances

$$d_0 = |JL| \text{ (red)} \dots 1$$

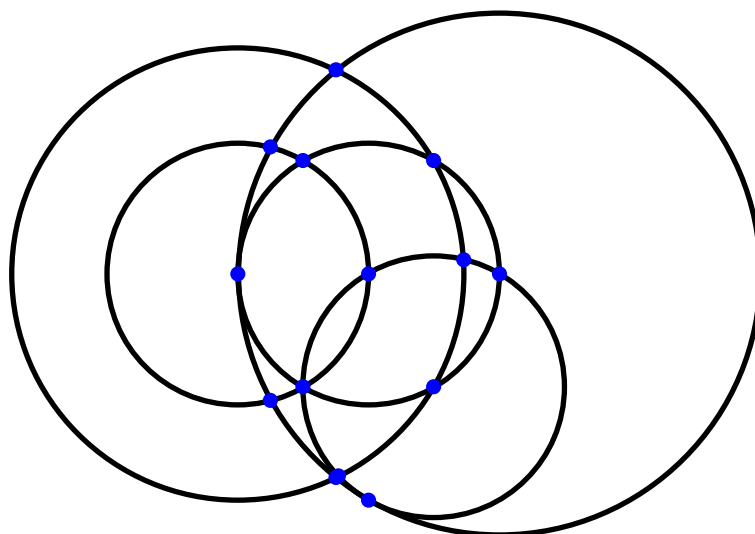
$$d_1 = |AJ| = |BE| = |BF| = |BH| = |BI| = |BK| = |CD| = |CG| = |DG| = |EF| \text{ (blue)} \dots 10$$

$$d_2 = |JM| \text{ (green)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (20 occurrences)}$$

## 2.65 Class of Similar Constructions No. 56

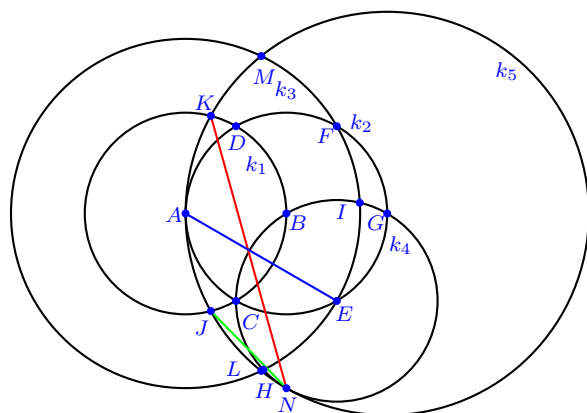


### Contained Constructions

MM90, MM93, MM120, MM123



## 2.65.1 Construction MM90



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(A, |CD|)$   
 $E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, |AB|)$   
 $G \in k_2 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  5.  $k_5 = k(G, |DE|)$   
 $J \in k_1 \cap k_5, K \in k_1 \cap k_5, L \in k_3 \cap k_5, M \in k_3 \cap k_5, N \in k_4 \cap k_5$

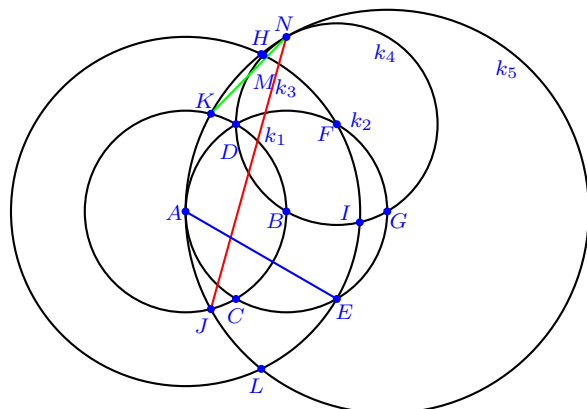
## Distances

$$\begin{aligned}
 d_0 &= |KN| \text{ (red)} \dots 1 \\
 d_1 &= |AE| = |AF| = |AH| = |AI| = |AL| = |AM| = |BN| = |CD| = \\
 &|CG| = |DG| = |EF| \text{ (blue)} \dots 11 \\
 d_2 &= |JN| \text{ (green)} \dots 1
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (22 occurrences)}$$

## 2.65.2 Construction MM93



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(A, |CD|)$   
 $E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, |AB|)$   
 $G \in k_2 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  5.  $k_5 = k(G, |DE|)$   
 $J \in k_1 \cap k_5, K \in k_1 \cap k_5, L \in k_3 \cap k_5, M \in k_3 \cap k_5, N \in k_4 \cap k_5$

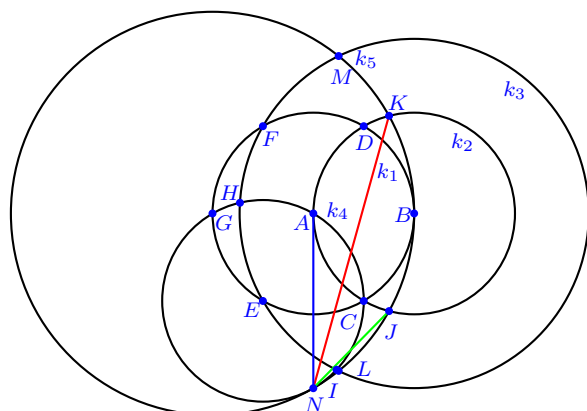
## Distances

$$\begin{aligned}
 d_0 &= |JN| \text{ (red)} \dots 1 \\
 d_1 &= |AE| = |AF| = |AH| = |AI| = |AL| = |AM| = |BN| = |CD| = \\
 &|CG| = |DG| = |EF| \text{ (blue)} \dots 11 \\
 d_2 &= |KN| \text{ (green)} \dots 1
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (22 occurrences)}$$

## 2.65.3 Construction MM120



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(B, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_1 \cap k_3$
  4.  $k_4 = k(E, |AB|)$   
 $G \in k_1 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  5.  $k_5 = k(G, |DE|)$   
 $J \in k_2 \cap k_5, K \in k_2 \cap k_5, L \in k_3 \cap k_5, M \in k_3 \cap k_5, N \in k_4 \cap k_5$

## Distances

$$d_0 = |KN| \text{ (red)} \dots 1$$

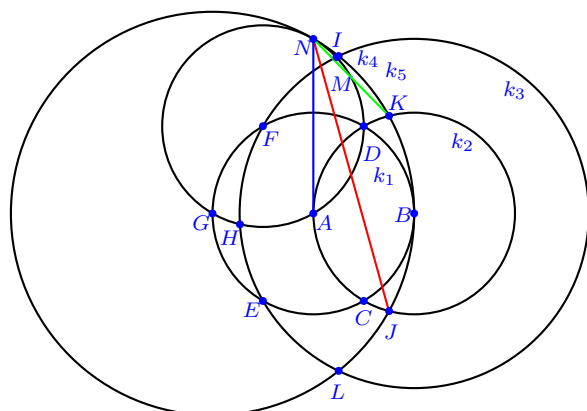
$$d_1 = |AN| = |BE| = |BF| = |BH| = |BI| = |BL| = |BM| = |CD| = |CG| = |DG| = |EF| \text{ (blue)} \dots 11$$

$$d_2 = |JN| \text{ (green)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (22 occurrences)}$$

## 2.65.4 Construction MM123



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(B, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_1 \cap k_3$
  4.  $k_4 = k(F, |AB|)$   
 $G \in k_1 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  5.  $k_5 = k(G, |DE|)$   
 $J \in k_2 \cap k_5, K \in k_2 \cap k_5, L \in k_3 \cap k_5, M \in k_3 \cap k_5, N \in k_4 \cap k_5$

## Distances

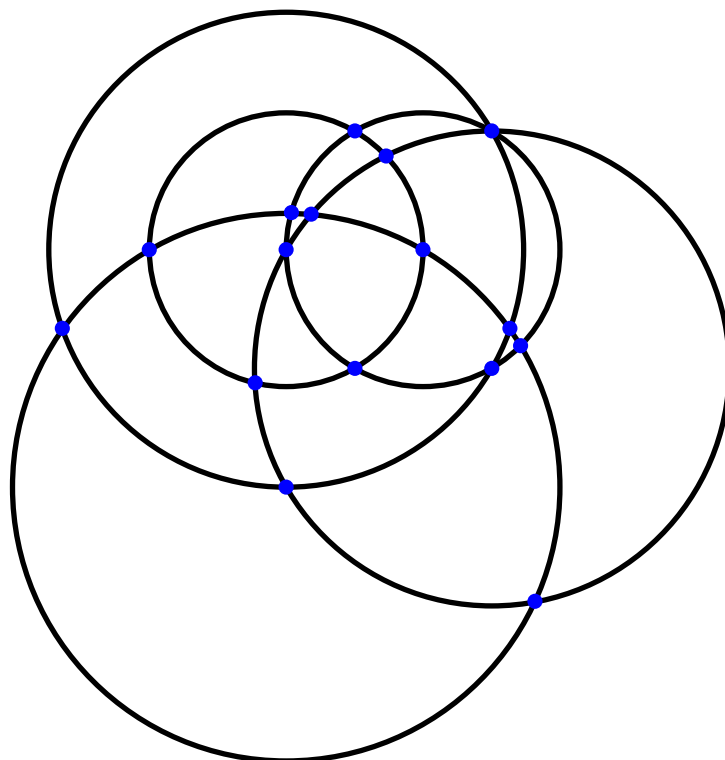
$$d_0 = |JN| \text{ (red) } \dots 1$$

$$d_1 = |AN| = |BE| = |BF| = |BH| = |BI| = |BL| = |BM| = |CD| = |CG| = |DG| = |EF| \text{ (blue) } \dots 11$$

$$d_2 = |KN| \text{ (green) } \dots 1$$

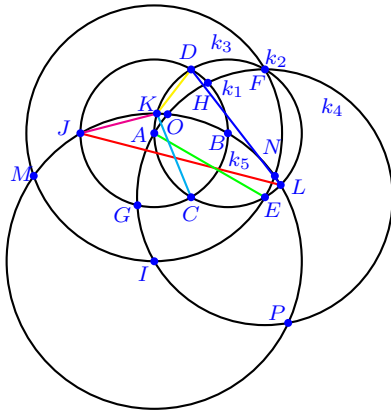
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (22 occurrences)}$$

**2.66 Class of Similar Constructions No. 57****Contained Constructions**

MM91, MM94, MM121, MM124

**2.66.1 Construction MM91**



**Construction Process**

- $A, B$  given initial points
- 1.  $k_1 = k(A, |AB|)$
- 2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
- 3.  $k_3 = k(A, |CD|)$   
 $E \in k_2 \cap k_3, F \in k_2 \cap k_3$
- 4.  $k_4 = k(E, |CD|)$   
 $G \in k_1 \cap k_4, H \in k_1 \cap k_4, I \in k_3 \cap k_4$
- 5.  $k_5 = k(I, |DE|)$   
 $J \in k_1 \cap k_5, K \in k_2 \cap k_5, L \in k_2 \cap k_5, M \in k_3 \cap k_5, N \in k_3 \cap k_5,$   
 $O \in k_4 \cap k_5, P \in k_4 \cap k_5$

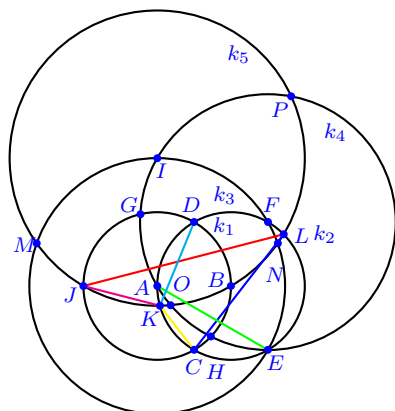
**Distances**

- $d_0 = |JL|$  (red) ... 1
- $d_1 = |DL|$  (blue) ... 1
- $d_2 = |AE| = |AF| = |AI| = |AM| = |AN| = |CD| = |CJ| = |DJ| = |EF| = |EG| = |EH| = |EI| = |EO| = |EP|$  (green) ... 14
- $d_3 = |CK| = |CL|$  (cyan) ... 2
- $d_4 = |JK|$  (magenta) ... 1
- $d_5 = |DK|$  (yellow) ... 1

**Occurrences of the Golden Ratio**

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (32 occurrences)}$$

## 2.66.2 Construction MM94



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(A, |CD|)$   
 $E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, |CD|)$   
 $G \in k_1 \cap k_4, H \in k_1 \cap k_4, I \in k_3 \cap k_4$
  5.  $k_5 = k(I, |DE|)$   
 $J \in k_1 \cap k_5, K \in k_2 \cap k_5, L \in k_2 \cap k_5, M \in k_3 \cap k_5, N \in k_3 \cap k_5,$   
 $O \in k_4 \cap k_5, P \in k_4 \cap k_5$

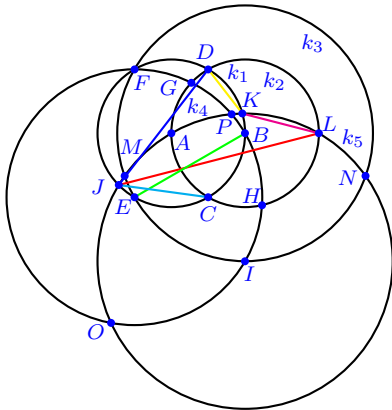
## Distances

$$\begin{aligned}
 d_0 &= |JL| \text{ (red)} \dots 1 \\
 d_1 &= |CL| \text{ (blue)} \dots 1 \\
 d_2 &= |AE| = |AF| = |AI| = |AM| = |AN| = |CD| = |CJ| = |DJ| = \\
 &= |EF| = |FG| = |FH| = |FI| = |FO| = |FP| \text{ (green)} \dots 14 \\
 d_3 &= |DK| = |DL| \text{ (cyan)} \dots 2 \\
 d_4 &= |JK| \text{ (magenta)} \dots 1 \\
 d_5 &= |CK| \text{ (yellow)} \dots 1
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (32 occurrences)}$$

2.66.3 Construction MM121



Construction Process

- $A, B$  given initial points
- 1.  $k_1 = k(A, |AB|)$
- 2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
- 3.  $k_3 = k(B, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_1 \cap k_3$
- 4.  $k_4 = k(E, |CD|)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4, I \in k_3 \cap k_4$
- 5.  $k_5 = k(I, |DE|)$   
 $J \in k_1 \cap k_5, K \in k_1 \cap k_5, L \in k_2 \cap k_5, M \in k_3 \cap k_5, N \in k_3 \cap k_5,$   
 $O \in k_4 \cap k_5, P \in k_4 \cap k_5$

Distances

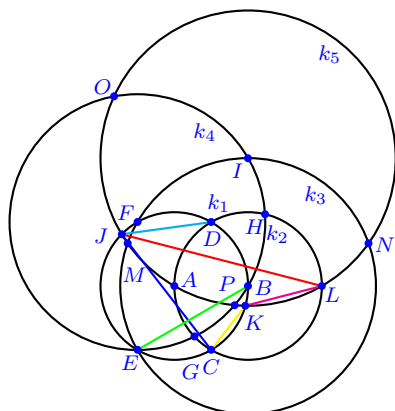
- $d_0 = |JL|$  (red) ... 1
- $d_1 = |DJ|$  (blue) ... 1
- $d_2 = |BE| = |BF| = |BI| = |BM| = |BN| = |CD| = |CL| = |DL| = |EF| = |EG| = |EH| = |EI| = |EO| = |EP|$  (green) ... 14
- $d_3 = |CJ| = |CK|$  (cyan) ... 2
- $d_4 = |KL|$  (magenta) ... 1
- $d_5 = |DK|$  (yellow) ... 1

Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (32 occurrences)}$$



## 2.66.4 Construction MM124



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(B, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_1 \cap k_3$
  4.  $k_4 = k(F, |CD|)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4, I \in k_3 \cap k_4$
  5.  $k_5 = k(I, |DE|)$   
 $J \in k_1 \cap k_5, K \in k_1 \cap k_5, L \in k_2 \cap k_5, M \in k_3 \cap k_5, N \in k_3 \cap k_5,$   
 $O \in k_4 \cap k_5, P \in k_4 \cap k_5$

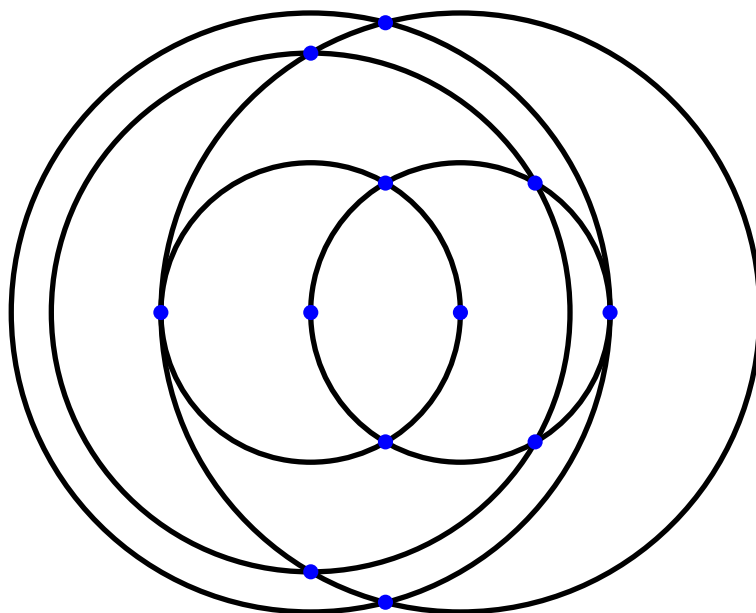
## Distances

$$\begin{aligned}
 d_0 &= |JL| \text{ (red)} \dots 1 \\
 d_1 &= |CJ| \text{ (blue)} \dots 1 \\
 d_2 &= |BE| = |BF| = |BI| = |BM| = |BN| = |CD| = |CL| = |DL| = \\
 &= |EF| = |FG| = |FH| = |FI| = |FO| = |FP| \text{ (green)} \dots 14 \\
 d_3 &= |DJ| = |DK| \text{ (cyan)} \dots 2 \\
 d_4 &= |KL| \text{ (magenta)} \dots 1 \\
 d_5 &= |CK| \text{ (yellow)} \dots 1
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (32 occurrences)}$$

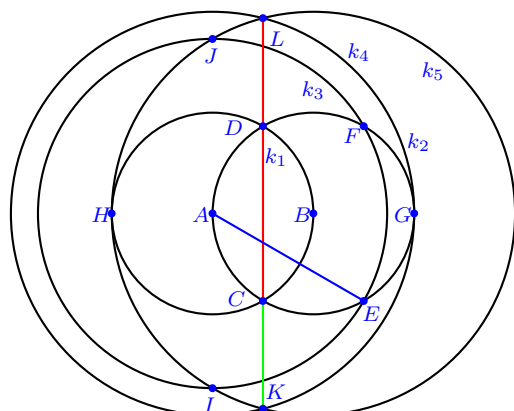
## 2.67 Class of Similar Constructions No. 58



### Contained Constructions

MM95, MM125

## 2.67.1 Construction MM95



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(A, |CD|)$   
 $E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(A, |DE|)$   
 $G \in k_2 \cap k_4$
  5.  $k_5 = k(B, |DE|)$   
 $H \in k_1 \cap k_5, I \in k_3 \cap k_5, J \in k_3 \cap k_5, K \in k_4 \cap k_5, L \in k_4 \cap k_5$

## Distances

$$d_0 = |CL| = |DK| \text{ (red) } \dots 2$$

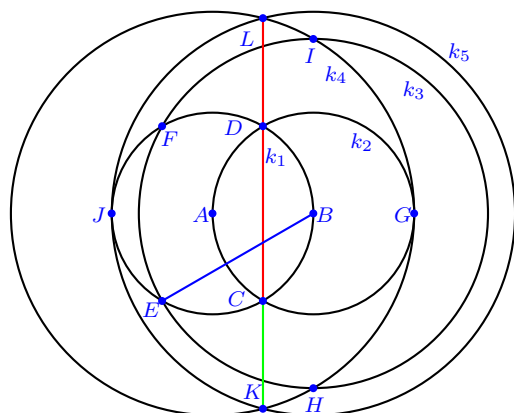
$$d_1 = |AE| = |AF| = |AI| = |AJ| = |CD| = |CG| = |CH| = |DG| = |DH| = |EF| = |EI| = |FJ| \text{ (blue) } \dots 12$$

$$d_2 = |CK| = |DL| \text{ (green) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (48 occurrences)}$$

## 2.67.2 Construction MM125



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(B, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_1 \cap k_3$
  4.  $k_4 = k(A, |DE|)$   
 $G \in k_2 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  5.  $k_5 = k(B, |DE|)$   
 $J \in k_1 \cap k_5, K \in k_4 \cap k_5, L \in k_4 \cap k_5$

## Distances

$$d_0 = |CL| = |DK| \text{ (red) } \dots 2$$

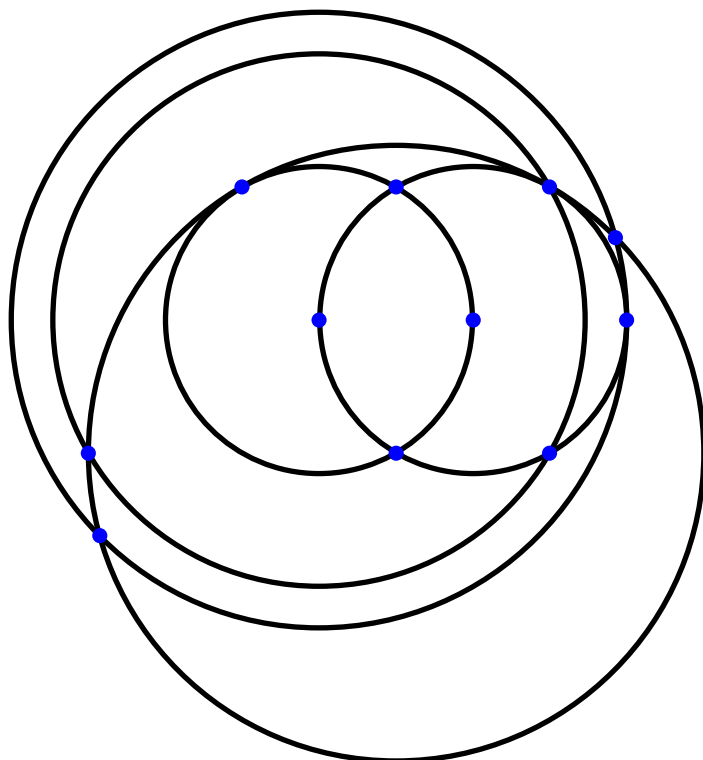
$$d_1 = |BE| = |BF| = |BH| = |BI| = |CD| = |CG| = |CJ| = |DG| = |DJ| = |EF| = |EH| = |FI| \text{ (blue) } \dots 12$$

$$d_2 = |CK| = |DL| \text{ (green) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (48 occurrences)}$$

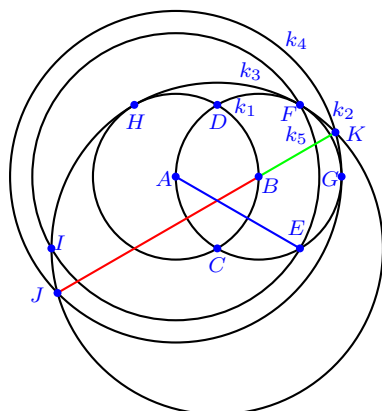
## 2.68 Class of Similar Constructions No. 59



### Contained Constructions

MM96, MM97, MM136, MM137

## 2.68.1 Construction MM96



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(A, |CD|)$   
 $E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(A, |DE|)$   
 $G \in k_2 \cap k_4$
  5.  $k_5 = k(C, |DE|)$   
 $H \in k_1 \cap k_5, I \in k_3 \cap k_5, J \in k_4 \cap k_5, K \in k_4 \cap k_5$

## Distances

$$d_0 = |BJ| \text{ (red) } \dots 1$$

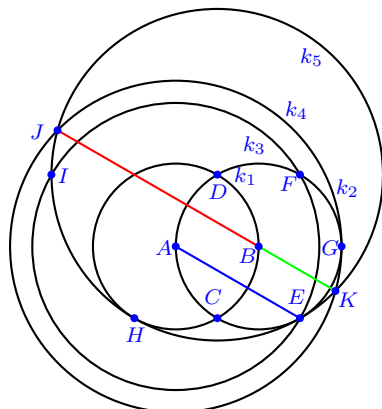
$$d_1 = |AE| = |AF| = |AI| = |BH| = |CD| = |CG| = |DG| = |EF| \text{ (blue) } \dots 8$$

$$d_2 = |BK| \text{ (green) } \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (16 occurrences)}$$

## 2.68.2 Construction MM97



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(A, |CD|)$   
 $E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(A, |DE|)$   
 $G \in k_2 \cap k_4$
  5.  $k_5 = k(D, |DE|)$   
 $H \in k_1 \cap k_5, I \in k_3 \cap k_5, J \in k_4 \cap k_5, K \in k_4 \cap k_5$

## Distances

$$d_0 = |BJ| \text{ (red) } \dots 1$$

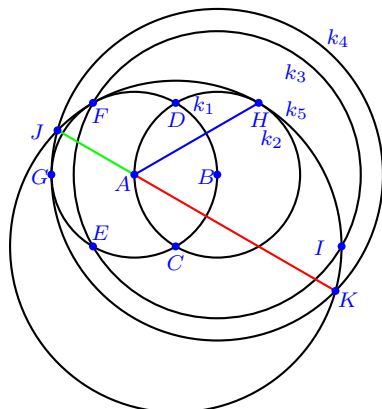
$$d_1 = |AE| = |AF| = |AI| = |BH| = |CD| = |CG| = |DG| = |EF| \text{ (blue) } \dots 8$$

$$d_2 = |BK| \text{ (green) } \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (16 occurrences)}$$

## 2.68.3 Construction MM136



## Construction Process

$A, B$  given initial points

1.  $k_1 = k(A, |AB|)$
2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
3.  $k_3 = k(B, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_1 \cap k_3$
4.  $k_4 = k(B, |DE|)$   
 $G \in k_1 \cap k_4$
5.  $k_5 = k(C, |DE|)$   
 $H \in k_2 \cap k_5, I \in k_3 \cap k_5, J \in k_4 \cap k_5, K \in k_4 \cap k_5$

## Distances

$$d_0 = |AK| \text{ (red) } \dots 1$$

$$d_1 = |AH| = |BE| = |BF| = |BI| = |CD| = |CG| = |DG| = |EF| \text{ (blue) }$$

$\dots 8$

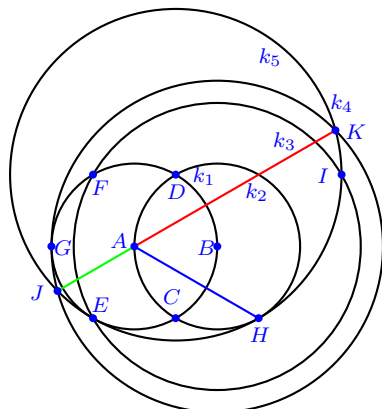
$$d_2 = |AJ| \text{ (green) } \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (16 occurrences)}$$



## 2.68.4 Construction MM137



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(B, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_1 \cap k_3$
  4.  $k_4 = k(B, |DE|)$   
 $G \in k_1 \cap k_4$
  5.  $k_5 = k(D, |DE|)$   
 $H \in k_2 \cap k_5, I \in k_3 \cap k_5, J \in k_4 \cap k_5, K \in k_4 \cap k_5$

## Distances

$$d_0 = |AK| \text{ (red) } \dots 1$$

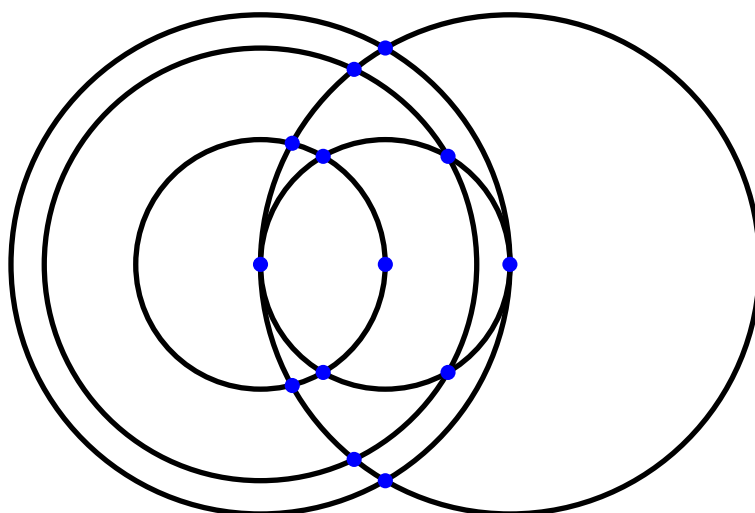
$$d_1 = |AH| = |BE| = |BF| = |BI| = |CD| = |CG| = |DG| = |EF| \text{ (blue) } \dots 8$$

$$d_2 = |AJ| \text{ (green) } \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (16 occurrences)}$$

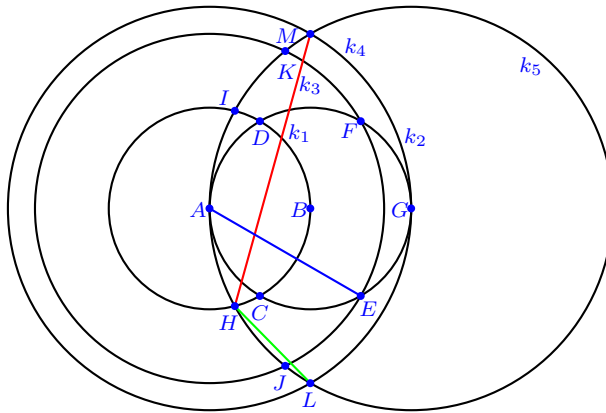
## 2.69 Class of Similar Constructions No. 60



### Contained Constructions

MM98, MM138

2.69.1 Construction MM98



Construction Process

- $A, B$  given initial points
- 1.  $k_1 = k(A, |AB|)$
- 2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
- 3.  $k_3 = k(A, |CD|)$   
 $E \in k_2 \cap k_3, F \in k_2 \cap k_3$
- 4.  $k_4 = k(A, |DE|)$   
 $G \in k_2 \cap k_4$
- 5.  $k_5 = k(G, |DE|)$   
 $H \in k_1 \cap k_5, I \in k_1 \cap k_5, J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5,$   
 $M \in k_4 \cap k_5$

Distances

$$d_0 = |HM| = |IL| \text{ (red) } \dots 2$$

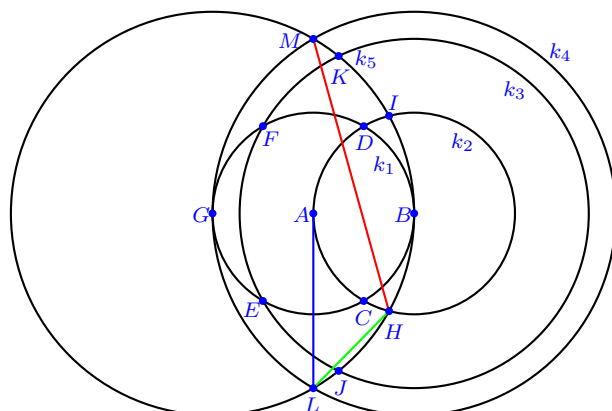
$$d_1 = |AE| = |AF| = |AJ| = |AK| = |BL| = |BM| = |CD| = |CG| = |DG| = |EF| \text{ (blue) } \dots 10$$

$$d_2 = |HL| = |IM| \text{ (green) } \dots 2$$

Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (40 occurrences)}$$

## 2.69.2 Construction MM138



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(B, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_1 \cap k_3$
  4.  $k_4 = k(B, |DE|)$   
 $G \in k_1 \cap k_4$
  5.  $k_5 = k(G, |DE|)$   
 $H \in k_2 \cap k_5, I \in k_2 \cap k_5, J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5,$   
 $M \in k_4 \cap k_5$

## Distances

$$d_0 = |HM| = |IL| \text{ (red) } \dots 2$$

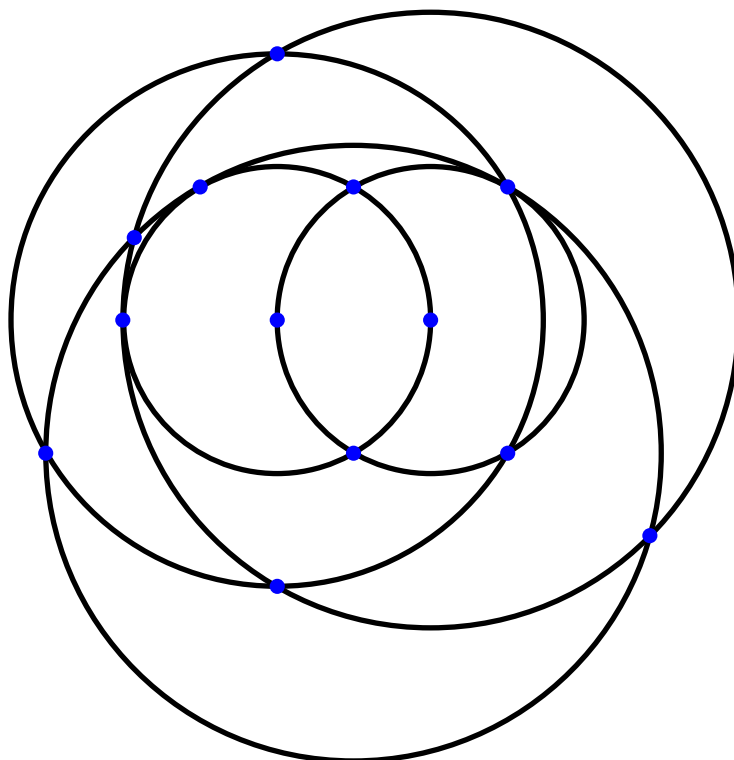
$$d_1 = |AL| = |AM| = |BE| = |BF| = |BJ| = |BK| = |CD| = |CG| = |DG| = |EF| \text{ (blue) } \dots 10$$

$$d_2 = |HL| = |IM| \text{ (green) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (40 occurrences)}$$

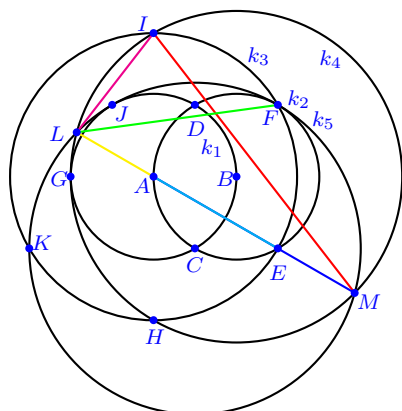
## 2.70 Class of Similar Constructions No. 61



### Contained Constructions

MM99, MM100, MM126, MM127

## 2.70.1 Construction MM99



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(A, |CD|)$   
 $E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(B, |DE|)$   
 $G \in k_1 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  5.  $k_5 = k(C, |DE|)$   
 $J \in k_1 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5, M \in k_4 \cap k_5$

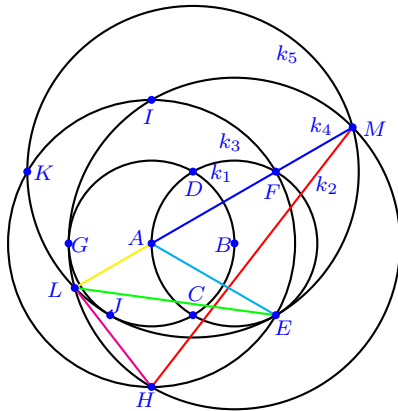
## Distances

$$\begin{aligned}
 d_0 &= |IM| = |KM| \text{ (red)} \dots 2 \\
 d_1 &= |AM| = |EL| \text{ (blue)} \dots 2 \\
 d_2 &= |FL| = |FM| = |HL| = |HM| \text{ (green)} \dots 4 \\
 d_3 &= |AE| = |AF| = |AH| = |AI| = |AK| = |BJ| = |CD| = |CG| = \\
 &|DG| = |EF| = |EH| = |FI| = |HK| \text{ (cyan)} \dots 13 \\
 d_4 &= |IL| = |KL| \text{ (magenta)} \dots 2 \\
 d_5 &= |AL| = |EM| \text{ (yellow)} \dots 2
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (68 occurrences)}$$

2.70.2 Construction MM100



Construction Process

- $A, B$  given initial points
- 1.  $k_1 = k(A, |AB|)$
- 2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
- 3.  $k_3 = k(A, |CD|)$   
 $E \in k_2 \cap k_3, F \in k_2 \cap k_3$
- 4.  $k_4 = k(B, |DE|)$   
 $G \in k_1 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
- 5.  $k_5 = k(D, |DE|)$   
 $J \in k_1 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5, M \in k_4 \cap k_5$

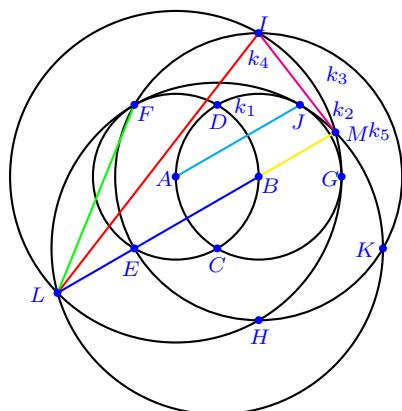
Distances

- $d_0 = |HM| = |KM|$  (red) ... 2
- $d_1 = |AM| = |FL|$  (blue) ... 2
- $d_2 = |EL| = |EM| = |IL| = |IM|$  (green) ... 4
- $d_3 = |AE| = |AF| = |AH| = |AI| = |AK| = |BJ| = |CD| = |CG| = |DG| = |EF| = |EH| = |FI| = |IK|$  (cyan) ... 13
- $d_4 = |HL| = |KL|$  (magenta) ... 2
- $d_5 = |AL| = |FM|$  (yellow) ... 2

Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (68 occurrences)}$$

## 2.70.3 Construction MM126



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(B, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_1 \cap k_3$
  4.  $k_4 = k(A, |DE|)$   
 $G \in k_2 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  5.  $k_5 = k(C, |DE|)$   
 $J \in k_2 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5, M \in k_4 \cap k_5$

## Distances

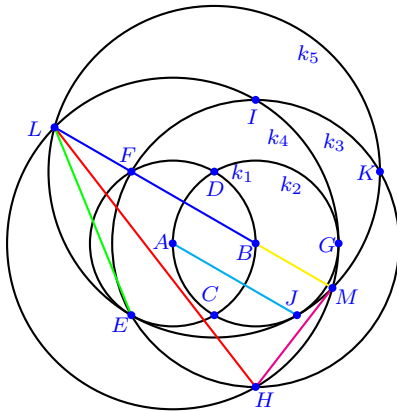
$$\begin{aligned}
 d_0 &= |IL| = |KL| \text{ (red)} \dots 2 \\
 d_1 &= |BL| = |EM| \text{ (blue)} \dots 2 \\
 d_2 &= |FL| = |FM| = |HL| = |HM| \text{ (green)} \dots 4 \\
 d_3 &= |AJ| = |BE| = |BF| = |BH| = |BI| = |BK| = |CD| = |CG| = \\
 &|DG| = |EF| = |EH| = |FI| = |HK| \text{ (cyan)} \dots 13 \\
 d_4 &= |IM| = |KM| \text{ (magenta)} \dots 2 \\
 d_5 &= |BM| = |EL| \text{ (yellow)} \dots 2
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (68 occurrences)}$$



2.70.4 Construction MM127



Construction Process

- $A, B$  given initial points
- 1.  $k_1 = k(A, |AB|)$
- 2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
- 3.  $k_3 = k(B, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_1 \cap k_3$
- 4.  $k_4 = k(A, |DE|)$   
 $G \in k_2 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
- 5.  $k_5 = k(D, |DE|)$   
 $J \in k_2 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5, M \in k_4 \cap k_5$

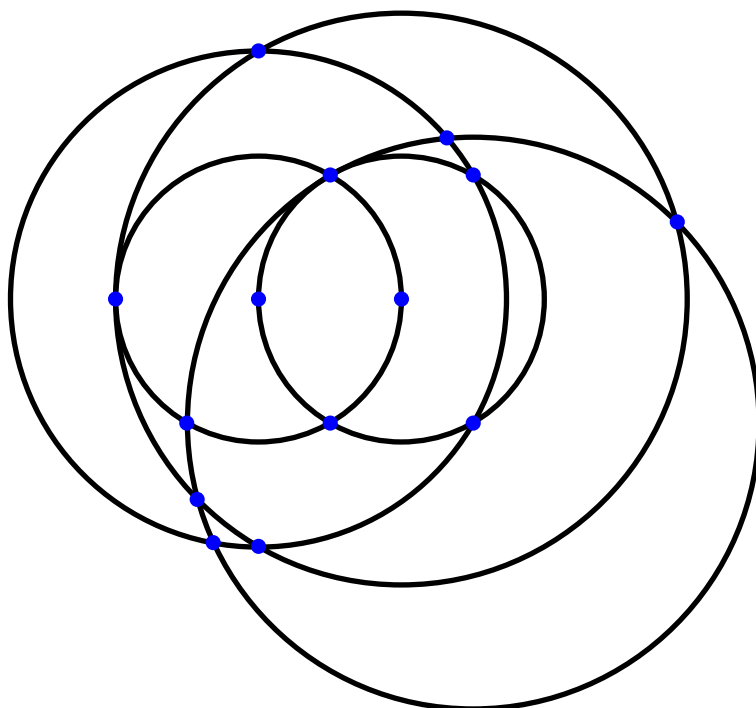
Distances

- $d_0 = |HL| = |KL|$  (red) ... 2
- $d_1 = |BL| = |FM|$  (blue) ... 2
- $d_2 = |EL| = |EM| = |IL| = |IM|$  (green) ... 4
- $d_3 = |AJ| = |BE| = |BF| = |BH| = |BI| = |BK| = |CD| = |CG| = |DG| = |EF| = |EH| = |FI| = |IK|$  (cyan) ... 13
- $d_4 = |HM| = |KM|$  (magenta) ... 2
- $d_5 = |BM| = |FL|$  (yellow) ... 2

Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (68 occurrences)}$$

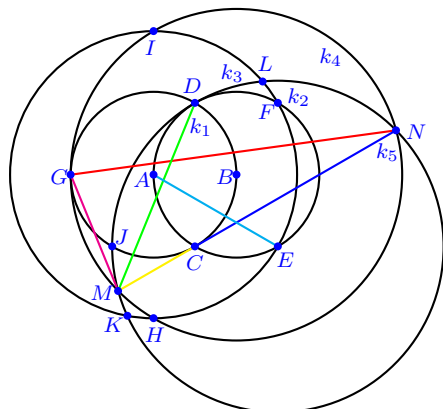
## 2.71 Class of Similar Constructions No. 62



### Contained Constructions

MM101, MM102, MM128, MM129

## 2.71.1 Construction MM101



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(A, |CD|)$   
 $E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(B, |DE|)$   
 $G \in k_1 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  5.  $k_5 = k(E, |DE|)$   
 $J \in k_1 \cap k_5, K \in k_3 \cap k_5, L \in k_3 \cap k_5, M \in k_4 \cap k_5, N \in k_4 \cap k_5$

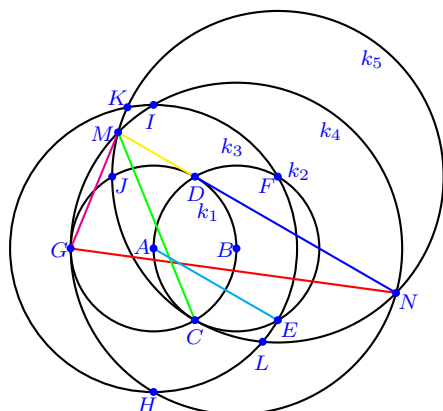
## Distances

$$\begin{aligned}
 d_0 &= |GN| \text{ (red)} \dots 1 \\
 d_1 &= |CN| \text{ (blue)} \dots 1 \\
 d_2 &= |DM| = |DN| \text{ (green)} \dots 2 \\
 d_3 &= |AE| = |AF| = |AH| = |AI| = |AK| = |AL| = |BJ| = |CD| = \\
 &|CG| = |DG| = |EF| = |EH| = |FI| \text{ (cyan)} \dots 13 \\
 d_4 &= |GM| \text{ (magenta)} \dots 1 \\
 d_5 &= |CM| \text{ (yellow)} \dots 1
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (30 occurrences)}$$

## 2.71.2 Construction MM102



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(A, |CD|)$   
 $E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(B, |DE|)$   
 $G \in k_1 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  5.  $k_5 = k(F, |DE|)$   
 $J \in k_1 \cap k_5, K \in k_3 \cap k_5, L \in k_3 \cap k_5, M \in k_4 \cap k_5, N \in k_4 \cap k_5$

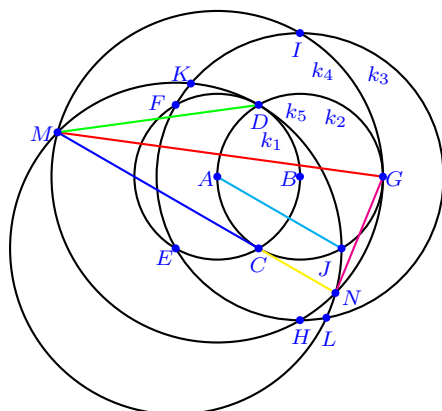
## Distances

$$\begin{aligned}
 d_0 &= |GN| \text{ (red)} \dots 1 \\
 d_1 &= |DN| \text{ (blue)} \dots 1 \\
 d_2 &= |CM| = |CN| \text{ (green)} \dots 2 \\
 d_3 &= |AE| = |AF| = |AH| = |AI| = |AK| = |AL| = |BJ| = |CD| = \\
 &|CG| = |DG| = |EF| = |EH| = |FI| \text{ (cyan)} \dots 13 \\
 d_4 &= |GM| \text{ (magenta)} \dots 1 \\
 d_5 &= |DM| \text{ (yellow)} \dots 1
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (30 occurrences)}$$

## 2.71.3 Construction MM128



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(B, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_1 \cap k_3$
  4.  $k_4 = k(A, |DE|)$   
 $G \in k_2 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  5.  $k_5 = k(E, |DE|)$   
 $J \in k_2 \cap k_5, K \in k_3 \cap k_5, L \in k_3 \cap k_5, M \in k_4 \cap k_5, N \in k_4 \cap k_5$

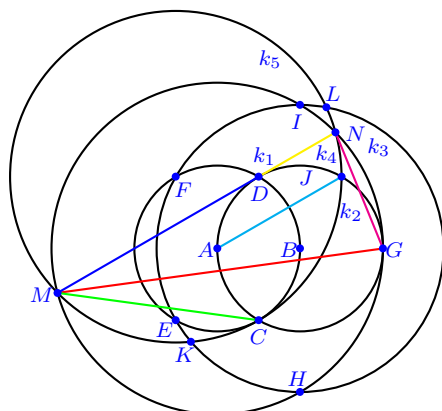
## Distances

$$\begin{aligned}
 d_0 &= |GM| \text{ (red)} \dots 1 \\
 d_1 &= |CM| \text{ (blue)} \dots 1 \\
 d_2 &= |DM| = |DN| \text{ (green)} \dots 2 \\
 d_3 &= |AJ| = |BE| = |BF| = |BH| = |BI| = |BK| = |BL| = |CD| = \\
 &= |CG| = |DG| = |EF| = |EH| = |FI| \text{ (cyan)} \dots 13 \\
 d_4 &= |GN| \text{ (magenta)} \dots 1 \\
 d_5 &= |CN| \text{ (yellow)} \dots 1
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (30 occurrences)}$$

## 2.71.4 Construction MM129



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(B, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_1 \cap k_3$
  4.  $k_4 = k(A, |DE|)$   
 $G \in k_2 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  5.  $k_5 = k(F, |DE|)$   
 $J \in k_2 \cap k_5, K \in k_3 \cap k_5, L \in k_3 \cap k_5, M \in k_4 \cap k_5, N \in k_4 \cap k_5$

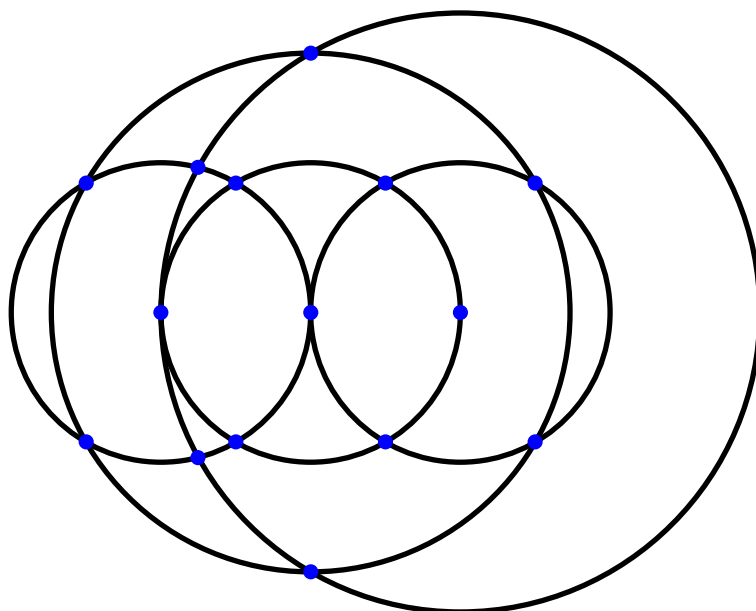
## Distances

$$\begin{aligned}
 d_0 &= |GM| \text{ (red)} \dots 1 \\
 d_1 &= |DM| \text{ (blue)} \dots 1 \\
 d_2 &= |CM| = |CN| \text{ (green)} \dots 2 \\
 d_3 &= |AJ| = |BE| = |BF| = |BH| = |BI| = |BK| = |BL| = |CD| = \\
 &|CG| = |DG| = |EF| = |EH| = |FI| \text{ (cyan)} \dots 13 \\
 d_4 &= |GN| \text{ (magenta)} \dots 1 \\
 d_5 &= |DN| \text{ (yellow)} \dots 1
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (30 occurrences)}$$

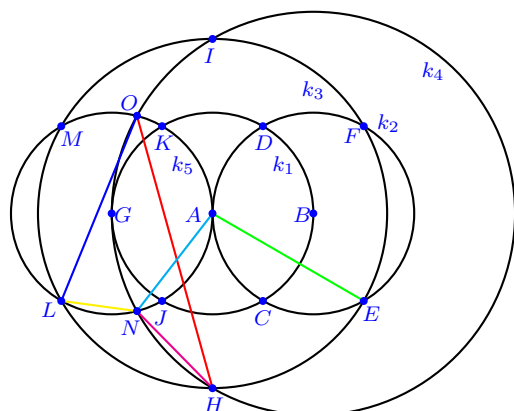
## 2.72 Class of Similar Constructions No. 63



### Contained Constructions

MM103, MM130

## 2.72.1 Construction MM103



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(A, |CD|)$   
 $E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(B, |DE|)$   
 $G \in k_1 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  5.  $k_5 = k(G, |AB|)$   
 $J \in k_1 \cap k_5, K \in k_1 \cap k_5, L \in k_3 \cap k_5, M \in k_3 \cap k_5, N \in k_4 \cap k_5,$   
 $O \in k_4 \cap k_5$

## Distances

$$d_0 = |HO| = |IN| \text{ (red) } \dots 2$$

$$d_1 = |LO| = |MN| \text{ (blue) } \dots 2$$

$$d_2 = |AE| = |AF| = |AH| = |AI| = |AL| = |AM| = |BJ| = |BK| = |CD| = |CG| = |DG| = |EF| = |EH| = |FI| = |HL| = |IM| = |JK| = |LM| \text{ (green) } \dots 18$$

$$d_3 = |AN| = |AO| \text{ (cyan) } \dots 2$$

$$d_4 = |HN| = |IO| \text{ (magenta) } \dots 2$$

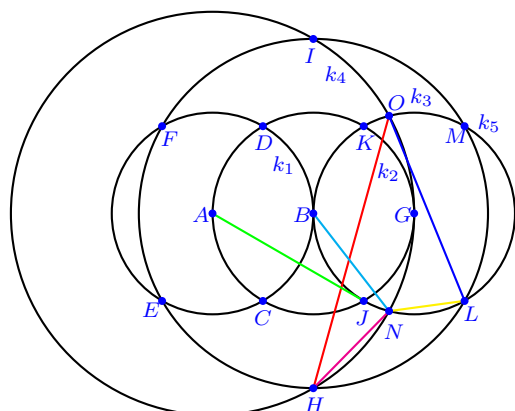
$$d_5 = |LN| = |MO| \text{ (yellow) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (80 occurrences)}$$



## 2.72.2 Construction MM130



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(B, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_1 \cap k_3$
  4.  $k_4 = k(A, |DE|)$   
 $G \in k_2 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  5.  $k_5 = k(G, |AB|)$   
 $J \in k_2 \cap k_5, K \in k_2 \cap k_5, L \in k_3 \cap k_5, M \in k_3 \cap k_5, N \in k_4 \cap k_5,$   
 $O \in k_4 \cap k_5$

## Distances

$$d_0 = |HO| = |IN| \text{ (red)} \dots 2$$

$$d_1 = |LO| = |MN| \text{ (blue)} \dots 2$$

$$d_2 = |AJ| = |AK| = |BE| = |BF| = |BH| = |BI| = |BL| = |BM| = |CD| = |CG| = |DG| = |EF| = |EH| = |FI| = |HL| = |IM| = |JK| = |LM| \text{ (green)} \dots 18$$

$$d_3 = |BN| = |BO| \text{ (cyan)} \dots 2$$

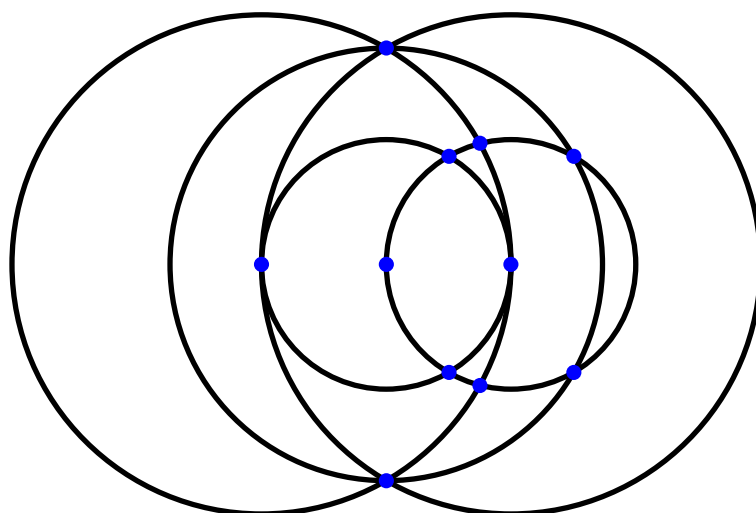
$$d_4 = |HN| = |IO| \text{ (magenta)} \dots 2$$

$$d_5 = |LN| = |MO| \text{ (yellow)} \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (80 occurrences)}$$

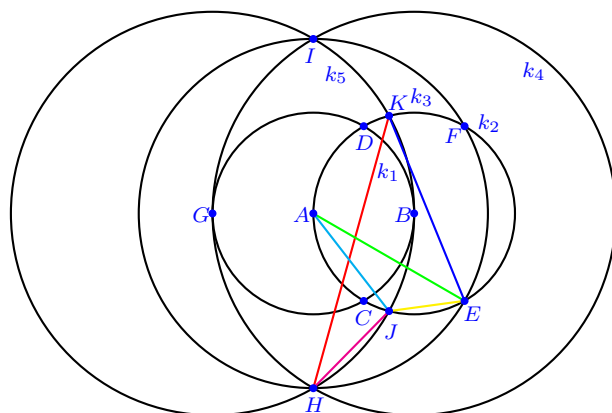
## 2.73 Class of Similar Constructions No. 64



### Contained Constructions

MM104, MM131

## 2.73.1 Construction MM104



## Construction Process

- $A, B$  given initial points
- $k_1 = k(A, |AB|)$
  - $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  - $k_3 = k(A, |CD|)$   
 $E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  - $k_4 = k(B, |DE|)$   
 $G \in k_1 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  - $k_5 = k(G, |DE|)$   
 $J \in k_2 \cap k_5, K \in k_2 \cap k_5$

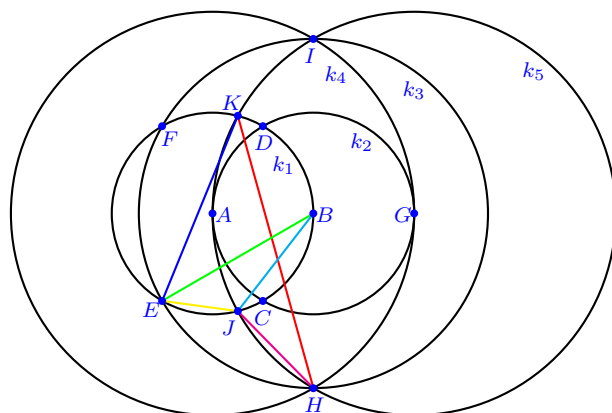
## Distances

$$\begin{aligned}
 d_0 &= |HK| = |IJ| \text{ (red)} \dots 2 \\
 d_1 &= |EK| = |FJ| \text{ (blue)} \dots 2 \\
 d_2 &= |AE| = |AF| = |AH| = |AI| = |CD| = |CG| = |DG| = |EF| = |EH| = |FI| \text{ (green)} \dots 10 \\
 d_3 &= |AJ| = |AK| \text{ (cyan)} \dots 2 \\
 d_4 &= |HJ| = |IK| \text{ (magenta)} \dots 2 \\
 d_5 &= |EJ| = |FK| \text{ (yellow)} \dots 2
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (48 occurrences)}$$

## 2.73.2 Construction MM131



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(B, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_1 \cap k_3$
  4.  $k_4 = k(A, |DE|)$   
 $G \in k_2 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  5.  $k_5 = k(G, |DE|)$   
 $J \in k_1 \cap k_5, K \in k_1 \cap k_5$

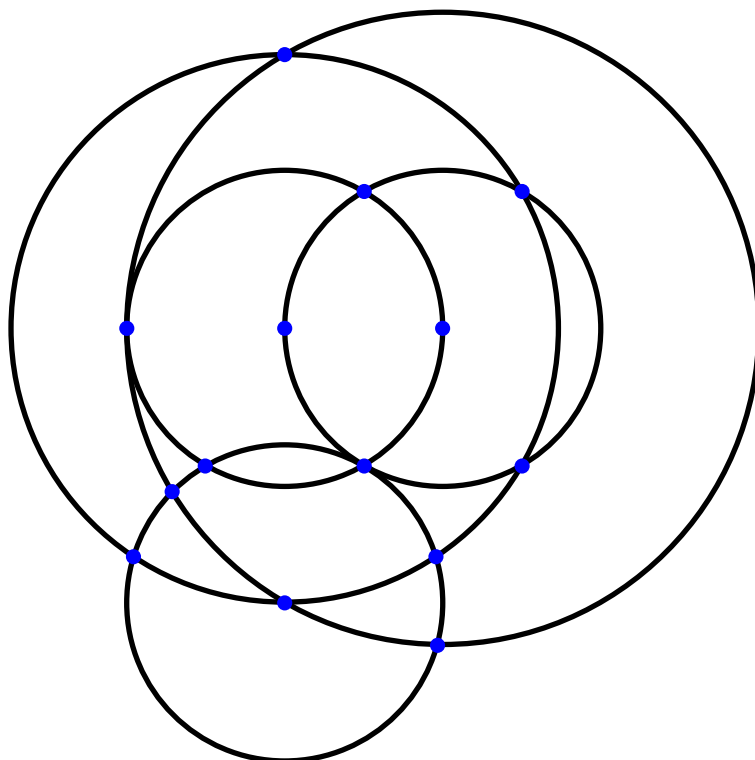
## Distances

$$\begin{aligned}
 d_0 &= |HK| = |IJ| \text{ (red)} \dots 2 \\
 d_1 &= |EK| = |FJ| \text{ (blue)} \dots 2 \\
 d_2 &= |BE| = |BF| = |BH| = |BI| = |CD| = |CG| = |DG| = |EF| = \\
 &|EH| = |FI| \text{ (green)} \dots 10 \\
 d_3 &= |BJ| = |BK| \text{ (cyan)} \dots 2 \\
 d_4 &= |HJ| = |IK| \text{ (magenta)} \dots 2 \\
 d_5 &= |EJ| = |FK| \text{ (yellow)} \dots 2
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (48 occurrences)}$$

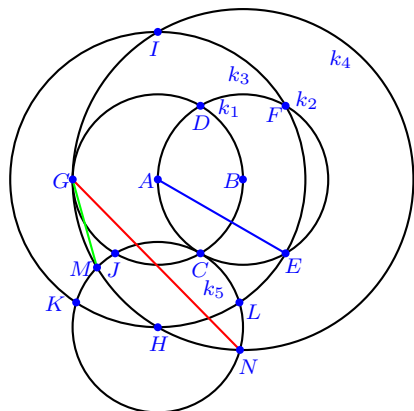
## 2.74 Class of Similar Constructions No. 65



### Contained Constructions

MM105, MM107, MM132, MM134

## 2.74.1 Construction MM105



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(A, |CD|)$   
 $E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(B, |DE|)$   
 $G \in k_1 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  5.  $k_5 = k(H, |AB|)$   
 $J \in k_1 \cap k_5, K \in k_3 \cap k_5, L \in k_3 \cap k_5, M \in k_4 \cap k_5, N \in k_4 \cap k_5$

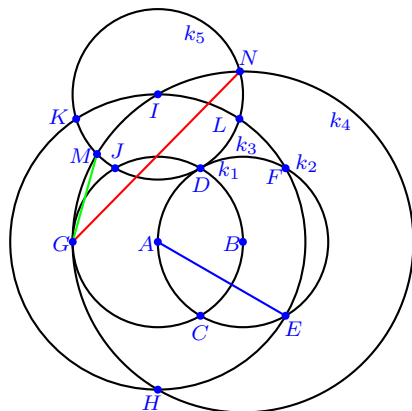
## Distances

$$\begin{aligned}
 d_0 &= |GN| \text{ (red)} \dots 1 \\
 d_1 &= |AE| = |AF| = |AH| = |AI| = |AK| = |AL| = |BJ| = |CD| = \\
 &|CG| = |DG| = |EF| = |EH| = |FI| \text{ (blue)} \dots 13 \\
 d_2 &= |GM| \text{ (green)} \dots 1
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (26 occurrences)}$$

## 2.74.2 Construction MM107



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(A, |CD|)$   
 $E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(B, |DE|)$   
 $G \in k_1 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  5.  $k_5 = k(I, |AB|)$   
 $J \in k_1 \cap k_5, K \in k_3 \cap k_5, L \in k_3 \cap k_5, M \in k_4 \cap k_5, N \in k_4 \cap k_5$

## Distances

$$d_0 = |GN| \text{ (red)} \dots 1$$

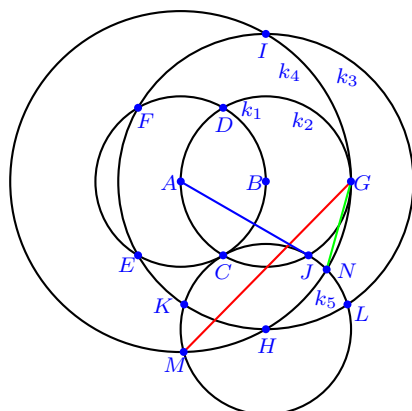
$$d_1 = |AE| = |AF| = |AH| = |AI| = |AK| = |AL| = |BJ| = |CD| = |CG| = |DG| = |EF| = |EH| = |FI| \text{ (blue)} \dots 13$$

$$d_2 = |GM| \text{ (green)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (26 occurrences)}$$

## 2.74.3 Construction MM132



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(B, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_1 \cap k_3$
  4.  $k_4 = k(A, |DE|)$   
 $G \in k_2 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  5.  $k_5 = k(H, |AB|)$   
 $J \in k_2 \cap k_5, K \in k_3 \cap k_5, L \in k_3 \cap k_5, M \in k_4 \cap k_5, N \in k_4 \cap k_5$

## Distances

$$d_0 = |GM| \text{ (red)} \dots 1$$

$$d_1 = |AJ| = |BE| = |BF| = |BH| = |BI| = |BK| = |BL| = |CD| = |CG| = |DG| = |EF| = |EH| = |FI| \text{ (blue)} \dots 13$$

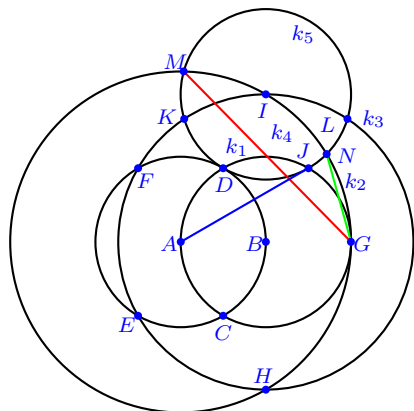
$$d_2 = |GN| \text{ (green)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (26 occurrences)}$$



## 2.74.4 Construction MM134



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(B, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_1 \cap k_3$
  4.  $k_4 = k(A, |DE|)$   
 $G \in k_2 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  5.  $k_5 = k(I, |AB|)$   
 $J \in k_2 \cap k_5, K \in k_3 \cap k_5, L \in k_3 \cap k_5, M \in k_4 \cap k_5, N \in k_4 \cap k_5$

## Distances

$$d_0 = |GM| \text{ (red)} \dots 1$$

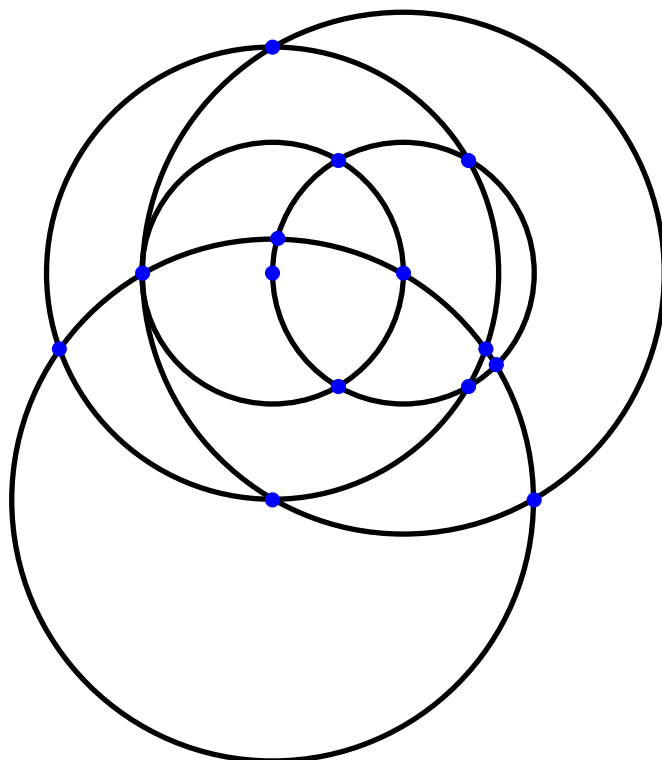
$$d_1 = |AJ| = |BE| = |BF| = |BH| = |BI| = |BK| = |BL| = |CD| = |CG| = |DG| = |EF| = |EH| = |FI| \text{ (blue)} \dots 13$$

$$d_2 = |GN| \text{ (green)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (26 occurrences)}$$

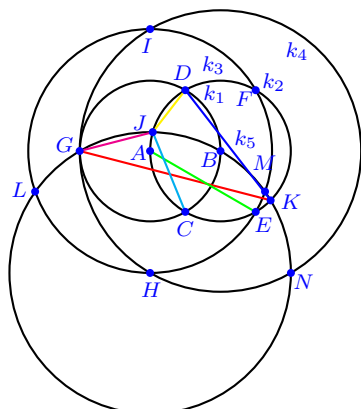
## 2.75 Class of Similar Constructions No. 66



### Contained Constructions

MM106, MM108, MM133, MM135

## 2.75.1 Construction MM106



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(A, |CD|)$   
 $E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(B, |DE|)$   
 $G \in k_1 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  5.  $k_5 = k(H, |DE|)$   
 $J \in k_2 \cap k_5, K \in k_2 \cap k_5, L \in k_3 \cap k_5, M \in k_3 \cap k_5, N \in k_4 \cap k_5$

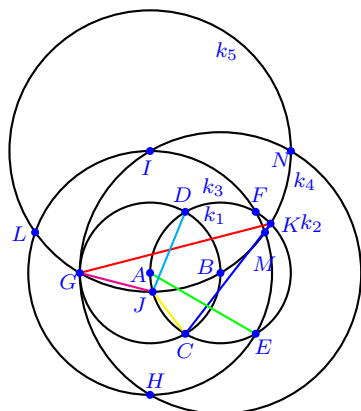
## Distances

$$\begin{aligned}
 d_0 &= |GK| = |JN| \text{ (red)} \dots 2 \\
 d_1 &= |DK| \text{ (blue)} \dots 1 \\
 d_2 &= |AE| = |AF| = |AH| = |AI| = |AL| = |AM| = |CD| = |CG| = \\
 &|CN| = |DG| = |EF| = |EH| = |FI| \text{ (green)} \dots 13 \\
 d_3 &= |CJ| = |CK| \text{ (cyan)} \dots 2 \\
 d_4 &= |GJ| = |KN| \text{ (magenta)} \dots 2 \\
 d_5 &= |DJ| \text{ (yellow)} \dots 1
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (56 occurrences)}$$

## 2.75.2 Construction MM108



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(A, |CD|)$   
 $E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(B, |DE|)$   
 $G \in k_1 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  5.  $k_5 = k(I, |DE|)$   
 $J \in k_2 \cap k_5, K \in k_2 \cap k_5, L \in k_3 \cap k_5, M \in k_3 \cap k_5, N \in k_4 \cap k_5$

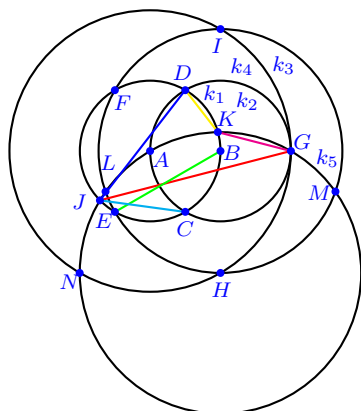
## Distances

$$\begin{aligned}
 d_0 &= |GK| = |JN| \text{ (red)} \dots 2 \\
 d_1 &= |CK| \text{ (blue)} \dots 1 \\
 d_2 &= |AE| = |AF| = |AH| = |AI| = |AL| = |AM| = |CD| = |CG| = \\
 &|DG| = |DN| = |EF| = |EH| = |FI| \text{ (green)} \dots 13 \\
 d_3 &= |DJ| = |DK| \text{ (cyan)} \dots 2 \\
 d_4 &= |GJ| = |KN| \text{ (magenta)} \dots 2 \\
 d_5 &= |CJ| \text{ (yellow)} \dots 1
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (56 occurrences)}$$

## 2.75.3 Construction MM133



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(B, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_1 \cap k_3$
  4.  $k_4 = k(A, |DE|)$   
 $G \in k_2 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  5.  $k_5 = k(H, |DE|)$   
 $J \in k_1 \cap k_5, K \in k_1 \cap k_5, L \in k_3 \cap k_5, M \in k_3 \cap k_5, N \in k_4 \cap k_5$

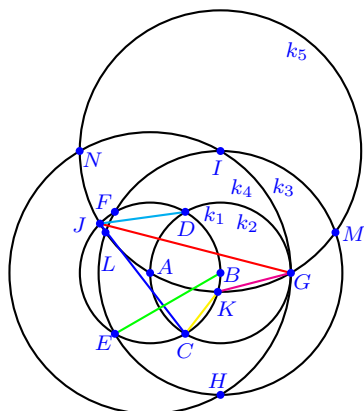
## Distances

$$\begin{aligned}
 d_0 &= |GJ| = |KN| \text{ (red)} \dots 2 \\
 d_1 &= |DJ| \text{ (blue)} \dots 1 \\
 d_2 &= |BE| = |BF| = |BH| = |BI| = |BL| = |BM| = |CD| = |CG| = \\
 &|CN| = |DG| = |EF| = |EH| = |FI| \text{ (green)} \dots 13 \\
 d_3 &= |CJ| = |CK| \text{ (cyan)} \dots 2 \\
 d_4 &= |GK| = |JN| \text{ (magenta)} \dots 2 \\
 d_5 &= |DK| \text{ (yellow)} \dots 1
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (56 occurrences)}$$

## 2.75.4 Construction MM135



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(B, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_1 \cap k_3$
  4.  $k_4 = k(A, |DE|)$   
 $G \in k_2 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  5.  $k_5 = k(I, |DE|)$   
 $J \in k_1 \cap k_5, K \in k_1 \cap k_5, L \in k_3 \cap k_5, M \in k_3 \cap k_5, N \in k_4 \cap k_5$

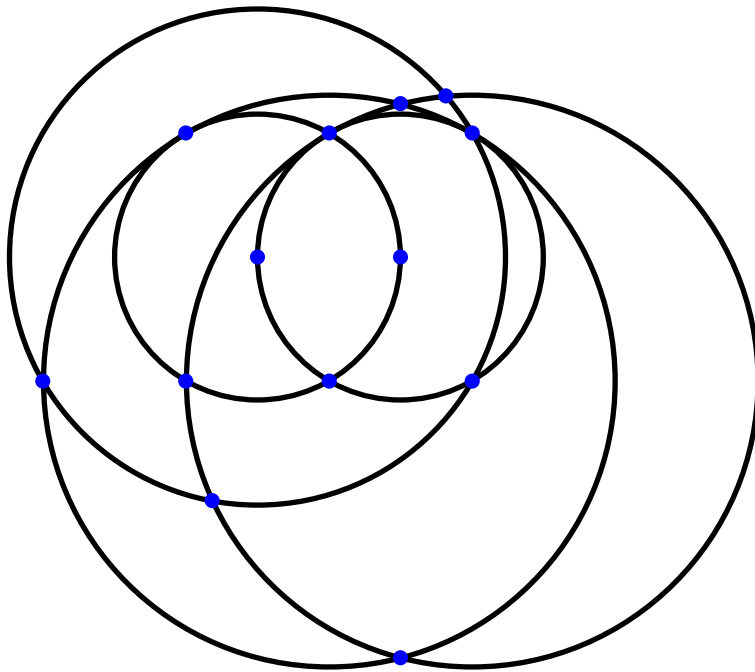
## Distances

$$\begin{aligned}
 d_0 &= |GJ| = |KN| \text{ (red)} \dots 2 \\
 d_1 &= |CJ| \text{ (blue)} \dots 1 \\
 d_2 &= |BE| = |BF| = |BH| = |BI| = |BL| = |BM| = |CD| = |CG| = \\
 &|DG| = |DN| = |EF| = |EH| = |FI| \text{ (green)} \dots 13 \\
 d_3 &= |DJ| = |DK| \text{ (cyan)} \dots 2 \\
 d_4 &= |GK| = |JN| \text{ (magenta)} \dots 2 \\
 d_5 &= |CK| \text{ (yellow)} \dots 1
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (56 occurrences)}$$

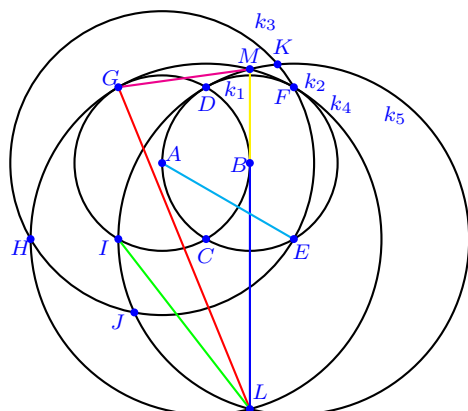
## 2.76 Class of Similar Constructions No. 67



### Contained Constructions

MM109, MM112, MM139, MM142

## 2.76.1 Construction MM109



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(A, |CD|)$   
 $E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(C, |DE|)$   
 $G \in k_1 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(E, |DE|)$   
 $I \in k_1 \cap k_5, J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5, M \in k_4 \cap k_5$

## Distances

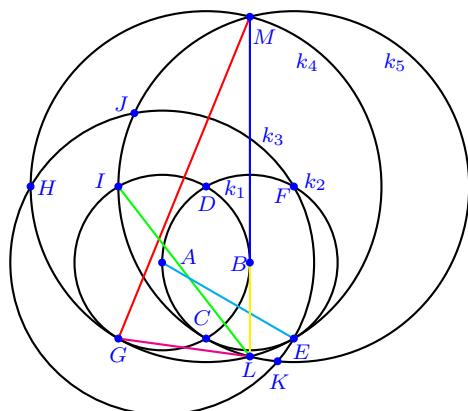
$$\begin{aligned}
 d_0 &= |GL| \text{ (red)} \dots 1 \\
 d_1 &= |BL| \text{ (blue)} \dots 1 \\
 d_2 &= |IL| = |IM| \text{ (green)} \dots 2 \\
 d_3 &= |AE| = |AF| = |AH| = |AJ| = |AK| = |BG| = |BI| = |CD| = \\
 &|EF| = |GI| \text{ (cyan)} \dots 10 \\
 d_4 &= |GM| \text{ (magenta)} \dots 1 \\
 d_5 &= |BM| \text{ (yellow)} \dots 1
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (24 occurrences)}$$



## 2.76.2 Construction MM112



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(A, |CD|)$   
 $E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(D, |DE|)$   
 $G \in k_1 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(F, |DE|)$   
 $I \in k_1 \cap k_5, J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5, M \in k_4 \cap k_5$

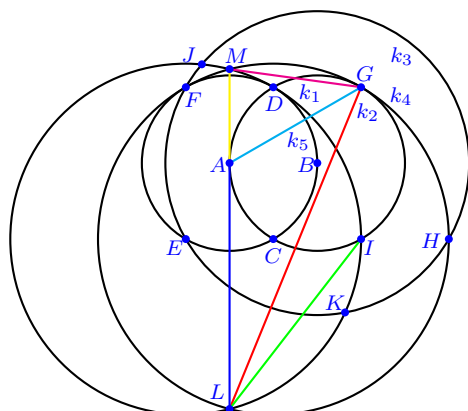
## Distances

$$\begin{aligned}
 d_0 &= |GM| \text{ (red)} \dots 1 \\
 d_1 &= |BM| \text{ (blue)} \dots 1 \\
 d_2 &= |IL| = |IM| \text{ (green)} \dots 2 \\
 d_3 &= |AE| = |AF| = |AH| = |AJ| = |AK| = |BG| = |BI| = |CD| = \\
 &|EF| = |GI| \text{ (cyan)} \dots 10 \\
 d_4 &= |GL| \text{ (magenta)} \dots 1 \\
 d_5 &= |BL| \text{ (yellow)} \dots 1
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (24 occurrences)}$$

## 2.76.3 Construction MM139



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2$ ,  $D \in k_1 \cap k_2$
  3.  $k_3 = k(B, |CD|)$   
 $E \in k_1 \cap k_3$ ,  $F \in k_1 \cap k_3$
  4.  $k_4 = k(C, |DE|)$   
 $G \in k_2 \cap k_4$ ,  $H \in k_3 \cap k_4$
  5.  $k_5 = k(E, |DE|)$   
 $I \in k_2 \cap k_5$ ,  $J \in k_3 \cap k_5$ ,  $K \in k_3 \cap k_5$ ,  $L \in k_4 \cap k_5$ ,  $M \in k_4 \cap k_5$

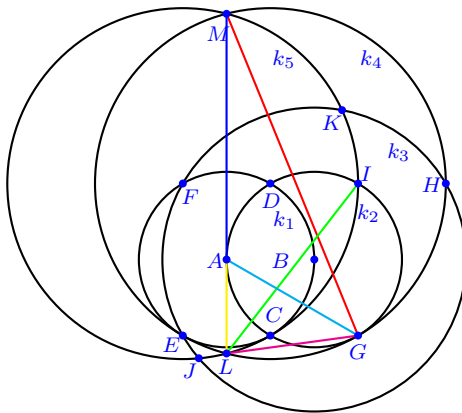
## Distances

$$\begin{aligned}
 d_0 &= |GL| \text{ (red)} \dots 1 \\
 d_1 &= |AL| \text{ (blue)} \dots 1 \\
 d_2 &= |IL| = |IM| \text{ (green)} \dots 2 \\
 d_3 &= |AG| = |AI| = |BE| = |BF| = |BH| = |BJ| = |BK| = |CD| = \\
 &= |EF| = |GI| \text{ (cyan)} \dots 10 \\
 d_4 &= |GM| \text{ (magenta)} \dots 1 \\
 d_5 &= |AM| \text{ (yellow)} \dots 1
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (24 occurrences)}$$

2.76.4 Construction MM142



Construction Process

- $A, B$  given initial points
- 1.  $k_1 = k(A, |AB|)$
- 2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
- 3.  $k_3 = k(B, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_1 \cap k_3$
- 4.  $k_4 = k(D, |DE|)$   
 $G \in k_2 \cap k_4, H \in k_3 \cap k_4$
- 5.  $k_5 = k(F, |DE|)$   
 $I \in k_2 \cap k_5, J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5, M \in k_4 \cap k_5$

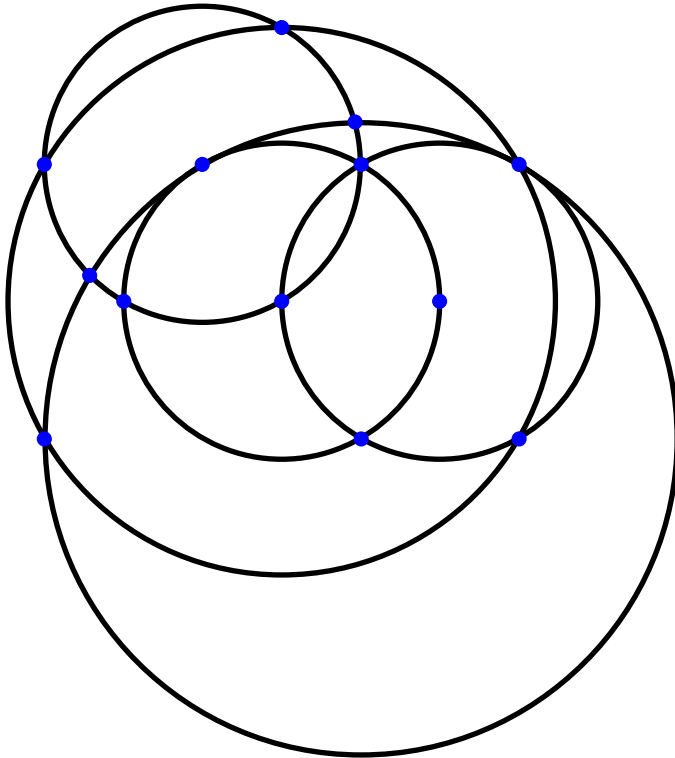
Distances

- $d_0 = |GM|$  (red) ... 1
- $d_1 = |AM|$  (blue) ... 1
- $d_2 = |IL| = |IM|$  (green) ... 2
- $d_3 = |AG| = |AI| = |BE| = |BF| = |BH| = |BJ| = |BK| = |CD| = |EF| = |GI|$  (cyan) ... 10
- $d_4 = |GL|$  (magenta) ... 1
- $d_5 = |AL|$  (yellow) ... 1

Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (24 occurrences)}$$

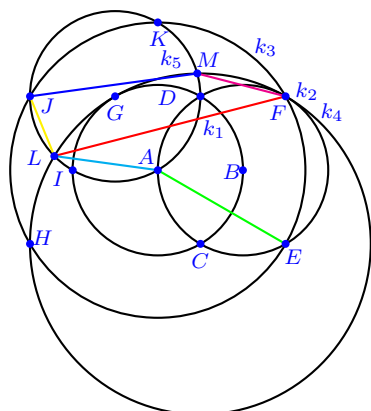
2.77 Class of Similar Constructions No. 68



**Contained Constructions**

MM110, MM113, MM140, MM143

## 2.77.1 Construction MM110



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(A, |CD|)$   
 $E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(C, |DE|)$   
 $G \in k_1 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(G, |AB|)$   
 $I \in k_1 \cap k_5, J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5, M \in k_4 \cap k_5$

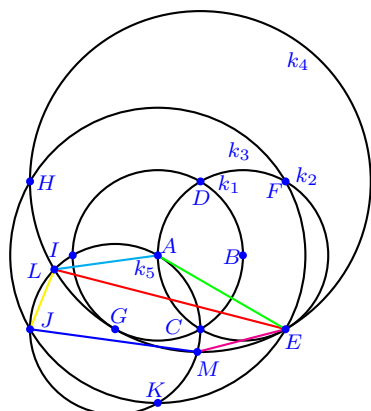
## Distances

$$\begin{aligned}
 d_0 &= |FL| = |HM| \text{ (red)} \dots 2 \\
 d_1 &= |JM| = |KL| \text{ (blue)} \dots 2 \\
 d_2 &= |AE| = |AF| = |AH| = |AJ| = |AK| = |BG| = |CD| = |CI| = \\
 &|DI| = |EF| = |FK| = |HJ| = |JK| \text{ (green)} \dots 13 \\
 d_3 &= |AL| = |AM| \text{ (cyan)} \dots 2 \\
 d_4 &= |FM| = |HL| \text{ (magenta)} \dots 2 \\
 d_5 &= |JL| = |KM| \text{ (yellow)} \dots 2
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (60 occurrences)}$$

## 2.77.2 Construction MM113



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(A, |CD|)$   
 $E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(D, |DE|)$   
 $G \in k_1 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(G, |AB|)$   
 $I \in k_1 \cap k_5, J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5, M \in k_4 \cap k_5$

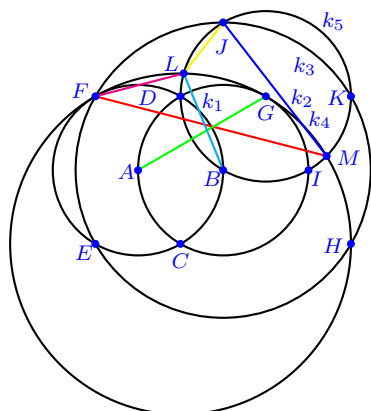
## Distances

$$\begin{aligned}
 d_0 &= |EL| = |HM| \text{ (red)} \dots 2 \\
 d_1 &= |JM| = |KL| \text{ (blue)} \dots 2 \\
 d_2 &= |AE| = |AF| = |AH| = |AJ| = |AK| = |BG| = |CD| = |CI| = \\
 &= |DI| = |EF| = |EK| = |HJ| = |JK| \text{ (green)} \dots 13 \\
 d_3 &= |AL| = |AM| \text{ (cyan)} \dots 2 \\
 d_4 &= |EM| = |HL| \text{ (magenta)} \dots 2 \\
 d_5 &= |JL| = |KM| \text{ (yellow)} \dots 2
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (60 occurrences)}$$

## 2.77.3 Construction MM140



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(B, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_1 \cap k_3$
  4.  $k_4 = k(C, |DE|)$   
 $G \in k_2 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(G, |AB|)$   
 $I \in k_2 \cap k_5, J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5, M \in k_4 \cap k_5$

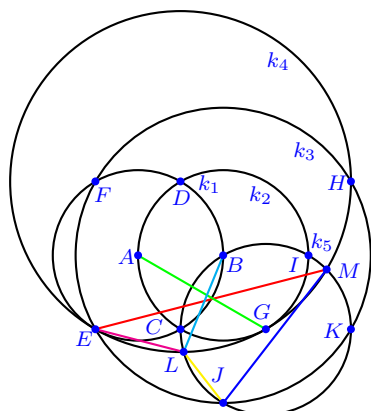
## Distances

$$\begin{aligned}
 d_0 &= |FM| = |HL| \text{ (red)} \dots 2 \\
 d_1 &= |JM| = |KL| \text{ (blue)} \dots 2 \\
 d_2 &= |AG| = |BE| = |BF| = |BH| = |BJ| = |BK| = |CD| = |CI| = \\
 &= |DI| = |EF| = |FJ| = |HK| = |JK| \text{ (green)} \dots 13 \\
 d_3 &= |BL| = |BM| \text{ (cyan)} \dots 2 \\
 d_4 &= |FL| = |HM| \text{ (magenta)} \dots 2 \\
 d_5 &= |JL| = |KM| \text{ (yellow)} \dots 2
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (60 occurrences)}$$

## 2.77.4 Construction MM143



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(B, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_1 \cap k_3$
  4.  $k_4 = k(D, |DE|)$   
 $G \in k_2 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(G, |AB|)$   
 $I \in k_2 \cap k_5, J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5, M \in k_4 \cap k_5$

## Distances

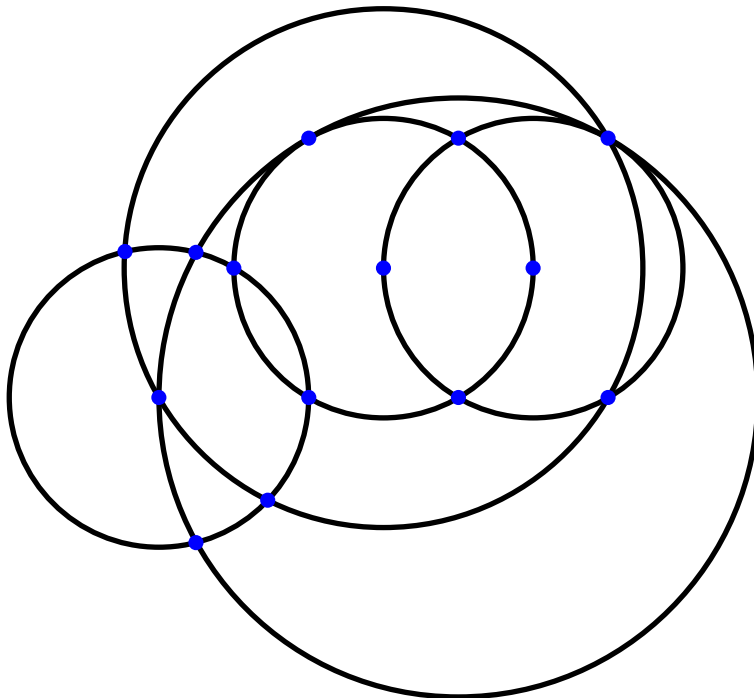
$$\begin{aligned}
 d_0 &= |EM| = |HL| \text{ (red)} \dots 2 \\
 d_1 &= |JM| = |KL| \text{ (blue)} \dots 2 \\
 d_2 &= |AG| = |BE| = |BF| = |BH| = |BJ| = |BK| = |CD| = |CI| = \\
 &= |DI| = |EF| = |EJ| = |HK| = |JK| \text{ (green)} \dots 13 \\
 d_3 &= |BL| = |BM| \text{ (cyan)} \dots 2 \\
 d_4 &= |EL| = |HM| \text{ (magenta)} \dots 2 \\
 d_5 &= |JL| = |KM| \text{ (yellow)} \dots 2
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (60 occurrences)}$$



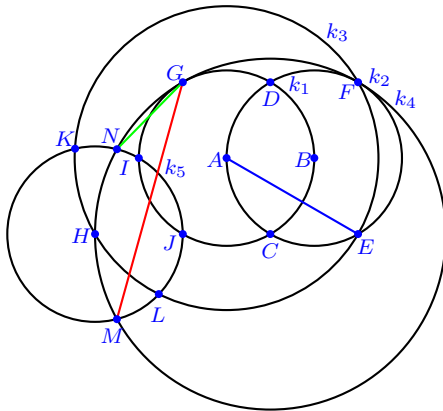
2.78 Class of Similar Constructions No. 69



**Contained Constructions**

MM111, MM114, MM141, MM144

2.78.1 Construction MM111



Construction Process

- $A, B$  given initial points
- 1.  $k_1 = k(A, |AB|)$
- 2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
- 3.  $k_3 = k(A, |CD|)$   
 $E \in k_2 \cap k_3, F \in k_2 \cap k_3$
- 4.  $k_4 = k(C, |DE|)$   
 $G \in k_1 \cap k_4, H \in k_3 \cap k_4$
- 5.  $k_5 = k(H, |AB|)$   
 $I \in k_1 \cap k_5, J \in k_1 \cap k_5, K \in k_3 \cap k_5, L \in k_3 \cap k_5, M \in k_4 \cap k_5,$   
 $N \in k_4 \cap k_5$

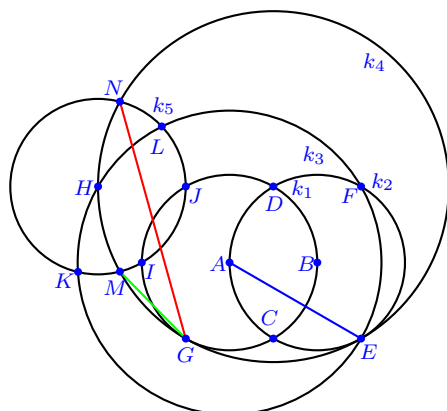
Distances

- $d_0 = |GM|$  (red) ... 1
- $d_1 = |AE| = |AF| = |AH| = |AK| = |AL| = |BG| = |BJ| = |CD| = |CI| = |DI| = |EF| = |GJ|$  (blue) ... 12
- $d_2 = |GN|$  (green) ... 1

Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (24 occurrences)}$$

## 2.78.2 Construction MM114



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(A, |CD|)$   
 $E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(D, |DE|)$   
 $G \in k_1 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(H, |AB|)$   
 $I \in k_1 \cap k_5, J \in k_1 \cap k_5, K \in k_3 \cap k_5, L \in k_3 \cap k_5, M \in k_4 \cap k_5,$   
 $N \in k_4 \cap k_5$

## Distances

$$d_0 = |GN| \text{ (red) } \dots 1$$

$$d_1 = |AE| = |AF| = |AH| = |AK| = |AL| = |BG| = |BJ| = |CD| =$$

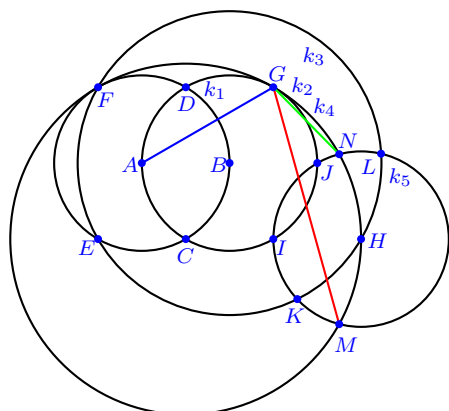
$$|CI| = |DI| = |EF| = |GJ| \text{ (blue) } \dots 12$$

$$d_2 = |GM| \text{ (green) } \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (24 occurrences)}$$

## 2.78.3 Construction MM141



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(B, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_1 \cap k_3$
  4.  $k_4 = k(C, |DE|)$   
 $G \in k_2 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(H, |AB|)$   
 $I \in k_2 \cap k_5, J \in k_2 \cap k_5, K \in k_3 \cap k_5, L \in k_3 \cap k_5, M \in k_4 \cap k_5,$   
 $N \in k_4 \cap k_5$

## Distances

$$d_0 = |GM| \text{ (red)} \dots 1$$

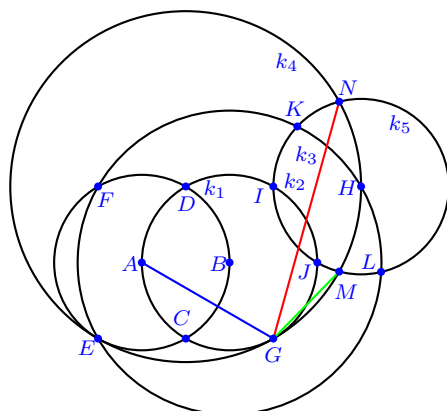
$$d_1 = |AG| = |AI| = |BE| = |BF| = |BH| = |BK| = |BL| = |CD| = |CJ| = |DJ| = |EF| = |GI| \text{ (blue)} \dots 12$$

$$d_2 = |GN| \text{ (green)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (24 occurrences)}$$

## 2.78.4 Construction MM144



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(B, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_1 \cap k_3$
  4.  $k_4 = k(D, |DE|)$   
 $G \in k_2 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(H, |AB|)$   
 $I \in k_2 \cap k_5, J \in k_2 \cap k_5, K \in k_3 \cap k_5, L \in k_3 \cap k_5, M \in k_4 \cap k_5,$   
 $N \in k_4 \cap k_5$

## Distances

$$d_0 = |GN| \text{ (red)} \dots 1$$

$$d_1 = |AG| = |AI| = |BE| = |BF| = |BH| = |BK| = |BL| = |CD| =$$

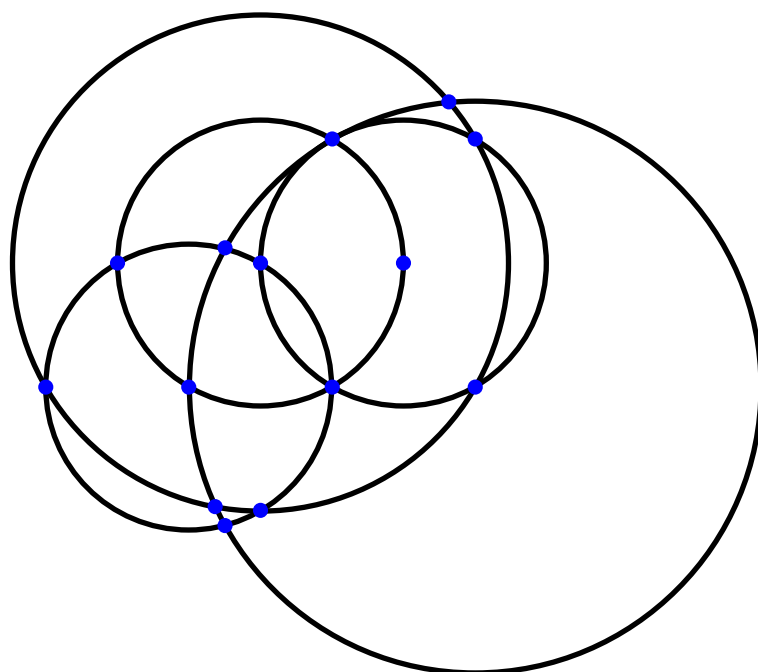
$$|CJ| = |DJ| = |EF| = |GI| \text{ (blue)} \dots 12$$

$$d_2 = |GM| \text{ (green)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (24 occurrences)}$$

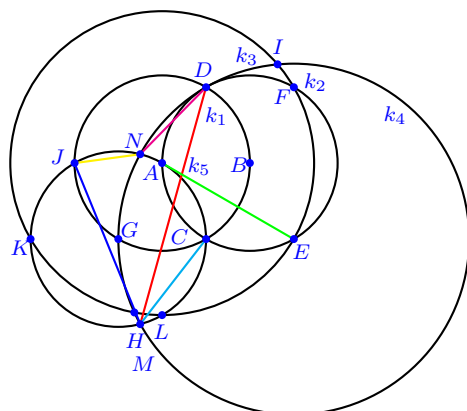
## 2.79 Class of Similar Constructions No. 70



### Contained Constructions

MM115, MM116, MM145, MM146

## 2.79.1 Construction MM115



## Construction Process

- $A, B$  given initial points
- $k_1 = k(A, |AB|)$
  - $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  - $k_3 = k(A, |CD|)$   
 $E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  - $k_4 = k(E, |DE|)$   
 $G \in k_1 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  - $k_5 = k(G, |AB|)$   
 $J \in k_1 \cap k_5, K \in k_3 \cap k_5, L \in k_3 \cap k_5, M \in k_4 \cap k_5, N \in k_4 \cap k_5$

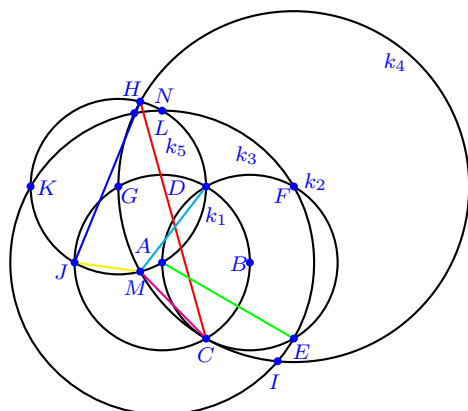
## Distances

$$\begin{aligned}
 d_0 &= |DM| \text{ (red)} \dots 1 \\
 d_1 &= |JM| \text{ (blue)} \dots 1 \\
 d_2 &= |AE| = |AF| = |AH| = |AI| = |AK| = |AL| = |BG| = |CD| = \\
 &|CJ| = |DJ| = |EF| = |EL| = |KL| \text{ (green)} \dots 13 \\
 d_3 &= |CM| = |CN| \text{ (cyan)} \dots 2 \\
 d_4 &= |DN| \text{ (magenta)} \dots 1 \\
 d_5 &= |JN| \text{ (yellow)} \dots 1
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (30 occurrences)}$$

## 2.79.2 Construction MM116



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(A, |CD|)$   
 $E \in k_2 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, |DE|)$   
 $G \in k_1 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  5.  $k_5 = k(G, |AB|)$   
 $J \in k_1 \cap k_5, K \in k_3 \cap k_5, L \in k_3 \cap k_5, M \in k_4 \cap k_5, N \in k_4 \cap k_5$

## Distances

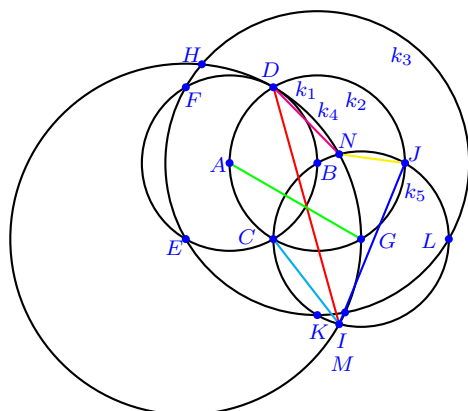
$$\begin{aligned}
 d_0 &= |CN| \text{ (red)} \dots 1 \\
 d_1 &= |JN| \text{ (blue)} \dots 1 \\
 d_2 &= |AE| = |AF| = |AH| = |AI| = |AK| = |AL| = |BG| = |CD| = \\
 &|CJ| = |DJ| = |EF| = |FL| = |KL| \text{ (green)} \dots 13 \\
 d_3 &= |DM| = |DN| \text{ (cyan)} \dots 2 \\
 d_4 &= |CM| \text{ (magenta)} \dots 1 \\
 d_5 &= |JM| \text{ (yellow)} \dots 1
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (30 occurrences)}$$



## 2.79.3 Construction MM145



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(B, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_1 \cap k_3$
  4.  $k_4 = k(E, |DE|)$   
 $G \in k_2 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  5.  $k_5 = k(G, |AB|)$   
 $J \in k_2 \cap k_5, K \in k_3 \cap k_5, L \in k_3 \cap k_5, M \in k_4 \cap k_5, N \in k_4 \cap k_5$

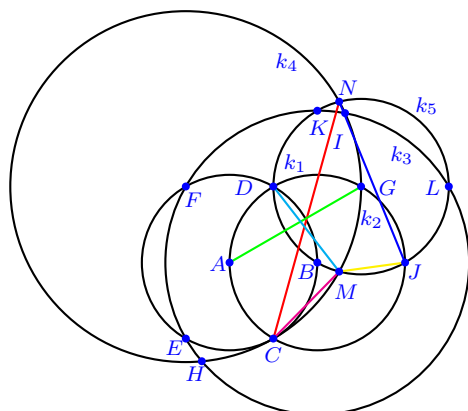
## Distances

$$\begin{aligned}
 d_0 &= |DM| \text{ (red)} \dots 1 \\
 d_1 &= |JM| \text{ (blue)} \dots 1 \\
 d_2 &= |AG| = |BE| = |BF| = |BH| = |BI| = |BK| = |BL| = |CD| = \\
 &= |CJ| = |DJ| = |EF| = |EK| = |KL| \text{ (green)} \dots 13 \\
 d_3 &= |CM| = |CN| \text{ (cyan)} \dots 2 \\
 d_4 &= |DN| \text{ (magenta)} \dots 1 \\
 d_5 &= |JN| \text{ (yellow)} \dots 1
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (30 occurrences)}$$

## 2.79.4 Construction MM146



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(B, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_1 \cap k_3$
  4.  $k_4 = k(F, |DE|)$   
 $G \in k_2 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  5.  $k_5 = k(G, |AB|)$   
 $J \in k_2 \cap k_5, K \in k_3 \cap k_5, L \in k_3 \cap k_5, M \in k_4 \cap k_5, N \in k_4 \cap k_5$

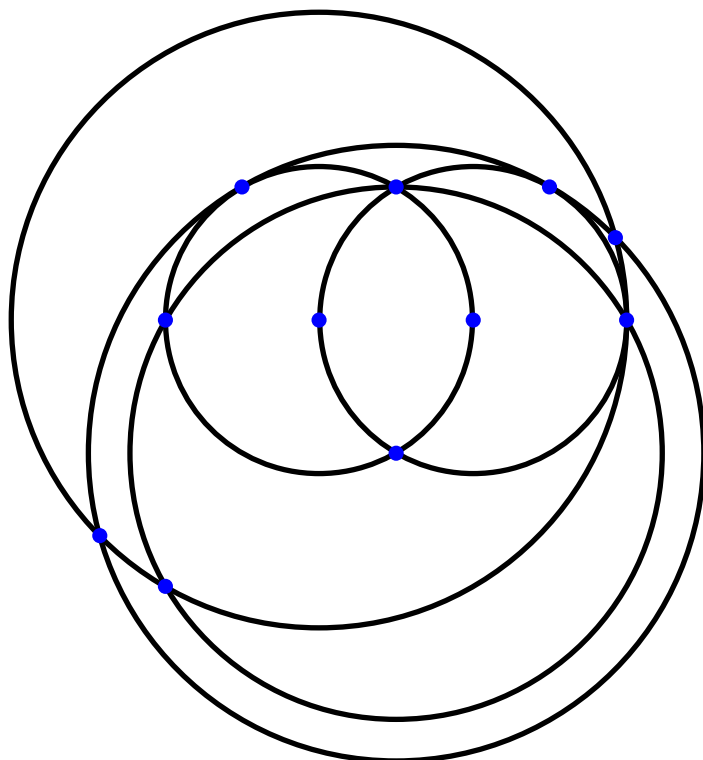
## Distances

$$\begin{aligned}
 d_0 &= |CN| \text{ (red)} \dots 1 \\
 d_1 &= |JN| \text{ (blue)} \dots 1 \\
 d_2 &= |AG| = |BE| = |BF| = |BH| = |BI| = |BK| = |BL| = |CD| = \\
 &= |CJ| = |DJ| = |EF| = |FK| = |KL| \text{ (green)} \dots 13 \\
 d_3 &= |DM| = |DN| \text{ (cyan)} \dots 2 \\
 d_4 &= |CM| \text{ (magenta)} \dots 1 \\
 d_5 &= |JM| \text{ (yellow)} \dots 1
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (30 occurrences)}$$

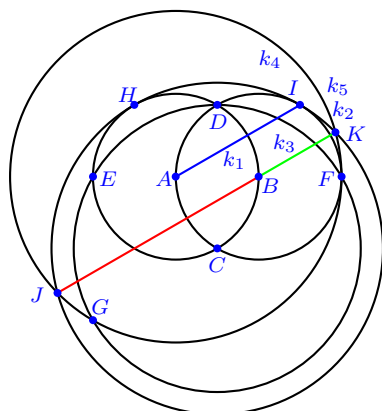
## 2.80 Class of Similar Constructions No. 71



### Contained Constructions

MM150, MM154, MM262, MM266

## 2.80.1 Construction MM150



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(A, |BE|)$   
 $G \in k_3 \cap k_4$
  5.  $k_5 = k(C, |BE|)$   
 $H \in k_1 \cap k_5, I \in k_2 \cap k_5, J \in k_4 \cap k_5, K \in k_4 \cap k_5$

## Distances

$$d_0 = |BJ| \text{ (red)} \dots 1$$

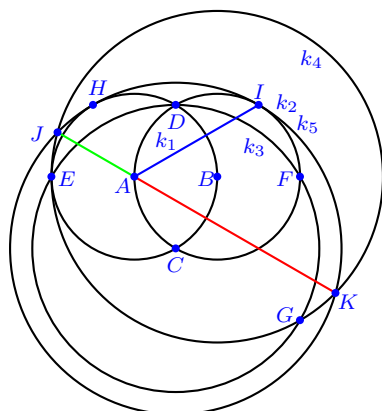
$$d_1 = |AI| = |BH| = |CD| = |CE| = |CF| = |CG| = |DE| = |DF| = |EG| \text{ (blue)} \dots 9$$

$$d_2 = |BK| \text{ (green)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (18 occurrences)}$$

## 2.80.2 Construction MM154



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(B, |BE|)$   
 $G \in k_3 \cap k_4$
  5.  $k_5 = k(C, |BE|)$   
 $H \in k_1 \cap k_5, I \in k_2 \cap k_5, J \in k_4 \cap k_5, K \in k_4 \cap k_5$

## Distances

$$d_0 = |AK| \text{ (red)} \dots 1$$

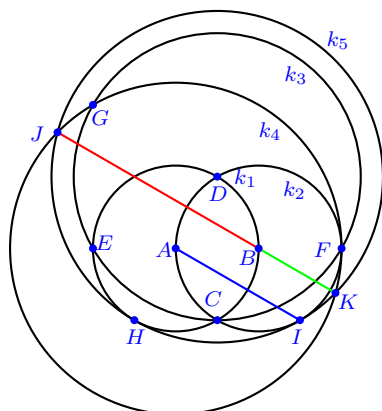
$$d_1 = |AI| = |BH| = |CD| = |CE| = |CF| = |CG| = |DE| = |DF| = |FG| \text{ (blue)} \dots 9$$

$$d_2 = |AJ| \text{ (green)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (18 occurrences)}$$

## 2.80.3 Construction MM262



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(A, |BE|)$   
 $G \in k_3 \cap k_4$
  5.  $k_5 = k(D, |BE|)$   
 $H \in k_1 \cap k_5, I \in k_2 \cap k_5, J \in k_4 \cap k_5, K \in k_4 \cap k_5$

## Distances

$$d_0 = |BJ| \text{ (red)} \dots 1$$

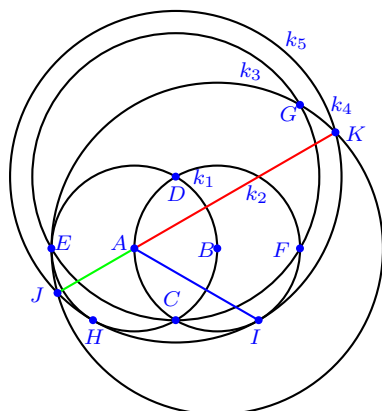
$$d_1 = |AI| = |BH| = |CD| = |CE| = |CF| = |DE| = |DF| = |DG| = |EG| \text{ (blue)} \dots 9$$

$$d_2 = |BK| \text{ (green)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (18 occurrences)}$$

## 2.80.4 Construction MM266



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(B, |BE|)$   
 $G \in k_3 \cap k_4$
  5.  $k_5 = k(D, |BE|)$   
 $H \in k_1 \cap k_5, I \in k_2 \cap k_5, J \in k_4 \cap k_5, K \in k_4 \cap k_5$

## Distances

$$d_0 = |AK| \text{ (red) } \dots 1$$

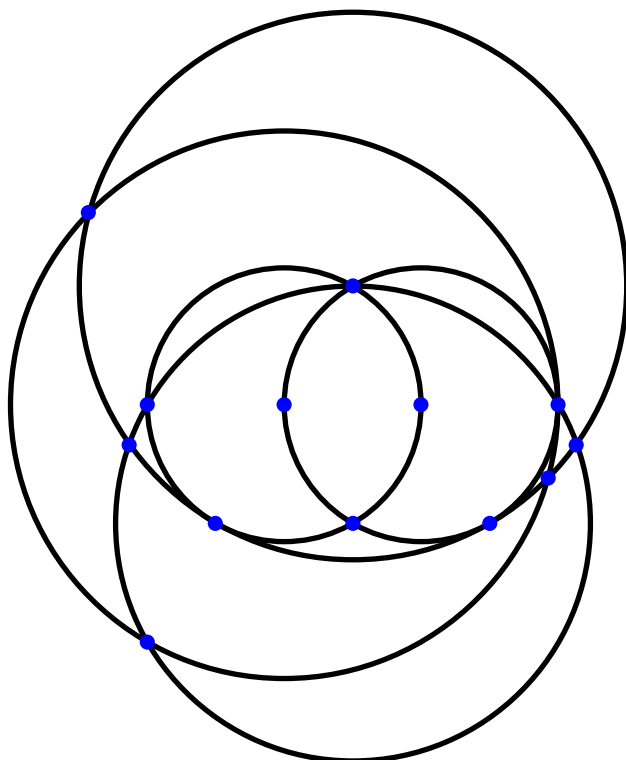
$$d_1 = |AI| = |BH| = |CD| = |CE| = |CF| = |DE| = |DF| = |DG| = |FG| \text{ (blue) } \dots 9$$

$$d_2 = |AJ| \text{ (green) } \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (18 occurrences)}$$

## 2.81 Class of Similar Constructions No. 72

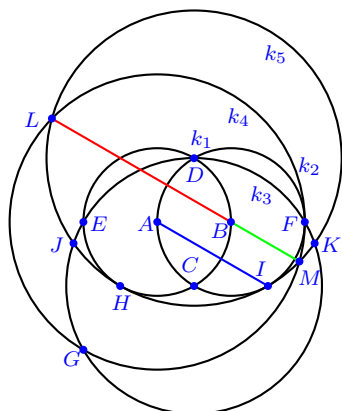


### Contained Constructions

MM151, MM155, MM261, MM265



## 2.81.1 Construction MM151



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(A, |BE|)$   
 $G \in k_3 \cap k_4$
  5.  $k_5 = k(D, |BE|)$   
 $H \in k_1 \cap k_5, I \in k_2 \cap k_5, J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5,$   
 $M \in k_4 \cap k_5$

## Distances

$$d_0 = |BL| \text{ (red) } \dots 1$$

$$d_1 = |AI| = |BH| = |CD| = |CE| = |CF| = |CG| = |CJ| = |CK| =$$

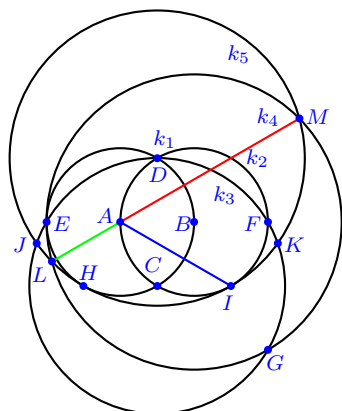
$$|DE| = |DF| = |EG| \text{ (blue) } \dots 11$$

$$d_2 = |BM| \text{ (green) } \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (22 occurrences)}$$

## 2.81.2 Construction MM155



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(B, |BE|)$   
 $G \in k_3 \cap k_4$
  5.  $k_5 = k(D, |BE|)$   
 $H \in k_1 \cap k_5, I \in k_2 \cap k_5, J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5,$   
 $M \in k_4 \cap k_5$

## Distances

$$d_0 = |AM| \text{ (red)} \dots 1$$

$$d_1 = |AI| = |BH| = |CD| = |CE| = |CF| = |CG| = |CJ| = |CK| =$$

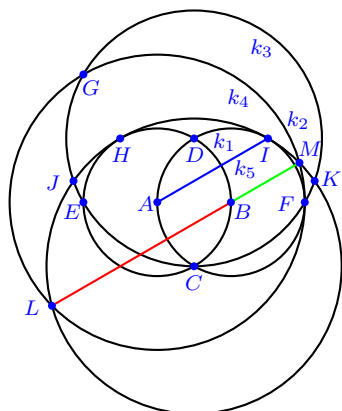
$$|DE| = |DF| = |FG| \text{ (blue)} \dots 11$$

$$d_2 = |AL| \text{ (green)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (22 occurrences)}$$

## 2.81.3 Construction MM261



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(A, |BE|)$   
 $G \in k_3 \cap k_4$
  5.  $k_5 = k(C, |BE|)$   
 $H \in k_1 \cap k_5, I \in k_2 \cap k_5, J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5,$   
 $M \in k_4 \cap k_5$

## Distances

$$d_0 = |BL| \text{ (red)} \dots 1$$

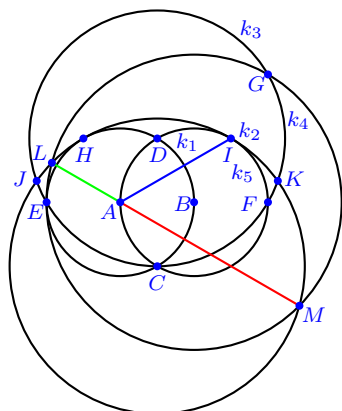
$$d_1 = |AI| = |BH| = |CD| = |CE| = |CF| = |DE| = |DF| = |DG| = |DJ| = |DK| = |EG| \text{ (blue)} \dots 11$$

$$d_2 = |BM| \text{ (green)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (22 occurrences)}$$

## 2.81.4 Construction MM265



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(B, |BE|)$   
 $G \in k_3 \cap k_4$
  5.  $k_5 = k(C, |BE|)$   
 $H \in k_1 \cap k_5, I \in k_2 \cap k_5, J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5,$   
 $M \in k_4 \cap k_5$

## Distances

$$d_0 = |AM| \text{ (red)} \dots 1$$

$$d_1 = |AI| = |BH| = |CD| = |CE| = |CF| = |DE| = |DF| = |DG| =$$

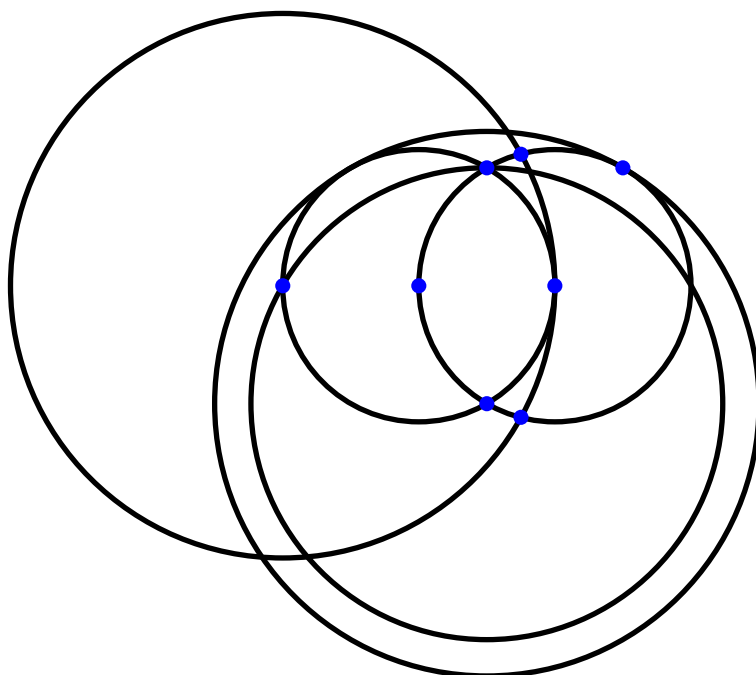
$$|DJ| = |DK| = |FG| \text{ (blue)} \dots 11$$

$$d_2 = |AL| \text{ (green)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (22 occurrences)}$$

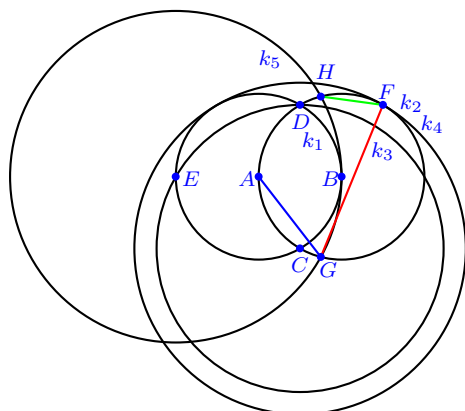
## 2.82 Class of Similar Constructions No. 73



### Contained Constructions

MM158, MM159, MM273, MM274

## 2.82.1 Construction MM158



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |CD|)$   
 $E \in k_1 \cap k_3$
  4.  $k_4 = k(C, |BE|)$   
 $F \in k_2 \cap k_4$
  5.  $k_5 = k(E, |BE|)$   
 $G \in k_2 \cap k_5, H \in k_2 \cap k_5$

## Distances

$$d_0 = |FG| \text{ (red) } \dots 1$$

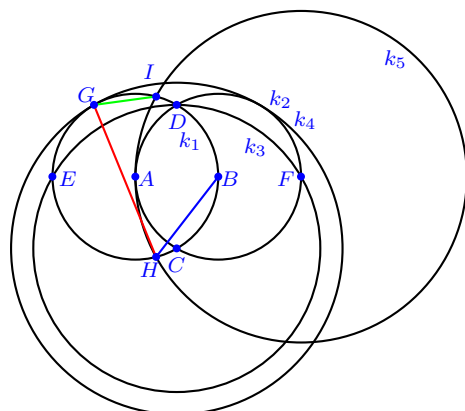
$$d_1 = |AG| = |AH| \text{ (blue) } \dots 2$$

$$d_2 = |FH| \text{ (green) } \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (4 occurrences)}$$

## 2.82.2 Construction MM159



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(C, |BE|)$   
 $G \in k_1 \cap k_4$
  5.  $k_5 = k(F, |BE|)$   
 $H \in k_1 \cap k_5, I \in k_1 \cap k_5$

## Distances

$$d_0 = |GH| \text{ (red) } \dots 1$$

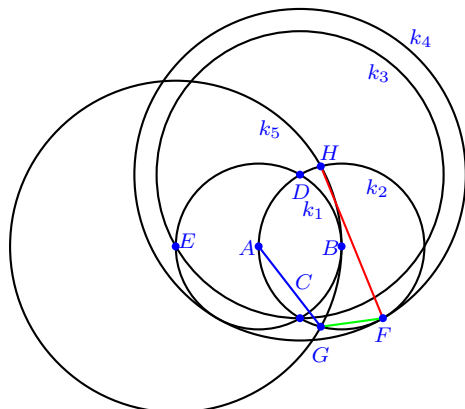
$$d_1 = |BH| = |BI| \text{ (blue) } \dots 2$$

$$d_2 = |GI| \text{ (green) } \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (4 occurrences)}$$

## 2.82.3 Construction MM273



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |CD|)$   
 $E \in k_1 \cap k_3$
  4.  $k_4 = k(D, |BE|)$   
 $F \in k_2 \cap k_4$
  5.  $k_5 = k(E, |BE|)$   
 $G \in k_2 \cap k_5, H \in k_2 \cap k_5$

## Distances

$$d_0 = |FH| \text{ (red)} \dots 1$$

$$d_1 = |AG| = |AH| \text{ (blue)} \dots 2$$

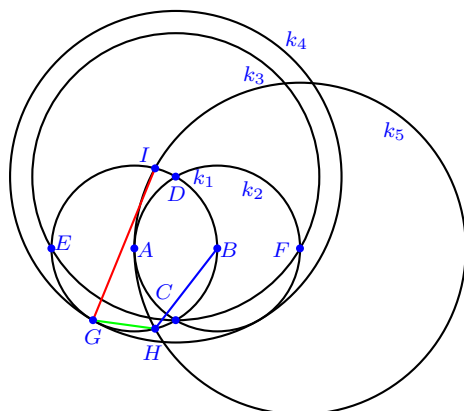
$$d_2 = |FG| \text{ (green)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (4 occurrences)}$$



## 2.82.4 Construction MM274



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(D, |BE|)$   
 $G \in k_1 \cap k_4$
  5.  $k_5 = k(F, |BE|)$   
 $H \in k_1 \cap k_5, I \in k_1 \cap k_5$

## Distances

$$d_0 = |GI| \text{ (red)} \dots 1$$

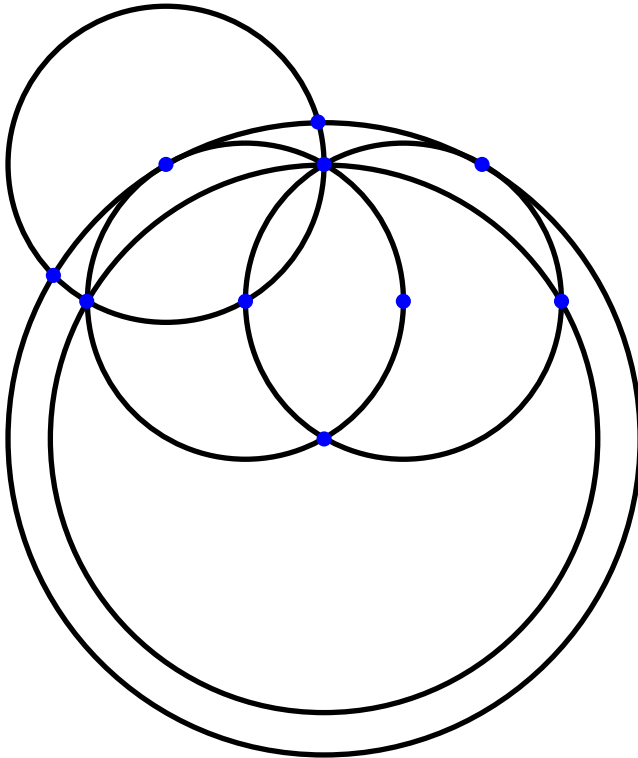
$$d_1 = |BH| = |BI| \text{ (blue)} \dots 2$$

$$d_2 = |GH| \text{ (green)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (4 occurrences)}$$

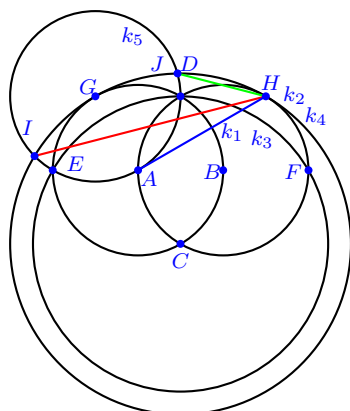
2.83 Class of Similar Constructions No. 74



**Contained Constructions**

MM160, MM161, MM275, MM276

## 2.83.1 Construction MM160



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(C, |BE|)$   
 $G \in k_1 \cap k_4, H \in k_2 \cap k_4$
  5.  $k_5 = k(G, |AB|)$   
 $I \in k_4 \cap k_5, J \in k_4 \cap k_5$

## Distances

$$d_0 = |HI| \text{ (red) } \dots 1$$

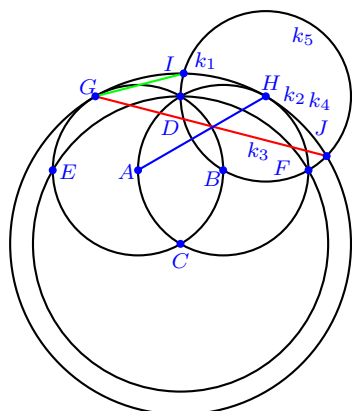
$$d_1 = |AH| = |BG| = |CD| = |CE| = |CF| = |DE| = |DF| \text{ (blue) } \dots 7$$

$$d_2 = |HJ| \text{ (green) } \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (14 occurrences)}$$

## 2.83.2 Construction MM161



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(C, |BE|)$   
 $G \in k_1 \cap k_4, H \in k_2 \cap k_4$
  5.  $k_5 = k(H, |AB|)$   
 $I \in k_4 \cap k_5, J \in k_4 \cap k_5$

## Distances

$$d_0 = |GJ| \text{ (red)} \dots 1$$

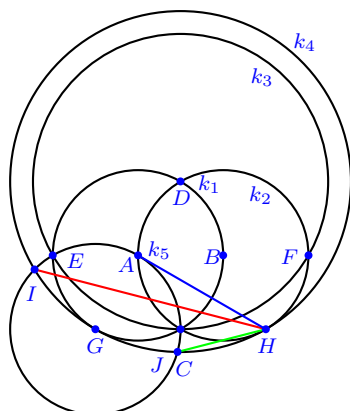
$$d_1 = |AH| = |BG| = |CD| = |CE| = |CF| = |DE| = |DF| \text{ (blue)} \dots 7$$

$$d_2 = |GI| \text{ (green)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (14 occurrences)}$$

## 2.83.3 Construction MM275



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(D, |BE|)$   
 $G \in k_1 \cap k_4, H \in k_2 \cap k_4$
  5.  $k_5 = k(G, |AB|)$   
 $I \in k_4 \cap k_5, J \in k_4 \cap k_5$

## Distances

$$d_0 = |HI| \text{ (red)} \dots 1$$

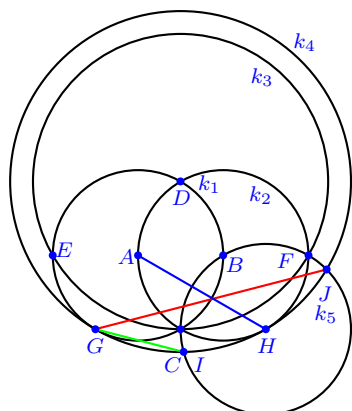
$$d_1 = |AH| = |BG| = |CD| = |CE| = |CF| = |DE| = |DF| \text{ (blue)} \dots 7$$

$$d_2 = |HJ| \text{ (green)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (14 occurrences)}$$

## 2.83.4 Construction MM276



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(D, |BE|)$   
 $G \in k_1 \cap k_4, H \in k_2 \cap k_4$
  5.  $k_5 = k(H, |AB|)$   
 $I \in k_4 \cap k_5, J \in k_4 \cap k_5$

## Distances

$$d_0 = |GJ| \text{ (red) } \dots 1$$

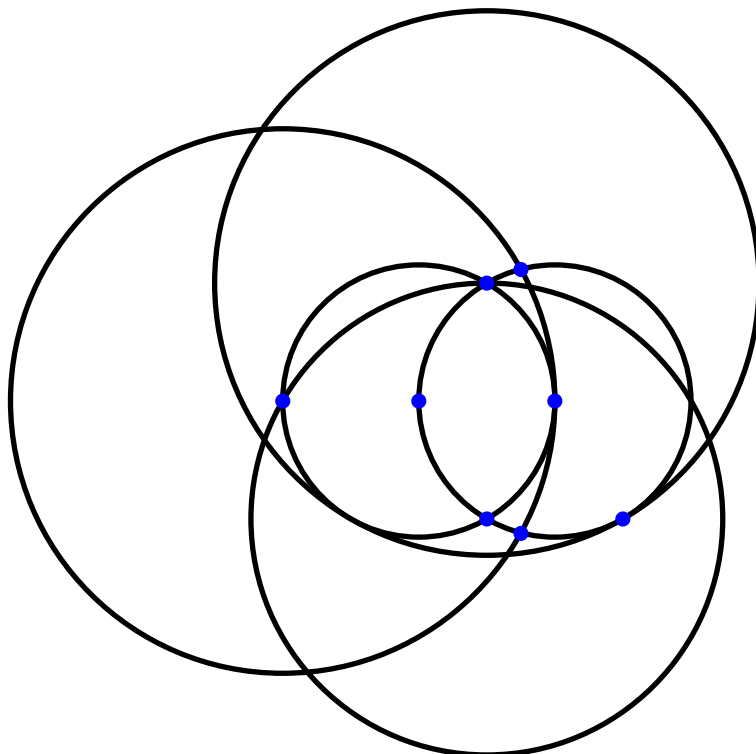
$$d_1 = |AH| = |BG| = |CD| = |CE| = |CF| = |DE| = |DF| \text{ (blue) } \dots 7$$

$$d_2 = |GI| \text{ (green) } \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (14 occurrences)}$$

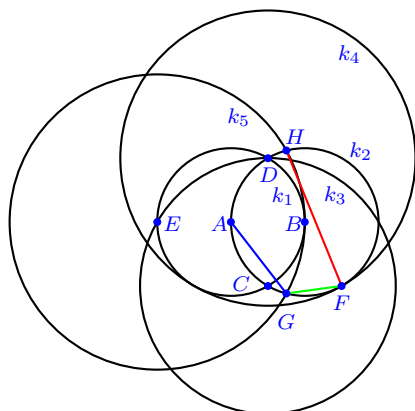
## 2.84 Class of Similar Constructions No. 75



### Contained Constructions

MM162, MM163, MM269, MM270

## 2.84.1 Construction MM162



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |CD|)$   
 $E \in k_1 \cap k_3$
  4.  $k_4 = k(D, |BE|)$   
 $F \in k_2 \cap k_4$
  5.  $k_5 = k(E, |BE|)$   
 $G \in k_2 \cap k_5, H \in k_2 \cap k_5$

## Distances

$$d_0 = |FH| \text{ (red)} \dots 1$$

$$d_1 = |AG| = |AH| \text{ (blue)} \dots 2$$

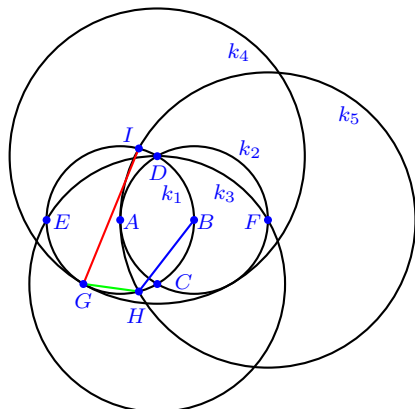
$$d_2 = |FG| \text{ (green)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (4 occurrences)}$$



## 2.84.2 Construction MM163



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(D, |BE|)$   
 $G \in k_1 \cap k_4$
  5.  $k_5 = k(F, |BE|)$   
 $H \in k_1 \cap k_5, I \in k_1 \cap k_5$

## Distances

$$d_0 = |GI| \text{ (red)} \dots 1$$

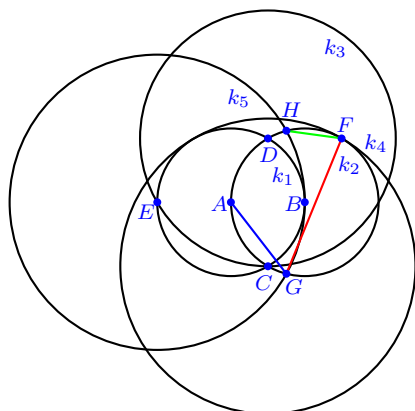
$$d_1 = |BH| = |BI| \text{ (blue)} \dots 2$$

$$d_2 = |GH| \text{ (green)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (4 occurrences)}$$

## 2.84.3 Construction MM269



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |CD|)$   
 $E \in k_1 \cap k_3$
  4.  $k_4 = k(C, |BE|)$   
 $F \in k_2 \cap k_4$
  5.  $k_5 = k(E, |BE|)$   
 $G \in k_2 \cap k_5, H \in k_2 \cap k_5$

## Distances

$$d_0 = |FG| \text{ (red)} \dots 1$$

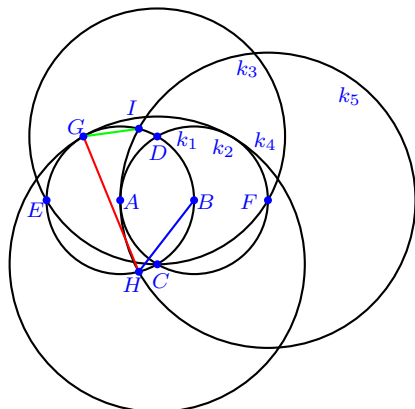
$$d_1 = |AG| = |AH| \text{ (blue)} \dots 2$$

$$d_2 = |FH| \text{ (green)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (4 occurrences)}$$

## 2.84.4 Construction MM270



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(C, |BE|)$   
 $G \in k_1 \cap k_4$
  5.  $k_5 = k(F, |BE|)$   
 $H \in k_1 \cap k_5, I \in k_1 \cap k_5$

## Distances

$$d_0 = |GH| \text{ (red) } \dots 1$$

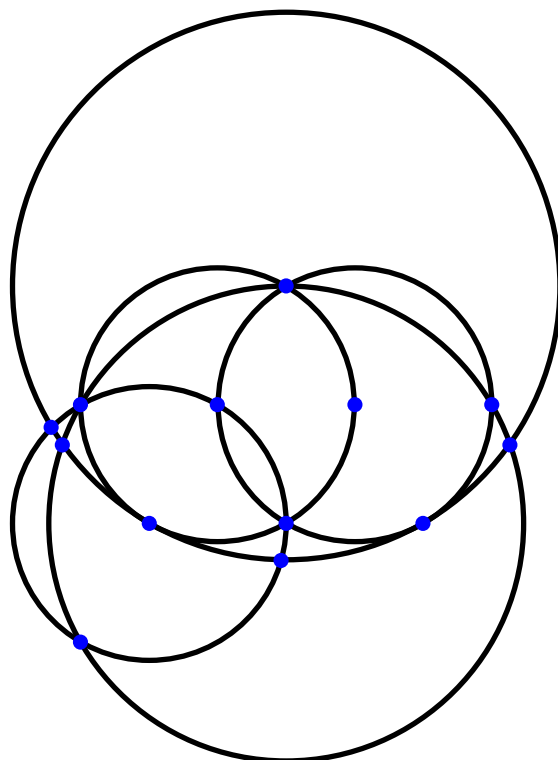
$$d_1 = |BH| = |BI| \text{ (blue) } \dots 2$$

$$d_2 = |GI| \text{ (green) } \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (4 occurrences)}$$

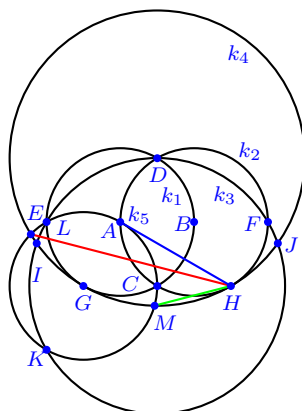
## 2.85 Class of Similar Constructions No. 76



### Contained Constructions

MM164, MM165, MM271, MM272

## 2.85.1 Construction MM164



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(D, |BE|)$   
 $G \in k_1 \cap k_4, H \in k_2 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
  5.  $k_5 = k(G, |AB|)$   
 $K \in k_3 \cap k_5, L \in k_4 \cap k_5, M \in k_4 \cap k_5$

## Distances

$$d_0 = |HL| \text{ (red) } \dots 1$$

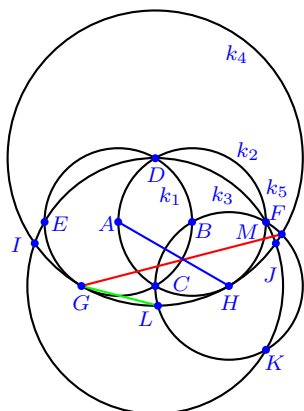
$$d_1 = |AH| = |BG| = |CD| = |CE| = |CF| = |CI| = |CJ| = |CK| = |DE| = |DF| = |EK| \text{ (blue) } \dots 11$$

$$d_2 = |HM| \text{ (green) } \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (22 occurrences)}$$

## 2.85.2 Construction MM165



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(D, |BE|)$   
 $G \in k_1 \cap k_4, H \in k_2 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
  5.  $k_5 = k(H, |AB|)$   
 $K \in k_3 \cap k_5, L \in k_4 \cap k_5, M \in k_4 \cap k_5$

## Distances

$$d_0 = |GM| \text{ (red)} \dots 1$$

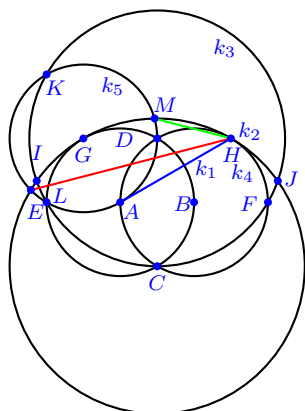
$$d_1 = |AH| = |BG| = |CD| = |CE| = |CF| = |CI| = |CJ| = |CK| = |DE| = |DF| = |FK| \text{ (blue)} \dots 11$$

$$d_2 = |GL| \text{ (green)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (22 occurrences)}$$

## 2.85.3 Construction MM271



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(C, |BE|)$   
 $G \in k_1 \cap k_4, H \in k_2 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
  5.  $k_5 = k(G, |AB|)$   
 $K \in k_3 \cap k_5, L \in k_4 \cap k_5, M \in k_4 \cap k_5$

## Distances

$$d_0 = |HL| \text{ (red)} \dots 1$$

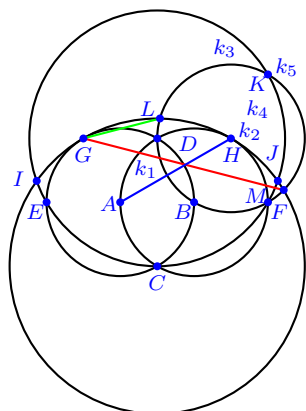
$$d_1 = |AH| = |BG| = |CD| = |CE| = |CF| = |DE| = |DF| = |DI| = |DJ| = |DK| = |EK| \text{ (blue)} \dots 11$$

$$d_2 = |HM| \text{ (green)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (22 occurrences)}$$

## 2.85.4 Construction MM272



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(C, |BE|)$   
 $G \in k_1 \cap k_4, H \in k_2 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
  5.  $k_5 = k(H, |AB|)$   
 $K \in k_3 \cap k_5, L \in k_4 \cap k_5, M \in k_4 \cap k_5$

## Distances

$$d_0 = |GM| \text{ (red)} \dots 1$$

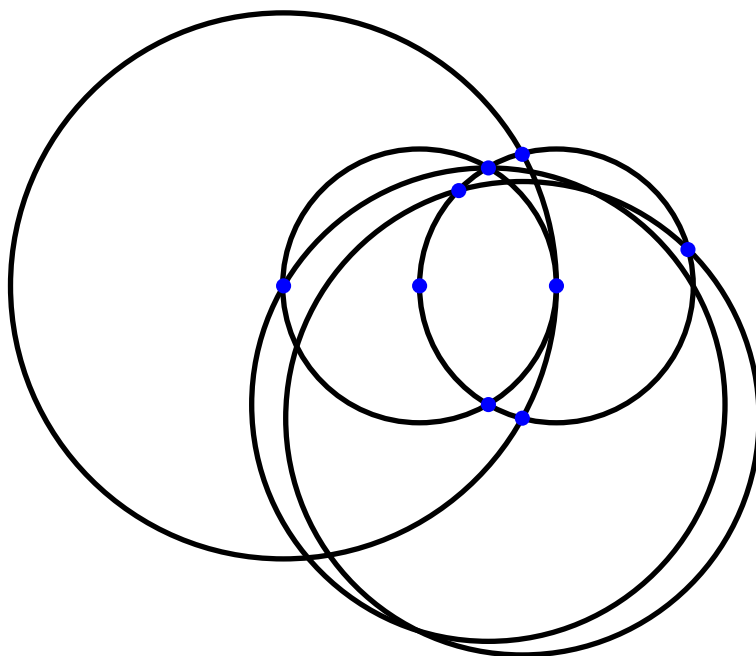
$$d_1 = |AH| = |BG| = |CD| = |CE| = |CF| = |DE| = |DF| = |DI| = |DJ| = |DK| = |FK| \text{ (blue)} \dots 11$$

$$d_2 = |GL| \text{ (green)} \dots 1$$

## Occurrences of the Golden Ratio

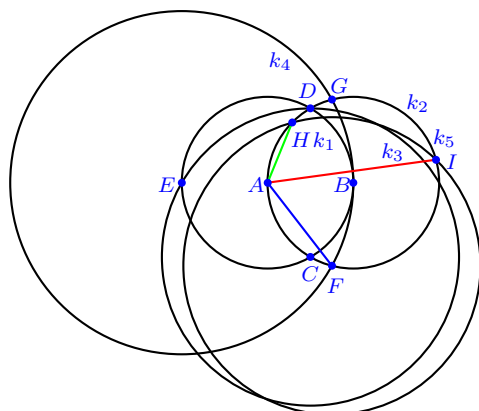
$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (22 occurrences)}$$



**2.86 Class of Similar Constructions No. 77****Contained Constructions**

MM167, MM213, MM281, MM327

## 2.86.1 Construction MM167



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |CD|)$   
 $E \in k_1 \cap k_3$
  4.  $k_4 = k(E, |BE|)$   
 $F \in k_2 \cap k_4, G \in k_2 \cap k_4$
  5.  $k_5 = k(F, |CD|)$   
 $H \in k_2 \cap k_5, I \in k_2 \cap k_5$

## Distances

$$d_0 = |AI| \text{ (red)} \dots 1$$

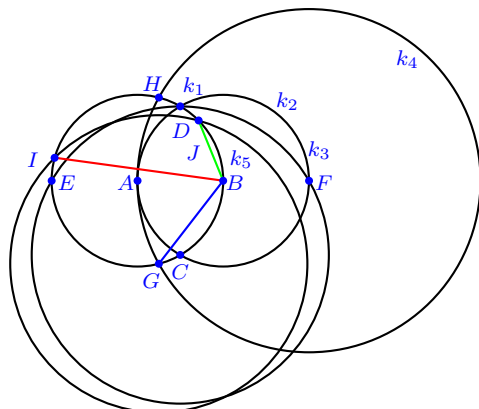
$$d_1 = |AF| = |AG| \text{ (blue)} \dots 2$$

$$d_2 = |AH| \text{ (green)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (4 occurrences)}$$

## 2.86.2 Construction MM213



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, |BE|)$   
 $G \in k_1 \cap k_4, H \in k_1 \cap k_4$
  5.  $k_5 = k(G, |CD|)$   
 $I \in k_1 \cap k_5, J \in k_1 \cap k_5$

## Distances

$$d_0 = |BI| \text{ (red) } \dots 1$$

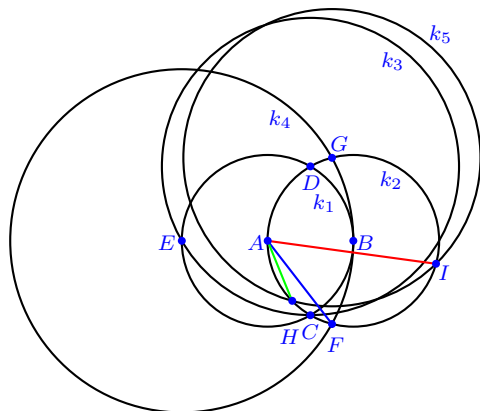
$$d_1 = |BG| = |BH| \text{ (blue) } \dots 2$$

$$d_2 = |BJ| \text{ (green) } \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (4 occurrences)}$$

## 2.86.3 Construction MM281



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |CD|)$   
 $E \in k_1 \cap k_3$
  4.  $k_4 = k(E, |BE|)$   
 $F \in k_2 \cap k_4, G \in k_2 \cap k_4$
  5.  $k_5 = k(G, |CD|)$   
 $H \in k_2 \cap k_5, I \in k_2 \cap k_5$

## Distances

$$d_0 = |AI| \text{ (red)} \dots 1$$

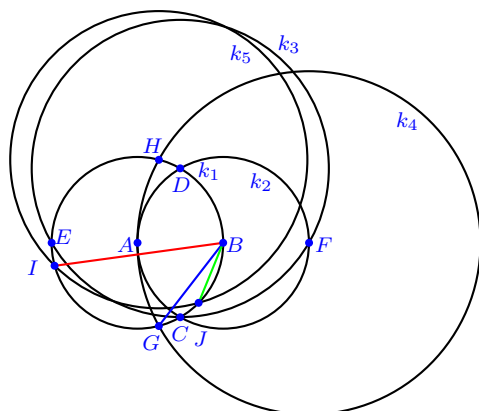
$$d_1 = |AF| = |AG| \text{ (blue)} \dots 2$$

$$d_2 = |AH| \text{ (green)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (4 occurrences)}$$

## 2.86.4 Construction MM327



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, |BE|)$   
 $G \in k_1 \cap k_4, H \in k_1 \cap k_4$
  5.  $k_5 = k(H, |CD|)$   
 $I \in k_1 \cap k_5, J \in k_1 \cap k_5$

## Distances

$$d_0 = |BI| \text{ (red)} \dots 1$$

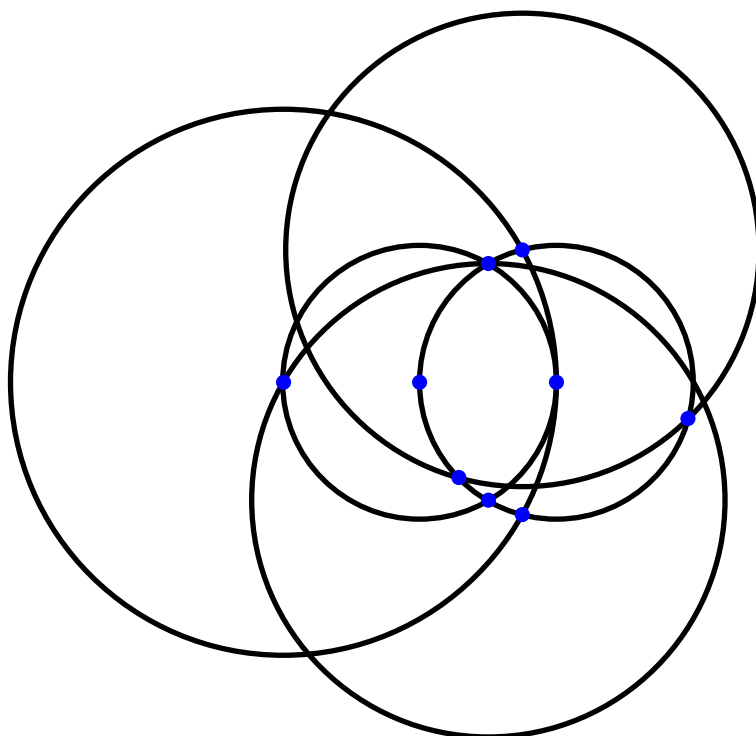
$$d_1 = |BG| = |BH| \text{ (blue)} \dots 2$$

$$d_2 = |BJ| \text{ (green)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (4 occurrences)}$$

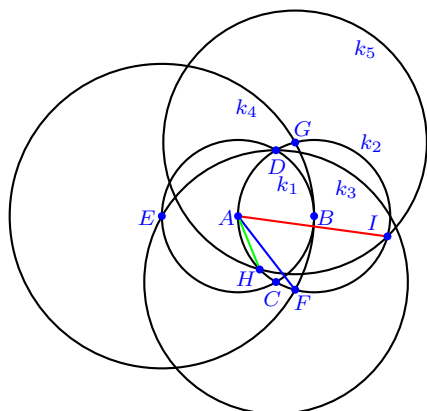
## 2.87 Class of Similar Constructions No. 78



### Contained Constructions

MM170, MM216, MM278, MM324

## 2.87.1 Construction MM170



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |CD|)$   
 $E \in k_1 \cap k_3$
  4.  $k_4 = k(E, |BE|)$   
 $F \in k_2 \cap k_4, G \in k_2 \cap k_4$
  5.  $k_5 = k(G, |CD|)$   
 $H \in k_2 \cap k_5, I \in k_2 \cap k_5$

## Distances

$$d_0 = |AI| \text{ (red)} \dots 1$$

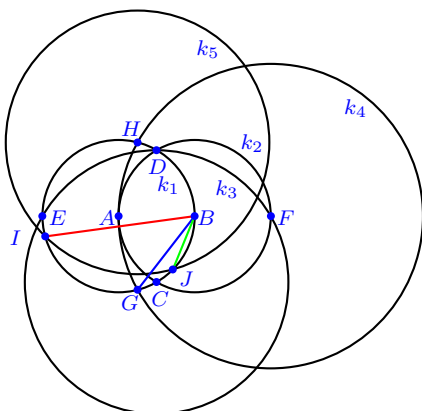
$$d_1 = |AF| = |AG| \text{ (blue)} \dots 2$$

$$d_2 = |AH| \text{ (green)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (4 occurrences)}$$

## 2.87.2 Construction MM216



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, |BE|)$   
 $G \in k_1 \cap k_4, H \in k_1 \cap k_4$
  5.  $k_5 = k(H, |CD|)$   
 $I \in k_1 \cap k_5, J \in k_1 \cap k_5$

## Distances

$$d_0 = |BI| \text{ (red) } \dots 1$$

$$d_1 = |BG| = |BH| \text{ (blue) } \dots 2$$

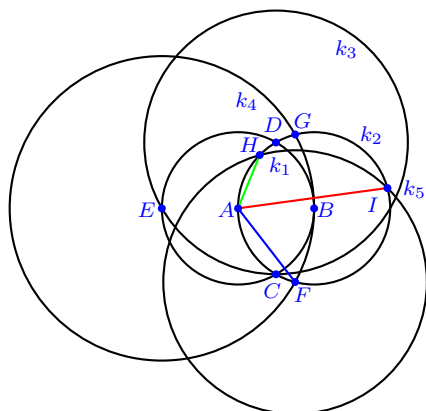
$$d_2 = |BJ| \text{ (green) } \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (4 occurrences)}$$



## 2.87.3 Construction MM278



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2$ ,  $D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |CD|)$   
 $E \in k_1 \cap k_3$
  4.  $k_4 = k(E, |BE|)$   
 $F \in k_2 \cap k_4$ ,  $G \in k_2 \cap k_4$
  5.  $k_5 = k(F, |CD|)$   
 $H \in k_2 \cap k_5$ ,  $I \in k_2 \cap k_5$

## Distances

$$d_0 = |AI| \text{ (red)} \dots 1$$

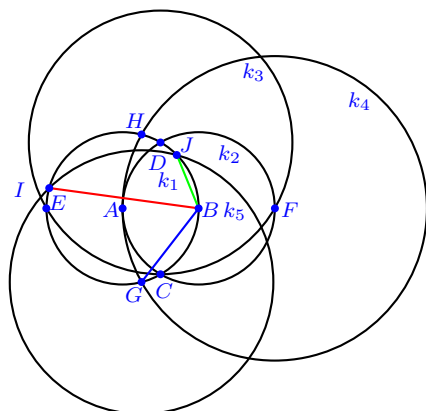
$$d_1 = |AF| = |AG| \text{ (blue)} \dots 2$$

$$d_2 = |AH| \text{ (green)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (4 occurrences)}$$

## 2.87.4 Construction MM324



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, |BE|)$   
 $G \in k_1 \cap k_4, H \in k_1 \cap k_4$
  5.  $k_5 = k(G, |CD|)$   
 $I \in k_1 \cap k_5, J \in k_1 \cap k_5$

## Distances

$$d_0 = |BI| \text{ (red)} \dots 1$$

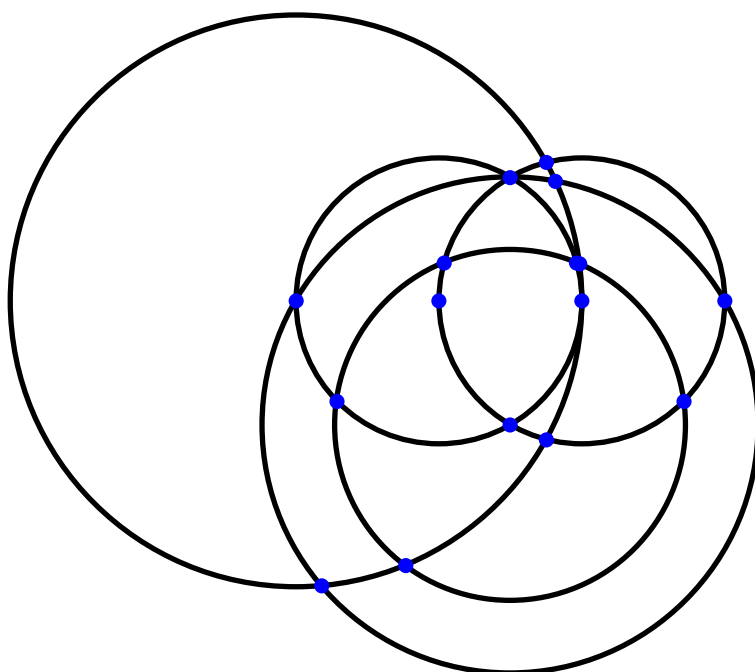
$$d_1 = |BG| = |BH| \text{ (blue)} \dots 2$$

$$d_2 = |BJ| \text{ (green)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (4 occurrences)}$$

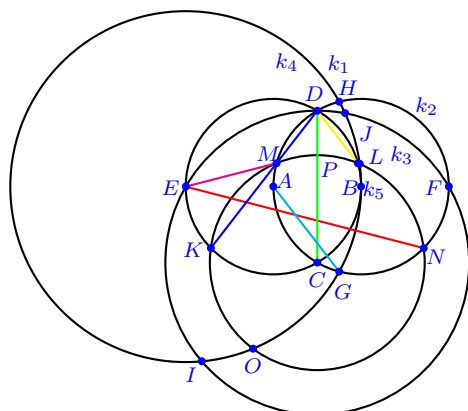
## 2.88 Class of Similar Constructions No. 79



### Contained Constructions

MM172, MM218, MM284, MM330

## 2.88.1 Construction MM172



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, |BE|)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
  5.  $k_5 = k(C, |AG|)$   
 $K \in k_1 \cap k_5, L \in k_1 \cap k_5, M \in k_2 \cap k_5, N \in k_2 \cap k_5, O \in k_4 \cap k_5,$   
 $P \in k_4 \cap k_5$

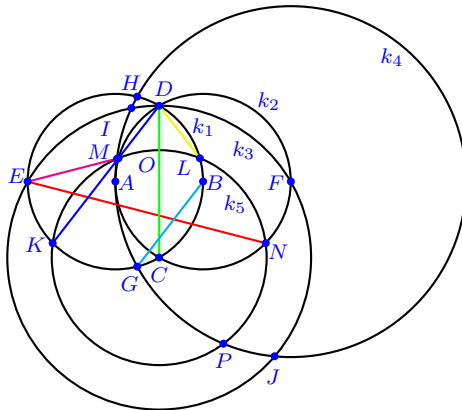
## Distances

$$\begin{aligned}
 d_0 &= |EN| = |FK| \text{ (red)} \dots 2 \\
 d_1 &= |DK| = |DN| = |EL| = |FM| \text{ (blue)} \dots 4 \\
 d_2 &= |CD| = |CE| = |CF| = |CI| = |CJ| = |DE| = |DF| \text{ (green)} \dots 7 \\
 d_3 &= |AG| = |AH| = |CK| = |CL| = |CM| = |CN| = |CO| = |CP| = |KM| = |LN| \text{ (cyan)} \dots 10 \\
 d_4 &= |EM| = |FL| \text{ (magenta)} \dots 2 \\
 d_5 &= |DL| = |DM| = |EK| = |FN| \text{ (yellow)} \dots 4
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (108 occurrences)}$$

2.88.2 Construction MM218



Construction Process

- $A, B$  given initial points
- 1.  $k_1 = k(A, |AB|)$
- 2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
- 3.  $k_3 = k(C, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
- 4.  $k_4 = k(F, |BE|)$   
 $G \in k_1 \cap k_4, H \in k_1 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
- 5.  $k_5 = k(C, |BG|)$   
 $K \in k_1 \cap k_5, L \in k_1 \cap k_5, M \in k_2 \cap k_5, N \in k_2 \cap k_5, O \in k_4 \cap k_5,$   
 $P \in k_4 \cap k_5$

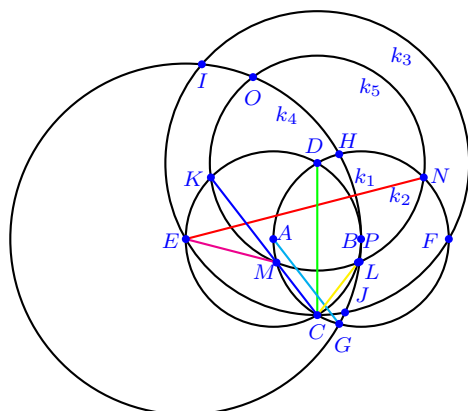
Distances

- $d_0 = |EN| = |FK|$  (red) ... 2
- $d_1 = |DK| = |DN| = |EL| = |FM|$  (blue) ... 4
- $d_2 = |CD| = |CE| = |CF| = |CI| = |CJ| = |DE| = |DF|$  (green) ... 7
- $d_3 = |BG| = |BH| = |CK| = |CL| = |CM| = |CN| = |CO| = |CP| = |KM| = |LN|$  (cyan) ... 10
- $d_4 = |EM| = |FL|$  (magenta) ... 2
- $d_5 = |DL| = |DM| = |EK| = |FN|$  (yellow) ... 4

Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (108 occurrences)}$$

## 2.88.3 Construction MM284



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, |BE|)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
  5.  $k_5 = k(D, |AG|)$   
 $K \in k_1 \cap k_5, L \in k_1 \cap k_5, M \in k_2 \cap k_5, N \in k_2 \cap k_5, O \in k_4 \cap k_5,$   
 $P \in k_4 \cap k_5$

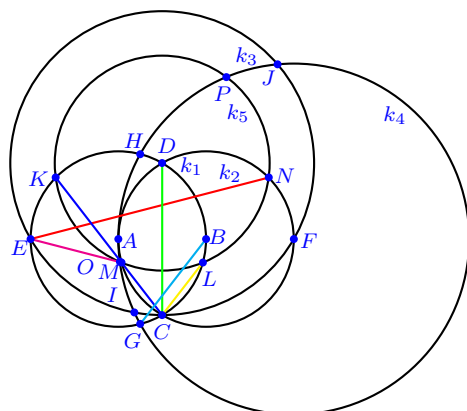
## Distances

$$\begin{aligned}
 d_0 &= |EN| = |FK| \text{ (red)} \dots 2 \\
 d_1 &= |CK| = |CN| = |EL| = |FM| \text{ (blue)} \dots 4 \\
 d_2 &= |CD| = |CE| = |CF| = |DE| = |DF| = |DI| = |DJ| \text{ (green)} \dots 7 \\
 d_3 &= |AG| = |AH| = |DK| = |DL| = |DM| = |DN| = |DO| = |DP| = \\
 &= |KM| = |LN| \text{ (cyan)} \dots 10 \\
 d_4 &= |EM| = |FL| \text{ (magenta)} \dots 2 \\
 d_5 &= |CL| = |CM| = |EK| = |FN| \text{ (yellow)} \dots 4
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (108 occurrences)}$$

## 2.88.4 Construction MM330



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, |BE|)$   
 $G \in k_1 \cap k_4, H \in k_1 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
  5.  $k_5 = k(D, |BG|)$   
 $K \in k_1 \cap k_5, L \in k_1 \cap k_5, M \in k_2 \cap k_5, N \in k_2 \cap k_5, O \in k_4 \cap k_5,$   
 $P \in k_4 \cap k_5$

## Distances

$$d_0 = |EN| = |FK| \text{ (red)} \dots 2$$

$$d_1 = |CK| = |CN| = |EL| = |FM| \text{ (blue)} \dots 4$$

$$d_2 = |CD| = |CE| = |CF| = |DE| = |DF| = |DI| = |DJ| \text{ (green)} \dots 7$$

$$d_3 = |BG| = |BH| = |DK| = |DL| = |DM| = |DN| = |DO| = |DP| = |KM| = |LN| \text{ (cyan)} \dots 10$$

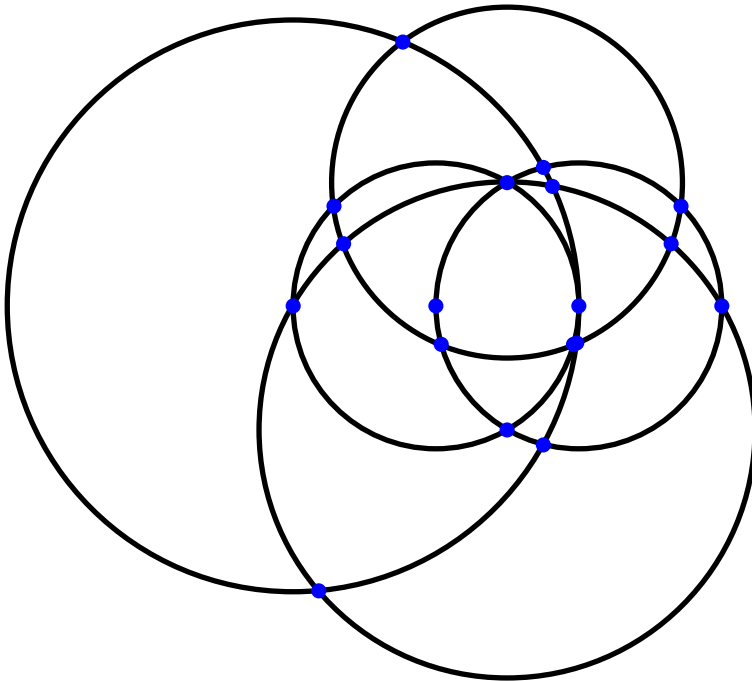
$$d_4 = |EM| = |FL| \text{ (magenta)} \dots 2$$

$$d_5 = |CL| = |CM| = |EK| = |FN| \text{ (yellow)} \dots 4$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (108 occurrences)}$$

## 2.89 Class of Similar Constructions No. 80

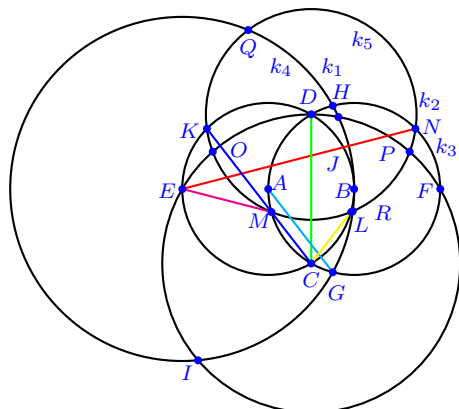


### Contained Constructions

MM173, MM219, MM283, MM329



## 2.89.1 Construction MM173



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, |BE|)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
  5.  $k_5 = k(D, |AG|)$   
 $K \in k_1 \cap k_5, L \in k_1 \cap k_5, M \in k_2 \cap k_5, N \in k_2 \cap k_5, O \in k_3 \cap k_5,$   
 $P \in k_3 \cap k_5, Q \in k_4 \cap k_5, R \in k_4 \cap k_5$

## Distances

$$d_0 = |EN| = |FK| \text{ (red)} \dots 2$$

$$d_1 = |CK| = |CN| = |EL| = |FM| \text{ (blue)} \dots 4$$

$$d_2 = |CD| = |CE| = |CF| = |CI| = |CJ| = |CO| = |CP| = |DE| = |DF| \text{ (green)} \dots 9$$

$$d_3 = |AG| = |AH| = |DK| = |DL| = |DM| = |DN| = |DO| = |DP| = |DQ| = |DR| = |KM| = |LN| \text{ (cyan)} \dots 12$$

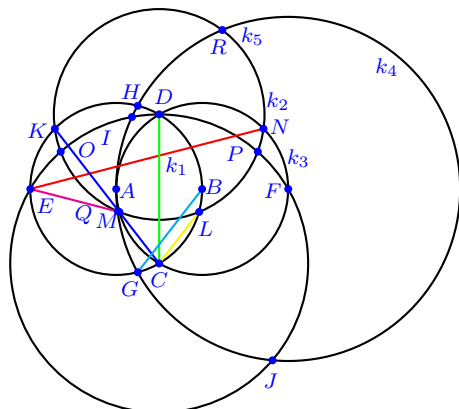
$$d_4 = |EM| = |FL| \text{ (magenta)} \dots 2$$

$$d_5 = |CL| = |CM| = |EK| = |FN| \text{ (yellow)} \dots 4$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (132 occurrences)}$$

## 2.89.2 Construction MM219



## Construction Process

- $A, B$  given initial points
- $k_1 = k(A, |AB|)$
  - $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  - $k_3 = k(C, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  - $k_4 = k(F, |BE|)$   
 $G \in k_1 \cap k_4, H \in k_1 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
  - $k_5 = k(D, |BG|)$   
 $K \in k_1 \cap k_5, L \in k_1 \cap k_5, M \in k_2 \cap k_5, N \in k_2 \cap k_5, O \in k_3 \cap k_5,$   
 $P \in k_3 \cap k_5, Q \in k_4 \cap k_5, R \in k_4 \cap k_5$

## Distances

$$d_0 = |EN| = |FK| \text{ (red) } \dots 2$$

$$d_1 = |CK| = |CN| = |EL| = |FM| \text{ (blue) } \dots 4$$

$$d_2 = |CD| = |CE| = |CF| = |CI| = |CJ| = |CO| = |CP| = |DE| = |DF| \text{ (green) } \dots 9$$

$$d_3 = |BG| = |BH| = |DK| = |DL| = |DM| = |DN| = |DO| = |DP| = |DQ| = |DR| = |KM| = |LN| \text{ (cyan) } \dots 12$$

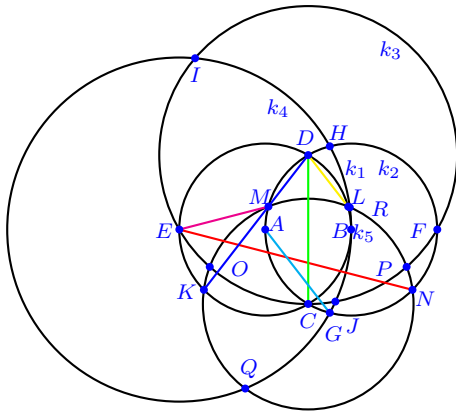
$$d_4 = |EM| = |FL| \text{ (magenta) } \dots 2$$

$$d_5 = |CL| = |CM| = |EK| = |FN| \text{ (yellow) } \dots 4$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (132 occurrences)}$$

2.89.3 Construction MM283



Construction Process

- $A, B$  given initial points
- 1.  $k_1 = k(A, |AB|)$
- 2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
- 3.  $k_3 = k(D, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
- 4.  $k_4 = k(E, |BE|)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
- 5.  $k_5 = k(C, |AG|)$   
 $K \in k_1 \cap k_5, L \in k_1 \cap k_5, M \in k_2 \cap k_5, N \in k_2 \cap k_5, O \in k_3 \cap k_5,$   
 $P \in k_3 \cap k_5, Q \in k_4 \cap k_5, R \in k_4 \cap k_5$

Distances

$$d_0 = |EN| = |FK| \text{ (red) } \dots 2$$

$$d_1 = |DK| = |DN| = |EL| = |FM| \text{ (blue) } \dots 4$$

$$d_2 = |CD| = |CE| = |CF| = |DE| = |DF| = |DI| = |DJ| = |DO| = |DP| \text{ (green) } \dots 9$$

$$d_3 = |AG| = |AH| = |CK| = |CL| = |CM| = |CN| = |CO| = |CP| = |CQ| = |CR| = |KM| = |LN| \text{ (cyan) } \dots 12$$

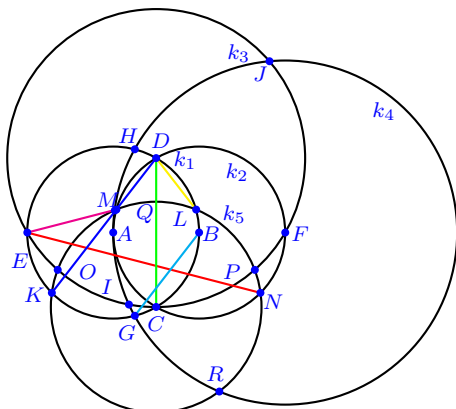
$$d_4 = |EM| = |FL| \text{ (magenta) } \dots 2$$

$$d_5 = |DL| = |DM| = |EK| = |FN| \text{ (yellow) } \dots 4$$

Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (132 occurrences)}$$

## 2.89.4 Construction MM329



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, |BE|)$   
 $G \in k_1 \cap k_4, H \in k_1 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
  5.  $k_5 = k(C, |BG|)$   
 $K \in k_1 \cap k_5, L \in k_1 \cap k_5, M \in k_2 \cap k_5, N \in k_2 \cap k_5, O \in k_3 \cap k_5,$   
 $P \in k_3 \cap k_5, Q \in k_4 \cap k_5, R \in k_4 \cap k_5$

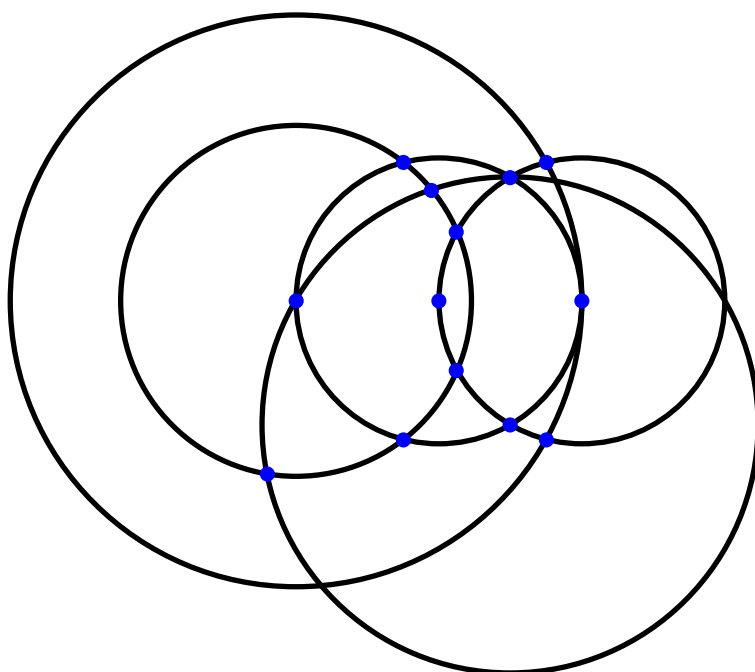
## Distances

$$\begin{aligned}
 d_0 &= |EN| = |FK| \text{ (red)} \dots 2 \\
 d_1 &= |DK| = |DN| = |EL| = |FM| \text{ (blue)} \dots 4 \\
 d_2 &= |CD| = |CE| = |CF| = |DE| = |DF| = |DI| = |DJ| = |DO| = |DP| \text{ (green)} \dots 9 \\
 d_3 &= |BG| = |BH| = |CK| = |CL| = |CM| = |CN| = |CO| = |CP| = |CQ| = |CR| = |KM| = |LN| \text{ (cyan)} \dots 12 \\
 d_4 &= |EM| = |FL| \text{ (magenta)} \dots 2 \\
 d_5 &= |DL| = |DM| = |EK| = |FN| \text{ (yellow)} \dots 4
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (132 occurrences)}$$

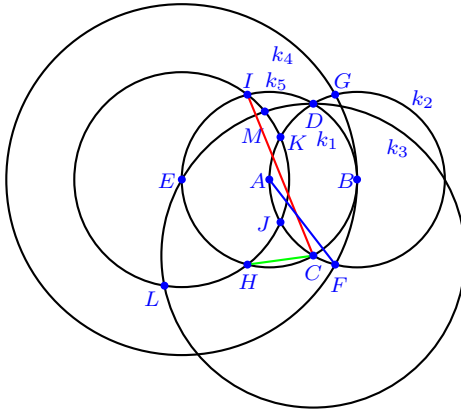
## 2.90 Class of Similar Constructions No. 81



### Contained Constructions

MM174, MM221, MM285, MM332

## 2.90.1 Construction MM174



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |CD|)$   
 $E \in k_1 \cap k_3$
  4.  $k_4 = k(E, |BE|)$   
 $F \in k_2 \cap k_4, G \in k_2 \cap k_4$
  5.  $k_5 = k(E, |AF|)$   
 $H \in k_1 \cap k_5, I \in k_1 \cap k_5, J \in k_2 \cap k_5, K \in k_2 \cap k_5, L \in k_3 \cap k_5,$   
 $M \in k_3 \cap k_5$

## Distances

$$d_0 = |CI| = |DH| \text{ (red) } \dots 2$$

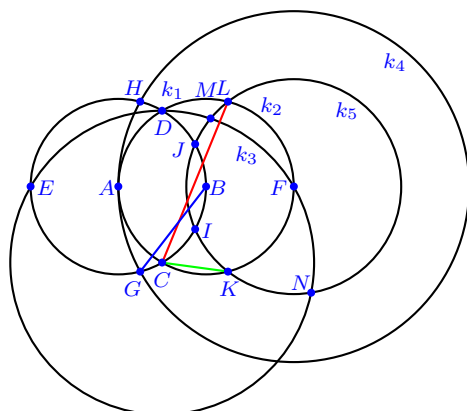
$$d_1 = |AF| = |AG| = |EH| = |EI| = |EJ| = |EK| = |EL| = |EM| \text{ (blue) } \dots 8$$

$$d_2 = |CH| = |DI| \text{ (green) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (32 occurrences)}$$

## 2.90.2 Construction MM221



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, |BE|)$   
 $G \in k_1 \cap k_4, H \in k_1 \cap k_4$
  5.  $k_5 = k(F, |BG|)$   
 $I \in k_1 \cap k_5, J \in k_1 \cap k_5, K \in k_2 \cap k_5, L \in k_2 \cap k_5, M \in k_3 \cap k_5,$   
 $N \in k_3 \cap k_5$

## Distances

$$d_0 = |CL| = |DK| \text{ (red) } \dots 2$$

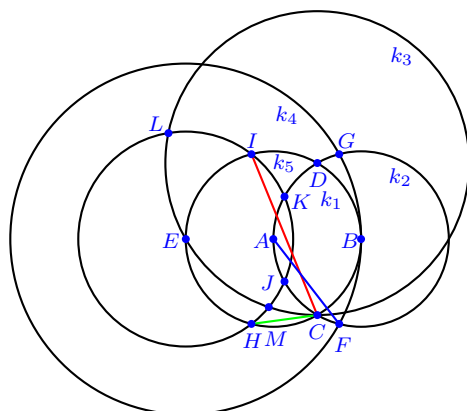
$$d_1 = |BG| = |BH| = |FI| = |FJ| = |FK| = |FL| = |FM| = |FN| \text{ (blue) } \dots 8$$

$$d_2 = |CK| = |DL| \text{ (green) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (32 occurrences)}$$

## 2.90.3 Construction MM285



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |CD|)$   
 $E \in k_1 \cap k_3$
  4.  $k_4 = k(E, |BE|)$   
 $F \in k_2 \cap k_4, G \in k_2 \cap k_4$
  5.  $k_5 = k(E, |AE|)$   
 $H \in k_1 \cap k_5, I \in k_1 \cap k_5, J \in k_2 \cap k_5, K \in k_2 \cap k_5, L \in k_3 \cap k_5,$   
 $M \in k_3 \cap k_5$

## Distances

$$d_0 = |CI| = |DH| \text{ (red) } \dots 2$$

$$d_1 = |AF| = |AG| = |EH| = |EI| = |EJ| = |EK| = |EL| = |EM| \text{ (blue) } \dots 8$$

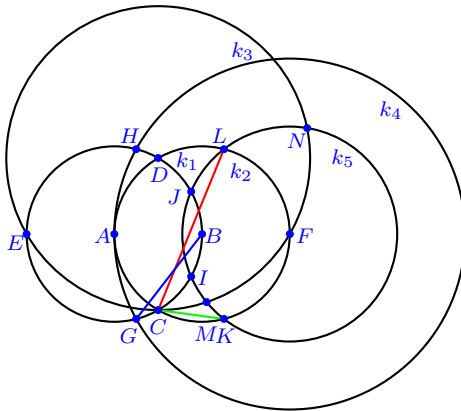
$$d_2 = |CH| = |DI| \text{ (green) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (32 occurrences)}$$



## 2.90.4 Construction MM332



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, |BE|)$   
 $G \in k_1 \cap k_4, H \in k_1 \cap k_4$
  5.  $k_5 = k(F, |BG|)$   
 $I \in k_1 \cap k_5, J \in k_1 \cap k_5, K \in k_2 \cap k_5, L \in k_2 \cap k_5, M \in k_3 \cap k_5,$   
 $N \in k_3 \cap k_5$

## Distances

$$d_0 = |CL| = |DK| \text{ (red) } \dots 2$$

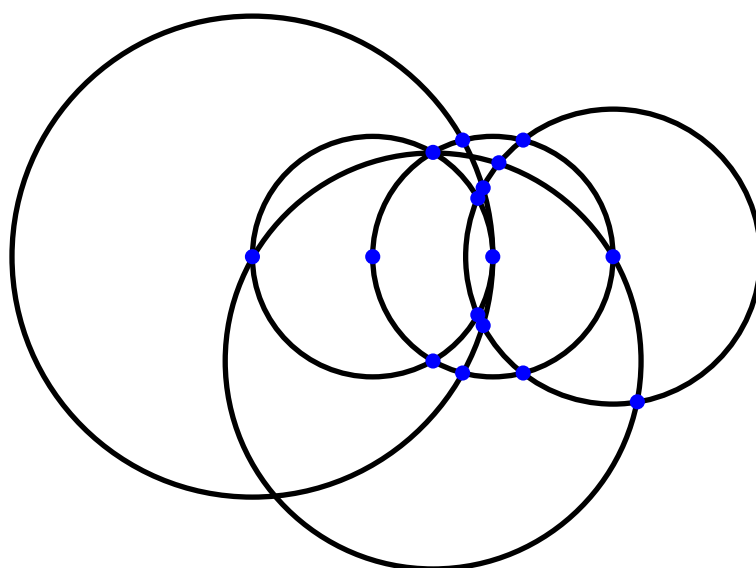
$$d_1 = |BG| = |BH| = |FI| = |FJ| = |FK| = |FL| = |FM| = |FN| \text{ (blue) } \dots 8$$

$$d_2 = |CK| = |DL| \text{ (green) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (32 occurrences)}$$

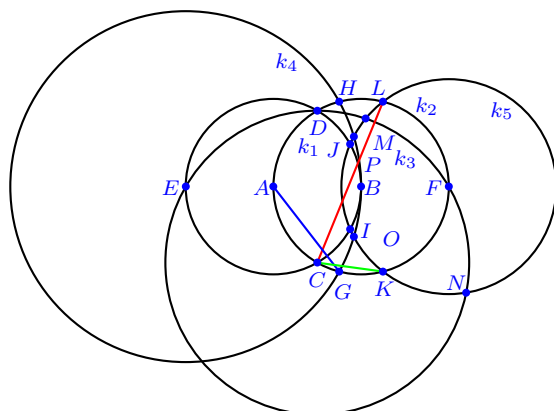
## 2.91 Class of Similar Constructions No. 82



### Contained Constructions

MM175, MM220, MM286, MM331

## 2.91.1 Construction MM175



## Construction Process

- $A, B$  given initial points
- $k_1 = k(A, |AB|)$
  - $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  - $k_3 = k(C, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  - $k_4 = k(E, |BE|)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
  - $k_5 = k(F, |AG|)$   
 $I \in k_1 \cap k_5, J \in k_1 \cap k_5, K \in k_2 \cap k_5, L \in k_2 \cap k_5, M \in k_3 \cap k_5,$   
 $N \in k_3 \cap k_5, O \in k_4 \cap k_5, P \in k_4 \cap k_5$

## Distances

$$d_0 = |CL| = |DK| \text{ (red) } \dots 2$$

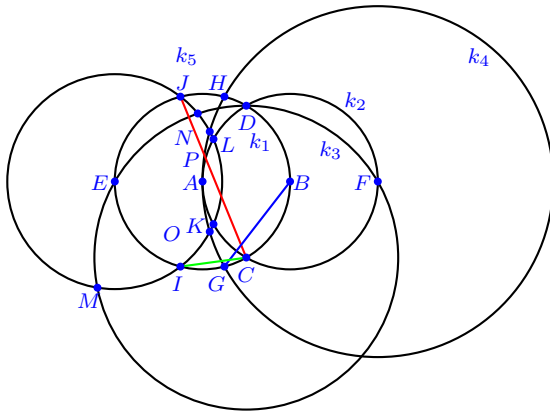
$$d_1 = |AG| = |AH| = |FI| = |FJ| = |FK| = |FL| = |FM| = |FN| = |FO| = |FP| \text{ (blue) } \dots 10$$

$$d_2 = |CK| = |DL| \text{ (green) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (40 occurrences)}$$

**2.91.2 Construction MM220**



**Construction Process**

- $A, B$  given initial points
- 1.  $k_1 = k(A, |AB|)$
- 2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
- 3.  $k_3 = k(C, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
- 4.  $k_4 = k(F, |BE|)$   
 $G \in k_1 \cap k_4, H \in k_1 \cap k_4$
- 5.  $k_5 = k(E, |BG|)$   
 $I \in k_1 \cap k_5, J \in k_1 \cap k_5, K \in k_2 \cap k_5, L \in k_2 \cap k_5, M \in k_3 \cap k_5,$   
 $N \in k_3 \cap k_5, O \in k_4 \cap k_5, P \in k_4 \cap k_5$

**Distances**

$$d_0 = |CJ| = |DI| \text{ (red) } \dots 2$$

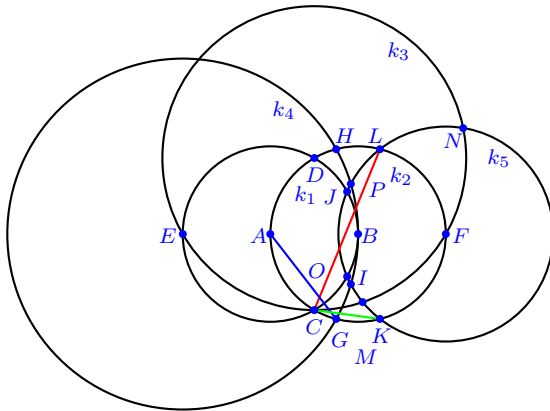
$$d_1 = |BG| = |BH| = |EI| = |EJ| = |EK| = |EL| = |EM| = |EN| = |EO| = |EP| \text{ (blue) } \dots 10$$

$$d_2 = |CI| = |DJ| \text{ (green) } \dots 2$$

**Occurrences of the Golden Ratio**

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (40 occurrences)}$$

2.91.3 Construction MM286



Construction Process

- $A, B$  given initial points
- 1.  $k_1 = k(A, |AB|)$
- 2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
- 3.  $k_3 = k(D, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
- 4.  $k_4 = k(E, |BE|)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
- 5.  $k_5 = k(F, |AG|)$   
 $I \in k_1 \cap k_5, J \in k_1 \cap k_5, K \in k_2 \cap k_5, L \in k_2 \cap k_5, M \in k_3 \cap k_5,$   
 $N \in k_3 \cap k_5, O \in k_4 \cap k_5, P \in k_4 \cap k_5$

Distances

$$d_0 = |CL| = |DK| \text{ (red) } \dots 2$$

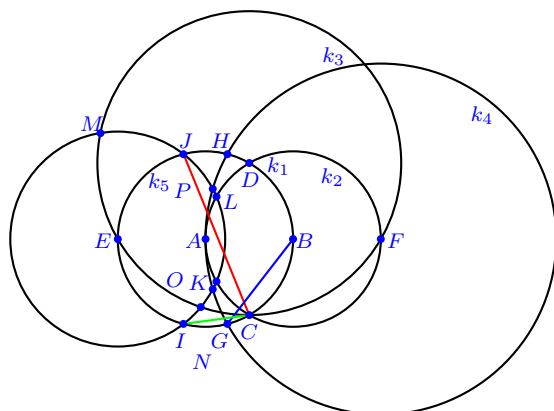
$$d_1 = |AG| = |AH| = |FI| = |FJ| = |FK| = |FL| = |FM| = |FN| = |FO| = |FP| \text{ (blue) } \dots 10$$

$$d_2 = |CK| = |DL| \text{ (green) } \dots 2$$

Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (40 occurrences)}$$

## 2.91.4 Construction MM331



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, |BE|)$   
 $G \in k_1 \cap k_4, H \in k_1 \cap k_4$
  5.  $k_5 = k(E, |BG|)$   
 $I \in k_1 \cap k_5, J \in k_1 \cap k_5, K \in k_2 \cap k_5, L \in k_2 \cap k_5, M \in k_3 \cap k_5,$   
 $N \in k_3 \cap k_5, O \in k_4 \cap k_5, P \in k_4 \cap k_5$

## Distances

$$d_0 = |CJ| = |DI| \text{ (red)} \dots 2$$

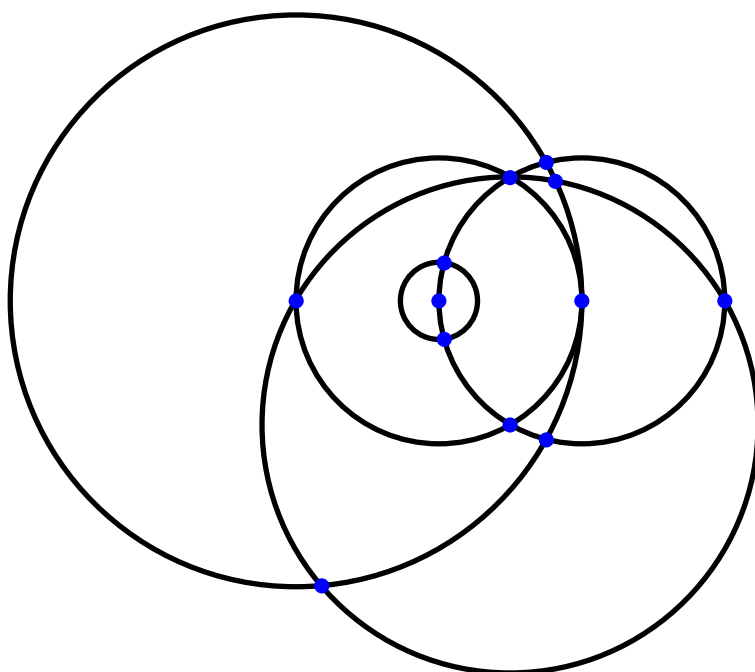
$$d_1 = |BG| = |BH| = |EI| = |EJ| = |EK| = |EL| = |EM| = |EN| = |EO| = |EP| \text{ (blue)} \dots 10$$

$$d_2 = |CI| = |DJ| \text{ (green)} \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (40 occurrences)}$$

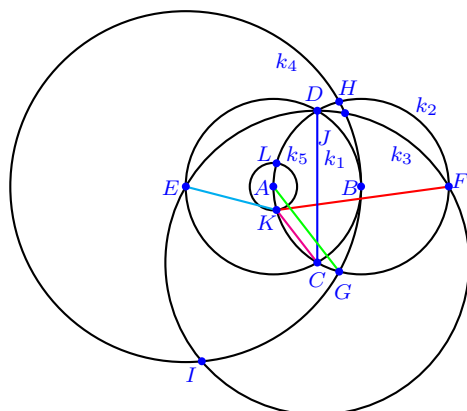
## 2.92 Class of Similar Constructions No. 83



### Contained Constructions

MM176, MM223, MM287, MM334

## 2.92.1 Construction MM176



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, |BE|)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
  5.  $k_5 = k(A, |CG|)$   
 $K \in k_2 \cap k_5, L \in k_2 \cap k_5$

## Distances

$$d_0 = |FK| = |FL| \text{ (red)} \dots 2$$

$$d_1 = |CD| = |CE| = |CF| = |CI| = |CJ| = |DE| = |DF| \text{ (blue)} \dots 7$$

$$d_2 = |AG| = |AH| = |CL| = |DK| \text{ (green)} \dots 4$$

$$d_3 = |EK| = |EL| \text{ (cyan)} \dots 2$$

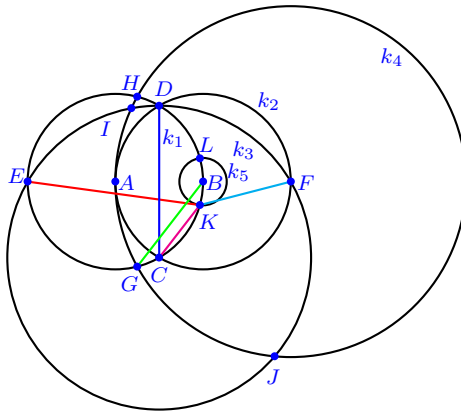
$$d_4 = |CK| = |DL| \text{ (magenta)} \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} \text{ (30 occurrences)}$$



## 2.92.2 Construction MM223



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, |BE|)$   
 $G \in k_1 \cap k_4, H \in k_1 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
  5.  $k_5 = k(B, |CG|)$   
 $K \in k_1 \cap k_5, L \in k_1 \cap k_5$

## Distances

$$d_0 = |EK| = |EL| \text{ (red) } \dots 2$$

$$d_1 = |CD| = |CE| = |CF| = |CI| = |CJ| = |DE| = |DF| \text{ (blue) } \dots 7$$

$$d_2 = |BG| = |BH| = |CL| = |DK| \text{ (green) } \dots 4$$

$$d_3 = |FK| = |FL| \text{ (cyan) } \dots 2$$

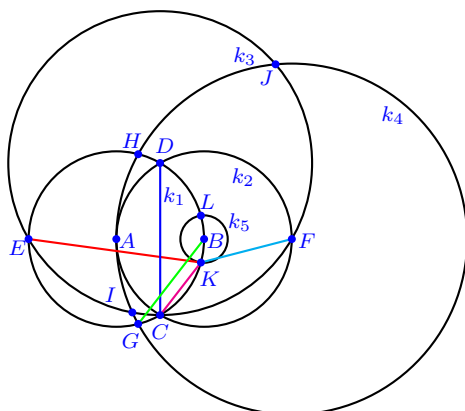
$$d_4 = |CK| = |DL| \text{ (magenta) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} \text{ (30 occurrences)}$$



## 2.92.4 Construction MM334



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, |BE|)$   
 $G \in k_1 \cap k_4, H \in k_1 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
  5.  $k_5 = k(B, |CG|)$   
 $K \in k_1 \cap k_5, L \in k_1 \cap k_5$

## Distances

$$d_0 = |EK| = |EL| \text{ (red) } \dots 2$$

$$d_1 = |CD| = |CE| = |CF| = |DE| = |DF| = |DI| = |DJ| \text{ (blue) } \dots 7$$

$$d_2 = |BG| = |BH| = |CL| = |DK| \text{ (green) } \dots 4$$

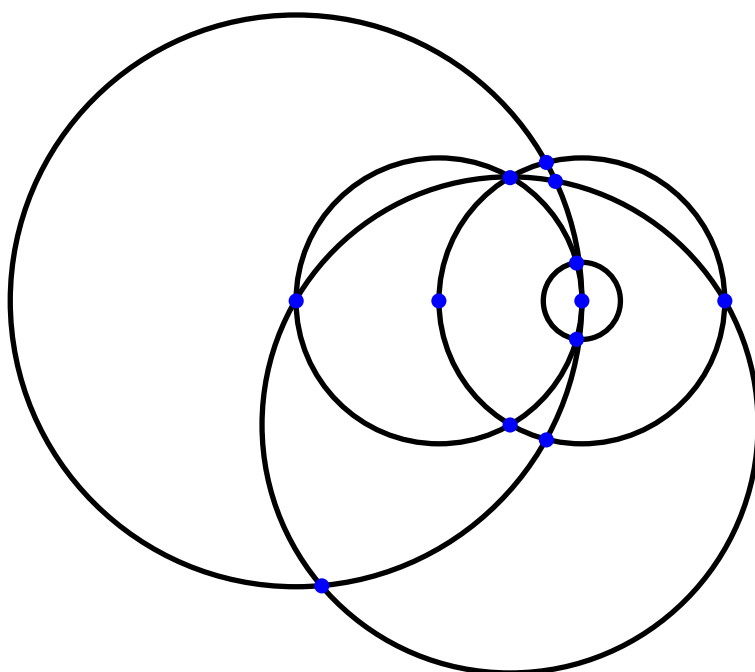
$$d_3 = |FK| = |FL| \text{ (cyan) } \dots 2$$

$$d_4 = |CK| = |DL| \text{ (magenta) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} \text{ (30 occurrences)}$$

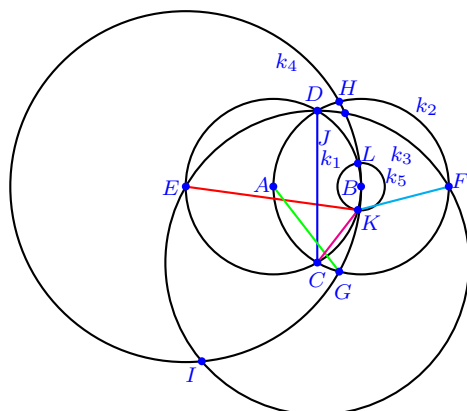
## 2.93 Class of Similar Constructions No. 84



### Contained Constructions

MM177, MM222, MM288, MM333

## 2.93.1 Construction MM177



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, |BE|)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
  5.  $k_5 = k(B, |CG|)$   
 $K \in k_1 \cap k_5, L \in k_1 \cap k_5$

## Distances

$$d_0 = |EK| = |EL| \text{ (red) } \dots 2$$

$$d_1 = |CD| = |CE| = |CF| = |CI| = |CJ| = |DE| = |DF| \text{ (blue) } \dots 7$$

$$d_2 = |AG| = |AH| = |CL| = |DK| \text{ (green) } \dots 4$$

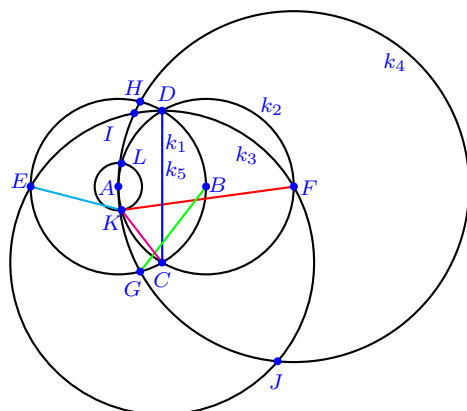
$$d_3 = |FK| = |FL| \text{ (cyan) } \dots 2$$

$$d_4 = |CK| = |DL| \text{ (magenta) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} \text{ (30 occurrences)}$$

## 2.93.2 Construction MM222



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, |BE|)$   
 $G \in k_1 \cap k_4, H \in k_1 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
  5.  $k_5 = k(A, |CG|)$   
 $K \in k_2 \cap k_5, L \in k_2 \cap k_5$

## Distances

$$d_0 = |FK| = |FL| \text{ (red)} \dots 2$$

$$d_1 = |CD| = |CE| = |CF| = |CI| = |CJ| = |DE| = |DF| \text{ (blue)} \dots 7$$

$$d_2 = |BG| = |BH| = |CL| = |DK| \text{ (green)} \dots 4$$

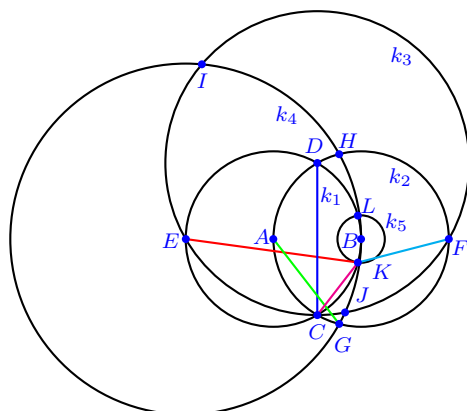
$$d_3 = |EK| = |EL| \text{ (cyan)} \dots 2$$

$$d_4 = |CK| = |DL| \text{ (magenta)} \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} \text{ (30 occurrences)}$$

## 2.93.3 Construction MM288



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, |BE|)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
  5.  $k_5 = k(B, |CG|)$   
 $K \in k_1 \cap k_5, L \in k_1 \cap k_5$

## Distances

$$d_0 = |EK| = |EL| \text{ (red)} \dots 2$$

$$d_1 = |CD| = |CE| = |CF| = |DE| = |DF| = |DI| = |DJ| \text{ (blue)} \dots 7$$

$$d_2 = |AG| = |AH| = |CL| = |DK| \text{ (green)} \dots 4$$

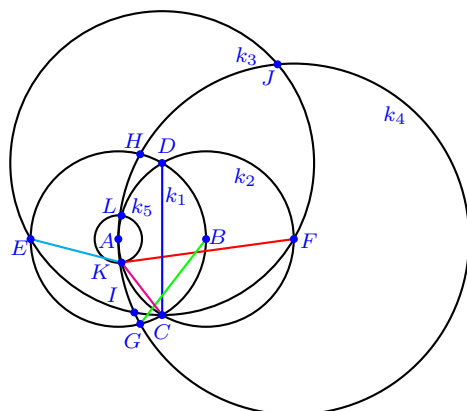
$$d_3 = |FK| = |FL| \text{ (cyan)} \dots 2$$

$$d_4 = |CK| = |DL| \text{ (magenta)} \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} \text{ (30 occurrences)}$$

## 2.93.4 Construction MM333



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, |BE|)$   
 $G \in k_1 \cap k_4, H \in k_1 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
  5.  $k_5 = k(A, |CG|)$   
 $K \in k_2 \cap k_5, L \in k_2 \cap k_5$

## Distances

$$d_0 = |FK| = |FL| \text{ (red) } \dots 2$$

$$d_1 = |CD| = |CE| = |CF| = |DE| = |DF| = |DI| = |DJ| \text{ (blue) } \dots 7$$

$$d_2 = |BG| = |BH| = |CL| = |DK| \text{ (green) } \dots 4$$

$$d_3 = |EK| = |EL| \text{ (cyan) } \dots 2$$

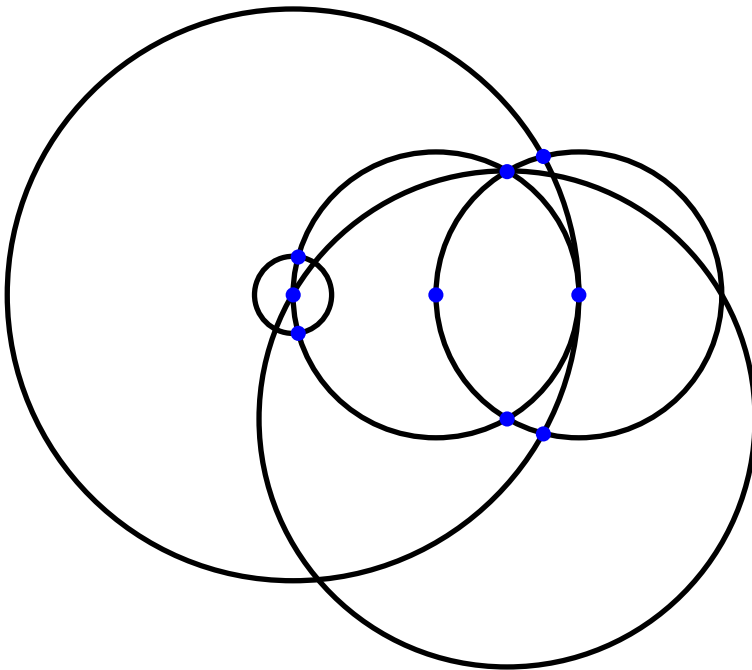
$$d_4 = |CK| = |DL| \text{ (magenta) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} \text{ (30 occurrences)}$$



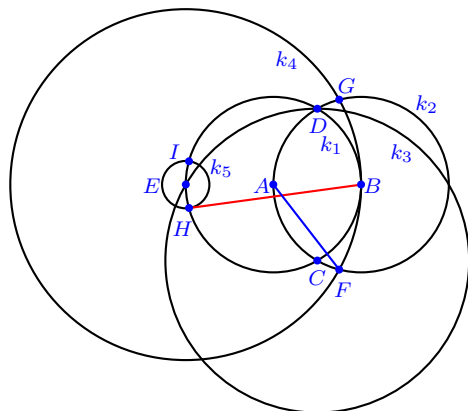
2.94 Class of Similar Constructions No. 85



**Contained Constructions**

MM180, MM227, MM291, MM338

## 2.94.1 Construction MM180



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |CD|)$   
 $E \in k_1 \cap k_3$
  4.  $k_4 = k(E, |BE|)$   
 $F \in k_2 \cap k_4, G \in k_2 \cap k_4$
  5.  $k_5 = k(E, |CF|)$   
 $H \in k_1 \cap k_5, I \in k_1 \cap k_5$

## Distances

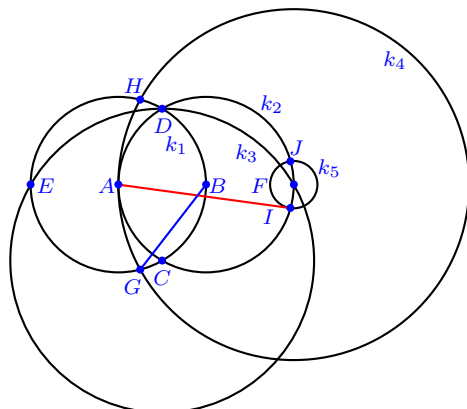
$$d_0 = |BH| = |BI| \text{ (red)} \dots 2$$

$$d_1 = |AF| = |AG| \text{ (blue)} \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (4 occurrences)}$$

## 2.94.2 Construction MM227



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, |BE|)$   
 $G \in k_1 \cap k_4, H \in k_1 \cap k_4$
  5.  $k_5 = k(F, |CG|)$   
 $I \in k_2 \cap k_5, J \in k_2 \cap k_5$

## Distances

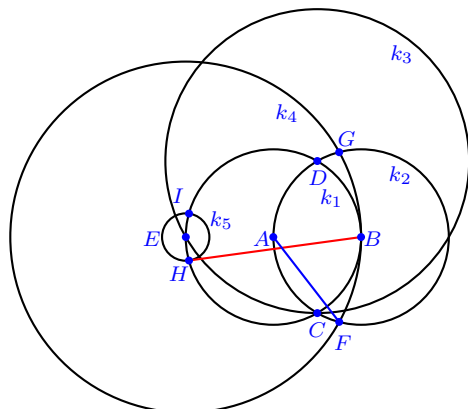
$$d_0 = |AI| = |AJ| \text{ (red)} \dots 2$$

$$d_1 = |BG| = |BH| \text{ (blue)} \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (4 occurrences)}$$

## 2.94.3 Construction MM291



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |CD|)$   
 $E \in k_1 \cap k_3$
  4.  $k_4 = k(E, |BE|)$   
 $F \in k_2 \cap k_4, G \in k_2 \cap k_4$
  5.  $k_5 = k(E, |CF|)$   
 $H \in k_1 \cap k_5, I \in k_1 \cap k_5$

## Distances

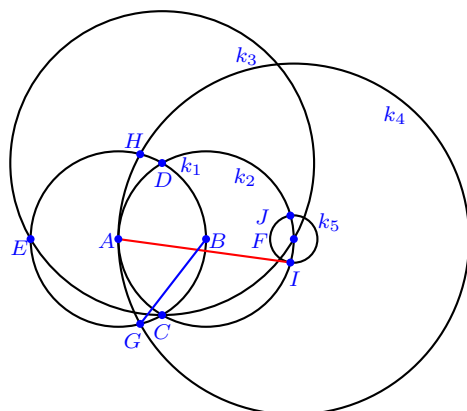
$$d_0 = |BH| = |BI| \text{ (red)} \dots 2$$

$$d_1 = |AF| = |AG| \text{ (blue)} \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (4 occurrences)}$$

## 2.94.4 Construction MM338



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, |BE|)$   
 $G \in k_1 \cap k_4, H \in k_1 \cap k_4$
  5.  $k_5 = k(F, |CG|)$   
 $I \in k_2 \cap k_5, J \in k_2 \cap k_5$

## Distances

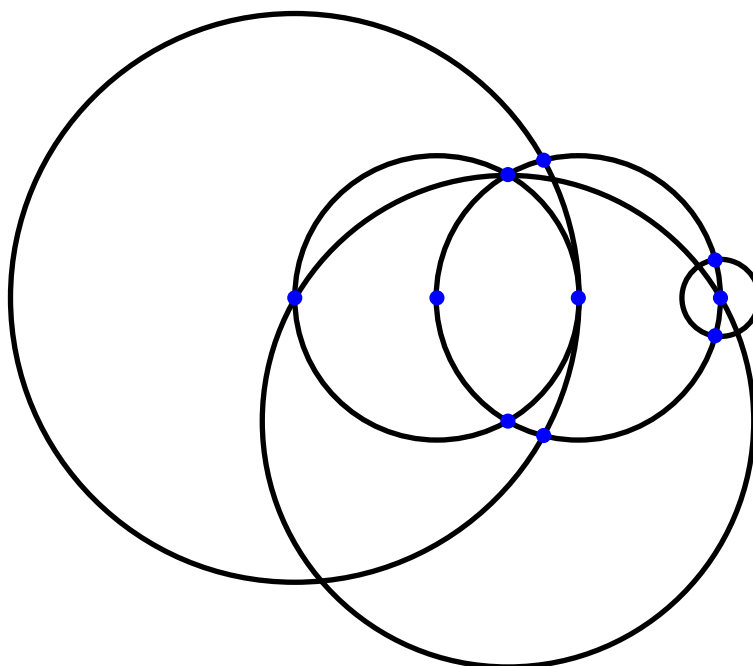
$$d_0 = |AI| = |AJ| \text{ (red)} \dots 2$$

$$d_1 = |BG| = |BH| \text{ (blue)} \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (4 occurrences)}$$

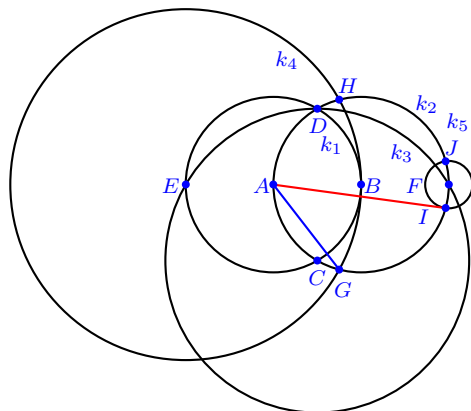
## 2.95 Class of Similar Constructions No. 86



### Contained Constructions

MM181, MM226, MM292, MM337

## 2.95.1 Construction MM181



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, |BE|)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
  5.  $k_5 = k(F, |CF|)$   
 $I \in k_2 \cap k_5, J \in k_2 \cap k_5$

## Distances

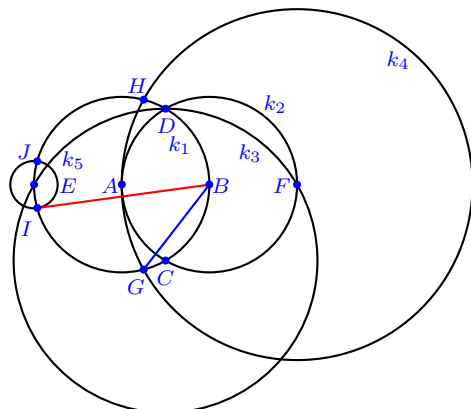
$$d_0 = |AI| = |AJ| \text{ (red) } \dots 2$$

$$d_1 = |AG| = |AH| \text{ (blue) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (4 occurrences)}$$

## 2.95.2 Construction MM226



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, |BE|)$   
 $G \in k_1 \cap k_4, H \in k_1 \cap k_4$
  5.  $k_5 = k(E, |CG|)$   
 $I \in k_1 \cap k_5, J \in k_1 \cap k_5$

## Distances

$$d_0 = |BI| = |BJ| \text{ (red) } \dots 2$$

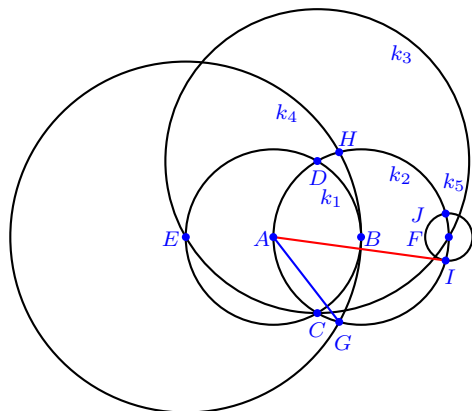
$$d_1 = |BG| = |BH| \text{ (blue) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (4 occurrences)}$$



## 2.95.3 Construction MM292



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, |BE|)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
  5.  $k_5 = k(F, |CG|)$   
 $I \in k_2 \cap k_5, J \in k_2 \cap k_5$

## Distances

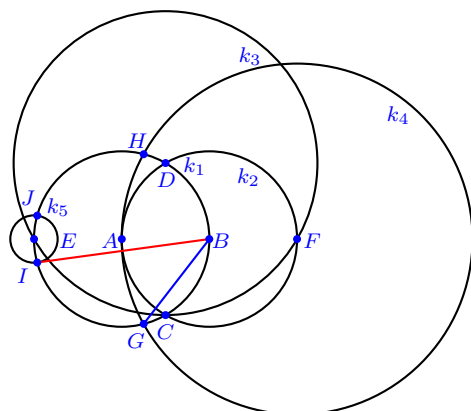
$$d_0 = |AI| = |AJ| \text{ (red) } \dots 2$$

$$d_1 = |AG| = |AH| \text{ (blue) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (4 occurrences)}$$

## 2.95.4 Construction MM337



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_3$
  3.  $k_3 = k(D, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, |BE|)$   
 $G \in k_1 \cap k_4, H \in k_1 \cap k_4$
  5.  $k_5 = k(E, |CG|)$   
 $I \in k_1 \cap k_5, J \in k_1 \cap k_5$

## Distances

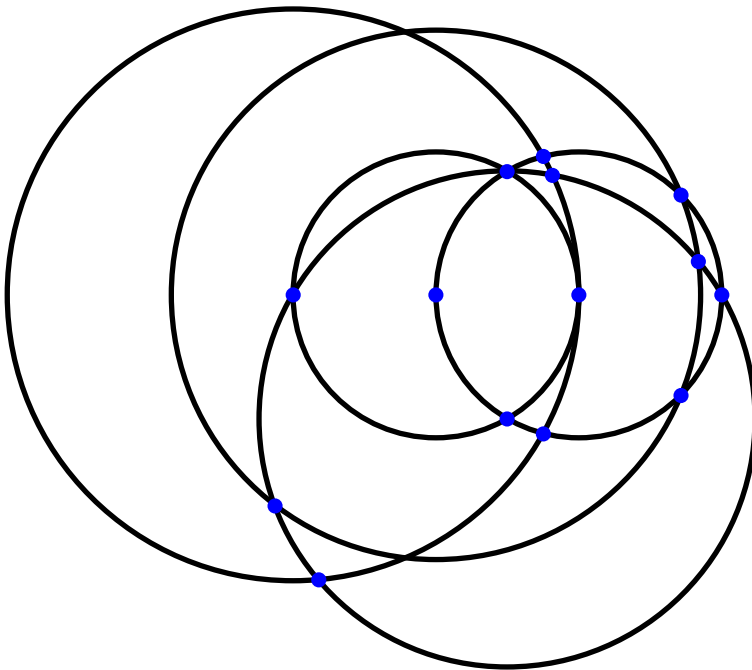
$$d_0 = |BI| = |BJ| \text{ (red)} \dots 2$$

$$d_1 = |BG| = |BH| \text{ (blue)} \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (4 occurrences)}$$

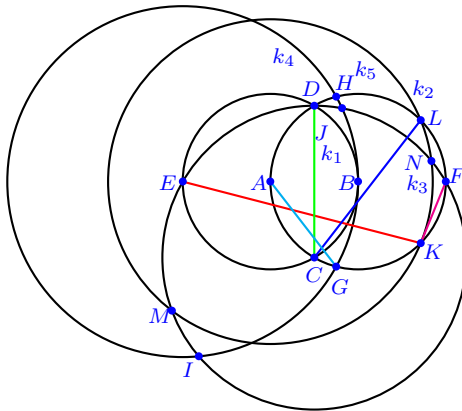
## 2.96 Class of Similar Constructions No. 87



### Contained Constructions

MM184, MM231, MM295, MM342

2.96.1 Construction MM184



Construction Process

- $A, B$  given initial points
- 1.  $k_1 = k(A, |AB|)$
- 2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
- 3.  $k_3 = k(C, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
- 4.  $k_4 = k(E, |BE|)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
- 5.  $k_5 = k(A, |DG|)$   
 $K \in k_2 \cap k_5, L \in k_2 \cap k_5, M \in k_3 \cap k_5, N \in k_3 \cap k_5$

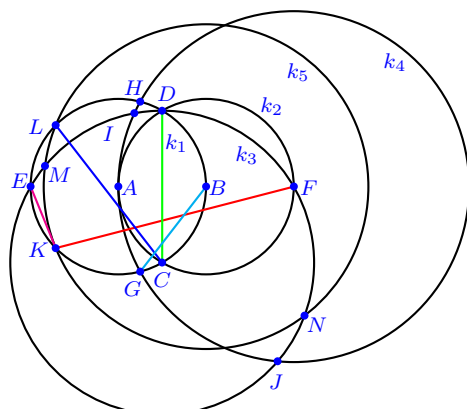
Distances

- $d_0 = |EK| = |EL|$  (red) ... 2
- $d_1 = |CL| = |DK|$  (blue) ... 2
- $d_2 = |CD| = |CE| = |CF| = |CI| = |CJ| = |CM| = |CN| = |DE| = |DF|$  (green) ... 9
- $d_3 = |AG| = |AH| = |CK| = |DL|$  (cyan) ... 4
- $d_4 = |FK| = |FL|$  (magenta) ... 2

Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_3}{d_4} \text{ (34 occurrences)}$$

## 2.96.2 Construction MM231



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, |BE|)$   
 $G \in k_1 \cap k_4, H \in k_1 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
  5.  $k_5 = k(B, |DG|)$   
 $K \in k_1 \cap k_5, L \in k_1 \cap k_5, M \in k_3 \cap k_5, N \in k_3 \cap k_5$

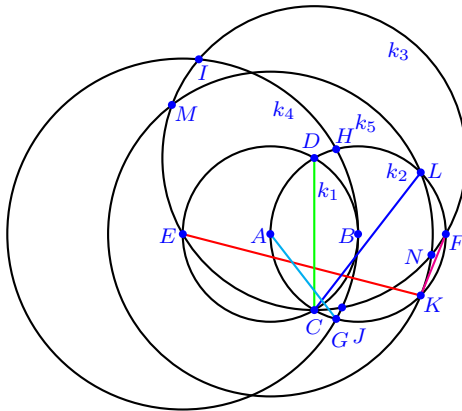
## Distances

$$\begin{aligned}
 d_0 &= |FK| = |FL| \text{ (red)} \dots 2 \\
 d_1 &= |CL| = |DK| \text{ (blue)} \dots 2 \\
 d_2 &= |CD| = |CE| = |CF| = |CI| = |CJ| = |CM| = |CN| = |DE| = |DF| \text{ (green)} \dots 9 \\
 d_3 &= |BG| = |BH| = |CK| = |DL| \text{ (cyan)} \dots 4 \\
 d_4 &= |EK| = |EL| \text{ (magenta)} \dots 2
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_3}{d_4} \text{ (34 occurrences)}$$

2.96.3 Construction MM295



Construction Process

- $A, B$  given initial points
- 1.  $k_1 = k(A, |AB|)$
- 2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
- 3.  $k_3 = k(D, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
- 4.  $k_4 = k(E, |BE|)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
- 5.  $k_5 = k(A, |DG|)$   
 $K \in k_2 \cap k_5, L \in k_2 \cap k_5, M \in k_3 \cap k_5, N \in k_3 \cap k_5$

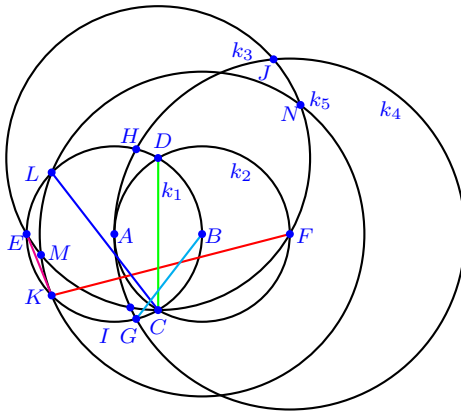
Distances

- $d_0 = |EK| = |EL|$  (red) ... 2
- $d_1 = |CL| = |DK|$  (blue) ... 2
- $d_2 = |CD| = |CE| = |CF| = |DE| = |DF| = |DI| = |DJ| = |DM| = |DN|$  (green) ... 9
- $d_3 = |AG| = |AH| = |CK| = |DL|$  (cyan) ... 4
- $d_4 = |FK| = |FL|$  (magenta) ... 2

Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_3}{d_4} \text{ (34 occurrences)}$$

## 2.96.4 Construction MM342



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, |BE|)$   
 $G \in k_1 \cap k_4, H \in k_1 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
  5.  $k_5 = k(B, |DG|)$   
 $K \in k_1 \cap k_5, L \in k_1 \cap k_5, M \in k_3 \cap k_5, N \in k_3 \cap k_5$

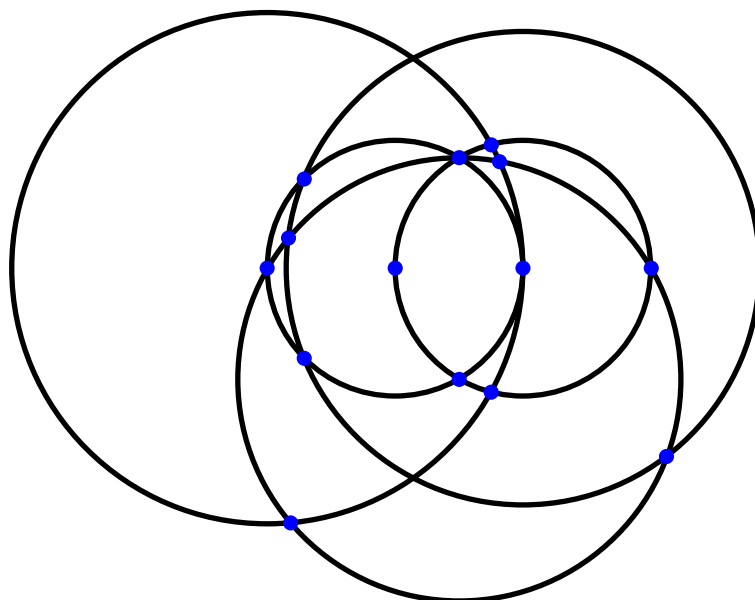
## Distances

$$\begin{aligned}
 d_0 &= |FK| = |FL| \text{ (red)} \dots 2 \\
 d_1 &= |CL| = |DK| \text{ (blue)} \dots 2 \\
 d_2 &= |CD| = |CE| = |CF| = |DE| = |DF| = |DI| = |DJ| = |DM| = |DN| \text{ (green)} \dots 9 \\
 d_3 &= |BG| = |BH| = |CK| = |DL| \text{ (cyan)} \dots 4 \\
 d_4 &= |EK| = |EL| \text{ (magenta)} \dots 2
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_3}{d_4} \text{ (34 occurrences)}$$

## 2.97 Class of Similar Constructions No. 88

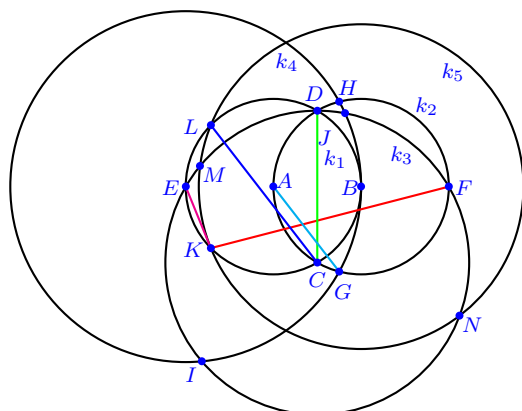


### Contained Constructions

MM185, MM230, MM296, MM341



## 2.97.1 Construction MM185



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, |BE|)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
  5.  $k_5 = k(B, |DG|)$   
 $K \in k_1 \cap k_5, L \in k_1 \cap k_5, M \in k_3 \cap k_5, N \in k_3 \cap k_5$

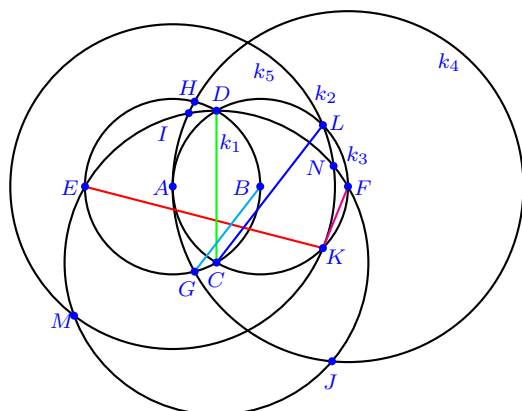
## Distances

$$\begin{aligned}
 d_0 &= |FK| = |FL| \text{ (red)} \dots 2 \\
 d_1 &= |CL| = |DK| \text{ (blue)} \dots 2 \\
 d_2 &= |CD| = |CE| = |CF| = |CI| = |CJ| = |CM| = |CN| = |DE| = |DF| \text{ (green)} \dots 9 \\
 d_3 &= |AG| = |AH| = |CK| = |DL| \text{ (cyan)} \dots 4 \\
 d_4 &= |EK| = |EL| \text{ (magenta)} \dots 2
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_3}{d_4} \text{ (34 occurrences)}$$

## 2.97.2 Construction MM230



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, |BE|)$   
 $G \in k_1 \cap k_4, H \in k_1 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
  5.  $k_5 = k(A, |DG|)$   
 $K \in k_2 \cap k_5, L \in k_2 \cap k_5, M \in k_3 \cap k_5, N \in k_3 \cap k_5$

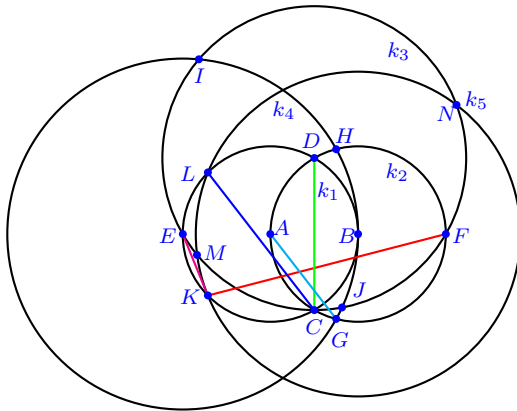
## Distances

$$\begin{aligned}
 d_0 &= |EK| = |EL| \text{ (red)} \dots 2 \\
 d_1 &= |CL| = |DK| \text{ (blue)} \dots 2 \\
 d_2 &= |CD| = |CE| = |CF| = |CI| = |CJ| = |CM| = |CN| = |DE| = |DF| \text{ (green)} \dots 9 \\
 d_3 &= |BG| = |BH| = |CK| = |DL| \text{ (cyan)} \dots 4 \\
 d_4 &= |FK| = |FL| \text{ (magenta)} \dots 2
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_3}{d_4} \text{ (34 occurrences)}$$

### 2.97.3 Construction MM296



#### Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, |BE|)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
  5.  $k_5 = k(B, |DG|)$   
 $K \in k_1 \cap k_5, L \in k_1 \cap k_5, M \in k_3 \cap k_5, N \in k_3 \cap k_5$

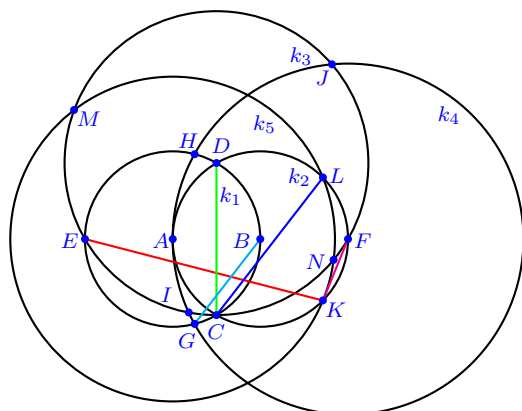
#### Distances

$$\begin{aligned}
 d_0 &= |FK| = |FL| \text{ (red)} \dots 2 \\
 d_1 &= |CL| = |DK| \text{ (blue)} \dots 2 \\
 d_2 &= |CD| = |CE| = |CF| = |DE| = |DF| = |DI| = |DJ| = |DM| = |DN| \text{ (green)} \dots 9 \\
 d_3 &= |AG| = |AH| = |CK| = |DL| \text{ (cyan)} \dots 4 \\
 d_4 &= |EK| = |EL| \text{ (magenta)} \dots 2
 \end{aligned}$$

#### Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_3}{d_4} \text{ (34 occurrences)}$$

## 2.97.4 Construction MM341



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, |BE|)$   
 $G \in k_1 \cap k_4, H \in k_1 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
  5.  $k_5 = k(A, |DG|)$   
 $K \in k_2 \cap k_5, L \in k_2 \cap k_5, M \in k_3 \cap k_5, N \in k_3 \cap k_5$

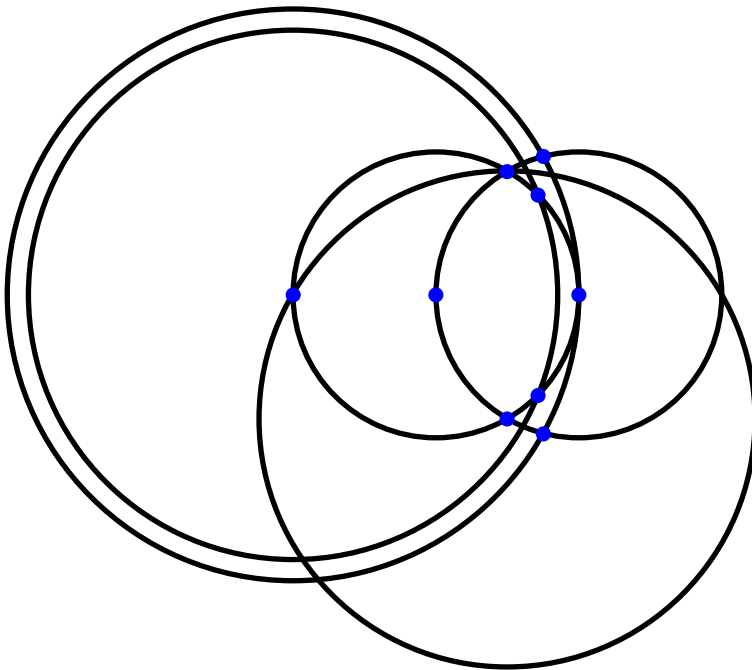
## Distances

$$\begin{aligned}
 d_0 &= |EK| = |EL| \text{ (red)} \dots 2 \\
 d_1 &= |CL| = |DK| \text{ (blue)} \dots 2 \\
 d_2 &= |CD| = |CE| = |CF| = |DE| = |DF| = |DI| = |DJ| = |DM| = |DN| \text{ (green)} \dots 9 \\
 d_3 &= |BG| = |BH| = |CK| = |DL| \text{ (cyan)} \dots 4 \\
 d_4 &= |FK| = |FL| \text{ (magenta)} \dots 2
 \end{aligned}$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_3}{d_4} \text{ (34 occurrences)}$$

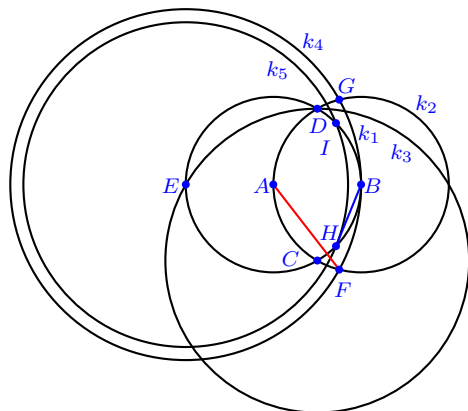
## 2.98 Class of Similar Constructions No. 89



### Contained Constructions

MM188, MM235, MM299, MM346

## 2.98.1 Construction MM188



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |CD|)$   
 $E \in k_1 \cap k_3$
  4.  $k_4 = k(E, |BE|)$   
 $F \in k_2 \cap k_4, G \in k_2 \cap k_4$
  5.  $k_5 = k(E, |DF|)$   
 $H \in k_1 \cap k_5, I \in k_1 \cap k_5$

## Distances

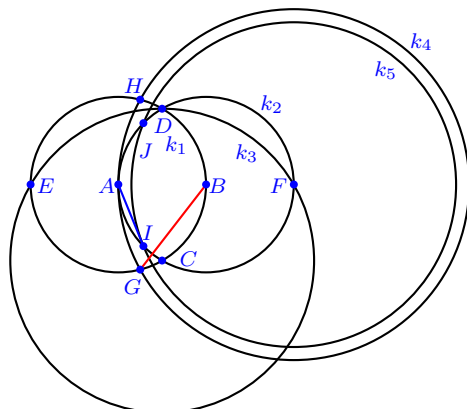
$$d_0 = |AF| = |AG| \text{ (red) } \dots 2$$

$$d_1 = |BH| = |BI| \text{ (blue) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (4 occurrences)}$$

## 2.98.2 Construction MM235



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, |BE|)$   
 $G \in k_1 \cap k_4, H \in k_1 \cap k_4$
  5.  $k_5 = k(F, |DG|)$   
 $I \in k_2 \cap k_5, J \in k_2 \cap k_5$

## Distances

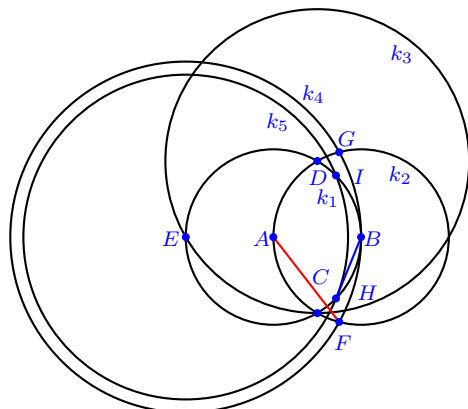
$$d_0 = |BG| = |BH| \text{ (red)} \dots 2$$

$$d_1 = |AI| = |AJ| \text{ (blue)} \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (4 occurrences)}$$

## 2.98.3 Construction MM299



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |CD|)$   
 $E \in k_1 \cap k_3$
  4.  $k_4 = k(E, |BE|)$   
 $F \in k_2 \cap k_4, G \in k_2 \cap k_4$
  5.  $k_5 = k(E, |DF|)$   
 $H \in k_1 \cap k_5, I \in k_1 \cap k_5$

## Distances

$$d_0 = |AF| = |AG| \text{ (red) } \dots 2$$

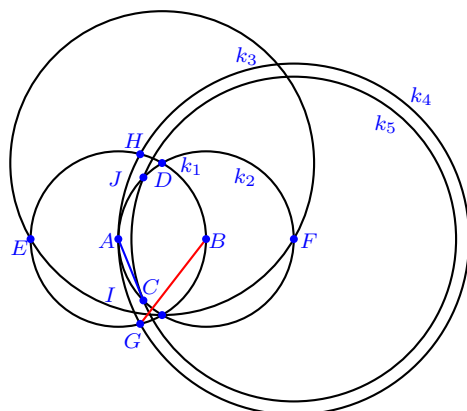
$$d_1 = |BH| = |BI| \text{ (blue) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (4 occurrences)}$$



## 2.98.4 Construction MM346



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, |BE|)$   
 $G \in k_1 \cap k_4, H \in k_1 \cap k_4$
  5.  $k_5 = k(F, |DG|)$   
 $I \in k_2 \cap k_5, J \in k_2 \cap k_5$

## Distances

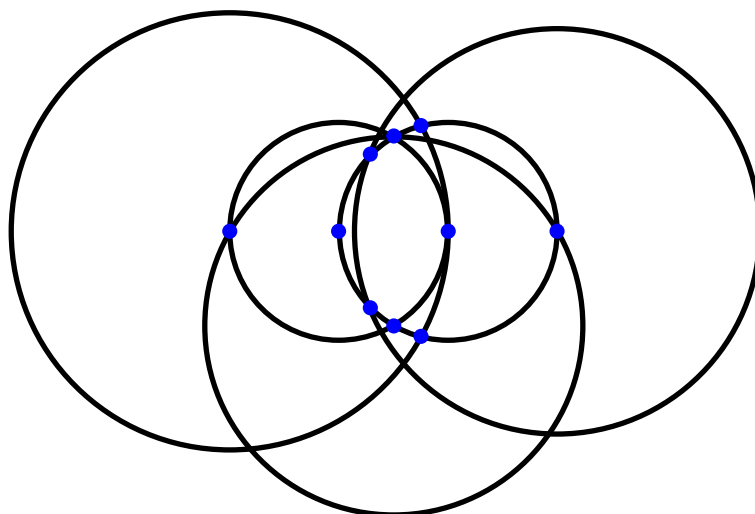
$$d_0 = |BG| = |BH| \text{ (red) } \dots 2$$

$$d_1 = |AI| = |AJ| \text{ (blue) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (4 occurrences)}$$

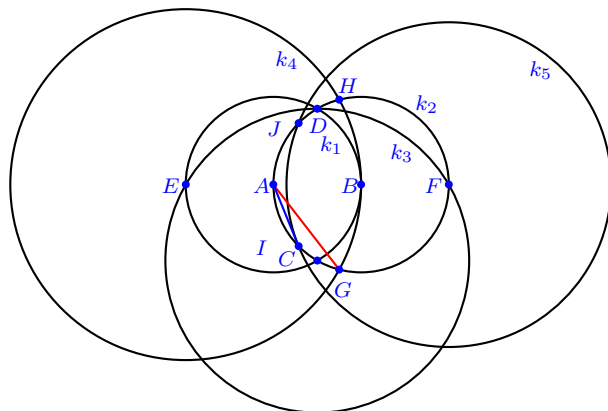
## 2.99 Class of Similar Constructions No. 90



### Contained Constructions

MM189, MM234, MM300, MM345

## 2.99.1 Construction MM189



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, |BE|)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
  5.  $k_5 = k(F, |DF|)$   
 $I \in k_2 \cap k_5, J \in k_2 \cap k_5$

## Distances

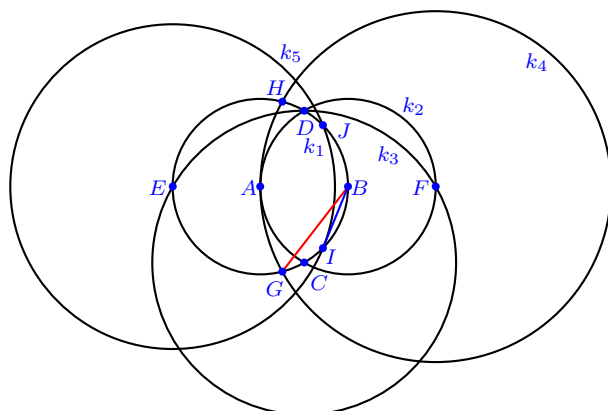
$$d_0 = |AG| = |AH| \text{ (red) } \dots 2$$

$$d_1 = |AI| = |AJ| \text{ (blue) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (4 occurrences)}$$

## 2.99.2 Construction MM234



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, |BE|)$   
 $G \in k_1 \cap k_4, H \in k_1 \cap k_4$
  5.  $k_5 = k(E, |DG|)$   
 $I \in k_1 \cap k_5, J \in k_1 \cap k_5$

## Distances

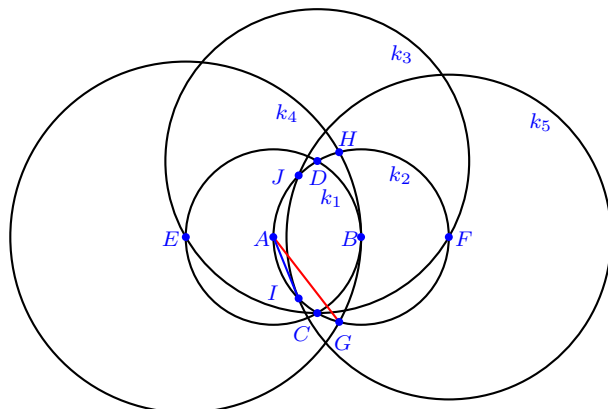
$$d_0 = |BG| = |BH| \text{ (red) } \dots 2$$

$$d_1 = |BI| = |BJ| \text{ (blue) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (4 occurrences)}$$

## 2.99.3 Construction MM300



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, |BE|)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
  5.  $k_5 = k(F, |DG|)$   
 $I \in k_2 \cap k_5, J \in k_2 \cap k_5$

## Distances

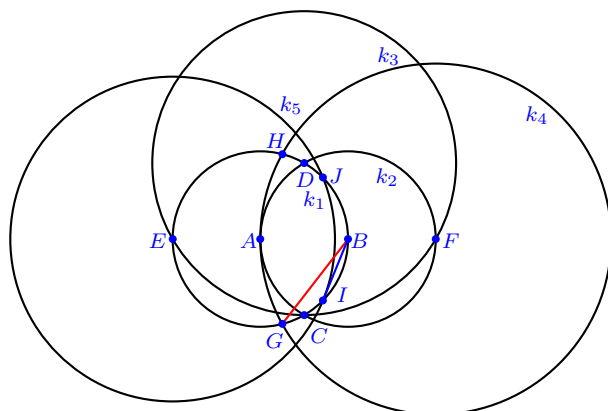
$$d_0 = |AG| = |AH| \text{ (red) } \dots 2$$

$$d_1 = |AI| = |AJ| \text{ (blue) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (4 occurrences)}$$

## 2.99.4 Construction MM345



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, |BE|)$   
 $G \in k_1 \cap k_4, H \in k_1 \cap k_4$
  5.  $k_5 = k(E, |DG|)$   
 $I \in k_1 \cap k_5, J \in k_1 \cap k_5$

## Distances

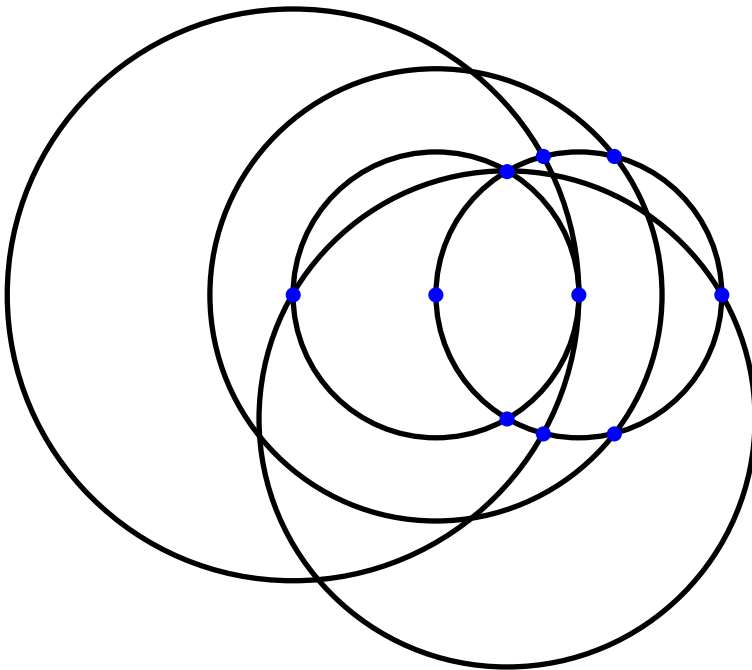
$$d_0 = |BG| = |BH| \text{ (red) } \dots 2$$

$$d_1 = |BI| = |BJ| \text{ (blue) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (4 occurrences)}$$

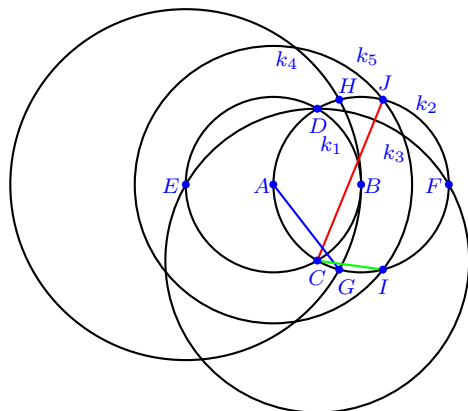
## 2.100 Class of Similar Constructions No. 91



### Contained Constructions

MM192, MM239, MM303, MM350

## 2.100.1 Construction MM192



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, |BE|)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
  5.  $k_5 = k(A, |FG|)$   
 $I \in k_2 \cap k_5, J \in k_2 \cap k_5$

## Distances

$$d_0 = |CJ| = |DI| \text{ (red)} \dots 2$$

$$d_1 = |AG| = |AH| = |FI| = |FJ| \text{ (blue)} \dots 4$$

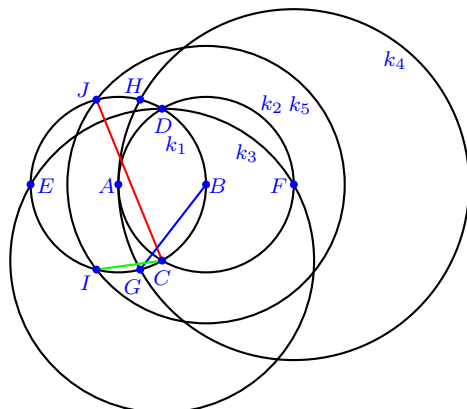
$$d_2 = |CI| = |DJ| \text{ (green)} \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (16 occurrences)}$$



## 2.100.2 Construction MM239



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, |BE|)$   
 $G \in k_1 \cap k_4, H \in k_1 \cap k_4$
  5.  $k_5 = k(B, |EG|)$   
 $I \in k_1 \cap k_5, J \in k_1 \cap k_5$

## Distances

$$d_0 = |CJ| = |DI| \text{ (red) } \dots 2$$

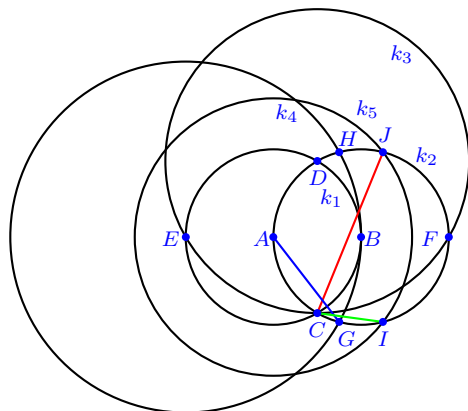
$$d_1 = |BG| = |BH| = |EI| = |EJ| \text{ (blue) } \dots 4$$

$$d_2 = |CI| = |DJ| \text{ (green) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (16 occurrences)}$$

## 2.100.3 Construction MM303



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, |BE|)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
  5.  $k_5 = k(A, |FG|)$   
 $I \in k_2 \cap k_5, J \in k_2 \cap k_5$

## Distances

$$d_0 = |CJ| = |DI| \text{ (red)} \dots 2$$

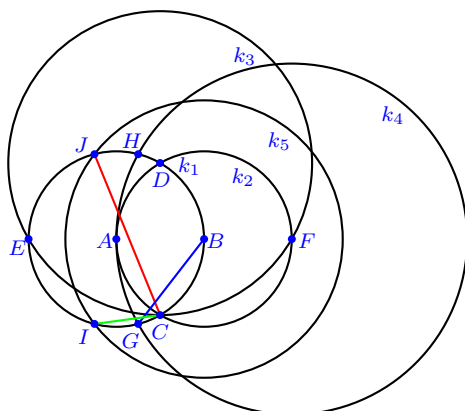
$$d_1 = |AG| = |AH| = |FI| = |FJ| \text{ (blue)} \dots 4$$

$$d_2 = |CI| = |DJ| \text{ (green)} \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (16 occurrences)}$$

## 2.100.4 Construction MM350



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, |BE|)$   
 $G \in k_1 \cap k_4, H \in k_1 \cap k_4$
  5.  $k_5 = k(B, |EG|)$   
 $I \in k_1 \cap k_5, J \in k_1 \cap k_5$

## Distances

$$d_0 = |CJ| = |DI| \text{ (red) } \dots 2$$

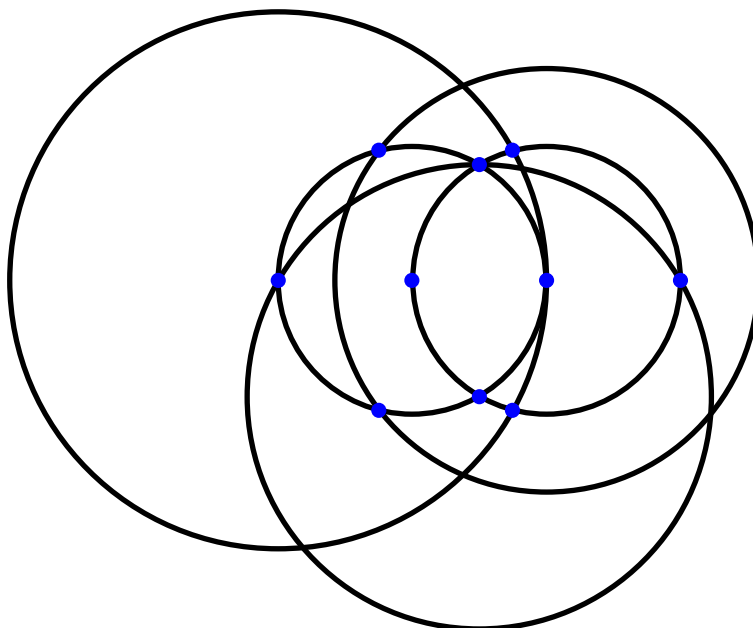
$$d_1 = |BG| = |BH| = |EI| = |EJ| \text{ (blue) } \dots 4$$

$$d_2 = |CI| = |DJ| \text{ (green) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (16 occurrences)}$$

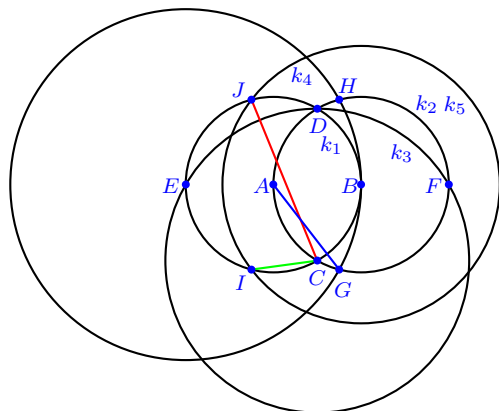
## 2.101 Class of Similar Constructions No. 92



### Contained Constructions

MM193, MM238, MM304, MM349

## 2.101.1 Construction MM193



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, |BE|)$   
 $G \in k_2 \cap k_4, H \in k_1 \cap k_4$
  5.  $k_5 = k(B, |FG|)$   
 $I \in k_1 \cap k_5, J \in k_1 \cap k_5$

## Distances

$$d_0 = |CJ| = |DI| \text{ (red)} \dots 2$$

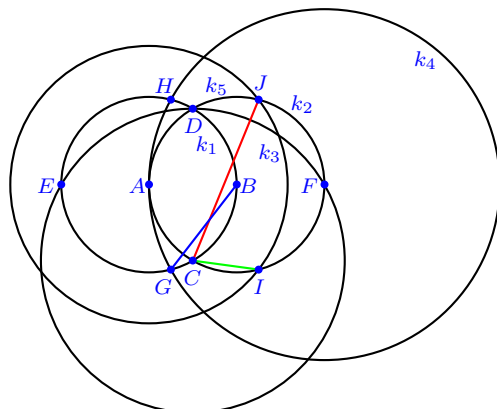
$$d_1 = |AG| = |AH| = |EI| = |EJ| \text{ (blue)} \dots 4$$

$$d_2 = |CI| = |DJ| \text{ (green)} \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (16 occurrences)}$$

## 2.101.2 Construction MM238



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, |BE|)$   
 $G \in k_1 \cap k_4, H \in k_1 \cap k_4$
  5.  $k_5 = k(A, |EG|)$   
 $I \in k_2 \cap k_5, J \in k_2 \cap k_5$

## Distances

$$d_0 = |CJ| = |DI| \text{ (red)} \dots 2$$

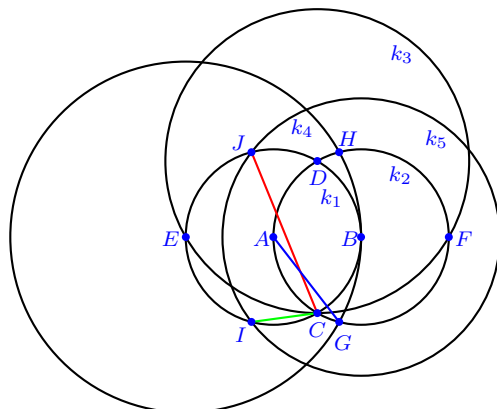
$$d_1 = |BG| = |BH| = |FI| = |FJ| \text{ (blue)} \dots 4$$

$$d_2 = |CI| = |DJ| \text{ (green)} \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (16 occurrences)}$$

## 2.101.3 Construction MM304



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, |BE|)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
  5.  $k_5 = k(B, |FG|)$   
 $I \in k_1 \cap k_5, J \in k_1 \cap k_5$

## Distances

$$d_0 = |CJ| = |DI| \text{ (red)} \dots 2$$

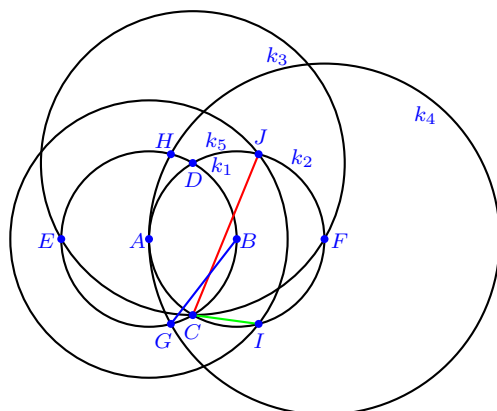
$$d_1 = |AG| = |AH| = |EI| = |EJ| \text{ (blue)} \dots 4$$

$$d_2 = |CI| = |DJ| \text{ (green)} \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (16 occurrences)}$$

## 2.101.4 Construction MM349



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, |BE|)$   
 $G \in k_1 \cap k_4, H \in k_1 \cap k_4$
  5.  $k_5 = k(A, |EG|)$   
 $I \in k_2 \cap k_5, J \in k_2 \cap k_5$

## Distances

$$d_0 = |CJ| = |DI| \text{ (red) } \dots 2$$

$$d_1 = |BG| = |BH| = |FI| = |FJ| \text{ (blue) } \dots 4$$

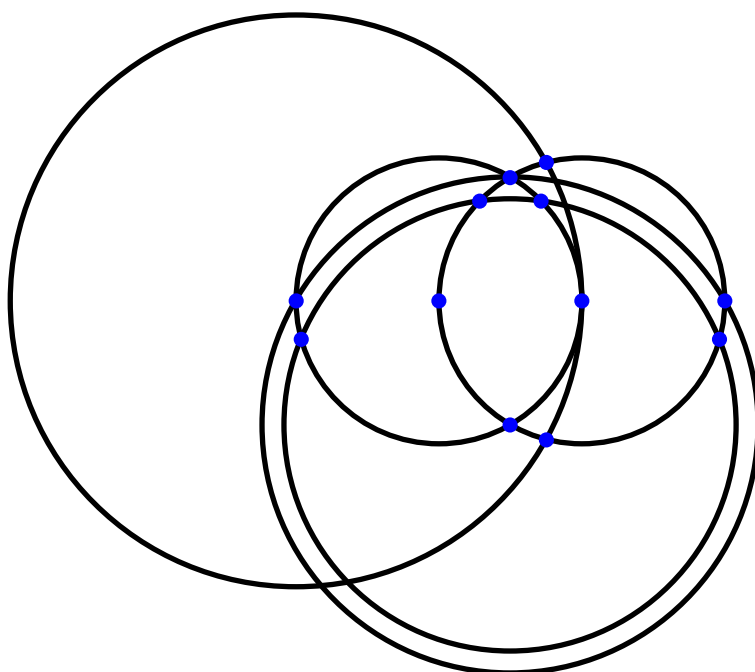
$$d_2 = |CI| = |DJ| \text{ (green) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (16 occurrences)}$$



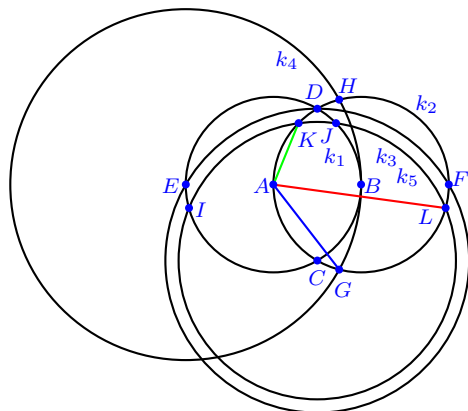
## 2.102 Class of Similar Constructions No. 93



### Contained Constructions

MM194, MM240, MM306, MM352

## 2.102.1 Construction MM194



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, |BE|)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
  5.  $k_5 = k(C, |FG|)$   
 $I \in k_1 \cap k_5, J \in k_1 \cap k_5, K \in k_2 \cap k_5, L \in k_2 \cap k_5$

## Distances

$$d_0 = |AL| = |BI| \text{ (red) } \dots 2$$

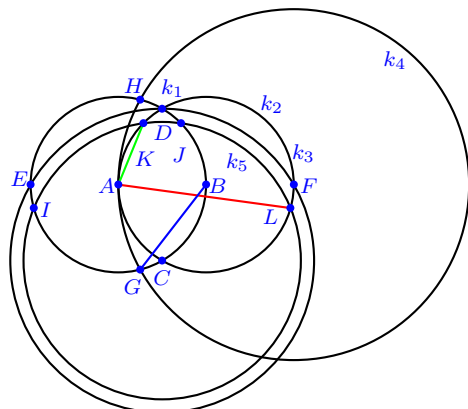
$$d_1 = |AG| = |AH| \text{ (blue) } \dots 2$$

$$d_2 = |AK| = |BJ| \text{ (green) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)}$$

## 2.102.2 Construction MM240



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, |BE|)$   
 $G \in k_1 \cap k_4, H \in k_1 \cap k_4$
  5.  $k_5 = k(C, |EG|)$   
 $I \in k_1 \cap k_5, J \in k_1 \cap k_5, K \in k_2 \cap k_5, L \in k_2 \cap k_5$

## Distances

$$d_0 = |AL| = |BI| \text{ (red)} \dots 2$$

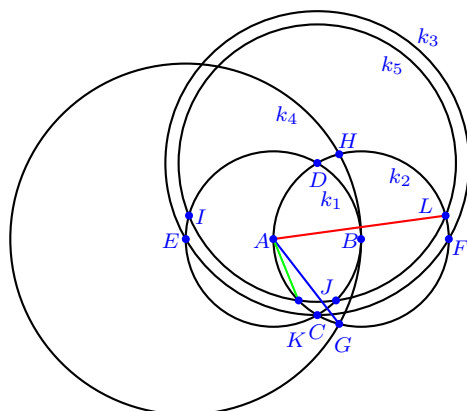
$$d_1 = |BG| = |BH| \text{ (blue)} \dots 2$$

$$d_2 = |AK| = |BJ| \text{ (green)} \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)}$$

## 2.102.3 Construction MM306



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, |BE|)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
  5.  $k_5 = k(D, |FG|)$   
 $I \in k_1 \cap k_5, J \in k_1 \cap k_5, K \in k_2 \cap k_5, L \in k_2 \cap k_5$

## Distances

$$d_0 = |AL| = |BI| \text{ (red)} \dots 2$$

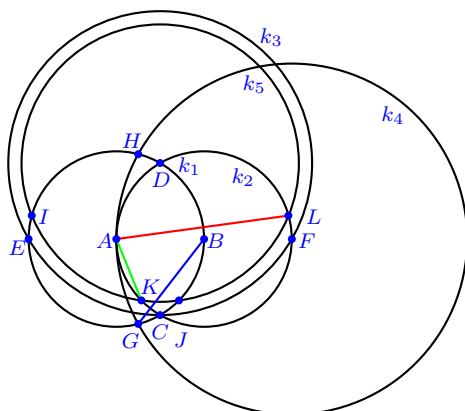
$$d_1 = |AG| = |AH| \text{ (blue)} \dots 2$$

$$d_2 = |AK| = |BJ| \text{ (green)} \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)}$$

## 2.102.4 Construction MM352



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, |BE|)$   
 $G \in k_1 \cap k_4, H \in k_1 \cap k_4$
  5.  $k_5 = k(D, |EG|)$   
 $I \in k_1 \cap k_5, J \in k_1 \cap k_5, K \in k_2 \cap k_5, L \in k_2 \cap k_5$

## Distances

$$d_0 = |AL| = |BI| \text{ (red) } \dots 2$$

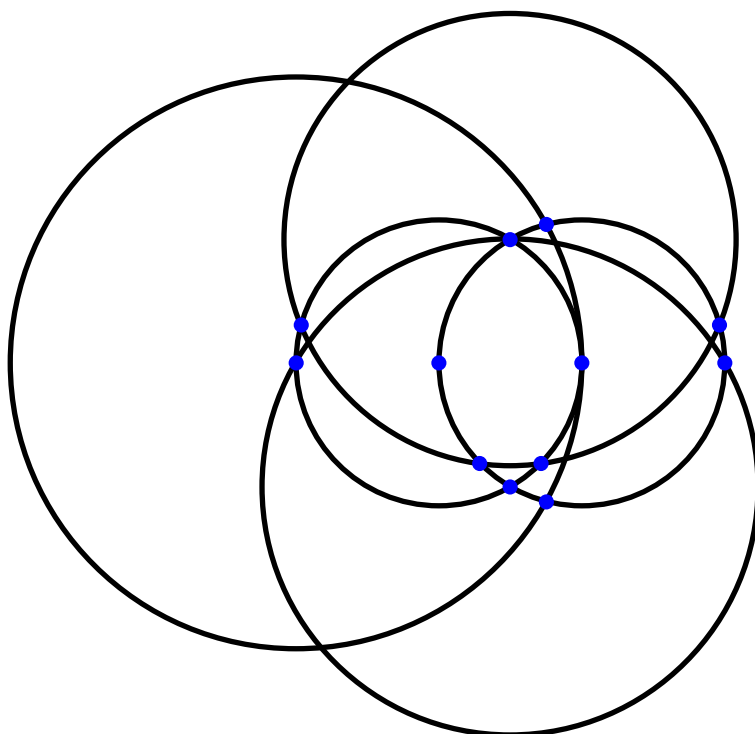
$$d_1 = |BG| = |BH| \text{ (blue) } \dots 2$$

$$d_2 = |AK| = |BJ| \text{ (green) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)}$$

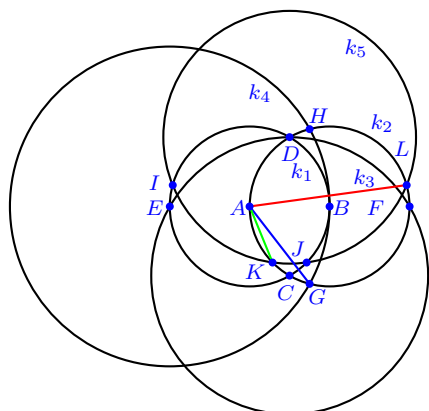
## 2.103 Class of Similar Constructions No. 94



### Contained Constructions

MM195, MM241, MM305, MM351

## 2.103.1 Construction MM195



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, |BE|)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
  5.  $k_5 = k(D, |FG|)$   
 $I \in k_1 \cap k_5, J \in k_1 \cap k_5, K \in k_2 \cap k_5, L \in k_2 \cap k_5$

## Distances

$$d_0 = |AL| = |BI| \text{ (red)} \dots 2$$

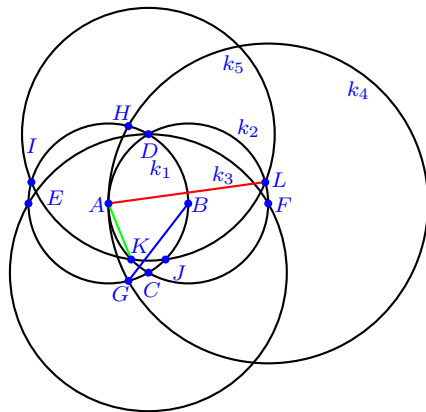
$$d_1 = |AG| = |AH| \text{ (blue)} \dots 2$$

$$d_2 = |AK| = |BJ| \text{ (green)} \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)}$$

## 2.103.2 Construction MM241



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, |BE|)$   
 $G \in k_1 \cap k_4, H \in k_1 \cap k_4$
  5.  $k_5 = k(D, |EG|)$   
 $I \in k_1 \cap k_5, J \in k_1 \cap k_5, K \in k_2 \cap k_5, L \in k_2 \cap k_5$

## Distances

$$d_0 = |AL| = |BI| \text{ (red) } \dots 2$$

$$d_1 = |BG| = |BH| \text{ (blue) } \dots 2$$

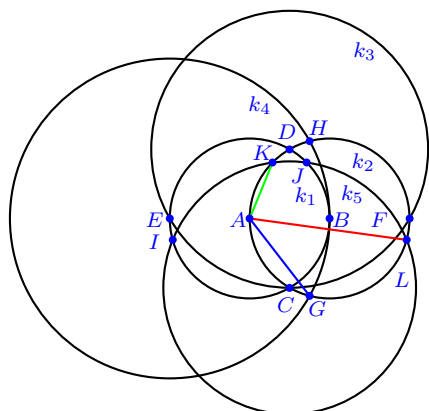
$$d_2 = |AK| = |BJ| \text{ (green) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)}$$



## 2.103.3 Construction MM305



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, |BE|)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
  5.  $k_5 = k(C, |FG|)$   
 $I \in k_1 \cap k_5, J \in k_1 \cap k_5, K \in k_2 \cap k_5, L \in k_2 \cap k_5$

## Distances

$$d_0 = |AL| = |BI| \text{ (red)} \dots 2$$

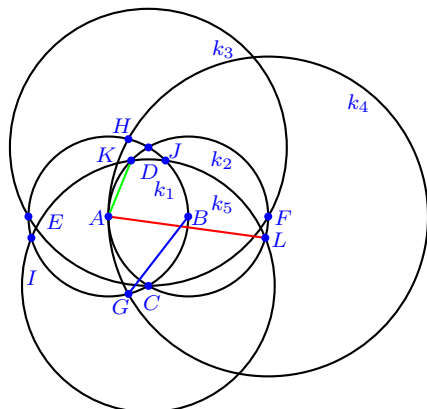
$$d_1 = |AG| = |AH| \text{ (blue)} \dots 2$$

$$d_2 = |AK| = |BJ| \text{ (green)} \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)}$$

## 2.103.4 Construction MM351



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, |BE|)$   
 $G \in k_1 \cap k_4, H \in k_1 \cap k_4$
  5.  $k_5 = k(C, |EG|)$   
 $I \in k_1 \cap k_5, J \in k_1 \cap k_5, K \in k_2 \cap k_5, L \in k_2 \cap k_5$

## Distances

$$d_0 = |AL| = |BI| \text{ (red) } \dots 2$$

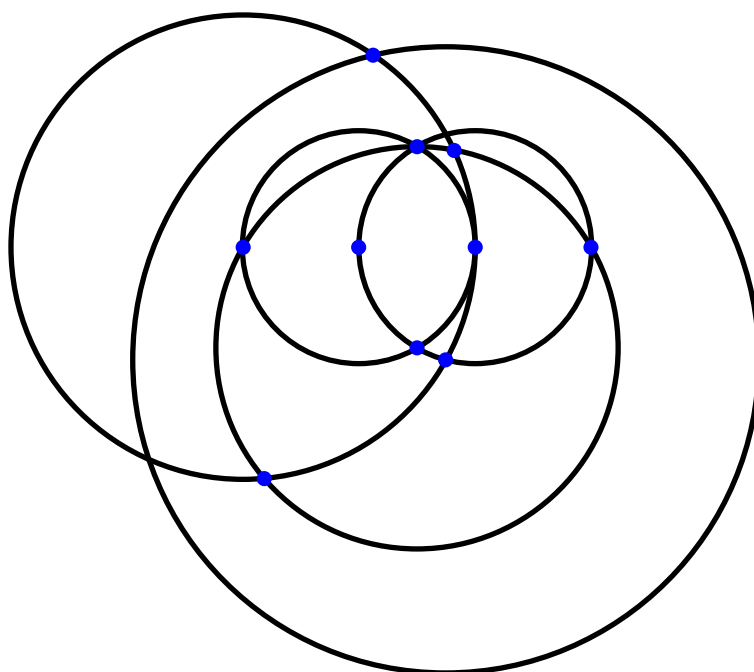
$$d_1 = |BG| = |BH| \text{ (blue) } \dots 2$$

$$d_2 = |AK| = |BJ| \text{ (green) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)}$$

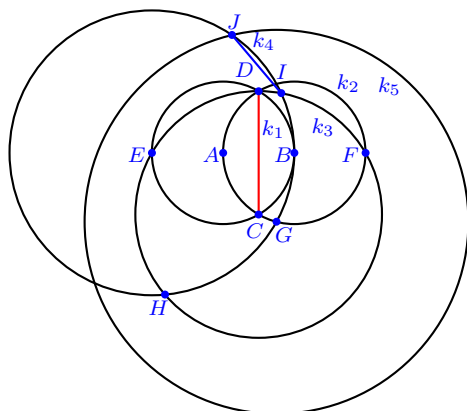
## 2.104 Class of Similar Constructions No. 95



### Contained Constructions

MM198, MM251, MM310, MM363

## 2.104.1 Construction MM198



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, |BE|)$   
 $G \in k_2 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  5.  $k_5 = k(G, |BH|)$   
 $J \in k_4 \cap k_5$

## Distances

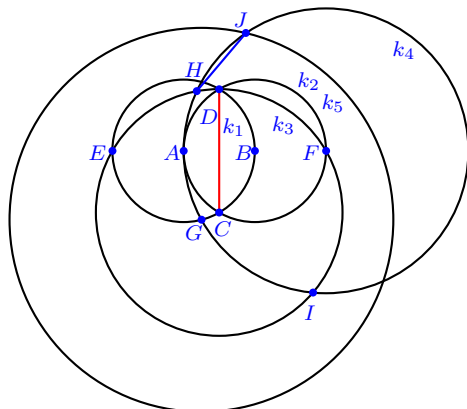
$$d_0 = |CD| = |CE| = |CF| = |CH| = |CI| = |DE| = |DF| \text{ (red)} \dots 7$$

$$d_1 = |IJ| \text{ (blue)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (7 occurrences)}$$

## 2.104.2 Construction MM251



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, |BE|)$   
 $G \in k_1 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  5.  $k_5 = k(G, |AI|)$   
 $J \in k_4 \cap k_5$

## Distances

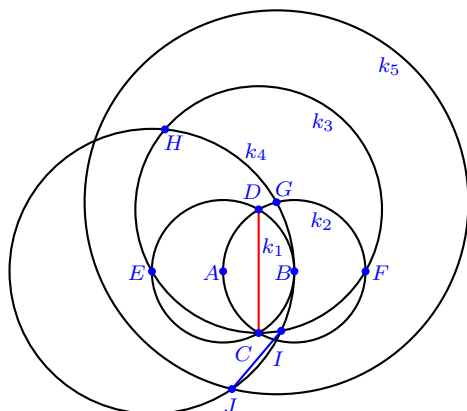
$$d_0 = |CD| = |CE| = |CF| = |CH| = |CI| = |DE| = |DF| \text{ (red)} \dots 7$$

$$d_1 = |HJ| \text{ (blue)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (7 occurrences)}$$

## 2.104.3 Construction MM310



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, |BE|)$   
 $G \in k_2 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  5.  $k_5 = k(G, |BH|)$   
 $J \in k_4 \cap k_5$

## Distances

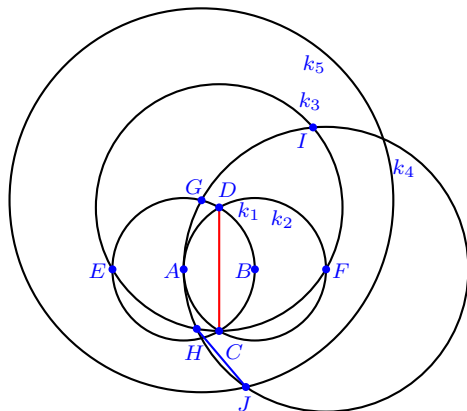
$$d_0 = |CD| = |CE| = |CF| = |DE| = |DF| = |DH| = |DI| \text{ (red) } \dots 7$$

$$d_1 = |IJ| \text{ (blue) } \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (7 occurrences)}$$

## 2.104.4 Construction MM363



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, |BE|)$   
 $G \in k_1 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  5.  $k_5 = k(G, |AI|)$   
 $J \in k_4 \cap k_5$

## Distances

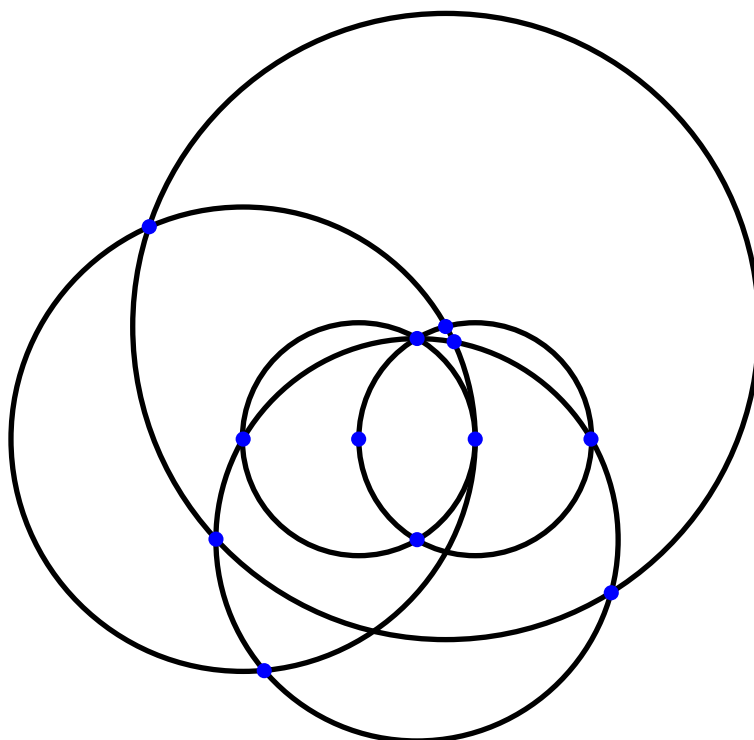
$$d_0 = |CD| = |CE| = |CF| = |DE| = |DF| = |DH| = |DI| \text{ (red)} \dots 7$$

$$d_1 = |HJ| \text{ (blue)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (7 occurrences)}$$

## 2.105 Class of Similar Constructions No. 96

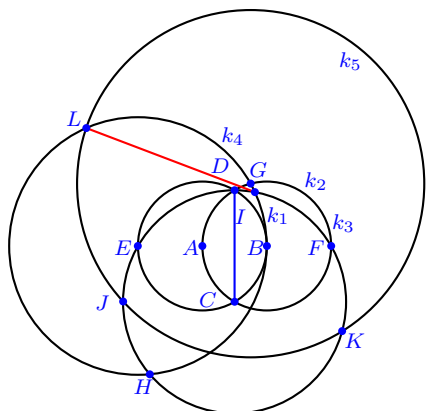


### Contained Constructions

MM199, MM252, MM309, MM362



## 2.105.1 Construction MM199



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, |BE|)$   
 $G \in k_2 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  5.  $k_5 = k(G, |BH|)$   
 $J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5$

## Distances

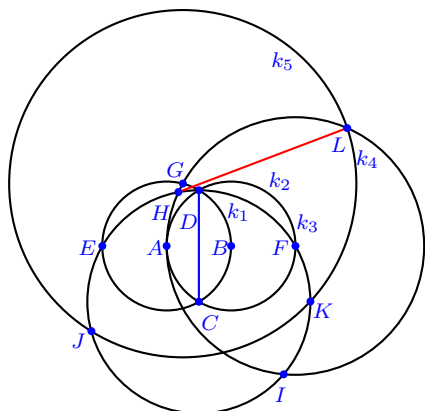
$$d_0 = |IL| \text{ (red)} \dots 1$$

$$d_1 = |CD| = |CE| = |CF| = |CH| = |CI| = |CJ| = |CK| = |DE| = |DF| \text{ (blue)} \dots 9$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (9 occurrences)}$$

## 2.105.2 Construction MM252



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, |BE|)$   
 $G \in k_1 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  5.  $k_5 = k(G, |AI|)$   
 $J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5$

## Distances

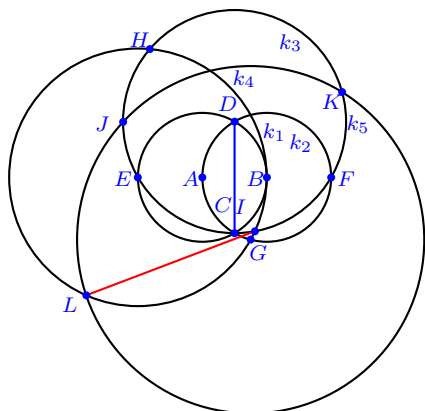
$$d_0 = |HL| \text{ (red)} \dots 1$$

$$d_1 = |CD| = |CE| = |CF| = |CH| = |CI| = |CJ| = |CK| = |DE| = |DF| \text{ (blue)} \dots 9$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (9 occurrences)}$$

## 2.105.3 Construction MM309



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, |BE|)$   
 $G \in k_2 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  5.  $k_5 = k(G, |BH|)$   
 $J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5$

## Distances

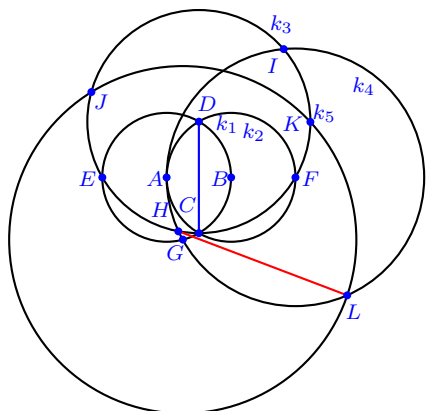
$$d_0 = |IL| \text{ (red)} \dots 1$$

$$d_1 = |CD| = |CE| = |CF| = |DE| = |DF| = |DH| = |DI| = |DJ| = |DK| \text{ (blue)} \dots 9$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (9 occurrences)}$$

## 2.105.4 Construction MM362



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, |BE|)$   
 $G \in k_1 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  5.  $k_5 = k(G, |AI|)$   
 $J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5$

## Distances

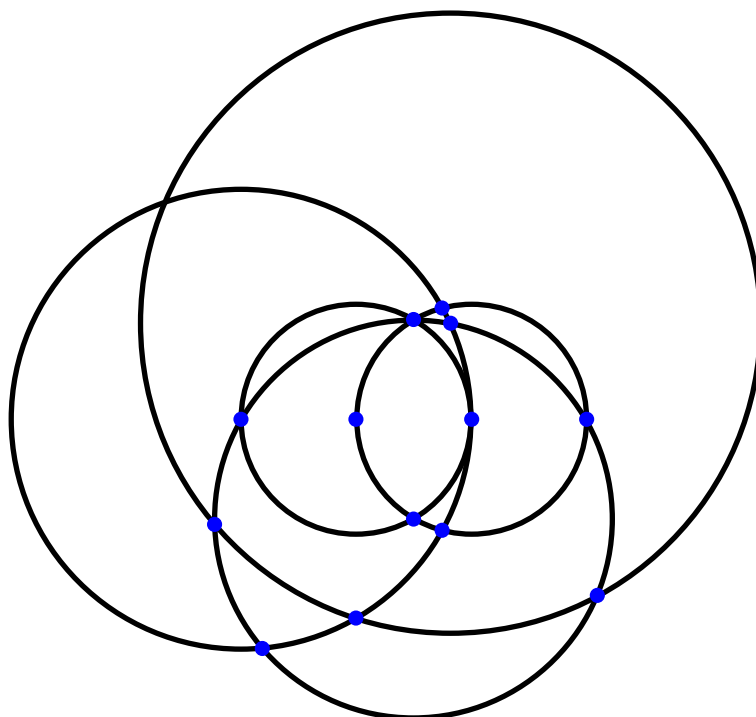
$$d_0 = |HL| \text{ (red)} \dots 1$$

$$d_1 = |CD| = |CE| = |CF| = |DE| = |DF| = |DH| = |DI| = |DJ| = |DK| \text{ (blue)} \dots 9$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (9 occurrences)}$$

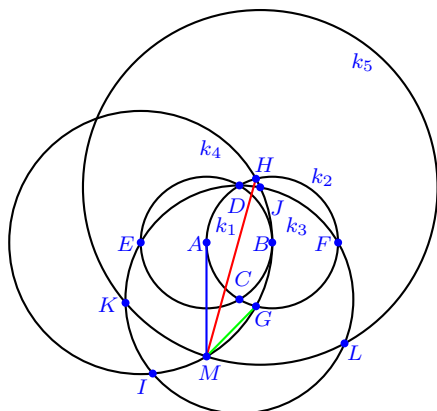
## 2.106 Class of Similar Constructions No. 97



### Contained Constructions

MM200, MM253, MM311, MM364

## 2.106.1 Construction MM200



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, |BE|)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
  5.  $k_5 = k(J, |BJ|)$   
 $K \in k_3 \cap k_5, L \in k_3 \cap k_5, M \in k_4 \cap k_5$

## Distances

$$d_0 = |HM| \text{ (red) } \dots 1$$

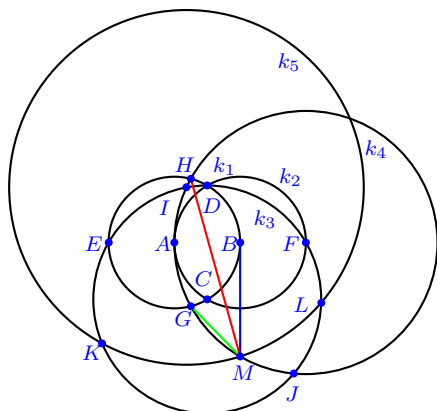
$$d_1 = |AM| = |CD| = |CE| = |CF| = |CI| = |CJ| = |CK| = |CL| = |DE| = |DF| \text{ (blue) } \dots 10$$

$$d_2 = |GM| \text{ (green) } \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (20 occurrences)}$$

## 2.106.2 Construction MM253



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, |BE|)$   
 $G \in k_1 \cap k_4, H \in k_1 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
  5.  $k_5 = k(I, |AJ|)$   
 $K \in k_3 \cap k_5, L \in k_3 \cap k_5, M \in k_4 \cap k_5$

## Distances

$$d_0 = |HM| \text{ (red)} \dots 1$$

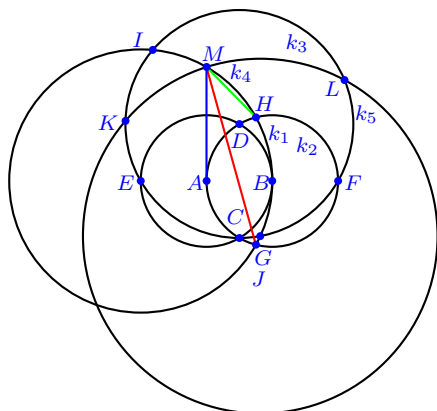
$$d_1 = |BM| = |CD| = |CE| = |CF| = |CI| = |CJ| = |CK| = |CL| = |DE| = |DF| \text{ (blue)} \dots 10$$

$$d_2 = |GM| \text{ (green)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (20 occurrences)}$$

## 2.106.3 Construction MM311



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, |BE|)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
  5.  $k_5 = k(J, |BI|)$   
 $K \in k_3 \cap k_5, L \in k_3 \cap k_5, M \in k_4 \cap k_5$

## Distances

$$d_0 = |GM| \text{ (red)} \dots 1$$

$$d_1 = |AM| = |CD| = |CE| = |CF| = |DE| = |DF| = |DI| = |DJ| = |DK| = |DL| \text{ (blue)} \dots 10$$

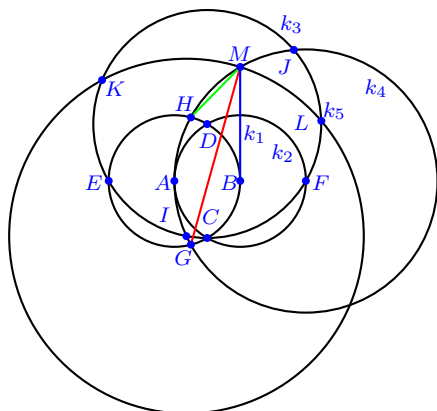
$$d_2 = |HM| \text{ (green)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (20 occurrences)}$$



## 2.106.4 Construction MM364



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, |BE|)$   
 $G \in k_1 \cap k_4, H \in k_1 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
  5.  $k_5 = k(I, |AJ|)$   
 $K \in k_3 \cap k_5, L \in k_3 \cap k_5, M \in k_4 \cap k_5$

## Distances

$$d_0 = |GM| \text{ (red)} \dots 1$$

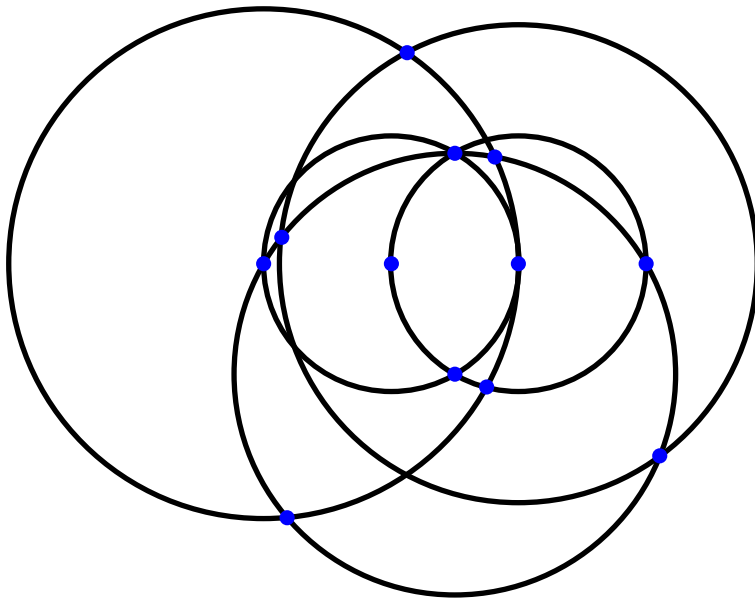
$$d_1 = |BM| = |CD| = |CE| = |CF| = |DE| = |DF| = |DI| = |DJ| = |DK| = |DL| \text{ (blue)} \dots 10$$

$$d_2 = |HM| \text{ (green)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (20 occurrences)}$$

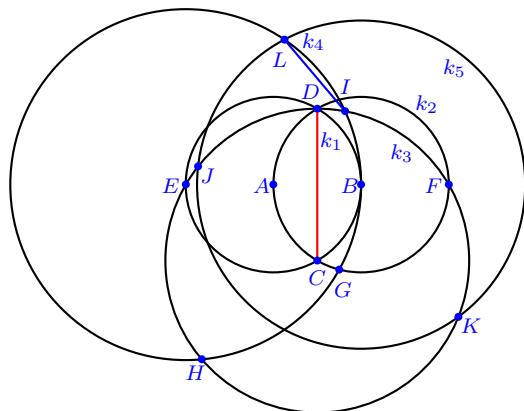
2.107 Class of Similar Constructions No. 98



**Contained Constructions**

MM201, MM254, MM314, MM367

## 2.107.1 Construction MM201



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, |BE|)$   
 $G \in k_2 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  5.  $k_5 = k(B, |GH|)$   
 $J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5$

## Distances

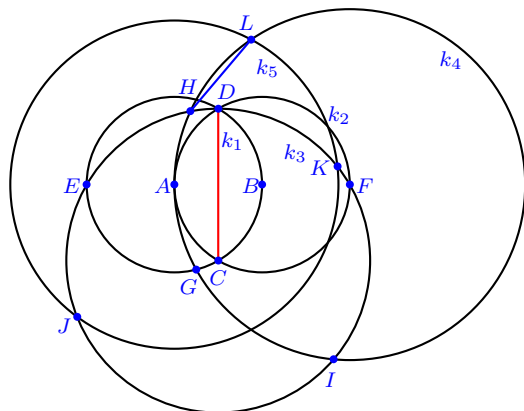
$$d_0 = |CD| = |CE| = |CF| = |CH| = |CI| = |CJ| = |CK| = |DE| = |DF| \text{ (red)} \dots 9$$

$$d_1 = |IL| \text{ (blue)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (9 occurrences)}$$

## 2.107.2 Construction MM254



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, |BE|)$   
 $G \in k_1 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  5.  $k_5 = k(A, |GI|)$   
 $J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5$

## Distances

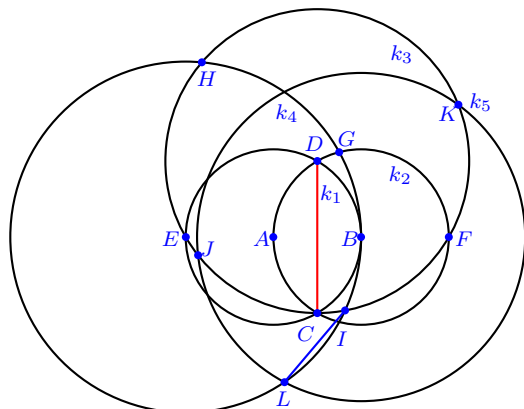
$$d_0 = |CD| = |CE| = |CF| = |CH| = |CI| = |CJ| = |CK| = |DE| = |DF| \text{ (red)} \dots 9$$

$$d_1 = |HL| \text{ (blue)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (9 occurrences)}$$

## 2.107.3 Construction MM314



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, |BE|)$   
 $G \in k_2 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  5.  $k_5 = k(B, |GH|)$   
 $J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5$

## Distances

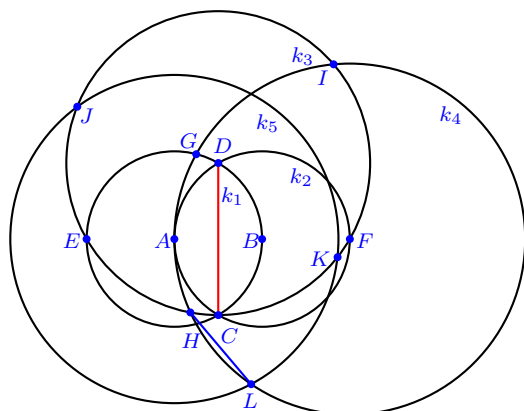
$$d_0 = |CD| = |CE| = |CF| = |DE| = |DF| = |DH| = |DI| = |DJ| = |DK| \text{ (red)} \dots 9$$

$$d_1 = |IL| \text{ (blue)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (9 occurrences)}$$

## 2.107.4 Construction MM367



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, |BE|)$   
 $G \in k_1 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  5.  $k_5 = k(A, |GI|)$   
 $J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5$

## Distances

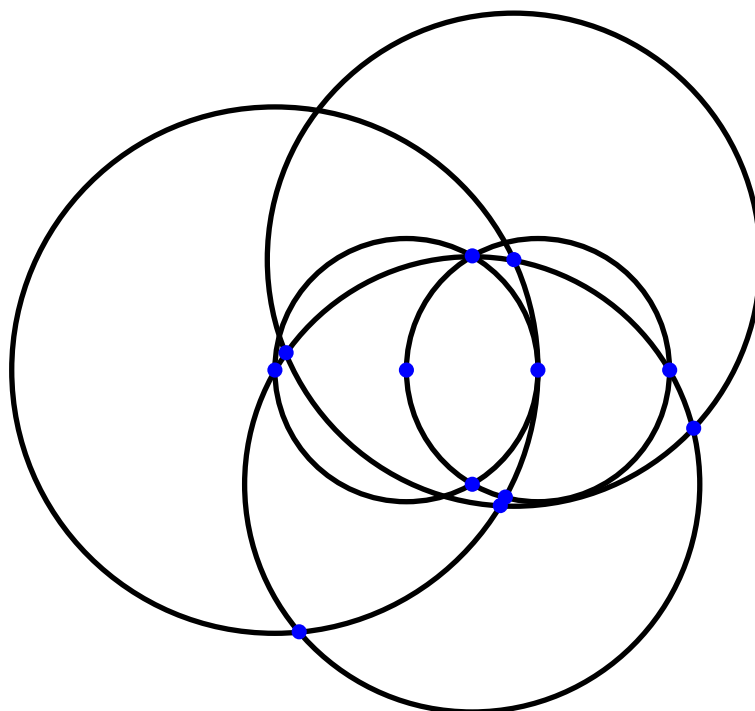
$$d_0 = |CD| = |CE| = |CF| = |DE| = |DF| = |DH| = |DI| = |DJ| = |DK| \text{ (red)} \dots 9$$

$$d_1 = |HL| \text{ (blue)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (9 occurrences)}$$

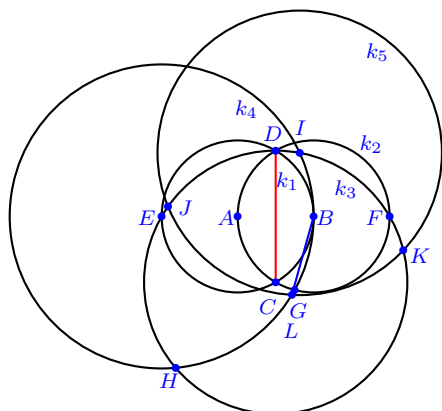
## 2.108 Class of Similar Constructions No. 99



### Contained Constructions

MM202, MM255, MM315, MM368

## 2.108.1 Construction MM202



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, |BE|)$   
 $G \in k_2 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  5.  $k_5 = k(I, |GH|)$   
 $J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5$

## Distances

$$d_0 = |CD| = |CE| = |CF| = |CH| = |CI| = |CJ| = |CK| = |DE| = |DF| \text{ (red)} \dots 9$$

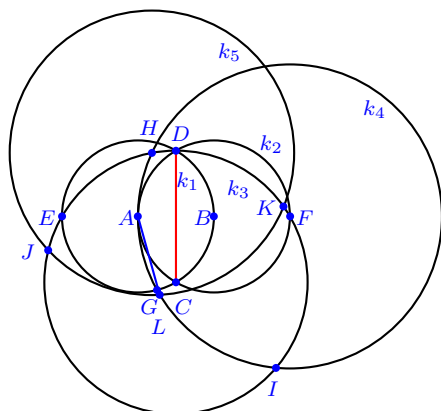
$$d_1 = |BL| \text{ (blue)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (9 occurrences)}$$



## 2.108.2 Construction MM255



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, |BE|)$   
 $G \in k_1 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  5.  $k_5 = k(H, |GI|)$   
 $J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5$

## Distances

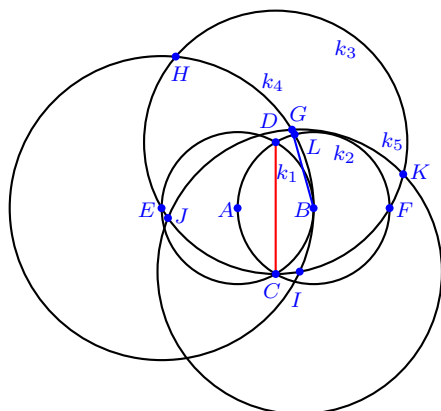
$$d_0 = |CD| = |CE| = |CF| = |CH| = |CI| = |CJ| = |CK| = |DE| = |DF| \text{ (red) } \dots 9$$

$$d_1 = |AL| \text{ (blue) } \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (9 occurrences)}$$

## 2.108.3 Construction MM315



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, |BE|)$   
 $G \in k_2 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  5.  $k_5 = k(I, |GH|)$   
 $J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5$

## Distances

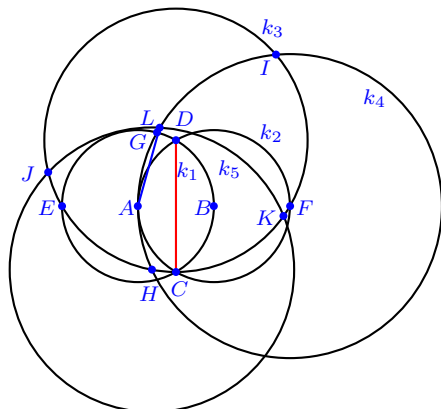
$$d_0 = |CD| = |CE| = |CF| = |DE| = |DF| = |DH| = |DI| = |DJ| = |DK| \text{ (red)} \dots 9$$

$$d_1 = |BL| \text{ (blue)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (9 occurrences)}$$

## 2.108.4 Construction MM368



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, |BE|)$   
 $G \in k_1 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  5.  $k_5 = k(H, |GI|)$   
 $J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5$

## Distances

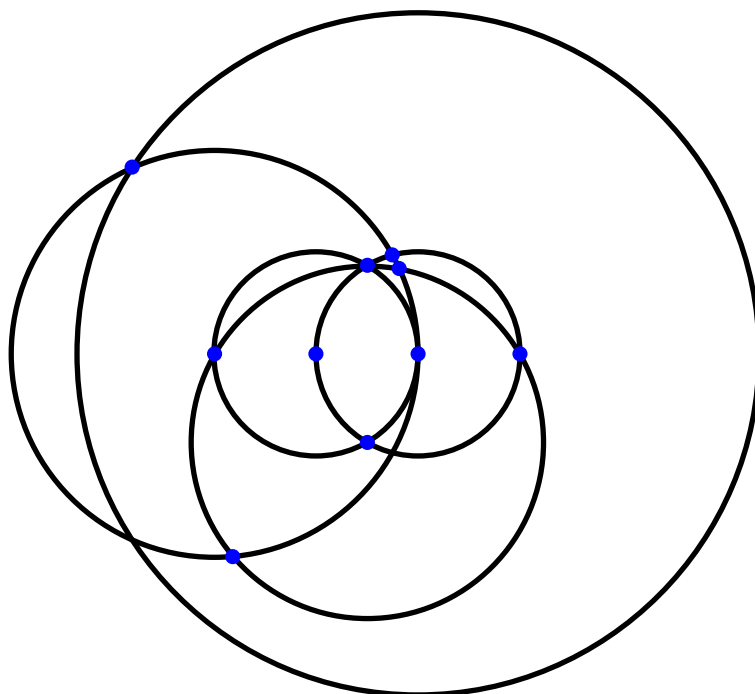
$$d_0 = |CD| = |CE| = |CF| = |DE| = |DF| = |DH| = |DI| = |DJ| = |DK| \text{ (red)} \dots 9$$

$$d_1 = |AL| \text{ (blue)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (9 occurrences)}$$

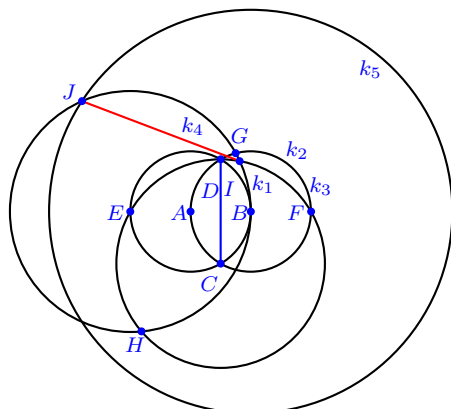
## 2.109 Class of Similar Constructions No. 100



### Contained Constructions

MM203, MM256, MM312, MM365

## 2.109.1 Construction MM203



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, |BE|)$   
 $G \in k_2 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  5.  $k_5 = k(B, |GH|)$   
 $J \in k_4 \cap k_5$

## Distances

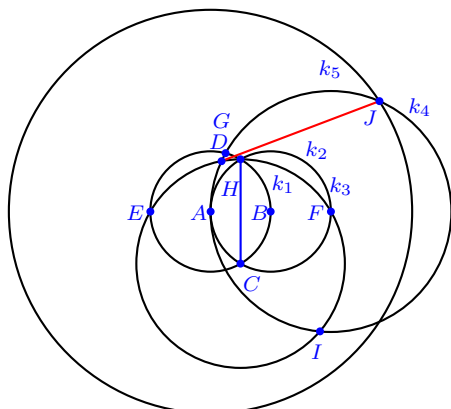
$$d_0 = |IJ| \text{ (red)} \dots 1$$

$$d_1 = |CD| = |CE| = |CF| = |CH| = |CI| = |DE| = |DF| \text{ (blue)} \dots 7$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (7 occurrences)}$$

## 2.109.2 Construction MM256



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, |BE|)$   
 $G \in k_1 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  5.  $k_5 = k(A, |GI|)$   
 $J \in k_4 \cap k_5$

## Distances

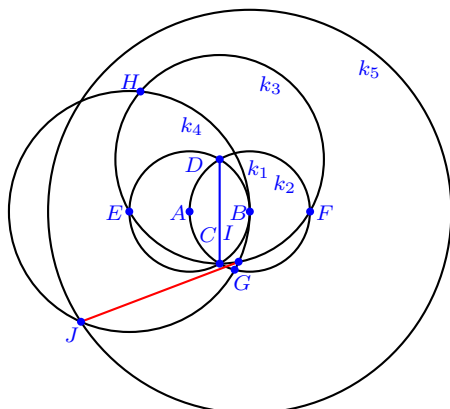
$$d_0 = |HJ| \text{ (red)} \dots 1$$

$$d_1 = |CD| = |CE| = |CF| = |CH| = |CI| = |DE| = |DF| \text{ (blue)} \dots 7$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (7 occurrences)}$$

## 2.109.3 Construction MM312



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, |BE|)$   
 $G \in k_2 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  5.  $k_5 = k(B, |GH|)$   
 $J \in k_4 \cap k_5$

## Distances

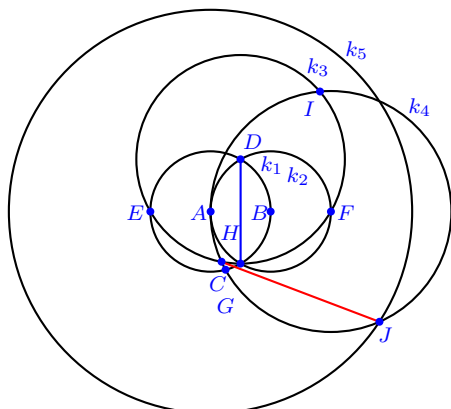
$$d_0 = |IJ| \text{ (red)} \dots 1$$

$$d_1 = |CD| = |CE| = |CF| = |DE| = |DF| = |DH| = |DI| \text{ (blue)} \dots 7$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (7 occurrences)}$$

## 2.109.4 Construction MM365



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, |BE|)$   
 $G \in k_1 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  5.  $k_5 = k(A, |GI|)$   
 $J \in k_4 \cap k_5$

## Distances

$$d_0 = |HJ| \text{ (red)} \dots 1$$

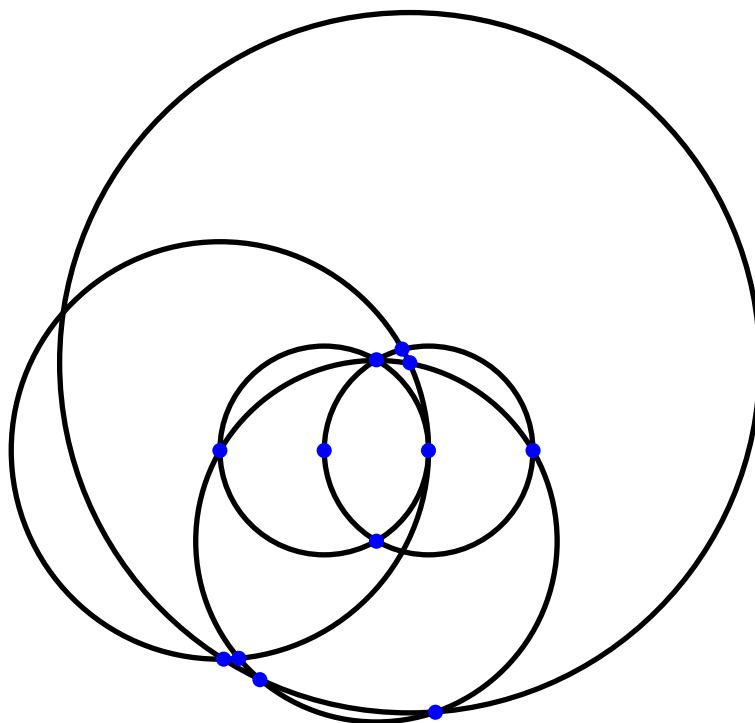
$$d_1 = |CD| = |CE| = |CF| = |DE| = |DF| = |DH| = |DI| \text{ (blue)} \dots 7$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (7 occurrences)}$$



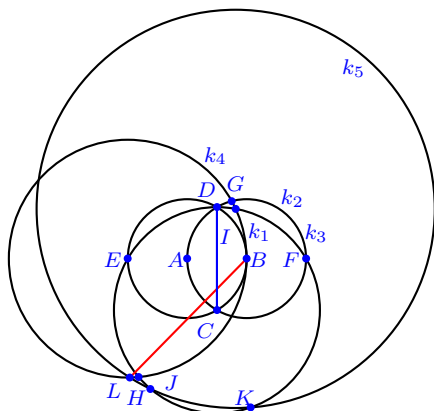
## 2.110 Class of Similar Constructions No. 101



### Contained Constructions

MM204, MM257, MM313, MM366

## 2.110.1 Construction MM204



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, |BE|)$   
 $G \in k_2 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  5.  $k_5 = k(I, |GH|)$   
 $J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5$

## Distances

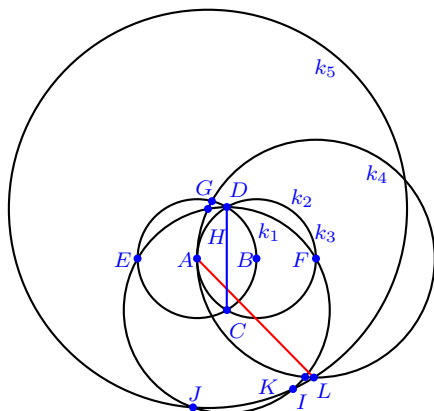
$$d_0 = |BL| \text{ (red)} \dots 1$$

$$d_1 = |CD| = |CE| = |CF| = |CH| = |CI| = |CJ| = |CK| = |DE| = |DF| \text{ (blue)} \dots 9$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (9 occurrences)}$$

## 2.110.2 Construction MM257



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, |BE|)$   
 $G \in k_1 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  5.  $k_5 = k(H, |GI|)$   
 $J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5$

## Distances

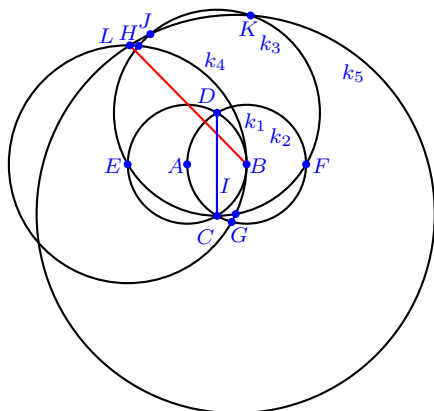
$$d_0 = |AL| \text{ (red)} \dots 1$$

$$d_1 = |CD| = |CE| = |CF| = |CH| = |CI| = |CJ| = |CK| = |DE| = |DF| \text{ (blue)} \dots 9$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (9 occurrences)}$$

## 2.110.3 Construction MM313



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, |BE|)$   
 $G \in k_2 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  5.  $k_5 = k(I, |GH|)$   
 $J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5$

## Distances

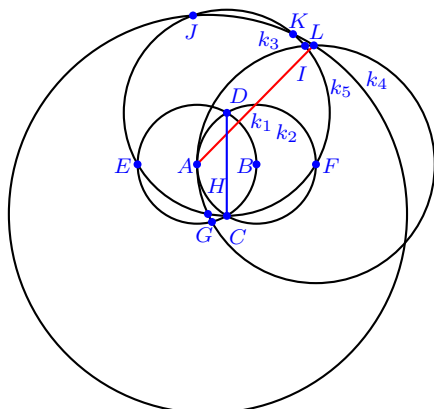
$$d_0 = |BL| \text{ (red)} \dots 1$$

$$d_1 = |CD| = |CE| = |CF| = |DE| = |DF| = |DH| = |DI| = |DJ| = |DK| \text{ (blue)} \dots 9$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (9 occurrences)}$$

## 2.110.4 Construction MM366



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, |BE|)$   
 $G \in k_1 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  5.  $k_5 = k(H, |GI|)$   
 $J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5$

## Distances

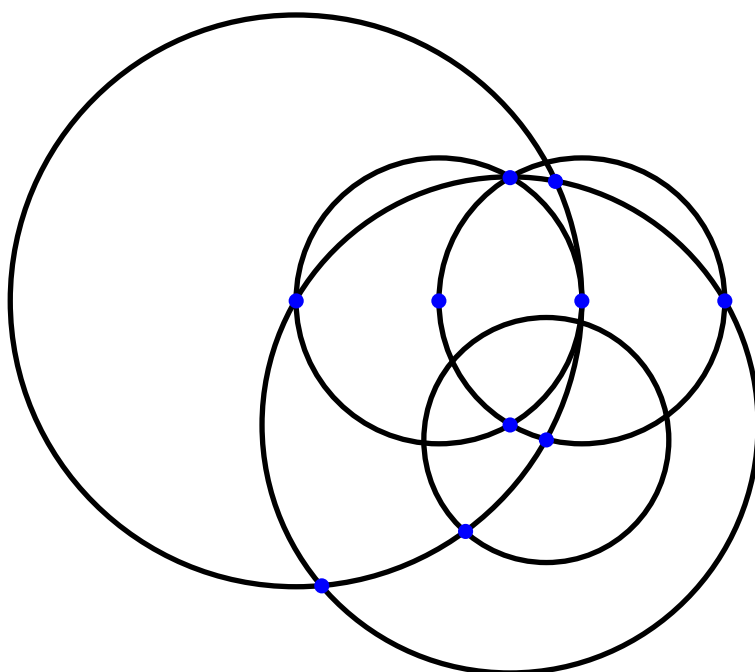
$$d_0 = |AL| \text{ (red)} \dots 1$$

$$d_1 = |CD| = |CE| = |CF| = |DE| = |DF| = |DH| = |DI| = |DJ| = |DK| \text{ (blue)} \dots 9$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (9 occurrences)}$$

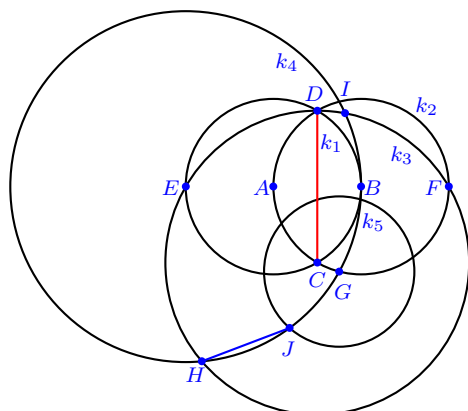
## 2.111 Class of Similar Constructions No. 102



### Contained Constructions

MM205, MM244, MM317, MM356

## 2.111.1 Construction MM205



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, |BE|)$   
 $G \in k_2 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  5.  $k_5 = k(G, |BI|)$   
 $J \in k_4 \cap k_5$

## Distances

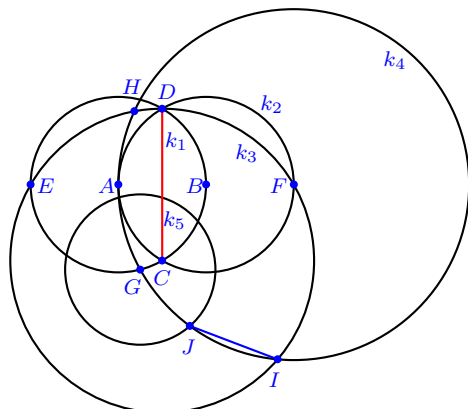
$$d_0 = |CD| = |CE| = |CF| = |CH| = |CI| = |DE| = |DF| \text{ (red) } \dots 7$$

$$d_1 = |HJ| \text{ (blue) } \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (7 occurrences)}$$

## 2.111.2 Construction MM244



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, |BE|)$   
 $G \in k_1 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  5.  $k_5 = k(G, |AH|)$   
 $J \in k_4 \cap k_5$

## Distances

$$d_0 = |CD| = |CE| = |CF| = |CH| = |CI| = |DE| = |DF| \text{ (red) } \dots 7$$

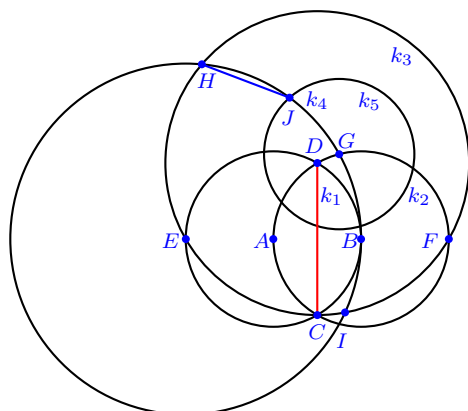
$$d_1 = |IJ| \text{ (blue) } \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (7 occurrences)}$$



## 2.111.3 Construction MM317



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, |BE|)$   
 $G \in k_2 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  5.  $k_5 = k(G, |BI|)$   
 $J \in k_4 \cap k_5$

## Distances

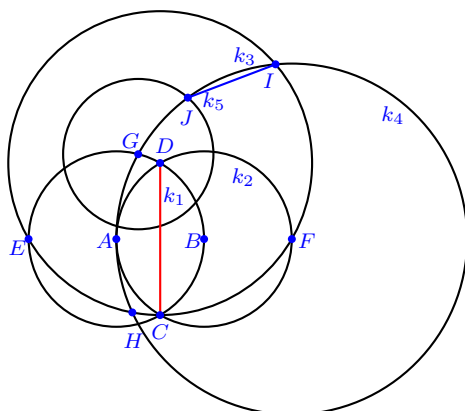
$$d_0 = |CD| = |CE| = |CF| = |DE| = |DF| = |DH| = |DI| \text{ (red)} \dots 7$$

$$d_1 = |HJ| \text{ (blue)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (7 occurrences)}$$

## 2.111.4 Construction MM356



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, |BE|)$   
 $G \in k_1 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  5.  $k_5 = k(G, |AH|)$   
 $J \in k_4 \cap k_5$

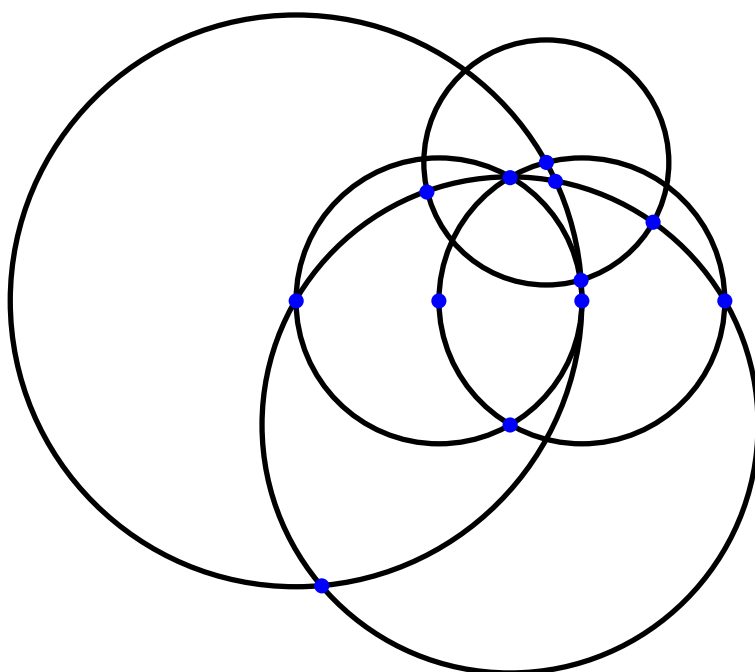
## Distances

$$d_0 = |CD| = |CE| = |CF| = |DE| = |DF| = |DH| = |DI| \text{ (red)} \dots 7$$

$$d_1 = |IJ| \text{ (blue)} \dots 1$$

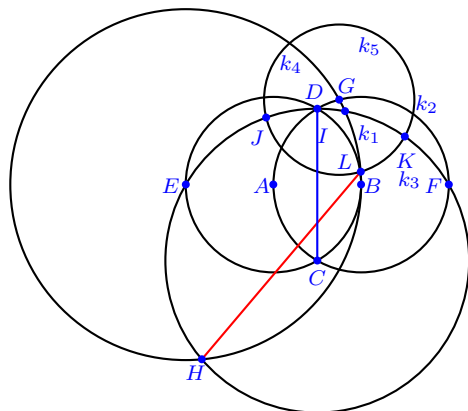
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (7 occurrences)}$$

**2.112 Class of Similar Constructions No. 103****Contained Constructions**

MM206, MM245, MM316, MM355

## 2.112.1 Construction MM206



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, |BE|)$   
 $G \in k_2 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  5.  $k_5 = k(G, |BI|)$   
 $J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5$

## Distances

$$d_0 = |HL| \text{ (red)} \dots 1$$

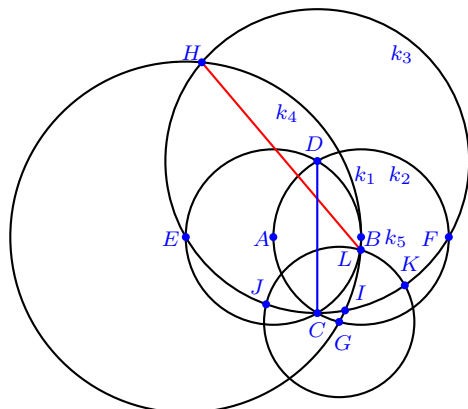
$$d_1 = |CD| = |CE| = |CF| = |CH| = |CI| = |CJ| = |CK| = |DE| = |DF| \text{ (blue)} \dots 9$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (9 occurrences)}$$



## 2.112.3 Construction MM316



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, |BE|)$   
 $G \in k_2 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  5.  $k_5 = k(G, |BI|)$   
 $J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5$

## Distances

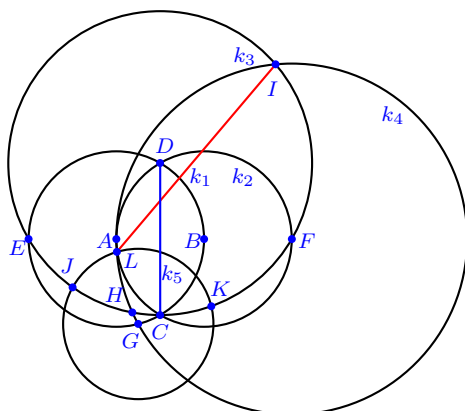
$$d_0 = |HL| \text{ (red)} \dots 1$$

$$d_1 = |CD| = |CE| = |CF| = |DE| = |DF| = |DH| = |DI| = |DJ| = |DK| \text{ (blue)} \dots 9$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (9 occurrences)}$$

## 2.112.4 Construction MM355



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, |BE|)$   
 $G \in k_1 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  5.  $k_5 = k(G, |AH|)$   
 $J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5$

## Distances

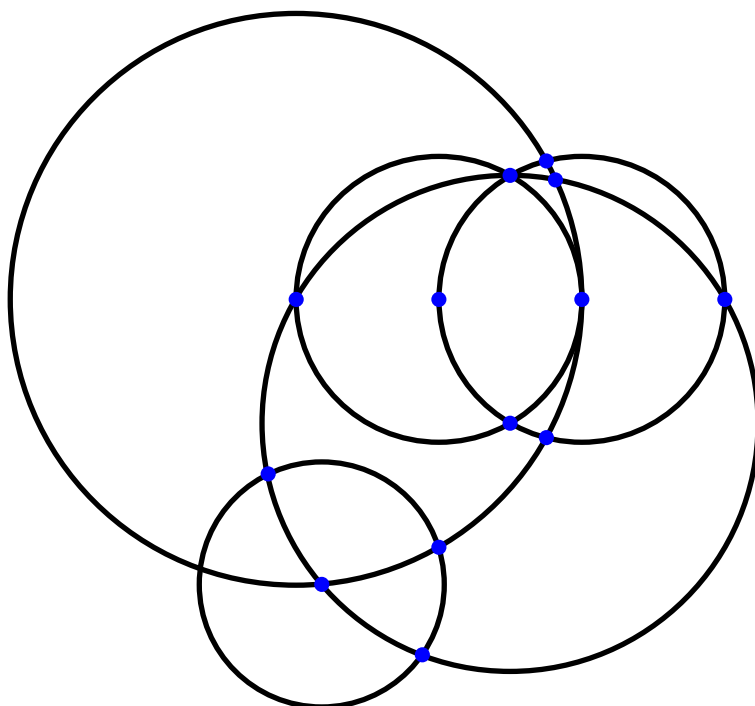
$$d_0 = |IL| \text{ (red)} \dots 1$$

$$d_1 = |CD| = |CE| = |CF| = |DE| = |DF| = |DH| = |DI| = |DJ| = |DK| \text{ (blue)} \dots 9$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (9 occurrences)}$$

### 2.113 Class of Similar Constructions No. 104

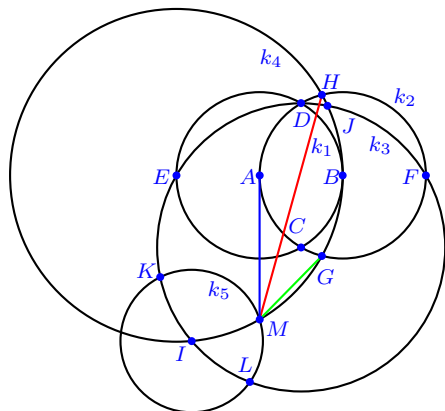


#### Contained Constructions

MM207, MM246, MM318, MM357



## 2.113.1 Construction MM207



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, |BE|)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
  5.  $k_5 = k(I, |BJ|)$   
 $K \in k_3 \cap k_5, L \in k_3 \cap k_5, M \in k_4 \cap k_5$

## Distances

$$d_0 = |HM| \text{ (red)} \dots 1$$

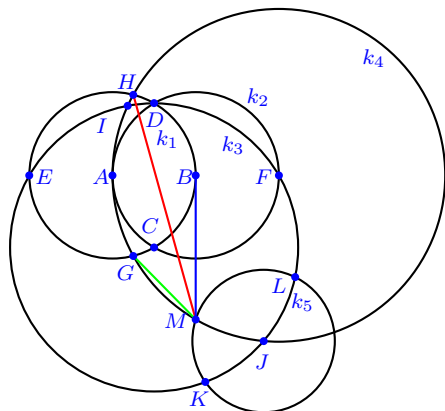
$$d_1 = |AM| = |CD| = |CE| = |CF| = |CI| = |CJ| = |CK| = |CL| = |DE| = |DF| \text{ (blue)} \dots 10$$

$$d_2 = |GM| \text{ (green)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (20 occurrences)}$$

## 2.113.2 Construction MM246



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, |BE|)$   
 $G \in k_1 \cap k_4, H \in k_1 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
  5.  $k_5 = k(J, |AI|)$   
 $K \in k_3 \cap k_5, L \in k_3 \cap k_5, M \in k_4 \cap k_5$

## Distances

$$d_0 = |HM| \text{ (red)} \dots 1$$

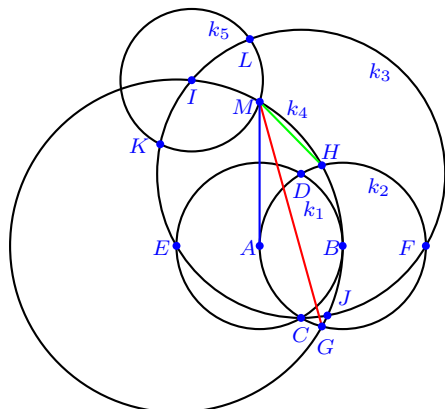
$$d_1 = |BM| = |CD| = |CE| = |CF| = |CI| = |CJ| = |CK| = |CL| = |DE| = |DF| \text{ (blue)} \dots 10$$

$$d_2 = |GM| \text{ (green)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (20 occurrences)}$$

## 2.113.3 Construction MM318



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, |BE|)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
  5.  $k_5 = k(I, |BJ|)$   
 $K \in k_3 \cap k_5, L \in k_3 \cap k_5, M \in k_4 \cap k_5$

## Distances

$$d_0 = |GM| \text{ (red)} \dots 1$$

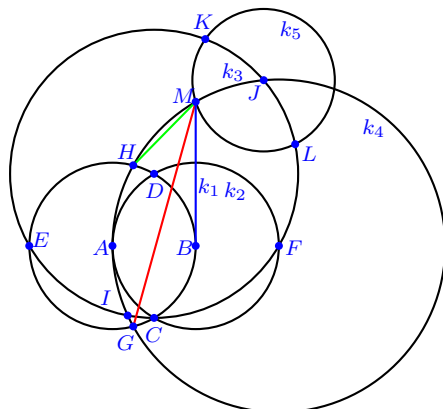
$$d_1 = |AM| = |CD| = |CE| = |CF| = |DE| = |DF| = |DI| = |DJ| = |DK| = |DL| \text{ (blue)} \dots 10$$

$$d_2 = |HM| \text{ (green)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (20 occurrences)}$$

## 2.113.4 Construction MM357



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, |BE|)$   
 $G \in k_1 \cap k_4, H \in k_1 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
  5.  $k_5 = k(J, |AI|)$   
 $K \in k_3 \cap k_5, L \in k_3 \cap k_5, M \in k_4 \cap k_5$

## Distances

$$d_0 = |GM| \text{ (red)} \dots 1$$

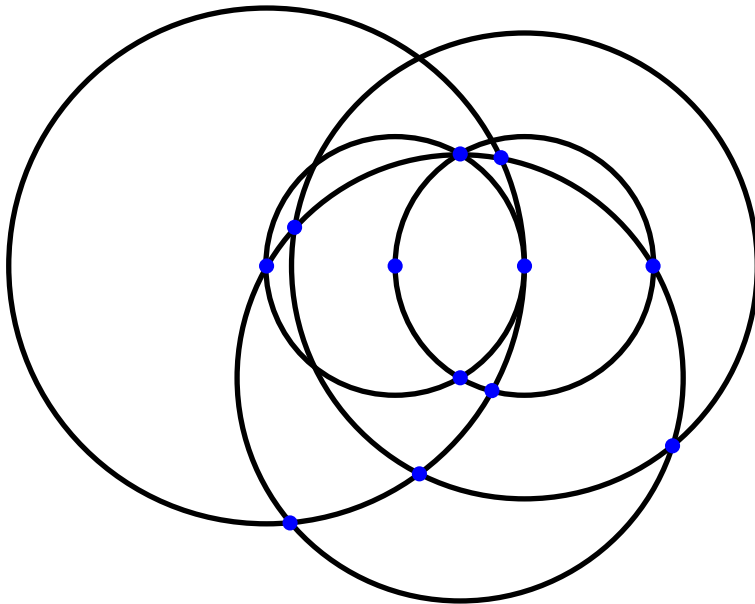
$$d_1 = |BM| = |CD| = |CE| = |CF| = |DE| = |DF| = |DI| = |DJ| = |DK| = |DL| \text{ (blue)} \dots 10$$

$$d_2 = |HM| \text{ (green)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (20 occurrences)}$$

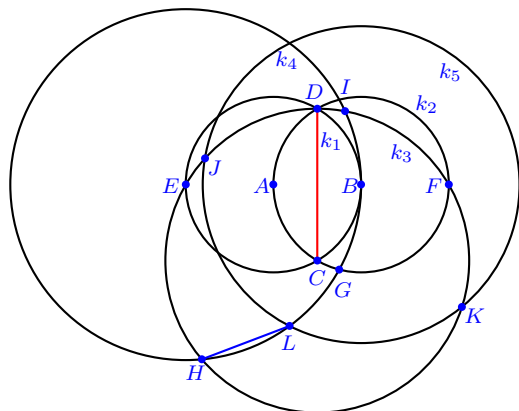
## 2.114 Class of Similar Constructions No. 105



### Contained Constructions

MM208, MM247, MM321, MM360

## 2.114.1 Construction MM208



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, |BE|)$   
 $G \in k_2 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  5.  $k_5 = k(B, |GI|)$   
 $J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5$

## Distances

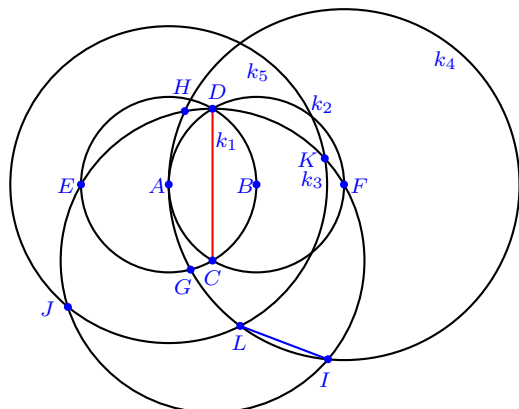
$$d_0 = |CD| = |CE| = |CF| = |CH| = |CI| = |CJ| = |CK| = |DE| = |DF| \text{ (red)} \dots 9$$

$$d_1 = |HL| \text{ (blue)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (9 occurrences)}$$

## 2.114.2 Construction MM247



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, |BE|)$   
 $G \in k_1 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  5.  $k_5 = k(A, |GH|)$   
 $J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5$

## Distances

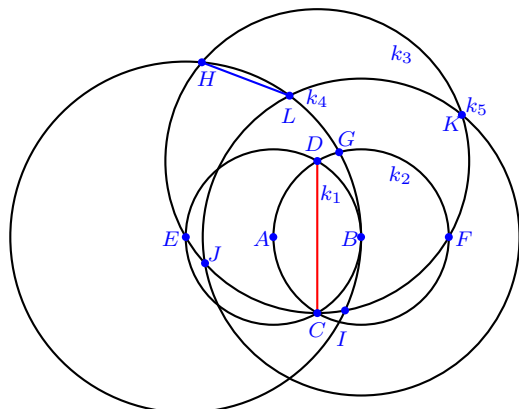
$$d_0 = |CD| = |CE| = |CF| = |CH| = |CI| = |CJ| = |CK| = |DE| = |DF| \text{ (red)} \dots 9$$

$$d_1 = |IL| \text{ (blue)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (9 occurrences)}$$

## 2.114.3 Construction MM321



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, |BE|)$   
 $G \in k_2 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  5.  $k_5 = k(B, |GI|)$   
 $J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5$

## Distances

$$d_0 = |CD| = |CE| = |CF| = |DE| = |DF| = |DH| = |DI| = |DJ| = |DK| \text{ (red)} \dots 9$$

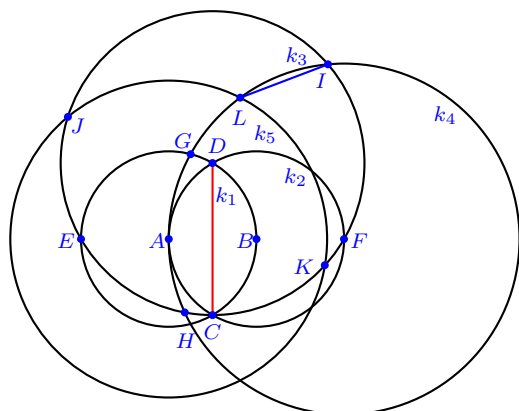
$$d_1 = |HL| \text{ (blue)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (9 occurrences)}$$



## 2.114.4 Construction MM360



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, |BE|)$   
 $G \in k_1 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  5.  $k_5 = k(A, |GH|)$   
 $J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5$

## Distances

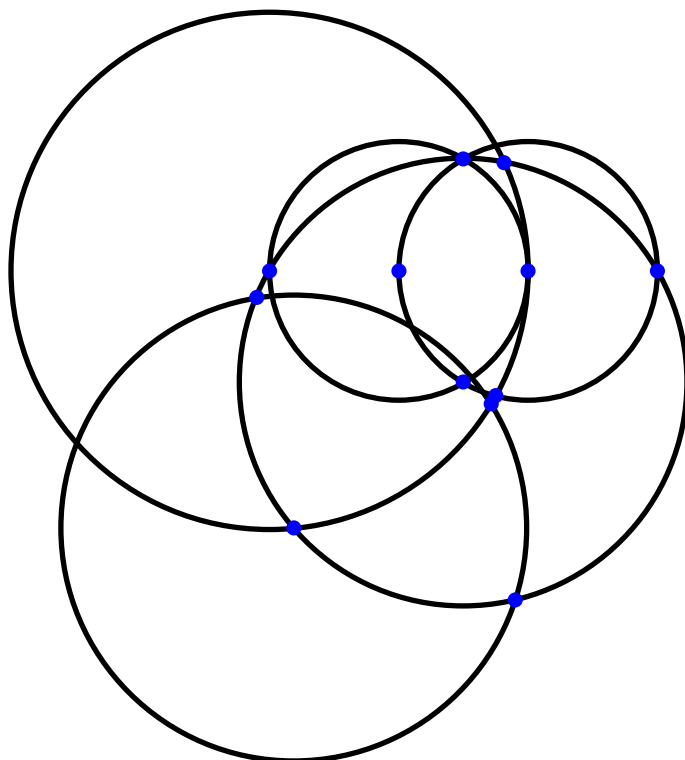
$$d_0 = |CD| = |CE| = |CF| = |DE| = |DF| = |DH| = |DI| = |DJ| = |DK| \text{ (red)} \dots 9$$

$$d_1 = |IL| \text{ (blue)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (9 occurrences)}$$

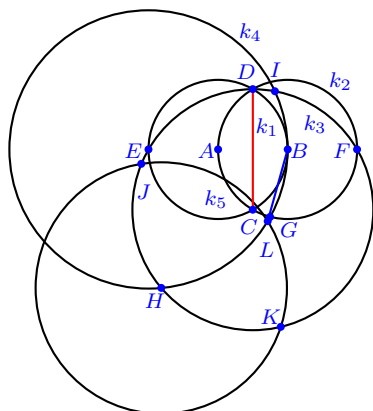
## 2.115 Class of Similar Constructions No. 106



### Contained Constructions

MM209, MM248, MM322, MM361

## 2.115.1 Construction MM209



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, |BE|)$   
 $G \in k_2 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  5.  $k_5 = k(H, |GI|)$   
 $J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5$

## Distances

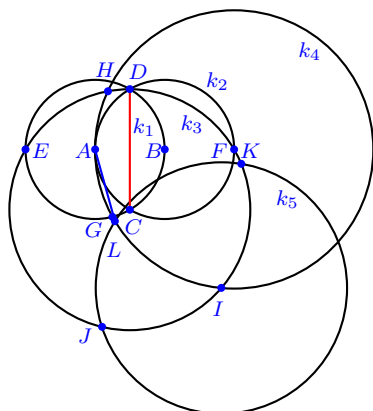
$$d_0 = |CD| = |CE| = |CF| = |CH| = |CI| = |CJ| = |CK| = |DE| = |DF| \text{ (red) } \dots 9$$

$$d_1 = |BL| \text{ (blue) } \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (9 occurrences)}$$

## 2.115.2 Construction MM248



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, |BE|)$   
 $G \in k_1 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  5.  $k_5 = k(I, |GH|)$   
 $J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5$

## Distances

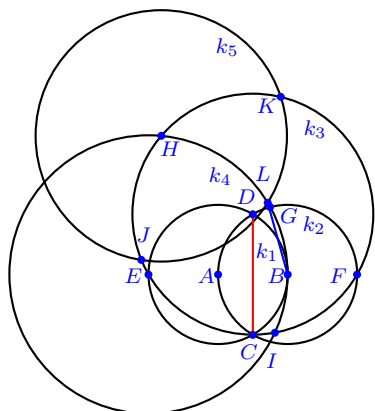
$$d_0 = |CD| = |CE| = |CF| = |CH| = |CI| = |CJ| = |CK| = |DE| = |DF| \text{ (red) } \dots 9$$

$$d_1 = |AL| \text{ (blue) } \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (9 occurrences)}$$

## 2.115.3 Construction MM322



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, |BE|)$   
 $G \in k_2 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  5.  $k_5 = k(H, |GI|)$   
 $J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5$

## Distances

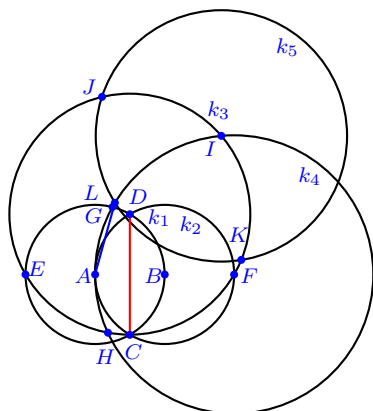
$$d_0 = |CD| = |CE| = |CF| = |DE| = |DF| = |DH| = |DI| = |DJ| = |DK| \text{ (red)} \dots 9$$

$$d_1 = |BL| \text{ (blue)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (9 occurrences)}$$

## 2.115.4 Construction MM361



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, |BE|)$   
 $G \in k_1 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  5.  $k_5 = k(I, |GH|)$   
 $J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5$

## Distances

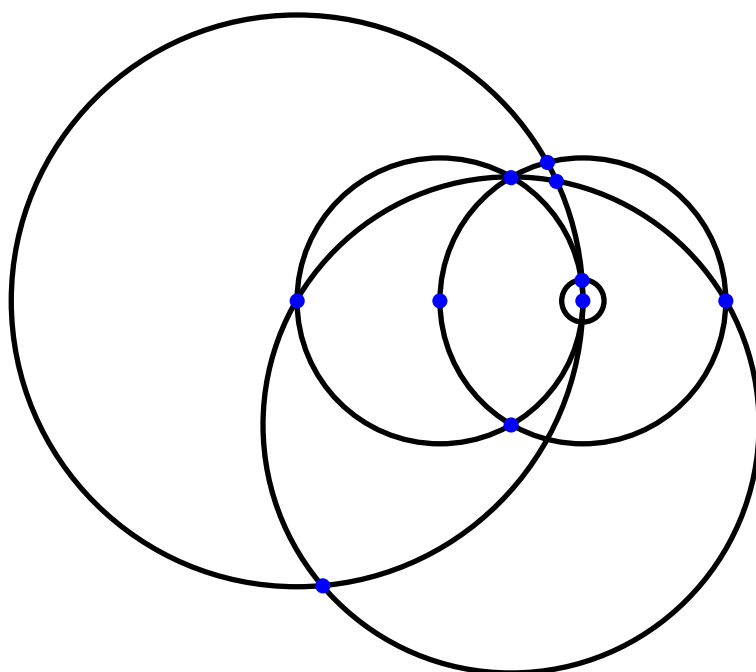
$$d_0 = |CD| = |CE| = |CF| = |DE| = |DF| = |DH| = |DI| = |DJ| = |DK| \text{ (red)} \dots 9$$

$$d_1 = |AL| \text{ (blue)} \dots 1$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (9 occurrences)}$$

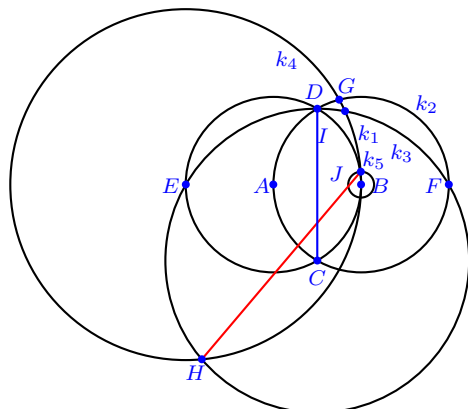
## 2.116 Class of Similar Constructions No. 107



### Contained Constructions

MM210, MM249, MM319, MM358

## 2.116.1 Construction MM210



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, |BE|)$   
 $G \in k_2 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  5.  $k_5 = k(B, |GI|)$   
 $J \in k_4 \cap k_5$

## Distances

$$d_0 = |HJ| \text{ (red)} \dots 1$$

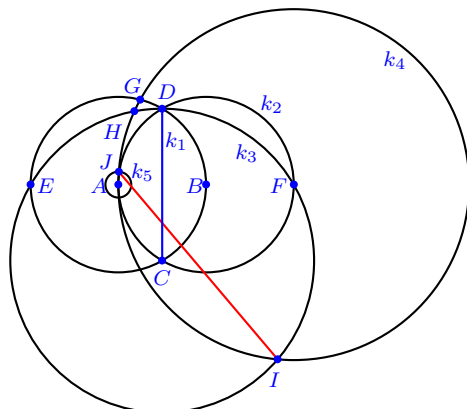
$$d_1 = |CD| = |CE| = |CF| = |CH| = |CI| = |DE| = |DF| \text{ (blue)} \dots 7$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (7 occurrences)}$$



## 2.116.2 Construction MM249



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, |BE|)$   
 $G \in k_1 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  5.  $k_5 = k(A, |GH|)$   
 $J \in k_4 \cap k_5$

## Distances

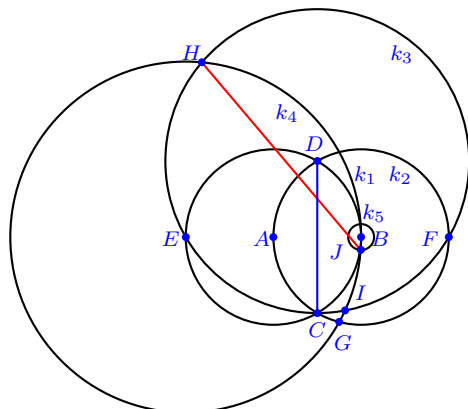
$$d_0 = |IJ| \text{ (red)} \dots 1$$

$$d_1 = |CD| = |CE| = |CF| = |CH| = |CI| = |DE| = |DF| \text{ (blue)} \dots 7$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (7 occurrences)}$$

## 2.116.3 Construction MM319



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, |BE|)$   
 $G \in k_2 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  5.  $k_5 = k(B, |GI|)$   
 $J \in k_4 \cap k_5$

## Distances

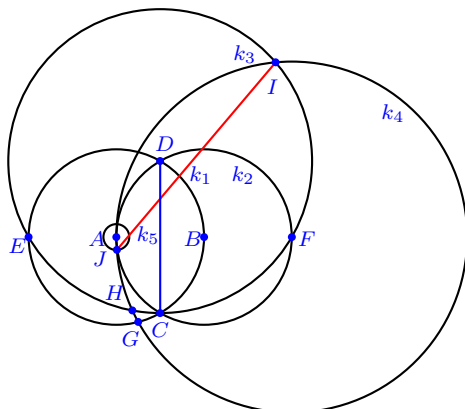
$$d_0 = |HJ| \text{ (red)} \dots 1$$

$$d_1 = |CD| = |CE| = |CF| = |DE| = |DF| = |DH| = |DI| \text{ (blue)} \dots 7$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (7 occurrences)}$$

## 2.116.4 Construction MM358



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, |BE|)$   
 $G \in k_1 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  5.  $k_5 = k(A, |GH|)$   
 $J \in k_4 \cap k_5$

## Distances

$$d_0 = |IJ| \text{ (red)} \dots 1$$

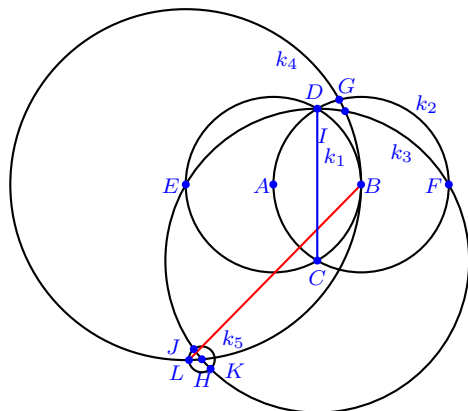
$$d_1 = |CD| = |CE| = |CF| = |DE| = |DF| = |DH| = |DI| \text{ (blue)} \dots 7$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (7 occurrences)}$$



## 2.117.1 Construction MM211



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, |BE|)$   
 $G \in k_2 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  5.  $k_5 = k(H, |GI|)$   
 $J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5$

## Distances

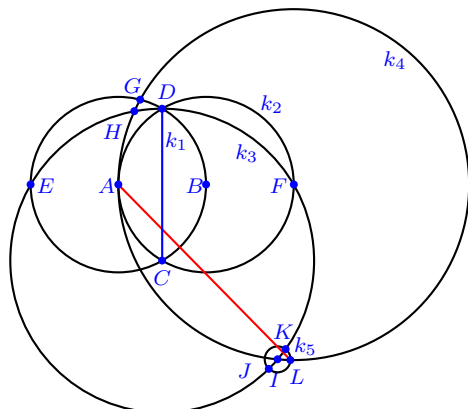
$$d_0 = |BL| \text{ (red)} \dots 1$$

$$d_1 = |CD| = |CE| = |CF| = |CH| = |CI| = |CJ| = |CK| = |DE| = |DF| \text{ (blue)} \dots 9$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (9 occurrences)}$$

## 2.117.2 Construction MM250



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, |BE|)$   
 $G \in k_1 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  5.  $k_5 = k(I, |GH|)$   
 $J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5$

## Distances

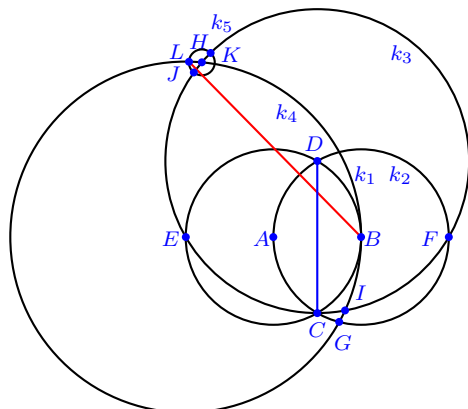
$$d_0 = |AL| \text{ (red)} \dots 1$$

$$d_1 = |CD| = |CE| = |CF| = |CH| = |CI| = |CJ| = |CK| = |DE| = |DF| \text{ (blue)} \dots 9$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (9 occurrences)}$$

## 2.117.3 Construction MM320



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, |BE|)$   
 $G \in k_2 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  5.  $k_5 = k(H, |GI|)$   
 $J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5$

## Distances

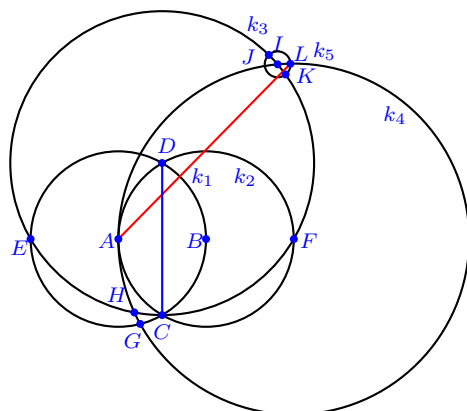
$$d_0 = |BL| \text{ (red)} \dots 1$$

$$d_1 = |CD| = |CE| = |CF| = |DE| = |DF| = |DH| = |DI| = |DJ| = |DK| \text{ (blue)} \dots 9$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (9 occurrences)}$$

## 2.117.4 Construction MM359



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(A, |AB|)$
  2.  $k_2 = k(B, |AB|)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, |CD|)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, |BE|)$   
 $G \in k_1 \cap k_4, H \in k_3 \cap k_4, I \in k_3 \cap k_4$
  5.  $k_5 = k(I, |GH|)$   
 $J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5$

## Distances

$$d_0 = |AL| \text{ (red)} \dots 1$$

$$d_1 = |CD| = |CE| = |CF| = |DE| = |DF| = |DH| = |DI| = |DJ| = |DK| \text{ (blue)} \dots 9$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (9 occurrences)}$$



## 2.118 CCO

The constructions presented in this section use only a collapsible compass. The collapsible compass cannot draw a circle with an arbitrary measured radius and only draws a circle given by a center and passing through another given point. The shortest construction of the golden ratio requires 5 steps, and 114 such constructions result from 1048 different processes. The constructions form 21 classes of mutually similar constructions.

The constructions are labeled CCO1 to CCO114.

In this section, we present list of these constructions. These constructions also belong to RCC and MM. Therefore, we also present the RCC and MM labels. CCO96 is identical with the second construction described by K. Hofstetter in [4].

### Remarks

Similar to MM, some constructions of CCO are analogous to RCC, in which several straight lines are replaced by a circle.

Replacing a ruler (straight line) by a compass (circle) is exemplified when comparing constructions RCC35 and CCO99. In RCC35, straight line  $p_1$  passes through points  $A$  and  $B$ . The circles  $k_2$  and  $k_3$  have centers at given points  $B$  and  $A$  and pass each through the other of the points. These steps could be performed in any order to construct points  $D$  and  $C$ , for which point  $D$  will be used to construct the final two circles.

In CCO99, circles with the center at the given points  $B$  and  $A$  passing through the other points are constructed ( $k_1$  and  $k_2$ ). The straight line passing through points  $A$  and  $B$  is replaced by the circle  $k_3$ , whose intersections with the circles  $k_1$  and  $k_2$  (at points  $F$  and  $E$ , respectively) are analogies to points  $D$  and  $C$  in RCC35, for which point  $F$  is used to construct the final two circles.

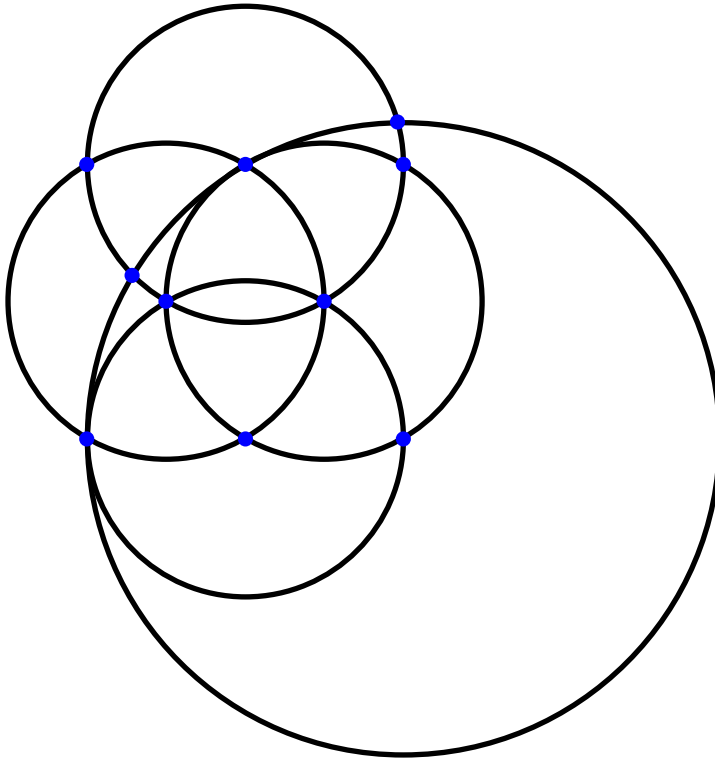
Similarly, we can compare for example constructions RCC2 and CCO46.

Comparing RCC306 and CCO18 demonstrates the difference caused by replacing the straight line  $p_3$  by the circle  $k_3$ .

## 2.119 Constructions Omitted in the Previous Kinds

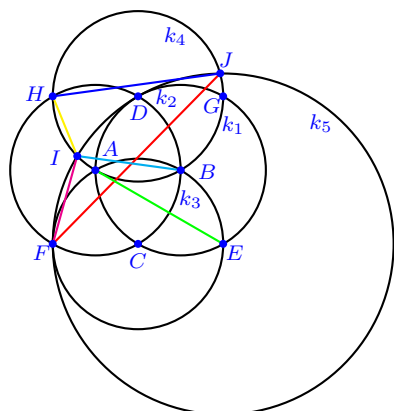
CCO1 (RCC266 MM2), CCO2 (RCC267 MM1), CCO3 (RCC268 MM4),  
 CCO4 (RCC269 MM3), CCO5 (RCC270 MM12), CCO6 (RCC271 MM11),  
 CCO7 (RCC272 MM10), CCO8 (RCC273 MM9), CCO9 (RCC274 MM23),  
 CCO10 (RCC275 MM24), CCO11 (RCC276 MM5), CCO12 (RCC277 MM6),  
 CCO13 (RCC278 MM19), CCO14 (RCC279 MM20), CCO15 (RCC280 MM21),  
 CCO16 (RCC281 MM22), CCO17 (RCC282 MM14), CCO18 (RCC283 MM13),  
 CCO19 (RCC284 MM15), CCO20 (RCC285 MM16), CCO21 (RCC286 MM17),  
 CCO22 (RCC287 MM18), CCO23 (RCC288 MM7), CCO24 (RCC289 MM8),  
 CCO25 (RCC314 MM53), CCO26 (RCC315 MM52), CCO27 (RCC316 MM51),  
 CCO28 (RCC317 MM50), CCO29 (RCC318 MM64), CCO30 (RCC319 MM65),  
 CCO31 (RCC320 MM46), CCO32 (RCC321 MM47), CCO33 (RCC322 MM60),  
 CCO34 (RCC323 MM61), CCO35 (RCC324 MM62), CCO36 (RCC325 MM63),  
 CCO37 (RCC326 MM55), CCO38 (RCC327 MM54), CCO39 (RCC328 MM56),  
 CCO40 (RCC329 MM57), CCO41 (RCC330 MM58), CCO42 (RCC331 MM59),  
 CCO43 (RCC332 MM48), CCO44 (RCC333 MM49), CCO45 (RCC348 MM263),  
 CCO46 (RCC349 MM258), CCO47 (RCC351 MM323), CCO48 (RCC353 MM339),  
 CCO49 (RCC354 MM335), CCO50 (RCC356 MM347), CCO51 (RCC357 MM344),  
 CCO52 (RCC358 MM325), CCO53 (RCC360 MM353), CCO54 (RCC362 MM326),  
 CCO55 (RCC364 MM348), CCO56 (RCC365 MM343), CCO57 (RCC367 MM340),  
 CCO58 (RCC368 MM336), CCO59 (RCC369 MM328), CCO60 (RCC371 MM354),  
 CCO61 (RCC372 MM260), CCO62 (RCC375 MM264), CCO63 (RCC376 MM259),  
 CCO64 (RCC379 MM267), CCO65 (RCC381 MM277), CCO66 (RCC383 MM293),  
 CCO67 (RCC384 MM289), CCO68 (RCC386 MM301), CCO69 (RCC387 MM298),  
 CCO70 (RCC389 MM307), CCO71 (RCC390 MM279), CCO72 (RCC392 MM280),  
 CCO73 (RCC394 MM302), CCO74 (RCC395 MM297), CCO75 (RCC397 MM294),  
 CCO76 (RCC398 MM290), CCO77 (RCC400 MM308), CCO78 (RCC401 MM282),  
 CCO79 (RCC402 MM268), CCO80 (RCC403 MM152), CCO81 (RCC404 MM147),  
 CCO82 (RCC406 MM212), CCO83 (RCC408 MM228), CCO84 (RCC409 MM224),  
 CCO85 (RCC411 MM236), CCO86 (RCC412 MM233), CCO87 (RCC413 MM214),  
 CCO88 (RCC415 MM242), CCO89 (RCC417 MM215), CCO90 (RCC419 MM237),  
 CCO91 (RCC420 MM232), CCO92 (RCC422 MM229), CCO93 (RCC423 MM225),  
 CCO94 (RCC424 MM217), CCO95 (RCC426 MM243), CCO96 (RCC427 MM149),  
 CCO97 (RCC430 MM153), CCO98 (RCC431 MM148), CCO99 (RCC434 MM156),  
 CCO100 (RCC436 MM166), CCO101 (RCC438 MM182), CCO102 (RCC439 MM178),  
 CCO103 (RCC441 MM190), CCO104 (RCC442 MM187), CCO105 (RCC444 MM196),  
 CCO106 (RCC445 MM168), CCO107 (RCC447 MM169), CCO108 (RCC449 MM191),  
 CCO109 (RCC450 MM186), CCO110 (RCC452 MM183), CCO111 (RCC453 MM179),  
 CCO112 (RCC455 MM197), CCO113 (RCC456 MM171), CCO114 (RCC457 MM157),

Contains 114 constructions.

**2.120 Class of Similar Constructions No. 109****Contained Constructions**

CCO1, CCO2, CCO3, CCO4, CCO13, CCO14, CCO15, CCO16, CCO17, CCO18,  
CCO19, CCO20, CCO33, CCO34, CCO35, CCO36, CCO37, CCO38, CCO39,  
CCO40

## 2.120.1 Construction CCO1



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, A)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(D, A)$   
 $G \in k_1 \cap k_4, H \in k_2 \cap k_4$
  5.  $k_5 = k(E, D)$   
 $I \in k_4 \cap k_5, J \in k_4 \cap k_5$

## Distances

- $d_0 = |FJ|$  (red) ... 1  
 $d_1 = |HJ|$  (blue) ... 1  
 $d_2 = |AE| = |AG| = |BF| = |BH| = |CD| = |EG| = |FH|$  (green) ... 7  
 $d_3 = |BI| = |BJ|$  (cyan) ... 2  
 $d_4 = |FI|$  (magenta) ... 1  
 $d_5 = |HI|$  (yellow) ... 1

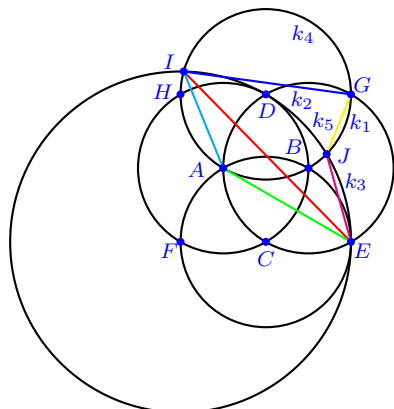
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (18 occurrences)}$$

## Identical Constructions

RCC266 MM2

## 2.120.2 Construction CCO2



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, A)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(D, A)$   
 $G \in k_1 \cap k_4, H \in k_2 \cap k_4$
  5.  $k_5 = k(F, D)$   
 $I \in k_4 \cap k_5, J \in k_4 \cap k_5$

## Distances

$$\begin{aligned}
 d_0 &= |EI| \text{ (red)} \dots 1 \\
 d_1 &= |GI| \text{ (blue)} \dots 1 \\
 d_2 &= |AE| = |AG| = |BF| = |BH| = |CD| = |EG| = |FH| \text{ (green)} \dots 7 \\
 d_3 &= |AI| = |AJ| \text{ (cyan)} \dots 2 \\
 d_4 &= |EJ| \text{ (magenta)} \dots 1 \\
 d_5 &= |GJ| \text{ (yellow)} \dots 1
 \end{aligned}$$

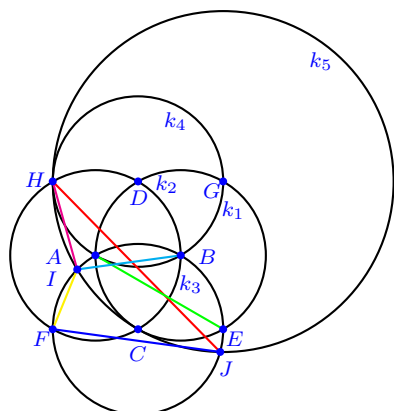
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (18 occurrences)}$$

## Identical Constructions

RCC267 MM1

## 2.120.3 Construction CCO3



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, A)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(D, A)$   
 $G \in k_1 \cap k_4, H \in k_2 \cap k_4$
  5.  $k_5 = k(G, C)$   
 $I \in k_3 \cap k_5, J \in k_3 \cap k_5$

## Distances

$$\begin{aligned}
 d_0 &= |HJ| \text{ (red)} \dots 1 \\
 d_1 &= |FJ| \text{ (blue)} \dots 1 \\
 d_2 &= |AE| = |AG| = |BF| = |BH| = |CD| = |EG| = |FH| \text{ (green)} \dots 7 \\
 d_3 &= |BI| = |BJ| \text{ (cyan)} \dots 2 \\
 d_4 &= |HI| \text{ (magenta)} \dots 1 \\
 d_5 &= |FI| \text{ (yellow)} \dots 1
 \end{aligned}$$

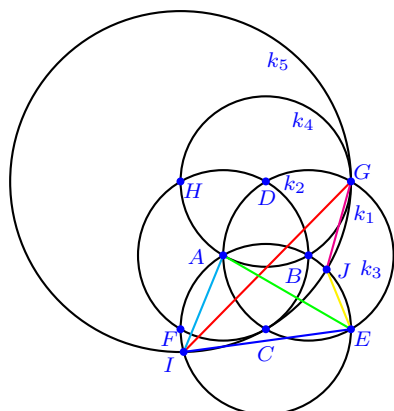
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (18 occurrences)}$$

## Identical Constructions

RCC268 MM4

## 2.120.4 Construction CCO4



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, A)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(D, A)$   
 $G \in k_1 \cap k_4, H \in k_2 \cap k_4$
  5.  $k_5 = k(H, C)$   
 $I \in k_3 \cap k_5, J \in k_3 \cap k_5$

## Distances

- $d_0 = |GI|$  (red) ... 1  
 $d_1 = |EI|$  (blue) ... 1  
 $d_2 = |AE| = |AG| = |BF| = |BH| = |CD| = |EG| = |FH|$  (green) ... 7  
 $d_3 = |AI| = |AJ|$  (cyan) ... 2  
 $d_4 = |GJ|$  (magenta) ... 1  
 $d_5 = |EJ|$  (yellow) ... 1

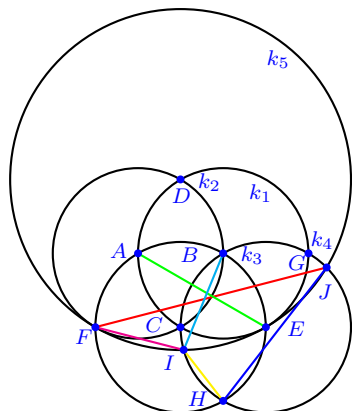
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \quad (18 \text{ occurrences})$$

## Identical Constructions

RCC269 MM3

## 2.120.5 Construction CCO13



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, A)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, B)$   
 $G \in k_1 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(D, E)$   
 $I \in k_4 \cap k_5, J \in k_4 \cap k_5$

## Distances

- $d_0 = |FJ|$  (red) ... 1  
 $d_1 = |HJ|$  (blue) ... 1  
 $d_2 = |AE| = |BF| = |BH| = |CD| = |CG| = |DG| = |FH|$  (green) ... 7  
 $d_3 = |BI| = |BJ|$  (cyan) ... 2  
 $d_4 = |FI|$  (magenta) ... 1  
 $d_5 = |HI|$  (yellow) ... 1

## Occurrences of the Golden Ratio

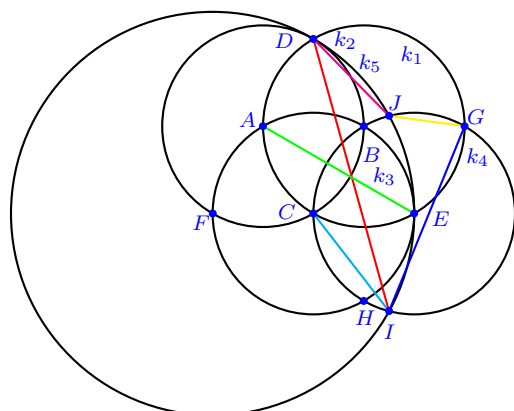
$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \quad (18 \text{ occurrences})$$

## Identical Constructions

RCC278 MM19



## 2.120.6 Construction CCO14



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, A)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, B)$   
 $G \in k_1 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(F, D)$   
 $I \in k_4 \cap k_5, J \in k_4 \cap k_5$

## Distances

$$\begin{aligned}
 d_0 &= |DI| \text{ (red)} \dots 1 \\
 d_1 &= |GI| \text{ (blue)} \dots 1 \\
 d_2 &= |AE| = |BF| = |BH| = |CD| = |CG| = |DG| = |FH| \text{ (green)} \dots 7 \\
 d_3 &= |CI| = |CJ| \text{ (cyan)} \dots 2 \\
 d_4 &= |DJ| \text{ (magenta)} \dots 1 \\
 d_5 &= |GJ| \text{ (yellow)} \dots 1
 \end{aligned}$$

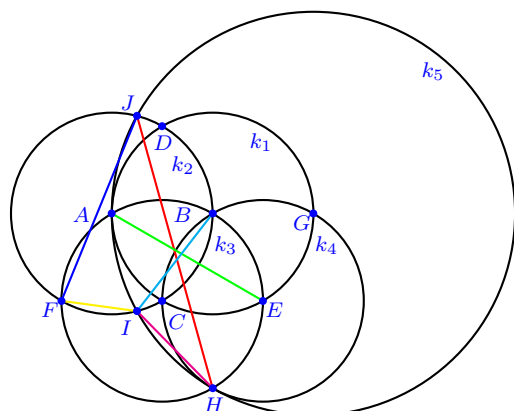
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (18 occurrences)}$$

## Identical Constructions

RCC279 MM20

## 2.120.7 Construction CCO15



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, A)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, B)$   
 $G \in k_1 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(G, A)$   
 $I \in k_2 \cap k_5, J \in k_2 \cap k_5$

## Distances

- $d_0 = |HJ|$  (red) ... 1  
 $d_1 = |FJ|$  (blue) ... 1  
 $d_2 = |AE| = |BF| = |BH| = |CD| = |CG| = |DG| = |FH|$  (green) ... 7  
 $d_3 = |BI| = |BJ|$  (cyan) ... 2  
 $d_4 = |HI|$  (magenta) ... 1  
 $d_5 = |FI|$  (yellow) ... 1

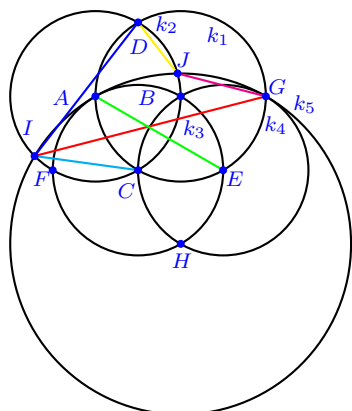
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (18 occurrences)}$$

## Identical Constructions

RCC280 MM21

## 2.120.8 Construction CCO16



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, A)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, B)$   
 $G \in k_1 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(H, A)$   
 $I \in k_2 \cap k_5, J \in k_2 \cap k_5$

## Distances

$$\begin{aligned}
 d_0 &= |GI| \text{ (red)} \dots 1 \\
 d_1 &= |DI| \text{ (blue)} \dots 1 \\
 d_2 &= |AE| = |BF| = |BH| = |CD| = |CG| = |DG| = |FH| \text{ (green)} \dots 7 \\
 d_3 &= |CI| = |CJ| \text{ (cyan)} \dots 2 \\
 d_4 &= |GJ| \text{ (magenta)} \dots 1 \\
 d_5 &= |DJ| \text{ (yellow)} \dots 1
 \end{aligned}$$

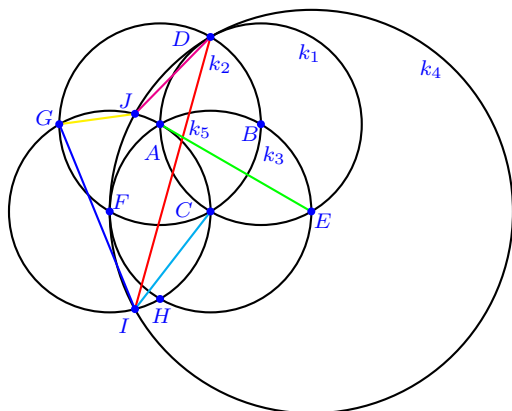
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (18 occurrences)}$$

## Identical Constructions

RCC281 MM22

## 2.120.9 Construction CCO17



## Construction Process

$A, B$  given initial points

1.  $k_1 = k(B, A)$
2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
3.  $k_3 = k(C, A)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
4.  $k_4 = k(E, D)$
5.  $k_5 = k(F, A)$   
 $G \in k_2 \cap k_5, H \in k_3 \cap k_5, I \in k_4 \cap k_5, J \in k_4 \cap k_5$

## Distances

- $d_0 = |DI|$  (red) ... 1  
 $d_1 = |GI|$  (blue) ... 1  
 $d_2 = |AE| = |AH| = |BF| = |CD| = |CG| = |DG| = |EH|$  (green) ... 7  
 $d_3 = |CI| = |CJ|$  (cyan) ... 2  
 $d_4 = |DJ|$  (magenta) ... 1  
 $d_5 = |GJ|$  (yellow) ... 1

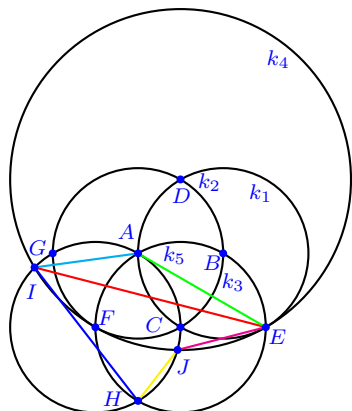
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \quad (18 \text{ occurrences})$$

## Identical Constructions

RCC282 MM14

## 2.120.10 Construction CCO18



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, A)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(D, E)$
  5.  $k_5 = k(F, A)$   
 $G \in k_2 \cap k_5, H \in k_3 \cap k_5, I \in k_4 \cap k_5, J \in k_4 \cap k_5$

## Distances

- $d_0 = |EI|$  (red) ... 1  
 $d_1 = |HI|$  (blue) ... 1  
 $d_2 = |AE| = |AH| = |BF| = |CD| = |CG| = |DG| = |EH|$  (green) ... 7  
 $d_3 = |AI| = |AJ|$  (cyan) ... 2  
 $d_4 = |EJ|$  (magenta) ... 1  
 $d_5 = |HJ|$  (yellow) ... 1

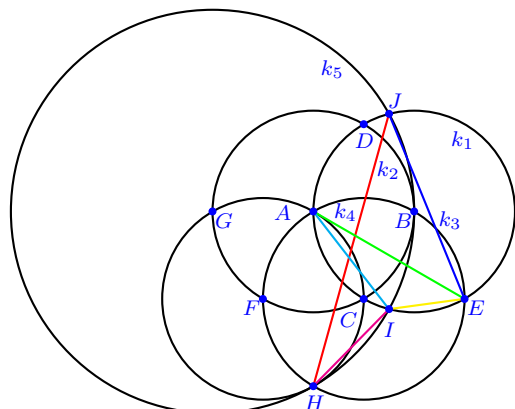
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \quad (18 \text{ occurrences})$$

## Identical Constructions

RCC283 MM13

## 2.120.11 Construction CCO19



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, A)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, A)$   
 $G \in k_2 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(G, B)$   
 $I \in k_1 \cap k_5, J \in k_1 \cap k_5$

## Distances

$$\begin{aligned}
 d_0 &= |HJ| \text{ (red)} \dots 1 \\
 d_1 &= |EJ| \text{ (blue)} \dots 1 \\
 d_2 &= |AE| = |AH| = |BF| = |CD| = |CG| = |DG| = |EH| \text{ (green)} \dots 7 \\
 d_3 &= |AI| = |AJ| \text{ (cyan)} \dots 2 \\
 d_4 &= |HI| \text{ (magenta)} \dots 1 \\
 d_5 &= |EI| \text{ (yellow)} \dots 1
 \end{aligned}$$

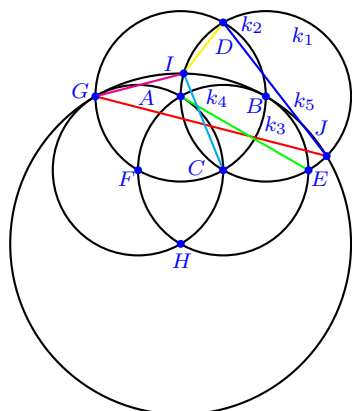
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (18 occurrences)}$$

## Identical Constructions

RCC284 MM15

## 2.120.12 Construction CCO20



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, A)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, A)$   
 $G \in k_2 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(H, B)$   
 $I \in k_1 \cap k_5, J \in k_1 \cap k_5$

## Distances

$$\begin{aligned}
 d_0 &= |GJ| \text{ (red)} \dots 1 \\
 d_1 &= |DJ| \text{ (blue)} \dots 1 \\
 d_2 &= |AE| = |AH| = |BF| = |CD| = |CG| = |DG| = |EH| \text{ (green)} \dots 7 \\
 d_3 &= |CI| = |CJ| \text{ (cyan)} \dots 2 \\
 d_4 &= |GI| \text{ (magenta)} \dots 1 \\
 d_5 &= |DI| \text{ (yellow)} \dots 1
 \end{aligned}$$

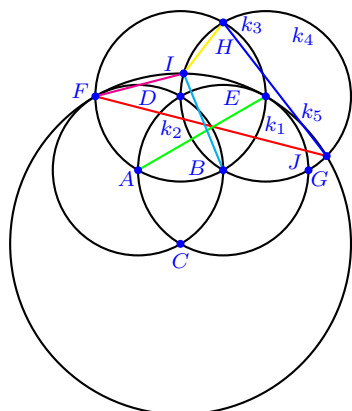
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (18 occurrences)}$$

## Identical Constructions

RCC285 MM16

## 2.120.13 Construction CCO33



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, A)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, B)$   
 $G \in k_1 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(C, E)$   
 $I \in k_4 \cap k_5, J \in k_4 \cap k_5$

## Distances

- $d_0 = |FJ|$  (red) ... 1  
 $d_1 = |HJ|$  (blue) ... 1  
 $d_2 = |AE| = |BF| = |BH| = |CD| = |CG| = |DG| = |FH|$  (green) ... 7  
 $d_3 = |BI| = |BJ|$  (cyan) ... 2  
 $d_4 = |FI|$  (magenta) ... 1  
 $d_5 = |HI|$  (yellow) ... 1

## Occurrences of the Golden Ratio

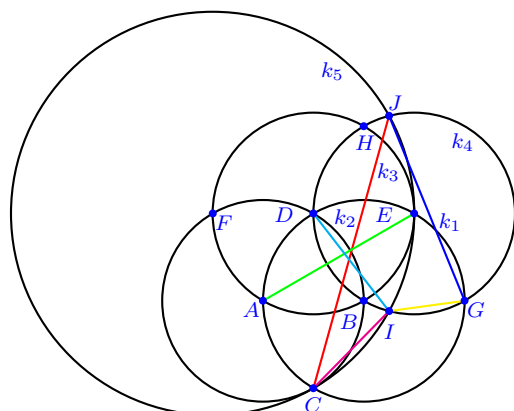
$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \quad (18 \text{ occurrences})$$

## Identical Constructions

RCC322 MM60



## 2.120.14 Construction CCO34



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, A)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, B)$   
 $G \in k_1 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(F, C)$   
 $I \in k_4 \cap k_5, J \in k_4 \cap k_5$

## Distances

$$\begin{aligned}
 d_0 &= |CJ| \text{ (red)} \dots 1 \\
 d_1 &= |GJ| \text{ (blue)} \dots 1 \\
 d_2 &= |AE| = |BF| = |BH| = |CD| = |CG| = |DG| = |FH| \text{ (green)} \dots 7 \\
 d_3 &= |DI| = |DJ| \text{ (cyan)} \dots 2 \\
 d_4 &= |CI| \text{ (magenta)} \dots 1 \\
 d_5 &= |GI| \text{ (yellow)} \dots 1
 \end{aligned}$$

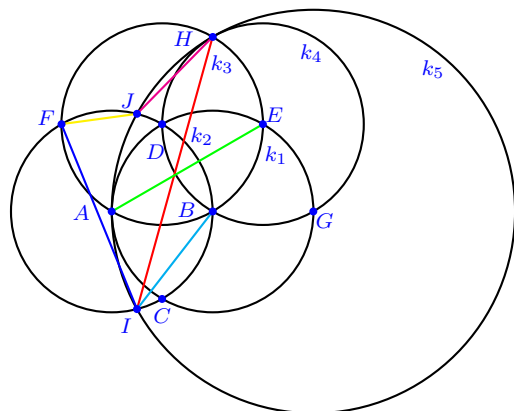
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (18 occurrences)}$$

## Identical Constructions

RCC323 MM61

## 2.120.15 Construction CCO35



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, A)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, B)$   
 $G \in k_1 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(G, A)$   
 $I \in k_2 \cap k_5, J \in k_2 \cap k_5$

## Distances

- $d_0 = |HI|$  (red) ... 1  
 $d_1 = |FI|$  (blue) ... 1  
 $d_2 = |AE| = |BF| = |BH| = |CD| = |CG| = |DG| = |FH|$  (green) ... 7  
 $d_3 = |BI| = |BJ|$  (cyan) ... 2  
 $d_4 = |HJ|$  (magenta) ... 1  
 $d_5 = |FJ|$  (yellow) ... 1

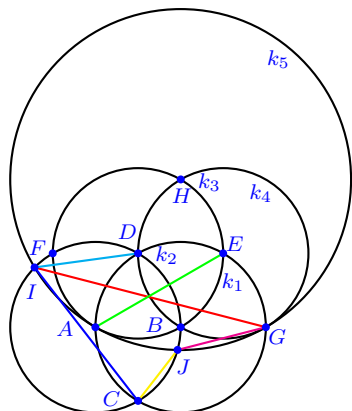
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (18 occurrences)}$$

## Identical Constructions

RCC324 MM62

## 2.120.16 Construction CCO36



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, A)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, B)$   
 $G \in k_1 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(H, A)$   
 $I \in k_2 \cap k_5, J \in k_2 \cap k_5$

## Distances

- $d_0 = |GI|$  (red) ... 1  
 $d_1 = |CI|$  (blue) ... 1  
 $d_2 = |AE| = |BF| = |BH| = |CD| = |CG| = |DG| = |FH|$  (green) ... 7  
 $d_3 = |DI| = |DJ|$  (cyan) ... 2  
 $d_4 = |GJ|$  (magenta) ... 1  
 $d_5 = |CJ|$  (yellow) ... 1

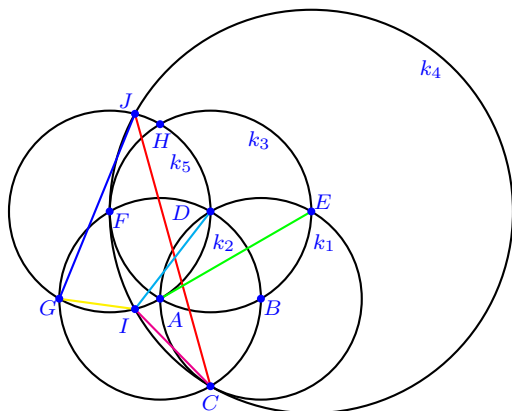
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \quad (18 \text{ occurrences})$$

## Identical Constructions

RCC325 MM63

## 2.120.17 Construction CCO37



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, A)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, C)$
  5.  $k_5 = k(F, A)$   
 $G \in k_2 \cap k_5, H \in k_3 \cap k_5, I \in k_4 \cap k_5, J \in k_4 \cap k_5$

## Distances

- $d_0 = |CJ|$  (red) ... 1  
 $d_1 = |GJ|$  (blue) ... 1  
 $d_2 = |AE| = |AH| = |BF| = |CD| = |CG| = |DG| = |EH|$  (green) ... 7  
 $d_3 = |DI| = |DJ|$  (cyan) ... 2  
 $d_4 = |CI|$  (magenta) ... 1  
 $d_5 = |GI|$  (yellow) ... 1

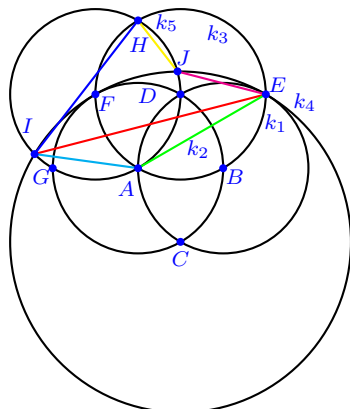
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \quad (18 \text{ occurrences})$$

## Identical Constructions

RCC326 MM55

## 2.120.18 Construction CCO38



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, A)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(C, E)$
  5.  $k_5 = k(F, A)$   
 $G \in k_2 \cap k_5, H \in k_3 \cap k_5, I \in k_4 \cap k_5, J \in k_4 \cap k_5$

## Distances

$$\begin{aligned}
 d_0 &= |EI| \text{ (red)} \dots 1 \\
 d_1 &= |HI| \text{ (blue)} \dots 1 \\
 d_2 &= |AE| = |AH| = |BF| = |CD| = |CG| = |DG| = |EH| \text{ (green)} \dots 7 \\
 d_3 &= |AI| = |AJ| \text{ (cyan)} \dots 2 \\
 d_4 &= |EJ| \text{ (magenta)} \dots 1 \\
 d_5 &= |HJ| \text{ (yellow)} \dots 1
 \end{aligned}$$

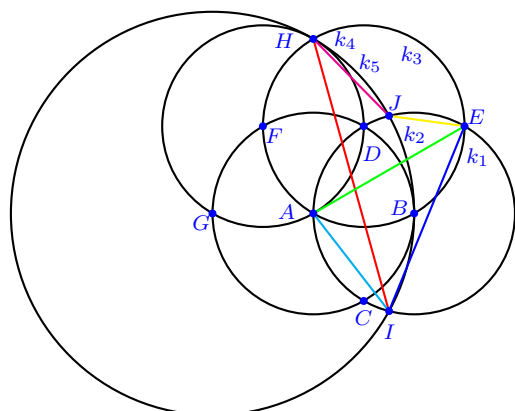
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (18 occurrences)}$$

## Identical Constructions

RCC327 MM54

## 2.120.19 Construction CCO39



## Construction Process

- $A, B$  given initial points
- $k_1 = k(B, A)$
  - $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  - $k_3 = k(D, A)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  - $k_4 = k(F, A)$   
 $G \in k_2 \cap k_4, H \in k_3 \cap k_4$
  - $k_5 = k(G, B)$   
 $I \in k_1 \cap k_5, J \in k_1 \cap k_5$

## Distances

$$\begin{aligned}
 d_0 &= |HI| \text{ (red)} \dots 1 \\
 d_1 &= |EI| \text{ (blue)} \dots 1 \\
 d_2 &= |AE| = |AH| = |BF| = |CD| = |CG| = |DG| = |EH| \text{ (green)} \dots 7 \\
 d_3 &= |AI| = |AJ| \text{ (cyan)} \dots 2 \\
 d_4 &= |HJ| \text{ (magenta)} \dots 1 \\
 d_5 &= |EJ| \text{ (yellow)} \dots 1
 \end{aligned}$$

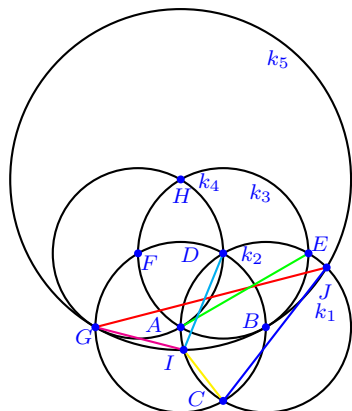
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (18 occurrences)}$$

## Identical Constructions

RCC328 MM56

## 2.120.20 Construction CCO40



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, A)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, A)$   
 $G \in k_2 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(H, B)$   
 $I \in k_1 \cap k_5, J \in k_1 \cap k_5$

## Distances

$$\begin{aligned}
 d_0 &= |GJ| \text{ (red)} \dots 1 \\
 d_1 &= |CJ| \text{ (blue)} \dots 1 \\
 d_2 &= |AE| = |AH| = |BF| = |CD| = |CG| = |DG| = |EH| \text{ (green)} \dots 7 \\
 d_3 &= |DI| = |DJ| \text{ (cyan)} \dots 2 \\
 d_4 &= |GI| \text{ (magenta)} \dots 1 \\
 d_5 &= |CI| \text{ (yellow)} \dots 1
 \end{aligned}$$

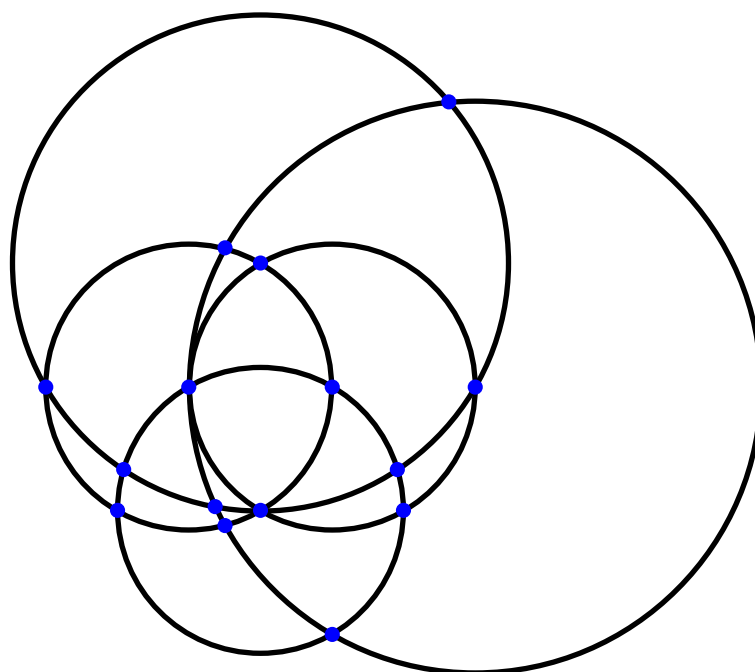
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (18 occurrences)}$$

## Identical Constructions

RCC329 MM57

## 2.121 Class of Similar Constructions No. 110

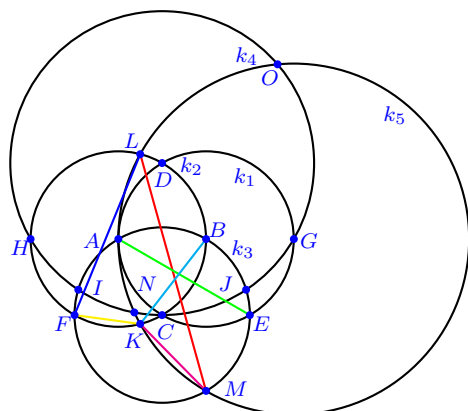


### Contained Constructions

CCO05, CCO06, CCO09, CCO10, CCO21, CCO22, CCO27, CCO28, CCO29,  
CCO30, CCO41, CCO42



## 2.121.1 Construction CCO5



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, A)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(D, C)$   
 $G \in k_1 \cap k_4, H \in k_2 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
  5.  $k_5 = k(G, A)$   
 $K \in k_2 \cap k_5, L \in k_2 \cap k_5, M \in k_3 \cap k_5, N \in k_4 \cap k_5, O \in k_4 \cap k_5$

## Distances

$$\begin{aligned}
 d_0 &= |LM| \text{ (red)} \dots 1 \\
 d_1 &= |FL| \text{ (blue)} \dots 1 \\
 d_2 &= |AE| = |BF| = |BM| = |CD| = |CG| = |CH| = |DG| = |DH| = \\
 &= |DI| = |DJ| = |DN| = |DO| = |FM| \text{ (green)} \dots 13 \\
 d_3 &= |BK| = |BL| \text{ (cyan)} \dots 2 \\
 d_4 &= |KM| \text{ (magenta)} \dots 1 \\
 d_5 &= |FK| \text{ (yellow)} \dots 1
 \end{aligned}$$

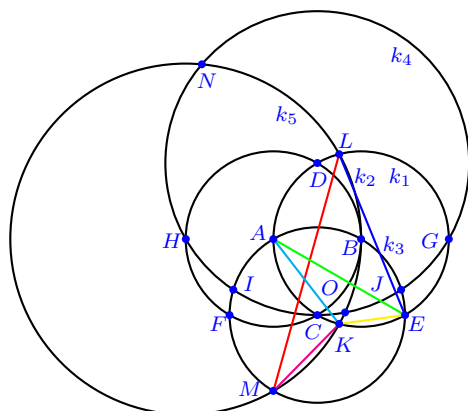
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (30 occurrences)}$$

## Identical Constructions

RCC270 MM12

## 2.121.2 Construction CCO6



## Construction Process

- $A, B$  given initial points
- $k_1 = k(B, A)$
  - $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  - $k_3 = k(C, A)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  - $k_4 = k(D, C)$   
 $G \in k_1 \cap k_4, H \in k_2 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
  - $k_5 = k(H, B)$   
 $K \in k_1 \cap k_5, L \in k_1 \cap k_5, M \in k_3 \cap k_5, N \in k_4 \cap k_5, O \in k_4 \cap k_5$

## Distances

$$\begin{aligned}
 d_0 &= |LM| \text{ (red)} \dots 1 \\
 d_1 &= |EL| \text{ (blue)} \dots 1 \\
 d_2 &= |AE| = |AM| = |BF| = |CD| = |CG| = |CH| = |DG| = |DH| = \\
 &= |DI| = |DJ| = |DN| = |DO| = |EM| \text{ (green)} \dots 13 \\
 d_3 &= |AK| = |AL| \text{ (cyan)} \dots 2 \\
 d_4 &= |KM| \text{ (magenta)} \dots 1 \\
 d_5 &= |EK| \text{ (yellow)} \dots 1
 \end{aligned}$$

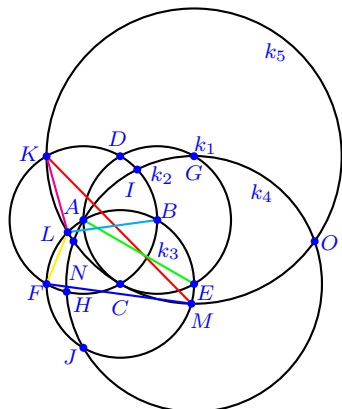
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (30 occurrences)}$$

## Identical Constructions

RCC271 MM11

## 2.121.3 Construction CCO9



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, A)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, A)$   
 $G \in k_1 \cap k_4, H \in k_2 \cap k_4, I \in k_2 \cap k_4, J \in k_3 \cap k_4$
  5.  $k_5 = k(G, C)$   
 $K \in k_2 \cap k_5, L \in k_3 \cap k_5, M \in k_3 \cap k_5, N \in k_4 \cap k_5, O \in k_4 \cap k_5$

## Distances

$$\begin{aligned}
 d_0 &= |KM| \text{ (red)} \dots 1 \\
 d_1 &= |FM| \text{ (blue)} \dots 1 \\
 d_2 &= |AE| = |AG| = |AJ| = |BF| = |BK| = |CD| = |EG| = |EH| = \\
 &= |EI| = |EJ| = |EN| = |EO| = |FK| \text{ (green)} \dots 13 \\
 d_3 &= |BL| = |BM| \text{ (cyan)} \dots 2 \\
 d_4 &= |KL| \text{ (magenta)} \dots 1 \\
 d_5 &= |FL| \text{ (yellow)} \dots 1
 \end{aligned}$$

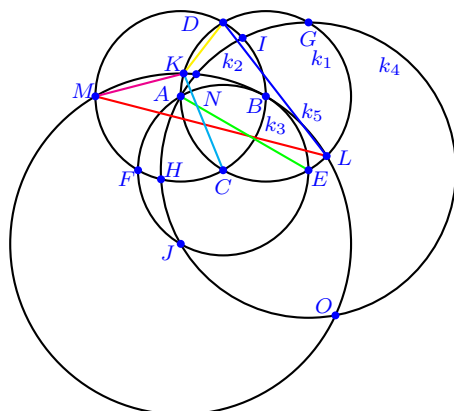
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (30 occurrences)}$$

## Identical Constructions

RCC274 MM23

## 2.121.4 Construction CCO10



## Construction Process

- $A, B$  given initial points
- $k_1 = k(B, A)$
  - $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  - $k_3 = k(C, A)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  - $k_4 = k(E, A)$   
 $G \in k_1 \cap k_4, H \in k_2 \cap k_4, I \in k_2 \cap k_4, J \in k_3 \cap k_4$
  - $k_5 = k(J, B)$   
 $K \in k_1 \cap k_5, L \in k_1 \cap k_5, M \in k_2 \cap k_5, N \in k_4 \cap k_5, O \in k_4 \cap k_5$

## Distances

$$\begin{aligned}
 d_0 &= |LM| \text{ (red)} \dots 1 \\
 d_1 &= |DL| \text{ (blue)} \dots 1 \\
 d_2 &= |AE| = |AG| = |AJ| = |BF| = |CD| = |CM| = |DM| = |EG| = \\
 &= |EH| = |EI| = |EJ| = |EN| = |EO| \text{ (green)} \dots 13 \\
 d_3 &= |CK| = |CL| \text{ (cyan)} \dots 2 \\
 d_4 &= |KM| \text{ (magenta)} \dots 1 \\
 d_5 &= |DK| \text{ (yellow)} \dots 1
 \end{aligned}$$

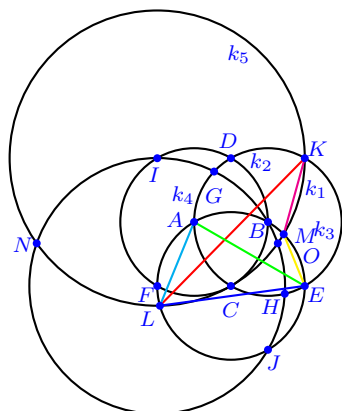
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (30 occurrences)}$$

## Identical Constructions

RCC275 MM24

## 2.121.5 Construction CCO21



## Construction Process

$A, B$  given initial points

1.  $k_1 = k(B, A)$

2.  $k_2 = k(A, B)$

$C \in k_1 \cap k_2, D \in k_1 \cap k_2$

3.  $k_3 = k(C, A)$

$E \in k_1 \cap k_3, F \in k_2 \cap k_3$

4.  $k_4 = k(F, B)$

$G \in k_1 \cap k_4, H \in k_1 \cap k_4, I \in k_2 \cap k_4, J \in k_3 \cap k_4$

5.  $k_5 = k(I, C)$

$K \in k_1 \cap k_5, L \in k_3 \cap k_5, M \in k_3 \cap k_5, N \in k_4 \cap k_5, O \in k_4 \cap k_5$

## Distances

$d_0 = |KL|$  (red) ... 1

$d_1 = |EL|$  (blue) ... 1

$d_2 = |AE| = |AK| = |BF| = |BI| = |BJ| = |CD| = |EK| = |FG| = |FH| = |FI| = |FJ| = |FN| = |FO|$  (green) ... 13

$d_3 = |AL| = |AM|$  (cyan) ... 2

$d_4 = |KM|$  (magenta) ... 1

$d_5 = |EM|$  (yellow) ... 1

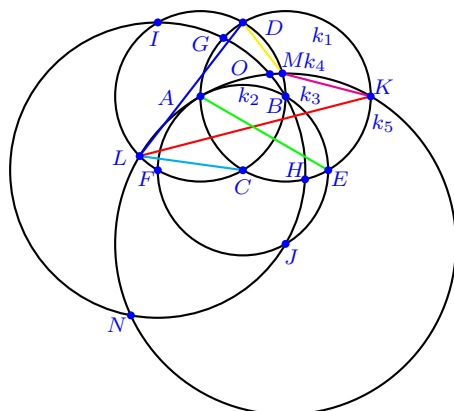
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (30 occurrences)}$$

## Identical Constructions

RCC286 MM17

## 2.121.6 Construction CCO22



## Construction Process

- $A, B$  given initial points
- $k_1 = k(B, A)$
  - $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  - $k_3 = k(C, A)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  - $k_4 = k(F, B)$   
 $G \in k_1 \cap k_4, H \in k_1 \cap k_4, I \in k_2 \cap k_4, J \in k_3 \cap k_4$
  - $k_5 = k(J, A)$   
 $K \in k_1 \cap k_5, L \in k_2 \cap k_5, M \in k_2 \cap k_5, N \in k_4 \cap k_5, O \in k_4 \cap k_5$

## Distances

$$\begin{aligned}
 d_0 &= |KL| \text{ (red) } \dots 1 \\
 d_1 &= |DL| \text{ (blue) } \dots 1 \\
 d_2 &= |AE| = |BF| = |BI| = |BJ| = |CD| = |CK| = |DK| = |FG| = \\
 &= |FH| = |FI| = |FJ| = |FN| = |FO| \text{ (green) } \dots 13 \\
 d_3 &= |CL| = |CM| \text{ (cyan) } \dots 2 \\
 d_4 &= |KM| \text{ (magenta) } \dots 1 \\
 d_5 &= |DM| \text{ (yellow) } \dots 1
 \end{aligned}$$

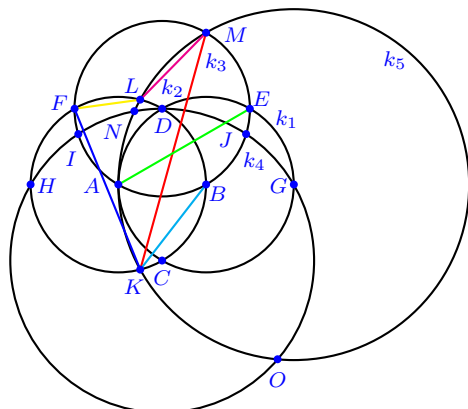
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (30 occurrences)}$$

## Identical Constructions

RCC287 MM18

## 2.121.7 Construction CCO27



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, A)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(C, D)$   
 $G \in k_1 \cap k_4, H \in k_2 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
  5.  $k_5 = k(G, A)$   
 $K \in k_2 \cap k_5, L \in k_2 \cap k_5, M \in k_3 \cap k_5, N \in k_4 \cap k_5, O \in k_4 \cap k_5$

## Distances

$$\begin{aligned}
 d_0 &= |KM| \text{ (red)} \dots 1 \\
 d_1 &= |FK| \text{ (blue)} \dots 1 \\
 d_2 &= |AE| = |BF| = |BM| = |CD| = |CG| = |CH| = |CI| = |CJ| = \\
 &= |CN| = |CO| = |DG| = |DH| = |FM| \text{ (green)} \dots 13 \\
 d_3 &= |BK| = |BL| \text{ (cyan)} \dots 2 \\
 d_4 &= |LM| \text{ (magenta)} \dots 1 \\
 d_5 &= |FL| \text{ (yellow)} \dots 1
 \end{aligned}$$

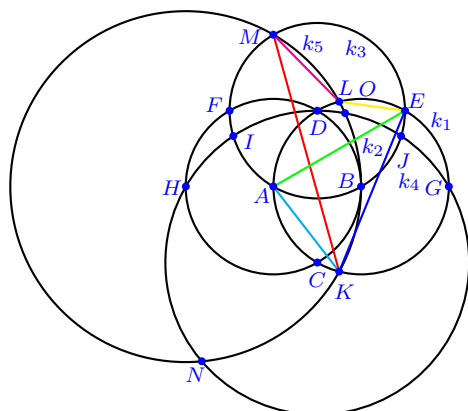
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (30 occurrences)}$$

## Identical Constructions

RCC316 MM51

## 2.121.8 Construction CCO28



## Construction Process

- $A, B$  given initial points
- $k_1 = k(B, A)$
  - $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  - $k_3 = k(D, A)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  - $k_4 = k(C, D)$   
 $G \in k_1 \cap k_4, H \in k_2 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
  - $k_5 = k(H, B)$   
 $K \in k_1 \cap k_5, L \in k_1 \cap k_5, M \in k_3 \cap k_5, N \in k_4 \cap k_5, O \in k_4 \cap k_5$

## Distances

$$\begin{aligned}
 d_0 &= |KM| \text{ (red)} \dots 1 \\
 d_1 &= |EK| \text{ (blue)} \dots 1 \\
 d_2 &= |AE| = |AM| = |BF| = |CD| = |CG| = |CH| = |CI| = |CJ| = \\
 &= |CN| = |CO| = |DG| = |DH| = |EM| \text{ (green)} \dots 13 \\
 d_3 &= |AK| = |AL| \text{ (cyan)} \dots 2 \\
 d_4 &= |LM| \text{ (magenta)} \dots 1 \\
 d_5 &= |EL| \text{ (yellow)} \dots 1
 \end{aligned}$$

## Occurrences of the Golden Ratio

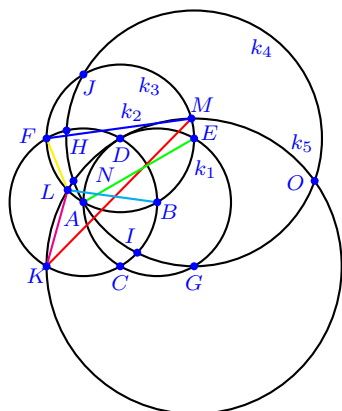
$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (30 occurrences)}$$

## Identical Constructions

RCC317 MM50



## 2.121.9 Construction CCO29



## Construction Process

- $A, B$  given initial points
- $k_1 = k(B, A)$
  - $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  - $k_3 = k(D, A)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  - $k_4 = k(E, A)$   
 $G \in k_1 \cap k_4, H \in k_2 \cap k_4, I \in k_2 \cap k_4, J \in k_3 \cap k_4$
  - $k_5 = k(G, D)$   
 $K \in k_2 \cap k_5, L \in k_3 \cap k_5, M \in k_3 \cap k_5, N \in k_4 \cap k_5, O \in k_4 \cap k_5$

## Distances

$$\begin{aligned}
 d_0 &= |KM| \text{ (red)} \dots 1 \\
 d_1 &= |FM| \text{ (blue)} \dots 1 \\
 d_2 &= |AE| = |AG| = |AJ| = |BF| = |BK| = |CD| = |EG| = |EH| = \\
 &= |EI| = |EJ| = |EN| = |EO| = |FK| \text{ (green)} \dots 13 \\
 d_3 &= |BL| = |BM| \text{ (cyan)} \dots 2 \\
 d_4 &= |KL| \text{ (magenta)} \dots 1 \\
 d_5 &= |FL| \text{ (yellow)} \dots 1
 \end{aligned}$$

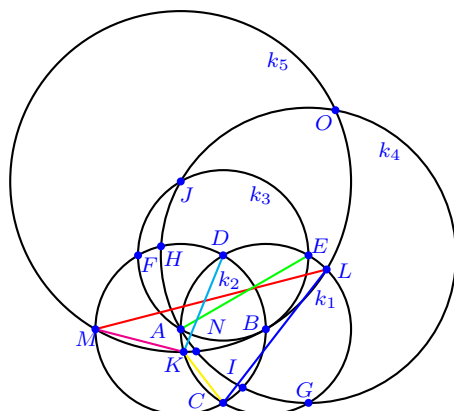
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (30 occurrences)}$$

## Identical Constructions

RCC318 MM64

## 2.121.10 Construction CCO30



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, A)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, A)$   
 $G \in k_1 \cap k_4, H \in k_2 \cap k_4, I \in k_2 \cap k_4, J \in k_3 \cap k_4$
  5.  $k_5 = k(J, B)$   
 $K \in k_1 \cap k_5, L \in k_1 \cap k_5, M \in k_2 \cap k_5, N \in k_4 \cap k_5, O \in k_4 \cap k_5$

## Distances

$$\begin{aligned}
 d_0 &= |LM| \text{ (red)} \dots 1 \\
 d_1 &= |CL| \text{ (blue)} \dots 1 \\
 d_2 &= |AE| = |AG| = |AJ| = |BF| = |CD| = |CM| = |DM| = |EG| = \\
 &= |EH| = |EI| = |EJ| = |EN| = |EO| \text{ (green)} \dots 13 \\
 d_3 &= |DK| = |DL| \text{ (cyan)} \dots 2 \\
 d_4 &= |KM| \text{ (magenta)} \dots 1 \\
 d_5 &= |CK| \text{ (yellow)} \dots 1
 \end{aligned}$$

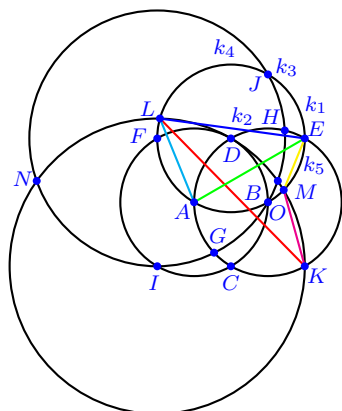
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (30 occurrences)}$$

## Identical Constructions

RCC319 MM65

## 2.121.11 Construction CCO41



## Construction Process

$A, B$  given initial points

1.  $k_1 = k(B, A)$
2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
3.  $k_3 = k(D, A)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
4.  $k_4 = k(F, B)$   
 $G \in k_1 \cap k_4, H \in k_1 \cap k_4, I \in k_2 \cap k_4, J \in k_3 \cap k_4$
5.  $k_5 = k(I, D)$   
 $K \in k_1 \cap k_5, L \in k_3 \cap k_5, M \in k_3 \cap k_5, N \in k_4 \cap k_5, O \in k_4 \cap k_5$

## Distances

- $d_0 = |KL|$  (red) ... 1  
 $d_1 = |EL|$  (blue) ... 1  
 $d_2 = |AE| = |AK| = |BF| = |BI| = |BJ| = |CD| = |EK| = |FG| = |FH| = |FI| = |FJ| = |FN| = |FO|$  (green) ... 13  
 $d_3 = |AL| = |AM|$  (cyan) ... 2  
 $d_4 = |KM|$  (magenta) ... 1  
 $d_5 = |EM|$  (yellow) ... 1

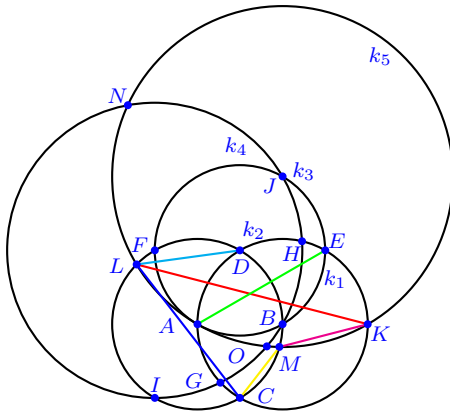
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \quad (30 \text{ occurrences})$$

## Identical Constructions

RCC330 MM58

## 2.121.12 Construction CCO42



## Construction Process

$A, B$  given initial points

1.  $k_1 = k(B, A)$

2.  $k_2 = k(A, B)$

$C \in k_1 \cap k_2, D \in k_1 \cap k_2$

3.  $k_3 = k(D, A)$

$E \in k_1 \cap k_3, F \in k_2 \cap k_3$

4.  $k_4 = k(F, B)$

$G \in k_1 \cap k_4, H \in k_1 \cap k_4, I \in k_2 \cap k_4, J \in k_3 \cap k_4$

5.  $k_5 = k(J, A)$

$K \in k_1 \cap k_5, L \in k_2 \cap k_5, M \in k_2 \cap k_5, N \in k_4 \cap k_5, O \in k_4 \cap k_5$

## Distances

$d_0 = |KL|$  (red) ... 1

$d_1 = |CL|$  (blue) ... 1

$d_2 = |AE| = |BF| = |BI| = |BJ| = |CD| = |CK| = |DK| = |FG| = |FH| = |FI| = |FJ| = |FN| = |FO|$  (green) ... 13

$d_3 = |DL| = |DM|$  (cyan) ... 2

$d_4 = |KM|$  (magenta) ... 1

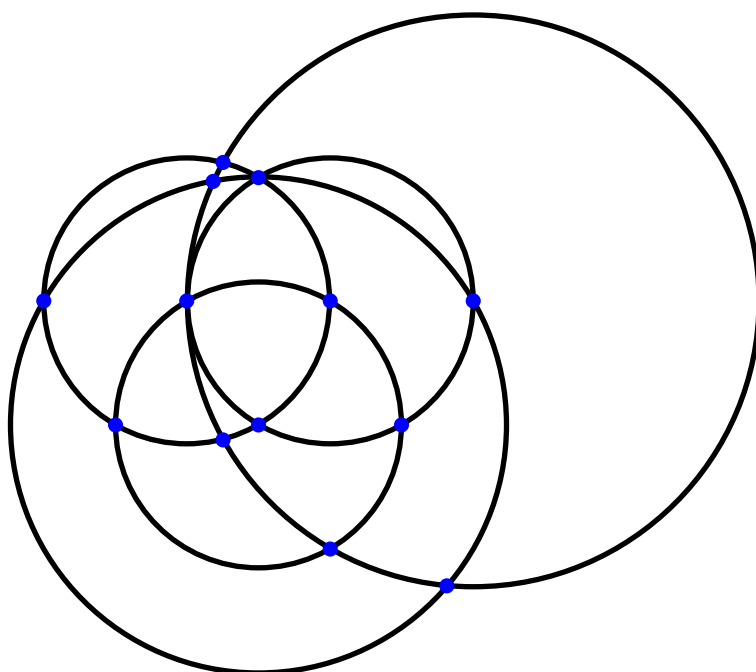
$d_5 = |CM|$  (yellow) ... 1

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (30 occurrences)}$$

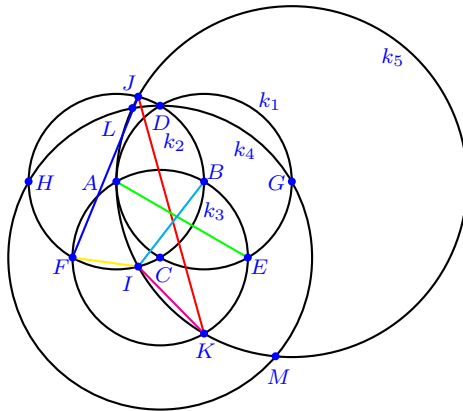
## Identical Constructions

RCC331 MM59

**2.122 Class of Similar Constructions No. 111****Contained Constructions**

CCO7, CCO8, CCO11, CCO12, CCO23, CCO24, CCO25, CCO26, CCO31,  
CCO32, CCO43, CCO44

## 2.122.1 Construction CCO7



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, A)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(C, D)$   
 $G \in k_1 \cap k_4, H \in k_2 \cap k_4$
  5.  $k_5 = k(G, A)$   
 $I \in k_2 \cap k_5, J \in k_2 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5, M \in k_4 \cap k_5$

## Distances

$$\begin{aligned}
 d_0 &= |JK| \text{ (red)} \dots 1 \\
 d_1 &= |FJ| \text{ (blue)} \dots 1 \\
 d_2 &= |AE| = |BF| = |BK| = |CD| = |CG| = |CH| = |CL| = |CM| = \\
 &= |DG| = |DH| = |FK| \text{ (green)} \dots 11 \\
 d_3 &= |BI| = |BJ| \text{ (cyan)} \dots 2 \\
 d_4 &= |IK| \text{ (magenta)} \dots 1 \\
 d_5 &= |FI| \text{ (yellow)} \dots 1
 \end{aligned}$$

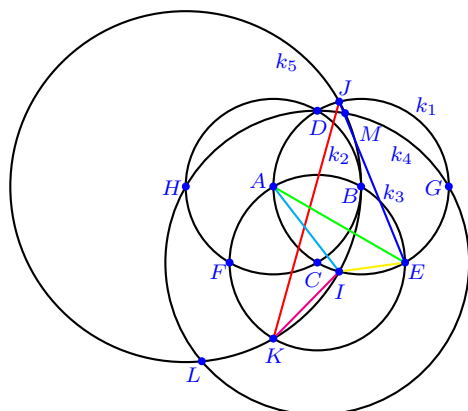
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (26 occurrences)}$$

## Identical Constructions

RCC272 MM10

## 2.122.2 Construction CCO8



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, A)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(C, D)$   
 $G \in k_1 \cap k_4, H \in k_2 \cap k_4$
  5.  $k_5 = k(H, B)$   
 $I \in k_1 \cap k_5, J \in k_1 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5, M \in k_4 \cap k_5$

## Distances

$$\begin{aligned}
 d_0 &= |JK| \text{ (red)} \dots 1 \\
 d_1 &= |EJ| \text{ (blue)} \dots 1 \\
 d_2 &= |AE| = |AK| = |BF| = |CD| = |CG| = |CH| = |CL| = |CM| = \\
 &= |DG| = |DH| = |EK| \text{ (green)} \dots 11 \\
 d_3 &= |AI| = |AJ| \text{ (cyan)} \dots 2 \\
 d_4 &= |IK| \text{ (magenta)} \dots 1 \\
 d_5 &= |EI| \text{ (yellow)} \dots 1
 \end{aligned}$$

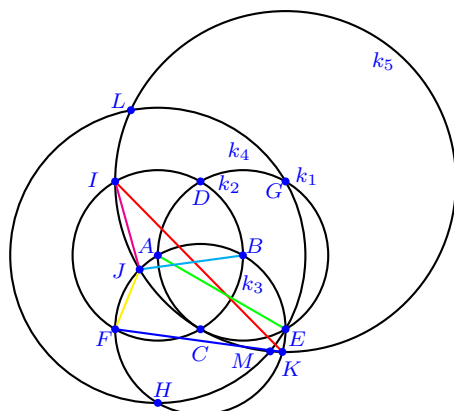
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (26 occurrences)}$$

## Identical Constructions

RCC273 MM9

## 2.122.3 Construction CCO11



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, A)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(A, E)$   
 $G \in k_1 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(G, C)$   
 $I \in k_2 \cap k_5, J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5, M \in k_4 \cap k_5$

## Distances

$$\begin{aligned}
 d_0 &= |IK| \text{ (red)} \dots 1 \\
 d_1 &= |FK| \text{ (blue)} \dots 1 \\
 d_2 &= |AE| = |AG| = |AH| = |AL| = |AM| = |BF| = |BI| = |CD| = \\
 &= |EG| = |EH| = |FI| \text{ (green)} \dots 11 \\
 d_3 &= |BJ| = |BK| \text{ (cyan)} \dots 2 \\
 d_4 &= |IJ| \text{ (magenta)} \dots 1 \\
 d_5 &= |FJ| \text{ (yellow)} \dots 1
 \end{aligned}$$

## Occurrences of the Golden Ratio

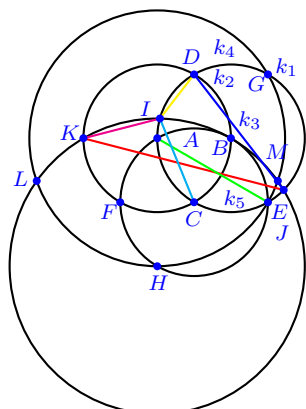
$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (26 occurrences)}$$

## Identical Constructions

RCC276 MM5



## 2.122.4 Construction CCO12



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, A)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(A, E)$   
 $G \in k_1 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(H, B)$   
 $I \in k_1 \cap k_5, J \in k_1 \cap k_5, K \in k_2 \cap k_5, L \in k_4 \cap k_5, M \in k_4 \cap k_5$

## Distances

$$\begin{aligned}
 d_0 &= |JK| \text{ (red)} \dots 1 \\
 d_1 &= |DJ| \text{ (blue)} \dots 1 \\
 d_2 &= |AE| = |AG| = |AH| = |AL| = |AM| = |BF| = |CD| = |CK| = \\
 &= |DK| = |EG| = |EH| \text{ (green)} \dots 11 \\
 d_3 &= |CI| = |CJ| \text{ (cyan)} \dots 2 \\
 d_4 &= |IK| \text{ (magenta)} \dots 1 \\
 d_5 &= |DI| \text{ (yellow)} \dots 1
 \end{aligned}$$

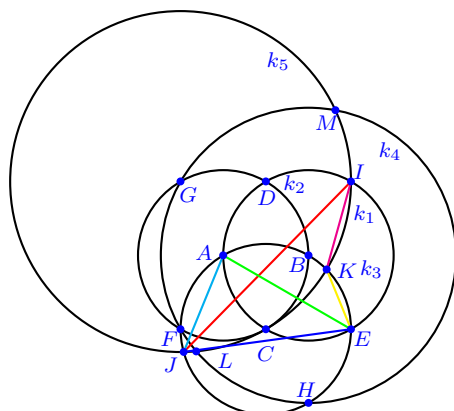
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (26 occurrences)}$$

## Identical Constructions

RCC277 MM6

## 2.122.5 Construction CCO23



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, A)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(B, F)$   
 $G \in k_2 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(G, C)$   
 $I \in k_1 \cap k_5, J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5, M \in k_4 \cap k_5$

## Distances

$$\begin{aligned}
 d_0 &= |IJ| \text{ (red)} \dots 1 \\
 d_1 &= |EJ| \text{ (blue)} \dots 1 \\
 d_2 &= |AE| = |AI| = |BF| = |BG| = |BH| = |BL| = |BM| = |CD| = \\
 &= |EI| = |FG| = |FH| \text{ (green)} \dots 11 \\
 d_3 &= |AJ| = |AK| \text{ (cyan)} \dots 2 \\
 d_4 &= |IK| \text{ (magenta)} \dots 1 \\
 d_5 &= |EK| \text{ (yellow)} \dots 1
 \end{aligned}$$

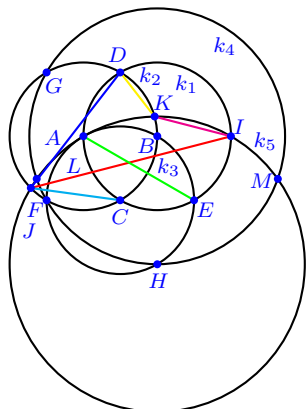
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (26 occurrences)}$$

## Identical Constructions

RCC288 MM7

## 2.122.6 Construction CCO24



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, A)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(B, F)$   
 $G \in k_2 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(H, A)$   
 $I \in k_1 \cap k_5, J \in k_2 \cap k_5, K \in k_2 \cap k_5, L \in k_4 \cap k_5, M \in k_4 \cap k_5$

## Distances

$$\begin{aligned}
 d_0 &= |IJ| \text{ (red)} \dots 1 \\
 d_1 &= |DJ| \text{ (blue)} \dots 1 \\
 d_2 &= |AE| = |BF| = |BG| = |BH| = |BL| = |BM| = |CD| = |CI| = \\
 &= |DI| = |FG| = |FH| \text{ (green)} \dots 11 \\
 d_3 &= |CJ| = |CK| \text{ (cyan)} \dots 2 \\
 d_4 &= |IK| \text{ (magenta)} \dots 1 \\
 d_5 &= |DK| \text{ (yellow)} \dots 1
 \end{aligned}$$

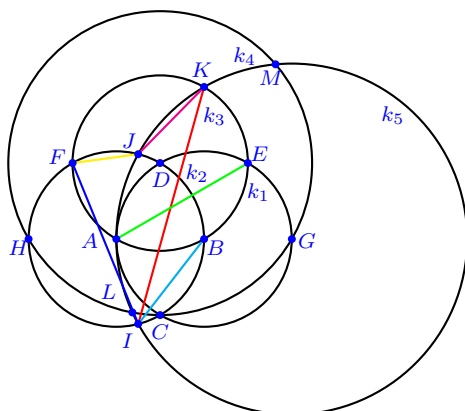
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (26 occurrences)}$$

## Identical Constructions

RCC289 MM8

## 2.122.7 Construction CCO25



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, A)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(D, C)$   
 $G \in k_1 \cap k_4, H \in k_2 \cap k_4$
  5.  $k_5 = k(G, A)$   
 $I \in k_2 \cap k_5, J \in k_2 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5, M \in k_4 \cap k_5$

## Distances

$$\begin{aligned}
 d_0 &= |IK| \text{ (red)} \dots 1 \\
 d_1 &= |FI| \text{ (blue)} \dots 1 \\
 d_2 &= |AE| = |BF| = |BK| = |CD| = |CG| = |CH| = |DG| = |DH| = \\
 &= |DL| = |DM| = |FK| \text{ (green)} \dots 11 \\
 d_3 &= |BI| = |BJ| \text{ (cyan)} \dots 2 \\
 d_4 &= |JK| \text{ (magenta)} \dots 1 \\
 d_5 &= |FJ| \text{ (yellow)} \dots 1
 \end{aligned}$$

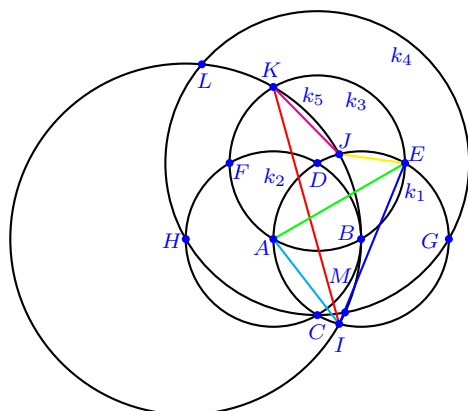
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (26 occurrences)}$$

## Identical Constructions

RCC314 MM53

## 2.122.8 Construction CCO26



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, A)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(D, C)$   
 $G \in k_1 \cap k_4, H \in k_2 \cap k_4$
  5.  $k_5 = k(H, B)$   
 $I \in k_1 \cap k_5, J \in k_1 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5, M \in k_4 \cap k_5$

## Distances

$$\begin{aligned}
 d_0 &= |IK| \text{ (red)} \dots 1 \\
 d_1 &= |EI| \text{ (blue)} \dots 1 \\
 d_2 &= |AE| = |AK| = |BF| = |CD| = |CG| = |CH| = |DG| = |DH| = \\
 &|DL| = |DM| = |EK| \text{ (green)} \dots 11 \\
 d_3 &= |AI| = |AJ| \text{ (cyan)} \dots 2 \\
 d_4 &= |JK| \text{ (magenta)} \dots 1 \\
 d_5 &= |EJ| \text{ (yellow)} \dots 1
 \end{aligned}$$

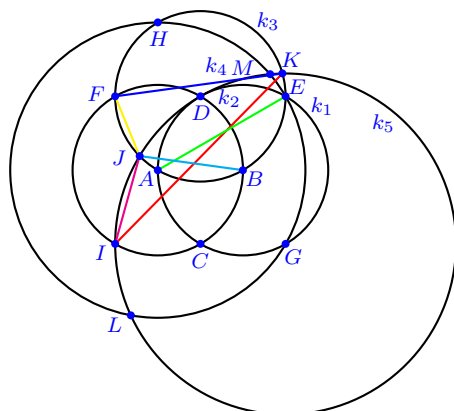
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (26 occurrences)}$$

## Identical Constructions

RCC315 MM52

## 2.122.9 Construction CCO31



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, A)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(A, E)$   
 $G \in k_1 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(G, D)$   
 $I \in k_2 \cap k_5, J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5, M \in k_4 \cap k_5$

## Distances

$$\begin{aligned}
 d_0 &= |IK| \text{ (red) } \dots 1 \\
 d_1 &= |FK| \text{ (blue) } \dots 1 \\
 d_2 &= |AE| = |AG| = |AH| = |AL| = |AM| = |BF| = |BI| = |CD| = \\
 &= |EG| = |EH| = |FI| \text{ (green) } \dots 11 \\
 d_3 &= |BJ| = |BK| \text{ (cyan) } \dots 2 \\
 d_4 &= |IJ| \text{ (magenta) } \dots 1 \\
 d_5 &= |FJ| \text{ (yellow) } \dots 1
 \end{aligned}$$

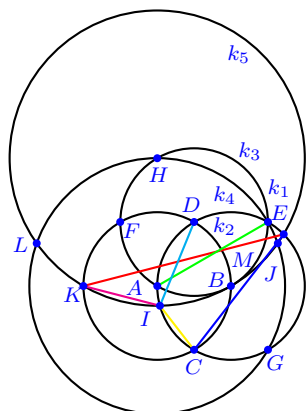
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (26 occurrences)}$$

## Identical Constructions

RCC320 MM46

## 2.122.10 Construction CCO32



## Construction Process

$A, B$  given initial points

1.  $k_1 = k(B, A)$
2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
3.  $k_3 = k(D, A)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
4.  $k_4 = k(A, E)$   
 $G \in k_1 \cap k_4, H \in k_3 \cap k_4$
5.  $k_5 = k(H, B)$   
 $I \in k_1 \cap k_5, J \in k_1 \cap k_5, K \in k_2 \cap k_5, L \in k_4 \cap k_5, M \in k_4 \cap k_5$

## Distances

$$\begin{aligned}
 d_0 &= |JK| \text{ (red)} \dots 1 \\
 d_1 &= |CJ| \text{ (blue)} \dots 1 \\
 d_2 &= |AE| = |AG| = |AH| = |AL| = |AM| = |BF| = |CD| = |CK| = \\
 &= |DK| = |EG| = |EH| \text{ (green)} \dots 11 \\
 d_3 &= |DI| = |DJ| \text{ (cyan)} \dots 2 \\
 d_4 &= |IK| \text{ (magenta)} \dots 1 \\
 d_5 &= |CI| \text{ (yellow)} \dots 1
 \end{aligned}$$

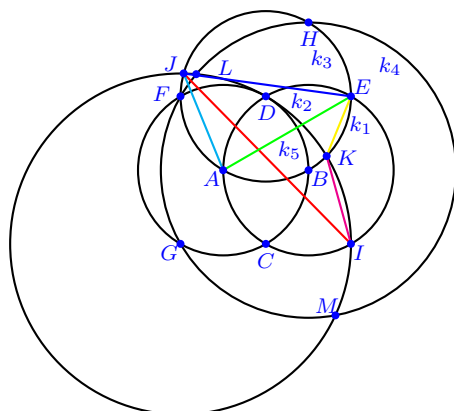
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (26 occurrences)}$$

## Identical Constructions

RCC321 MM47

## 2.122.11 Construction CCO43



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, A)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(B, F)$   
 $G \in k_2 \cap k_4, H \in k_3 \cap k_4$
  5.  $k_5 = k(G, D)$   
 $I \in k_1 \cap k_5, J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5, M \in k_4 \cap k_5$

## Distances

$$\begin{aligned}
 d_0 &= |IJ| \text{ (red)} \dots 1 \\
 d_1 &= |EJ| \text{ (blue)} \dots 1 \\
 d_2 &= |AE| = |AI| = |BF| = |BG| = |BH| = |BL| = |BM| = |CD| = \\
 &= |EI| = |FG| = |FH| \text{ (green)} \dots 11 \\
 d_3 &= |AJ| = |AK| \text{ (cyan)} \dots 2 \\
 d_4 &= |IK| \text{ (magenta)} \dots 1 \\
 d_5 &= |EK| \text{ (yellow)} \dots 1
 \end{aligned}$$

## Occurrences of the Golden Ratio

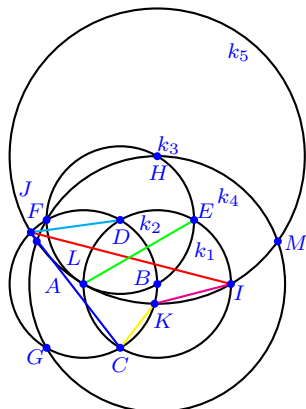
$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (26 occurrences)}$$

## Identical Constructions

RCC332 MM48



## 2.122.12 Construction CCO44



## Construction Process

$A, B$  given initial points

1.  $k_1 = k(B, A)$
2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
3.  $k_3 = k(D, A)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
4.  $k_4 = k(B, F)$   
 $G \in k_2 \cap k_4, H \in k_3 \cap k_4$
5.  $k_5 = k(H, A)$   
 $I \in k_1 \cap k_5, J \in k_2 \cap k_5, K \in k_2 \cap k_5, L \in k_4 \cap k_5, M \in k_4 \cap k_5$

## Distances

$$\begin{aligned}
 d_0 &= |IJ| \text{ (red)} \dots 1 \\
 d_1 &= |CJ| \text{ (blue)} \dots 1 \\
 d_2 &= |AE| = |BF| = |BG| = |BH| = |BL| = |BM| = |CD| = |CI| = \\
 &= |DI| = |FG| = |FH| \text{ (green)} \dots 11 \\
 d_3 &= |DJ| = |DK| \text{ (cyan)} \dots 2 \\
 d_4 &= |IK| \text{ (magenta)} \dots 1 \\
 d_5 &= |CK| \text{ (yellow)} \dots 1
 \end{aligned}$$

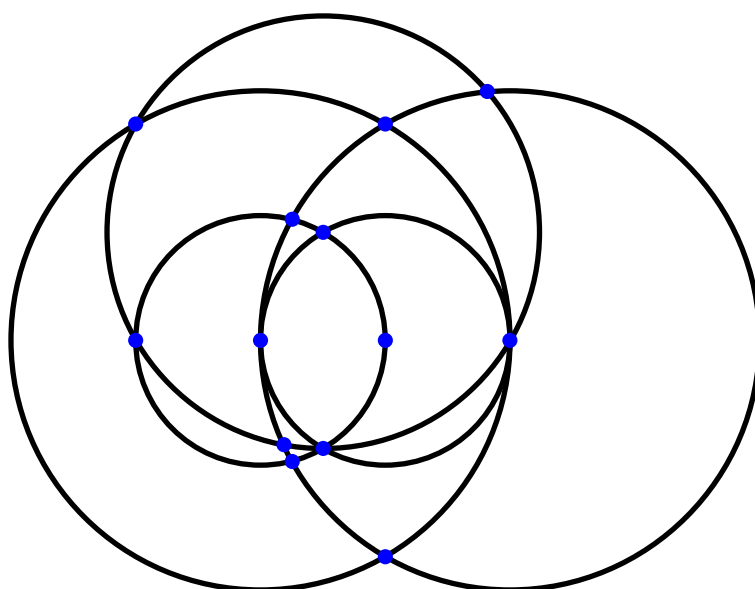
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (26 occurrences)}$$

## Identical Constructions

RCC333 MM49

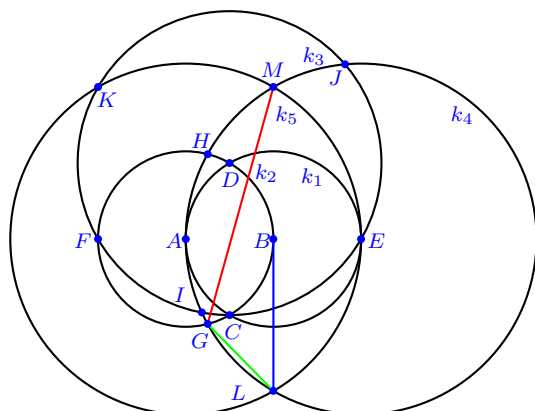
## 2.123 Class of Similar Constructions No. 112



### Contained Constructions

CCO45, CCO64, CCO80, CCO99

## 2.123.1 Construction CCO45



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, C)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, A)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
  5.  $k_5 = k(A, E)$   
 $K \in k_3 \cap k_5, L \in k_4 \cap k_5, M \in k_4 \cap k_5$

## Distances

$$d_0 = |GM| = |HL| \text{ (red)} \dots 2$$

$$d_1 = |BL| = |BM| = |CD| = |CE| = |CF| = |DE| = |DF| = |DI| = |DJ| = |DK| = |FK| \text{ (blue)} \dots 11$$

$$d_2 = |GL| = |HM| \text{ (green)} \dots 2$$

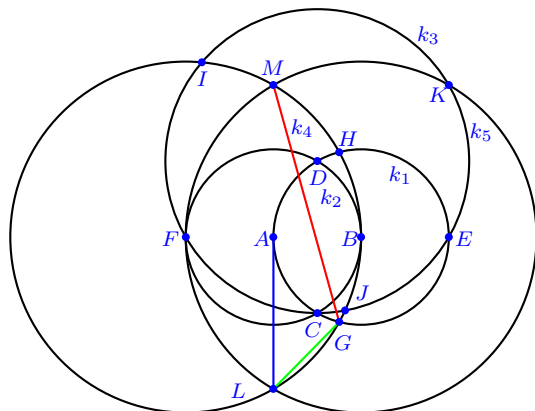
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (44 occurrences)}$$

## Identical Constructions

RCC348 MM263

## 2.123.2 Construction CCO64



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, C)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, B)$   
 $G \in k_1 \cap k_4, H \in k_1 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
  5.  $k_5 = k(B, F)$   
 $K \in k_3 \cap k_5, L \in k_4 \cap k_5, M \in k_4 \cap k_5$

## Distances

$$d_0 = |GM| = |HL| \text{ (red) } \dots 2$$

$$d_1 = |AL| = |AM| = |CD| = |CE| = |CF| = |DE| = |DF| = |DI| = |DJ| = |DK| = |EK| \text{ (blue) } \dots 11$$

$$d_2 = |GL| = |HM| \text{ (green) } \dots 2$$

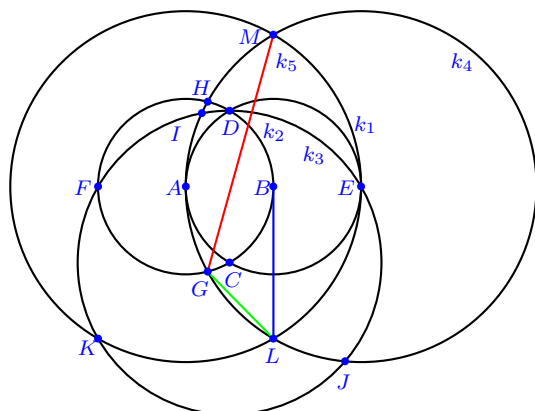
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (44 occurrences)}$$

## Identical Constructions

RCC379 MM267

## 2.123.3 Construction CCO80



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, D)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, A)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
  5.  $k_5 = k(A, E)$   
 $K \in k_3 \cap k_5, L \in k_4 \cap k_5, M \in k_4 \cap k_5$

## Distances

$$d_0 = |GM| = |HL| \text{ (red) } \dots 2$$

$$d_1 = |BL| = |BM| = |CD| = |CE| = |CF| = |CI| = |CJ| = |CK| = |DE| = |DF| = |FK| \text{ (blue) } \dots 11$$

$$d_2 = |GL| = |HM| \text{ (green) } \dots 2$$

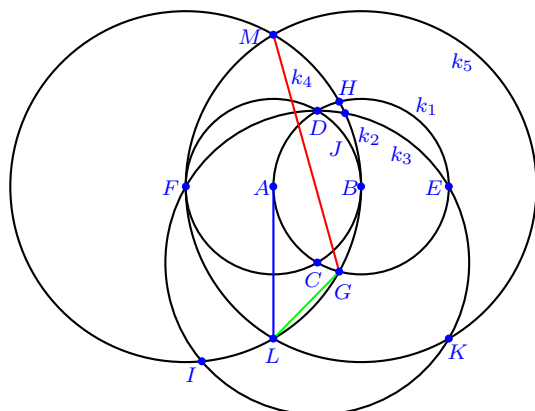
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (44 occurrences)}$$

## Identical Constructions

RCC403 MM152

## 2.123.4 Construction CCO99



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, D)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, B)$   
 $G \in k_1 \cap k_4, H \in k_1 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
  5.  $k_5 = k(B, F)$   
 $K \in k_3 \cap k_5, L \in k_4 \cap k_5, M \in k_4 \cap k_5$

## Distances

$$d_0 = |GM| = |HL| \text{ (red)} \dots 2$$

$$d_1 = |AL| = |AM| = |CD| = |CE| = |CF| = |CI| = |CJ| = |CK| = |DE| = |DF| = |EK| \text{ (blue)} \dots 11$$

$$d_2 = |GL| = |HM| \text{ (green)} \dots 2$$

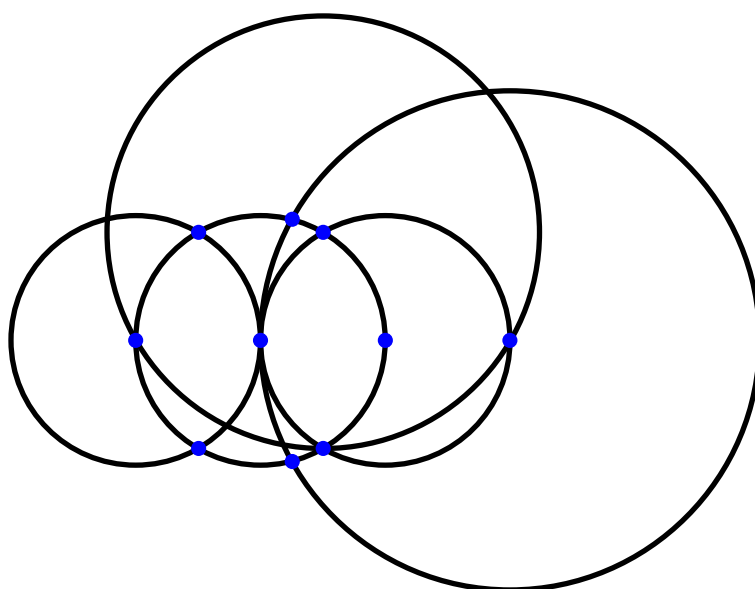
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (44 occurrences)}$$

## Identical Constructions

RCC434 MM156

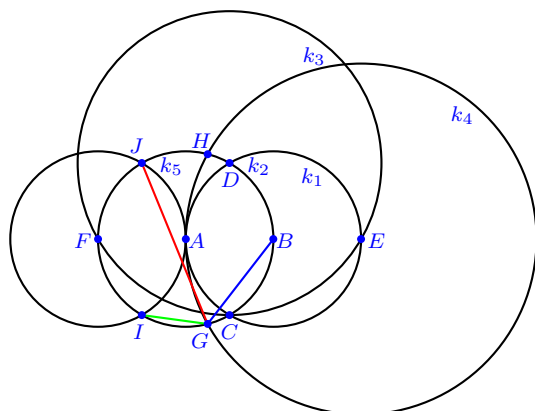
## 2.124 Class of Similar Constructions No. 113



### Contained Constructions

CCO46, CCO63, CCO81, CCO98

## 2.124.1 Construction CCO46



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, C)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, A)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
  5.  $k_5 = k(F, A)$   
 $I \in k_2 \cap k_5, J \in k_2 \cap k_5$

## Distances

$$d_0 = |GJ| = |HI| \text{ (red) } \dots 2$$

$$d_1 = |BG| = |BH| \text{ (blue) } \dots 2$$

$$d_2 = |GI| = |HJ| \text{ (green) } \dots 2$$

## Occurrences of the Golden Ratio

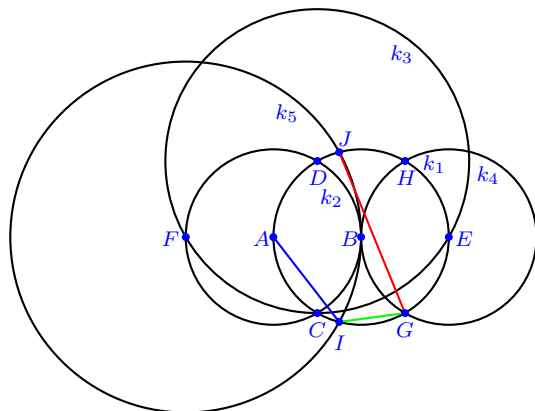
$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)}$$

## Identical Constructions

RCC349 MM258



## 2.124.2 Construction CCO63



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, C)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, B)$   
 $G \in k_1 \cap k_4, H \in k_1 \cap k_4$
  5.  $k_5 = k(F, B)$   
 $I \in k_1 \cap k_5, J \in k_1 \cap k_5$

## Distances

$$d_0 = |GJ| = |HI| \text{ (red) } \dots 2$$

$$d_1 = |AI| = |AJ| \text{ (blue) } \dots 2$$

$$d_2 = |GI| = |HJ| \text{ (green) } \dots 2$$

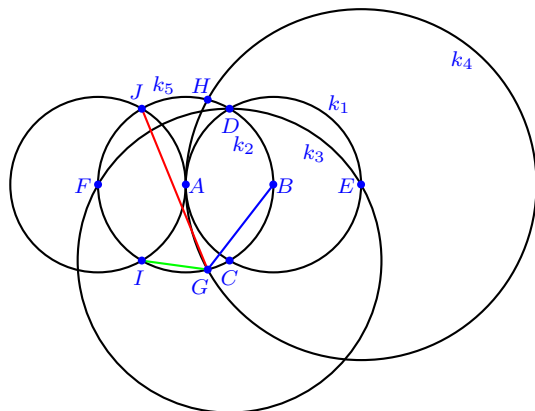
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)}$$

## Identical Constructions

RCC376 MM259

## 2.124.3 Construction CCO81



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, D)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, A)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
  5.  $k_5 = k(F, A)$   
 $I \in k_2 \cap k_5, J \in k_2 \cap k_5$

## Distances

$$d_0 = |GJ| = |HI| \text{ (red) } \dots 2$$

$$d_1 = |BG| = |BH| \text{ (blue) } \dots 2$$

$$d_2 = |GI| = |HJ| \text{ (green) } \dots 2$$

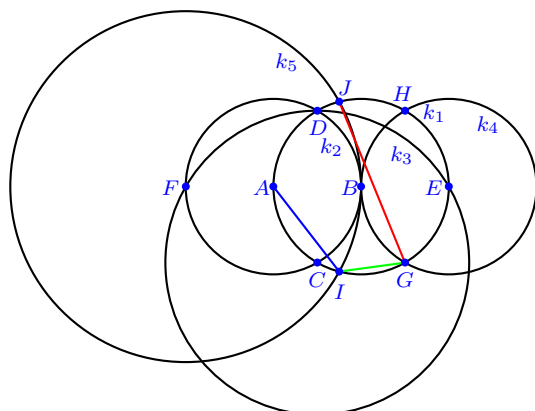
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)}$$

## Identical Constructions

RCC404 MM147

## 2.124.4 Construction CCO98



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, D)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, B)$   
 $G \in k_1 \cap k_4, H \in k_1 \cap k_4$
  5.  $k_5 = k(F, B)$   
 $I \in k_1 \cap k_5, J \in k_1 \cap k_5$

## Distances

$$d_0 = |GJ| = |HI| \text{ (red) } \dots 2$$

$$d_1 = |AI| = |AJ| \text{ (blue) } \dots 2$$

$$d_2 = |GI| = |HJ| \text{ (green) } \dots 2$$

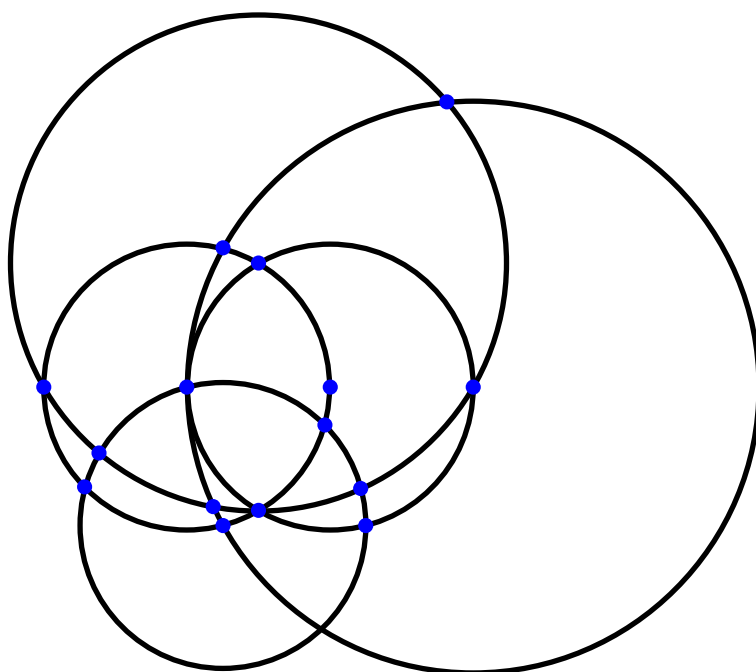
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (8 occurrences)}$$

## Identical Constructions

RCC431 MM148

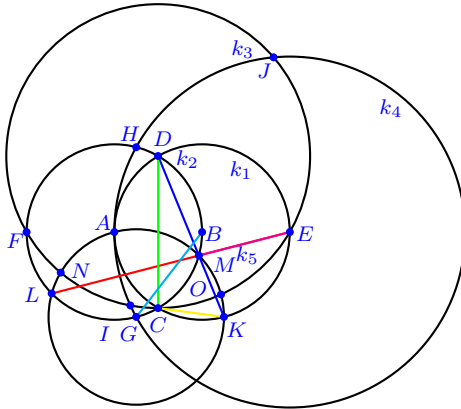
## 2.125 Class of Similar Constructions No. 114



### Contained Constructions

CCO47, CCO65, CCO89, CCO107

## 2.125.1 Construction CCO47



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, C)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, A)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
  5.  $k_5 = k(G, A)$   
 $K \in k_1 \cap k_5, L \in k_2 \cap k_5, M \in k_2 \cap k_5, N \in k_3 \cap k_5, O \in k_3 \cap k_5$

## Distances

$$\begin{aligned}
 d_0 &= |EL| \text{ (red)} \dots 1 \\
 d_1 &= |DK| = |DL| = |FM| = |KL| \text{ (blue)} \dots 4 \\
 d_2 &= |CD| = |CE| = |CF| = |DE| = |DF| = |DI| = |DJ| = |DN| = |DO| = |LM| \text{ (green)} \dots 10 \\
 d_3 &= |BG| = |BH| = |CL| = |DM| = |EK| \text{ (cyan)} \dots 5 \\
 d_4 &= |EM| \text{ (magenta)} \dots 1 \\
 d_5 &= |CK| = |CM| = |FL| = |KM| \text{ (yellow)} \dots 4
 \end{aligned}$$

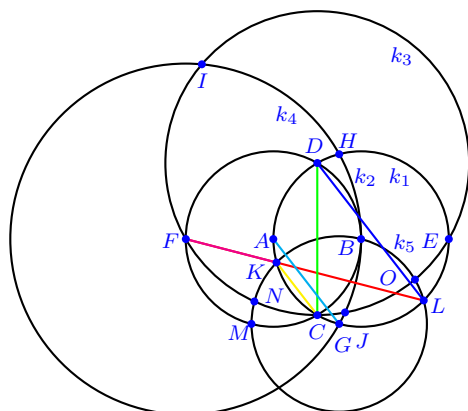
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (60 occurrences)}$$

## Identical Constructions

RCC351 MM323

## 2.125.2 Construction CCO65



## Construction Process

- $A, B$  given initial points
- $k_1 = k(B, A)$
  - $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  - $k_3 = k(D, C)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  - $k_4 = k(F, B)$   
 $G \in k_1 \cap k_4, H \in k_1 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
  - $k_5 = k(G, B)$   
 $K \in k_1 \cap k_5, L \in k_1 \cap k_5, M \in k_2 \cap k_5, N \in k_3 \cap k_5, O \in k_3 \cap k_5$

## Distances

$$\begin{aligned}
 d_0 &= |FL| \text{ (red)} \dots 1 \\
 d_1 &= |DL| = |DM| = |EK| = |LM| \text{ (blue)} \dots 4 \\
 d_2 &= |CD| = |CE| = |CF| = |DE| = |DF| = |DI| = |DJ| = |DN| = |DO| = |KL| \text{ (green)} \dots 10 \\
 d_3 &= |AG| = |AH| = |CL| = |DK| = |FM| \text{ (cyan)} \dots 5 \\
 d_4 &= |FK| \text{ (magenta)} \dots 1 \\
 d_5 &= |CK| = |CM| = |EL| = |KM| \text{ (yellow)} \dots 4
 \end{aligned}$$

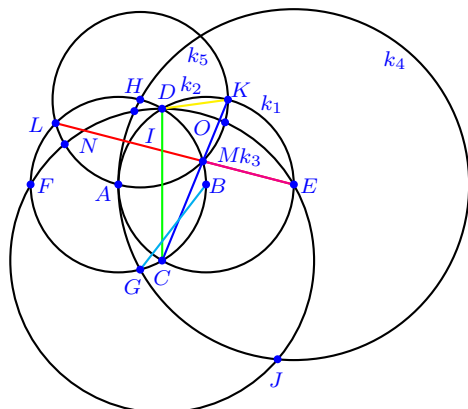
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (60 occurrences)}$$

## Identical Constructions

RCC381 MM277

## 2.125.3 Construction CCO89



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, D)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, A)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
  5.  $k_5 = k(H, A)$   
 $K \in k_1 \cap k_5, L \in k_2 \cap k_5, M \in k_2 \cap k_5, N \in k_3 \cap k_5, O \in k_3 \cap k_5$

## Distances

$$\begin{aligned}
 d_0 &= |EL| \text{ (red)} \dots 1 \\
 d_1 &= |CK| = |CL| = |FM| = |KL| \text{ (blue)} \dots 4 \\
 d_2 &= |CD| = |CE| = |CF| = |CI| = |CJ| = |CN| = |CO| = |DE| = \\
 &= |DF| = |LM| \text{ (green)} \dots 10 \\
 d_3 &= |BG| = |BH| = |CM| = |DL| = |EK| \text{ (cyan)} \dots 5 \\
 d_4 &= |EM| \text{ (magenta)} \dots 1 \\
 d_5 &= |DK| = |DM| = |FL| = |KM| \text{ (yellow)} \dots 4
 \end{aligned}$$

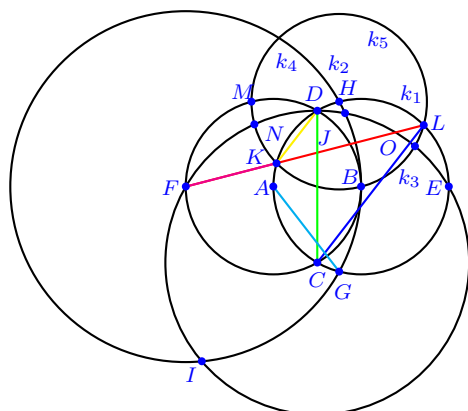
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (60 occurrences)}$$

## Identical Constructions

RCC417 MM215

## 2.125.4 Construction CCO107



## Construction Process

- $A, B$  given initial points
- $k_1 = k(B, A)$
  - $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  - $k_3 = k(C, D)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  - $k_4 = k(F, B)$   
 $G \in k_1 \cap k_4, H \in k_1 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
  - $k_5 = k(H, B)$   
 $K \in k_1 \cap k_5, L \in k_1 \cap k_5, M \in k_2 \cap k_5, N \in k_3 \cap k_5, O \in k_3 \cap k_5$

## Distances

$$\begin{aligned}
 d_0 &= |FL| \text{ (red)} \dots 1 \\
 d_1 &= |CL| = |CM| = |EK| = |LM| \text{ (blue)} \dots 4 \\
 d_2 &= |CD| = |CE| = |CF| = |CI| = |CJ| = |CN| = |CO| = |DE| = \\
 &= |DF| = |KL| \text{ (green)} \dots 10 \\
 d_3 &= |AG| = |AH| = |CK| = |DL| = |FM| \text{ (cyan)} \dots 5 \\
 d_4 &= |FK| \text{ (magenta)} \dots 1 \\
 d_5 &= |DK| = |DM| = |EL| = |KM| \text{ (yellow)} \dots 4
 \end{aligned}$$

## Occurrences of the Golden Ratio

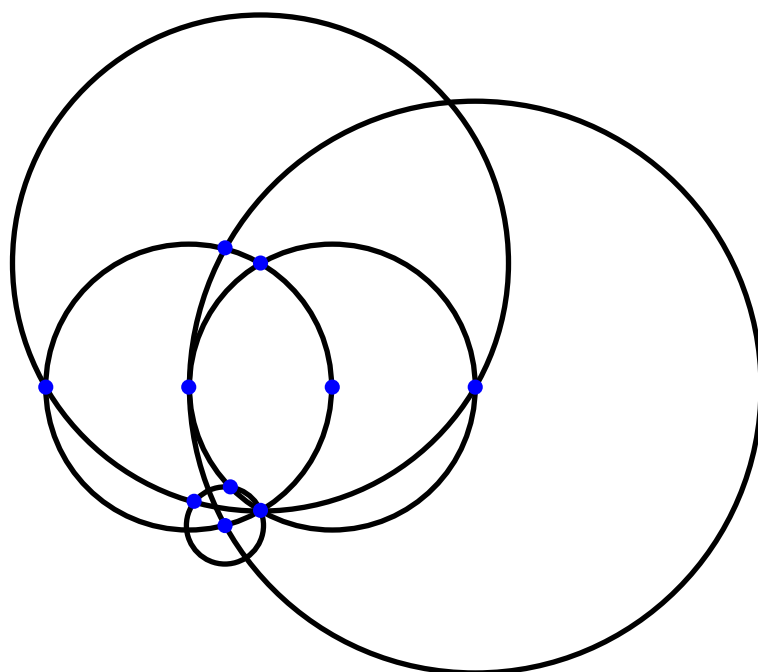
$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (60 occurrences)}$$

## Identical Constructions

RCC447 MM169



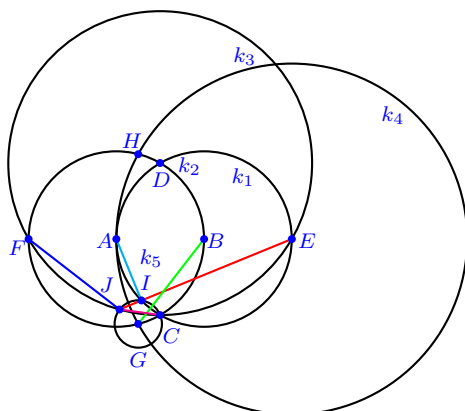
## 2.126 Class of Similar Constructions No. 115



### Contained Constructions

CCO48, CCO66, CCO92, CCO110

## 2.126.1 Construction CCO48



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, C)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, A)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
  5.  $k_5 = k(G, C)$   
 $I \in k_1 \cap k_5, J \in k_3 \cap k_5$

## Distances

$$d_0 = |EJ| \text{ (red)} \dots 1$$

$$d_1 = |FJ| \text{ (blue)} \dots 1$$

$$d_2 = |BG| = |BH| \text{ (green)} \dots 2$$

$$d_3 = |AI| \text{ (cyan)} \dots 1$$

$$d_4 = |CJ| \text{ (magenta)} \dots 1$$

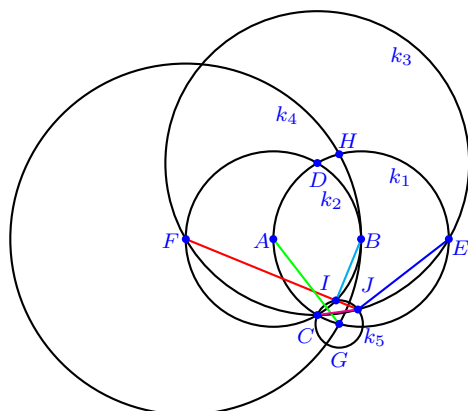
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_2}{d_3} = \frac{d_3}{d_4} \text{ (4 occurrences)}$$

## Identical Constructions

RCC353 MM339

## 2.126.2 Construction CCO66



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, C)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, B)$   
 $G \in k_1 \cap k_4, H \in k_1 \cap k_4$
  5.  $k_5 = k(G, C)$   
 $I \in k_2 \cap k_5, J \in k_3 \cap k_5$

## Distances

$$d_0 = |FJ| \text{ (red)} \dots 1$$

$$d_1 = |EJ| \text{ (blue)} \dots 1$$

$$d_2 = |AG| = |AH| \text{ (green)} \dots 2$$

$$d_3 = |BI| \text{ (cyan)} \dots 1$$

$$d_4 = |CJ| \text{ (magenta)} \dots 1$$

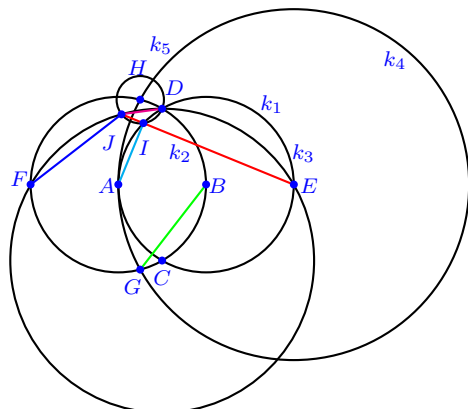
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_2}{d_3} = \frac{d_3}{d_4} \text{ (4 occurrences)}$$

## Identical Constructions

RCC383 MM293

## 2.126.3 Construction CCO92



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, D)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, A)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
  5.  $k_5 = k(H, D)$   
 $I \in k_1 \cap k_5, J \in k_3 \cap k_5$

## Distances

$$d_0 = |EJ| \text{ (red)} \dots 1$$

$$d_1 = |FJ| \text{ (blue)} \dots 1$$

$$d_2 = |BG| = |BH| \text{ (green)} \dots 2$$

$$d_3 = |AI| \text{ (cyan)} \dots 1$$

$$d_4 = |DJ| \text{ (magenta)} \dots 1$$

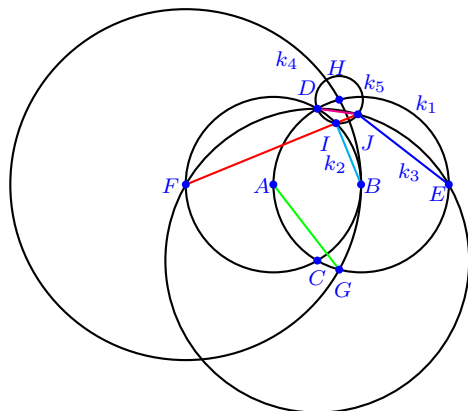
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_2}{d_3} = \frac{d_3}{d_4} \text{ (4 occurrences)}$$

## Identical Constructions

RCC422 MM229

## 2.126.4 Construction CCO110



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, D)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, B)$   
 $G \in k_1 \cap k_4, H \in k_1 \cap k_4$
  5.  $k_5 = k(H, D)$   
 $I \in k_2 \cap k_5, J \in k_3 \cap k_5$

## Distances

$$d_0 = |FJ| \text{ (red)} \dots 1$$

$$d_1 = |EJ| \text{ (blue)} \dots 1$$

$$d_2 = |AG| = |AH| \text{ (green)} \dots 2$$

$$d_3 = |BI| \text{ (cyan)} \dots 1$$

$$d_4 = |DJ| \text{ (magenta)} \dots 1$$

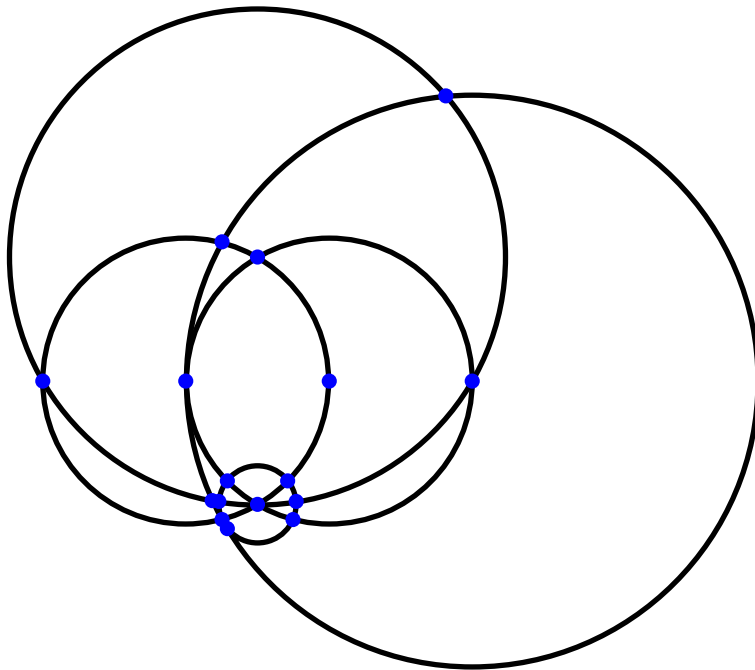
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_2}{d_3} = \frac{d_3}{d_4} \text{ (4 occurrences)}$$

## Identical Constructions

RCC452 MM183

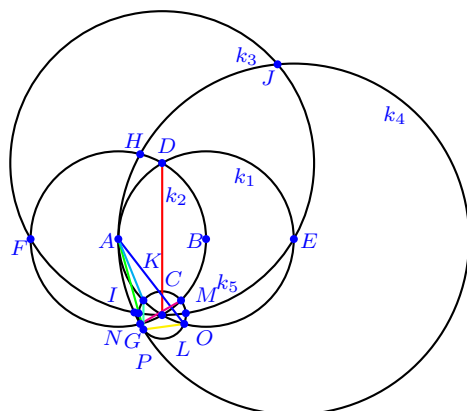
**2.127 Class of Similar Constructions No. 116**



**Contained Constructions**

CCO49, CCO67, CCO93, CCO111

## 2.127.1 Construction CCO49



## Construction Process

- $A, B$  given initial points
- $k_1 = k(B, A)$
  - $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  - $k_3 = k(D, C)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  - $k_4 = k(E, A)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
  - $k_5 = k(C, G)$   
 $K \in k_1 \cap k_5, L \in k_1 \cap k_5, M \in k_2 \cap k_5, N \in k_3 \cap k_5, O \in k_3 \cap k_5,$   
 $P \in k_4 \cap k_5$

## Distances

$$d_0 = |CD| = |CE| = |CF| = |DE| = |DF| = |DI| = |DJ| = |DN| = |DO| = |HM| \text{ (red)} \dots 10$$

$$d_1 = |AL| = |BG| = |BH| \text{ (blue)} \dots 3$$

$$d_2 = |AP| \text{ (green)} \dots 1$$

$$d_3 = |AK| = |BM| \text{ (cyan)} \dots 2$$

$$d_4 = |GM| = |KL| \text{ (magenta)} \dots 2$$

$$d_5 = |LP| \text{ (yellow)} \dots 1$$

$$d_6 = |KP| \text{ (grass)} \dots 1$$

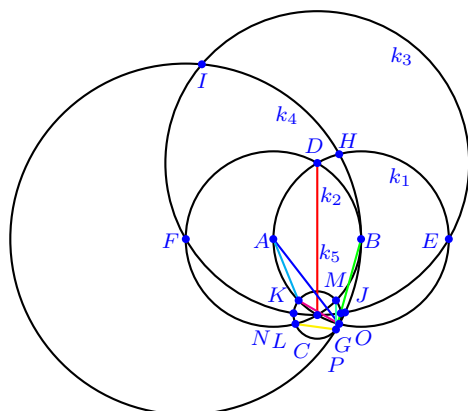
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_3}{d_5} = \frac{d_4}{d_6} \text{ (20 occurrences)}$$

## Identical Constructions

RCC354 MM335

## 2.127.2 Construction CCO67



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, C)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, B)$   
 $G \in k_1 \cap k_4, H \in k_1 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
  5.  $k_5 = k(C, G)$   
 $K \in k_1 \cap k_5, L \in k_2 \cap k_5, M \in k_2 \cap k_5, N \in k_3 \cap k_5, O \in k_3 \cap k_5,$   
 $P \in k_4 \cap k_5$

## Distances

$$d_0 = |CD| = |CE| = |CF| = |DE| = |DF| = |DI| = |DJ| = |DN| = |DO| = |HK| \text{ (red)} \dots 10$$

$$d_1 = |AG| = |AH| = |BL| \text{ (blue)} \dots 3$$

$$d_2 = |BP| \text{ (green)} \dots 1$$

$$d_3 = |AK| = |BM| \text{ (cyan)} \dots 2$$

$$d_4 = |GK| = |LM| \text{ (magenta)} \dots 2$$

$$d_5 = |LP| \text{ (yellow)} \dots 1$$

$$d_6 = |MP| \text{ (grass)} \dots 1$$

## Occurrences of the Golden Ratio

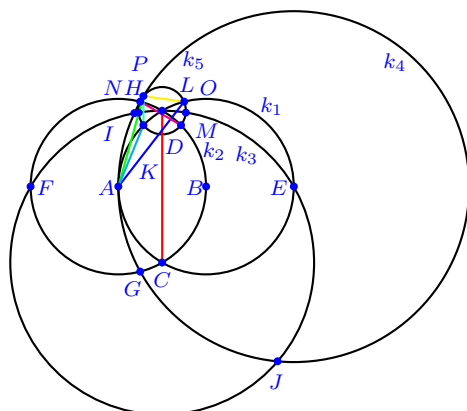
$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_3}{d_5} = \frac{d_4}{d_6} \text{ (20 occurrences)}$$

## Identical Constructions

RCC384 MM289



## 2.127.3 Construction CCO93



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, D)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, A)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
  5.  $k_5 = k(D, H)$   
 $K \in k_1 \cap k_5, L \in k_1 \cap k_5, M \in k_2 \cap k_5, N \in k_3 \cap k_5, O \in k_3 \cap k_5,$   
 $P \in k_4 \cap k_5$

## Distances

$$d_0 = |CD| = |CE| = |CF| = |CI| = |CJ| = |CN| = |CO| = |DE| = |DF| = |GM| \text{ (red)} \dots 10$$

$$d_1 = |AL| = |BG| = |BH| \text{ (blue)} \dots 3$$

$$d_2 = |AP| \text{ (green)} \dots 1$$

$$d_3 = |AK| = |BM| \text{ (cyan)} \dots 2$$

$$d_4 = |HM| = |KL| \text{ (magenta)} \dots 2$$

$$d_5 = |LP| \text{ (yellow)} \dots 1$$

$$d_6 = |KP| \text{ (grass)} \dots 1$$

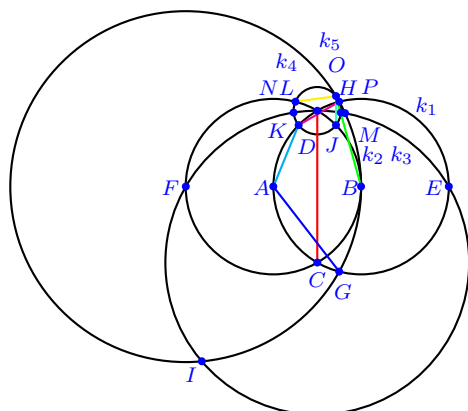
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_3}{d_5} = \frac{d_4}{d_6} \text{ (20 occurrences)}$$

## Identical Constructions

RCC423 MM225

## 2.127.4 Construction CCO111



## Construction Process

- $A, B$  given initial points
- $k_1 = k(B, A)$
  - $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  - $k_3 = k(C, D)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  - $k_4 = k(F, B)$   
 $G \in k_1 \cap k_4, H \in k_1 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
  - $k_5 = k(D, H)$   
 $K \in k_1 \cap k_5, L \in k_2 \cap k_5, M \in k_2 \cap k_5, N \in k_3 \cap k_5, O \in k_3 \cap k_5,$   
 $P \in k_4 \cap k_5$

## Distances

$$d_0 = |CD| = |CE| = |CF| = |CI| = |CJ| = |CN| = |CO| = |DE| = |DF| = |GK| \text{ (red)} \dots 10$$

$$d_1 = |AG| = |AH| = |BL| \text{ (blue)} \dots 3$$

$$d_2 = |BP| \text{ (green)} \dots 1$$

$$d_3 = |AK| = |BM| \text{ (cyan)} \dots 2$$

$$d_4 = |HK| = |LM| \text{ (magenta)} \dots 2$$

$$d_5 = |LP| \text{ (yellow)} \dots 1$$

$$d_6 = |MP| \text{ (grass)} \dots 1$$

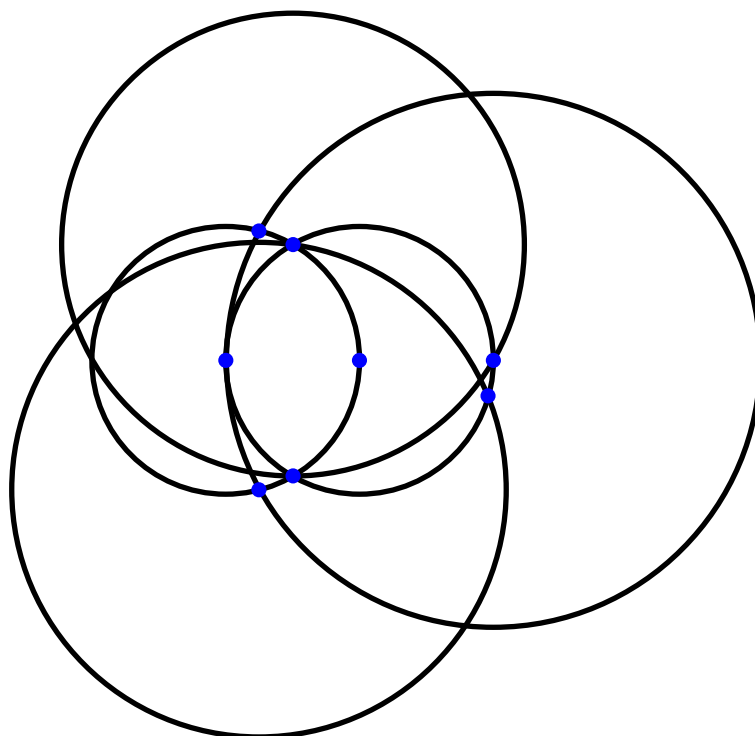
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_3}{d_5} = \frac{d_4}{d_6} \text{ (20 occurrences)}$$

## Identical Constructions

RCC453 MM179

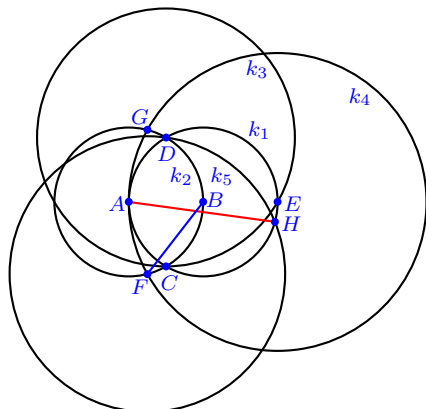
## 2.128 Class of Similar Constructions No. 117



### Contained Constructions

CCO50, CCO68, CCO90, CCO108

## 2.128.1 Construction CCO50



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, C)$   
 $E \in k_1 \cap k_3$
  4.  $k_4 = k(E, A)$   
 $F \in k_2 \cap k_4, G \in k_2 \cap k_4$
  5.  $k_5 = k(F, D)$   
 $H \in k_1 \cap k_5$

## Distances

$$d_0 = |AH| \text{ (red)} \dots 1$$

$$d_1 = |BF| = |BG| \text{ (blue)} \dots 2$$

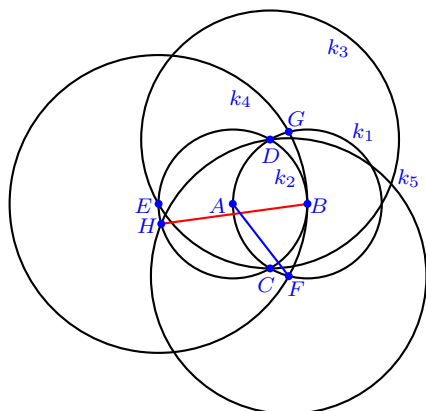
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (2 occurrences)}$$

## Identical Constructions

RCC356 MM347

## 2.128.2 Construction CCO68



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, C)$   
 $E \in k_2 \cap k_3$
  4.  $k_4 = k(E, B)$   
 $F \in k_1 \cap k_4, G \in k_1 \cap k_4$
  5.  $k_5 = k(F, D)$   
 $H \in k_2 \cap k_5$

## Distances

$$d_0 = |BH| \text{ (red) } \dots 1$$

$$d_1 = |AF| = |AG| \text{ (blue) } \dots 2$$

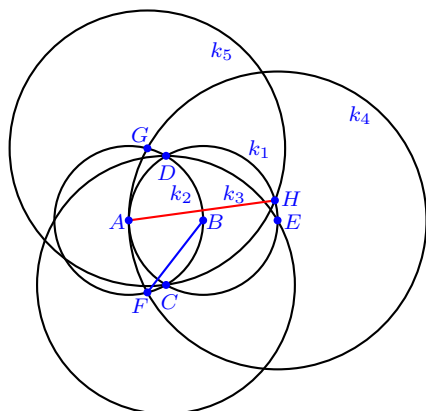
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (2 occurrences)}$$

## Identical Constructions

RCC386 MM301

## 2.128.3 Construction CCO90



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, D)$   
 $E \in k_1 \cap k_3$
  4.  $k_4 = k(E, A)$   
 $F \in k_2 \cap k_4, G \in k_2 \cap k_4$
  5.  $k_5 = k(G, C)$   
 $H \in k_1 \cap k_5$

## Distances

$$d_0 = |AH| \text{ (red) } \dots 1$$

$$d_1 = |BF| = |BG| \text{ (blue) } \dots 2$$

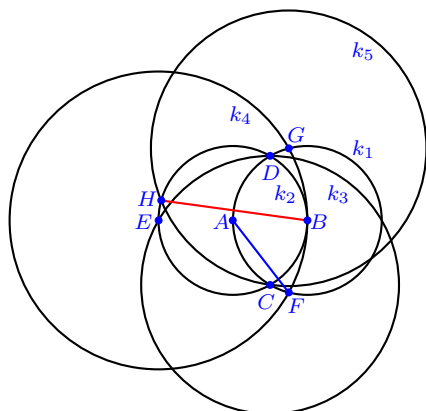
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (2 occurrences)}$$

## Identical Constructions

RCC419 MM237

## 2.128.4 Construction CCO108



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, D)$   
 $E \in k_2 \cap k_3$
  4.  $k_4 = k(E, B)$   
 $F \in k_1 \cap k_4, G \in k_1 \cap k_4$
  5.  $k_5 = k(G, C)$   
 $H \in k_2 \cap k_5$

## Distances

$$d_0 = |BH| \text{ (red)} \dots 1$$

$$d_1 = |AF| = |AG| \text{ (blue)} \dots 2$$

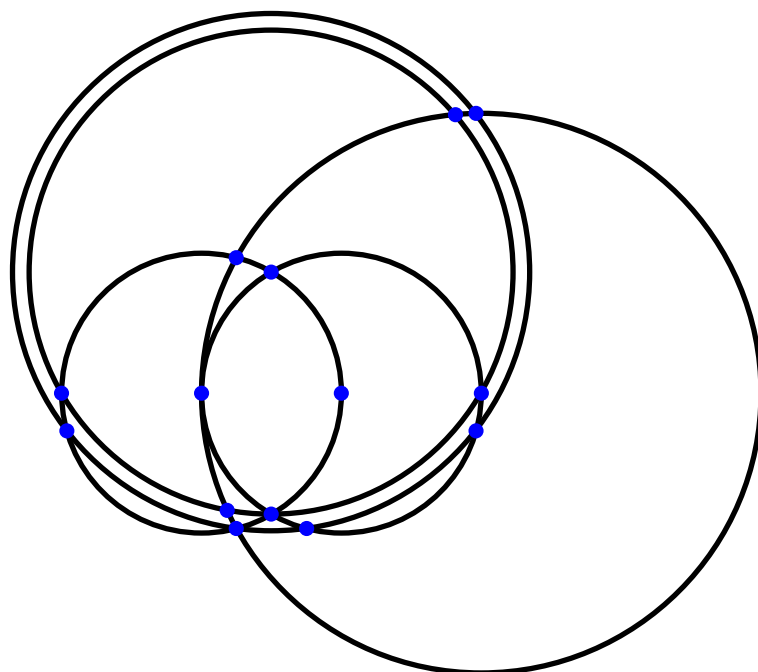
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (2 occurrences)}$$

## Identical Constructions

RCC449 MM191

## 2.129 Class of Similar Constructions No. 118

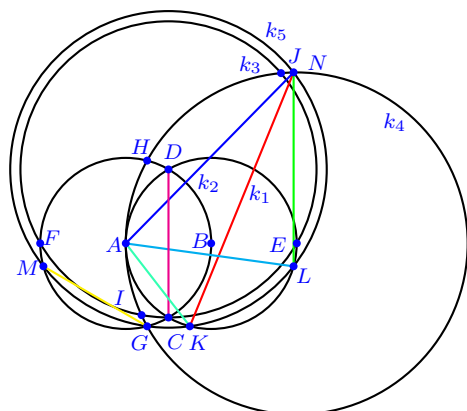


### Contained Constructions

CCO51, CCO69, CCO91, CCO109



## 2.129.1 Construction CCO51



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, C)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, A)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
  5.  $k_5 = k(D, G)$   
 $K \in k_1 \cap k_5, L \in k_1 \cap k_5, M \in k_2 \cap k_5, N \in k_4 \cap k_5$

## Distances

- $d_0 = |KN|$  (red) ... 1  
 $d_1 = |AN|$  (blue) ... 1  
 $d_2 = |LN|$  (green) ... 1  
 $d_3 = |AL| = |BM|$  (cyan) ... 2  
 $d_4 = |CD| = |CE| = |CF| = |DE| = |DF| = |DI| = |DJ| = |HM|$  (magenta) ... 8  
 $d_5 = |GM| = |KL|$  (yellow) ... 2  
 $d_6 = |AK| = |BG| = |BH|$  (grass) ... 3

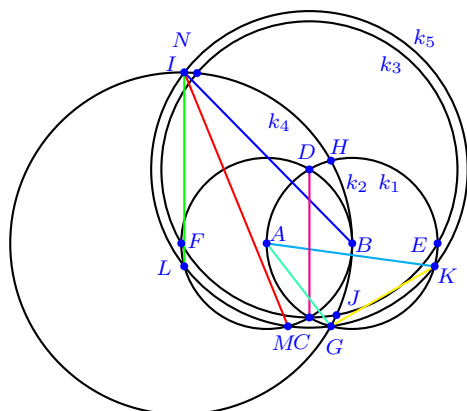
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_3} = \frac{d_1}{d_4} = \frac{d_2}{d_5} = \frac{d_3}{d_6} \quad (18 \text{ occurrences})$$

## Identical Constructions

RCC357 MM344

## 2.129.2 Construction CCO69



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_3$
  3.  $k_3 = k(D, C)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, B)$   
 $G \in k_1 \cap k_4, H \in k_1 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
  5.  $k_5 = k(D, G)$   
 $K \in k_1 \cap k_5, L \in k_2 \cap k_5, M \in k_2 \cap k_5, N \in k_4 \cap k_5$

## Distances

- $d_0 = |MN|$  (red) ... 1  
 $d_1 = |BN|$  (blue) ... 1  
 $d_2 = |LN|$  (green) ... 1  
 $d_3 = |AK| = |BL|$  (cyan) ... 2  
 $d_4 = |CD| = |CE| = |CF| = |DE| = |DF| = |DI| = |DJ| = |HK|$  (magenta) ... 8  
 $d_5 = |GK| = |LM|$  (yellow) ... 2  
 $d_6 = |AG| = |AH| = |BM|$  (grass) ... 3

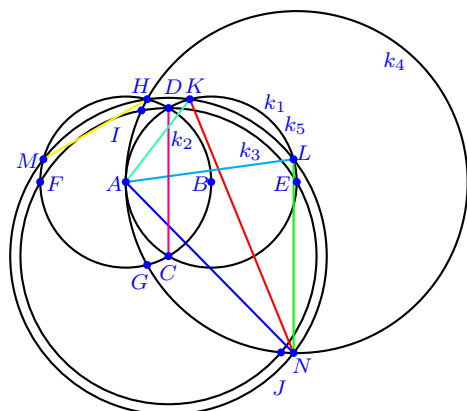
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_3} = \frac{d_1}{d_4} = \frac{d_2}{d_5} = \frac{d_3}{d_6} \quad (18 \text{ occurrences})$$

## Identical Constructions

RCC387 MM298

## 2.129.3 Construction CCO91



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, D)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, A)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
  5.  $k_5 = k(C, H)$   
 $K \in k_1 \cap k_5, L \in k_1 \cap k_5, M \in k_2 \cap k_5, N \in k_4 \cap k_5$

## Distances

- $d_0 = |KN|$  (red) ... 1  
 $d_1 = |AN|$  (blue) ... 1  
 $d_2 = |LN|$  (green) ... 1  
 $d_3 = |AL| = |BM|$  (cyan) ... 2  
 $d_4 = |CD| = |CE| = |CF| = |CI| = |CJ| = |DE| = |DF| = |GM|$  (magenta) ... 8  
 $d_5 = |HM| = |KL|$  (yellow) ... 2  
 $d_6 = |AK| = |BG| = |BH|$  (grass) ... 3

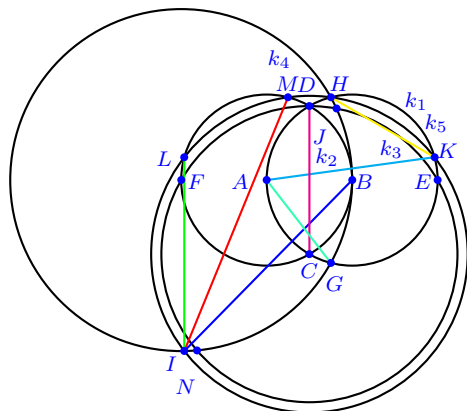
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_3} = \frac{d_1}{d_4} = \frac{d_2}{d_5} = \frac{d_3}{d_6} \quad (18 \text{ occurrences})$$

## Identical Constructions

RCC420 MM232

## 2.129.4 Construction CCO109



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, D)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, B)$   
 $G \in k_1 \cap k_4, H \in k_1 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
  5.  $k_5 = k(C, H)$   
 $K \in k_1 \cap k_5, L \in k_2 \cap k_5, M \in k_2 \cap k_5, N \in k_4 \cap k_5$

## Distances

- $d_0 = |MN|$  (red) ... 1  
 $d_1 = |BN|$  (blue) ... 1  
 $d_2 = |LN|$  (green) ... 1  
 $d_3 = |AK| = |BL|$  (cyan) ... 2  
 $d_4 = |CD| = |CE| = |CF| = |CI| = |CJ| = |DE| = |DF| = |GK|$  (magenta) ... 8  
 $d_5 = |HK| = |LM|$  (yellow) ... 2  
 $d_6 = |AG| = |AH| = |BM|$  (grass) ... 3

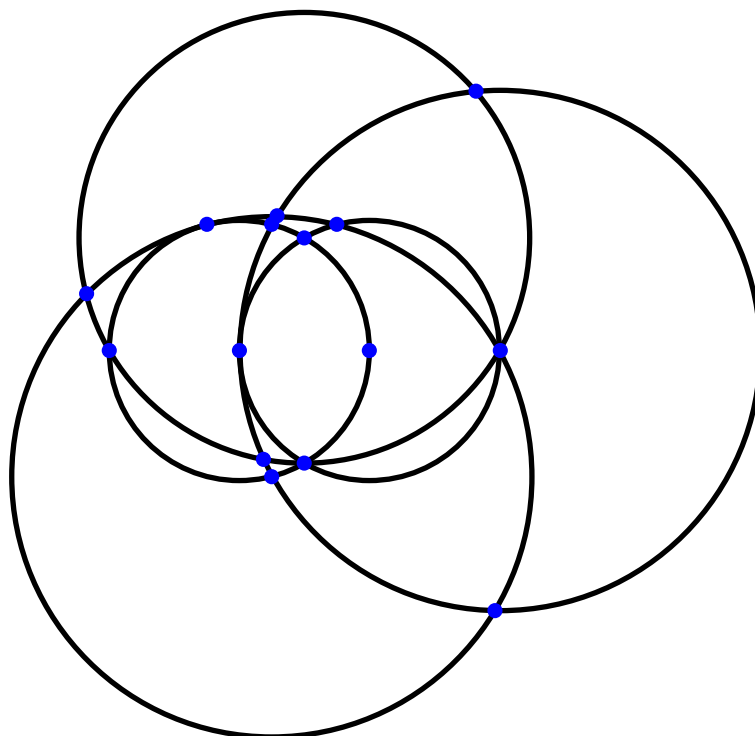
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_3} = \frac{d_1}{d_4} = \frac{d_2}{d_5} = \frac{d_3}{d_6} \quad (18 \text{ occurrences})$$

## Identical Constructions

RCC450 MM186

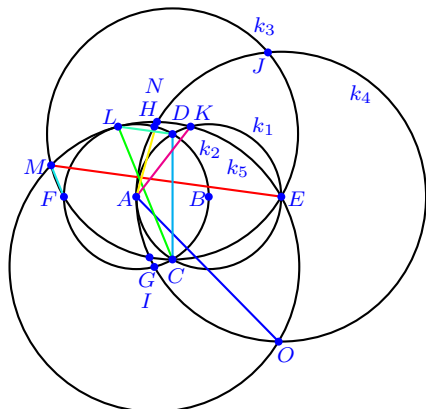
## 2.130 Class of Similar Constructions No. 119



### Contained Constructions

CCO52, CCO71, CCO94, CCO113

## 2.130.1 Construction CCO52



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, C)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, A)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
  5.  $k_5 = k(G, E)$   
 $K \in k_1 \cap k_5, L \in k_2 \cap k_5, M \in k_3 \cap k_5, N \in k_4 \cap k_5, O \in k_4 \cap k_5$

## Distances

- $d_0 = |EM| = |KO|$  (red) ... 2  
 $d_1 = |AO|$  (blue) ... 1  
 $d_2 = |CL|$  (green) ... 1  
 $d_3 = |CD| = |CE| = |CF| = |DE| = |DF| = |DI| = |DJ| = |DM|$  (cyan) ... 8  
 $d_4 = |AK| = |BG| = |BH| = |FL|$  (magenta) ... 4  
 $d_5 = |AN| = |LM|$  (yellow) ... 2  
 $d_6 = |DL|$  (grass) ... 1  
 $d_7 = |FM| = |KN|$  (sea) ... 2

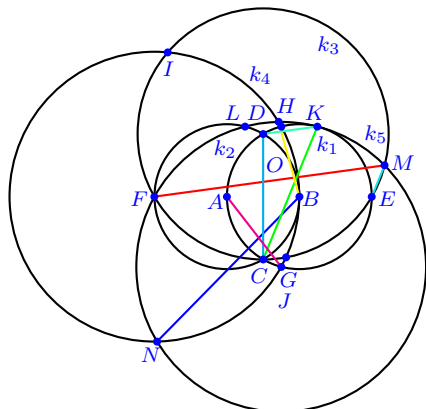
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} = \frac{d_4}{d_6} = \frac{d_6}{d_7} \quad (36 \text{ occurrences})$$

## Identical Constructions

RCC358 MM325

## 2.130.2 Construction CCO71



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, C)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, B)$   
 $G \in k_1 \cap k_4, H \in k_1 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
  5.  $k_5 = k(G, F)$   
 $K \in k_1 \cap k_5, L \in k_2 \cap k_5, M \in k_3 \cap k_5, N \in k_4 \cap k_5, O \in k_4 \cap k_5$

## Distances

$$\begin{aligned}
 d_0 &= |FM| = |LN| \text{ (red)} \dots 2 \\
 d_1 &= |BN| \text{ (blue)} \dots 1 \\
 d_2 &= |CK| \text{ (green)} \dots 1 \\
 d_3 &= |CD| = |CE| = |CF| = |DE| = |DF| = |DI| = |DJ| = |DM| \\
 &\text{ (cyan)} \dots 8 \\
 d_4 &= |AG| = |AH| = |BL| = |EK| \text{ (magenta)} \dots 4 \\
 d_5 &= |BO| = |KM| \text{ (yellow)} \dots 2 \\
 d_6 &= |DK| \text{ (grass)} \dots 1 \\
 d_7 &= |EM| = |LO| \text{ (sea)} \dots 2
 \end{aligned}$$

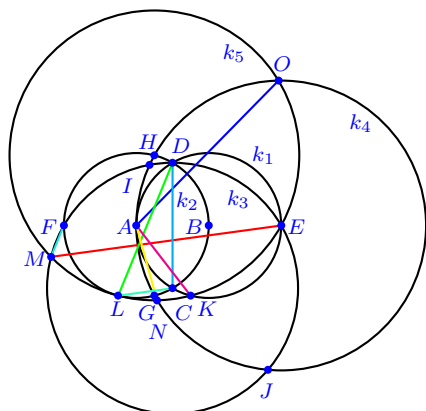
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} = \frac{d_4}{d_6} = \frac{d_6}{d_7} \text{ (36 occurrences)}$$

## Identical Constructions

RCC390 MM279

## 2.130.3 Construction CCO94



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, D)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, A)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
  5.  $k_5 = k(H, E)$   
 $K \in k_1 \cap k_5, L \in k_2 \cap k_5, M \in k_3 \cap k_5, N \in k_4 \cap k_5, O \in k_4 \cap k_5$

## Distances

- $d_0 = |EM| = |KO|$  (red) ... 2  
 $d_1 = |AO|$  (blue) ... 1  
 $d_2 = |DL|$  (green) ... 1  
 $d_3 = |CD| = |CE| = |CF| = |CI| = |CJ| = |CM| = |DE| = |DF|$  (cyan) ... 8  
 $d_4 = |AK| = |BG| = |BH| = |FL|$  (magenta) ... 4  
 $d_5 = |AN| = |LM|$  (yellow) ... 2  
 $d_6 = |CL|$  (grass) ... 1  
 $d_7 = |FM| = |KN|$  (sea) ... 2

## Occurrences of the Golden Ratio

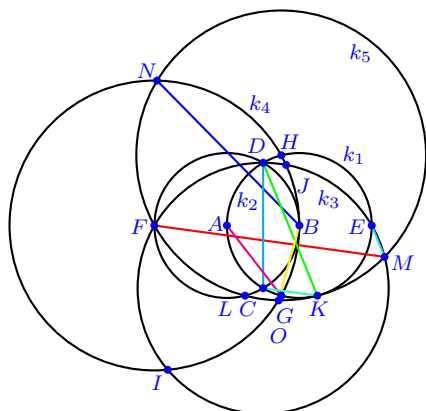
$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} = \frac{d_4}{d_6} = \frac{d_6}{d_7} \quad (36 \text{ occurrences})$$

## Identical Constructions

RCC424 MM217



## 2.130.4 Construction CCO113



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, D)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, B)$   
 $G \in k_1 \cap k_4, H \in k_1 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
  5.  $k_5 = k(H, F)$   
 $K \in k_1 \cap k_5, L \in k_2 \cap k_5, M \in k_3 \cap k_5, N \in k_4 \cap k_5, O \in k_4 \cap k_5$

## Distances

- $d_0 = |FM| = |LN|$  (red) ... 2  
 $d_1 = |BN|$  (blue) ... 1  
 $d_2 = |DK|$  (green) ... 1  
 $d_3 = |CD| = |CE| = |CF| = |CI| = |CJ| = |CM| = |DE| = |DF|$  (cyan) ... 8  
 $d_4 = |AG| = |AH| = |BL| = |EK|$  (magenta) ... 4  
 $d_5 = |BO| = |KM|$  (yellow) ... 2  
 $d_6 = |CK|$  (grass) ... 1  
 $d_7 = |EM| = |LO|$  (sea) ... 2

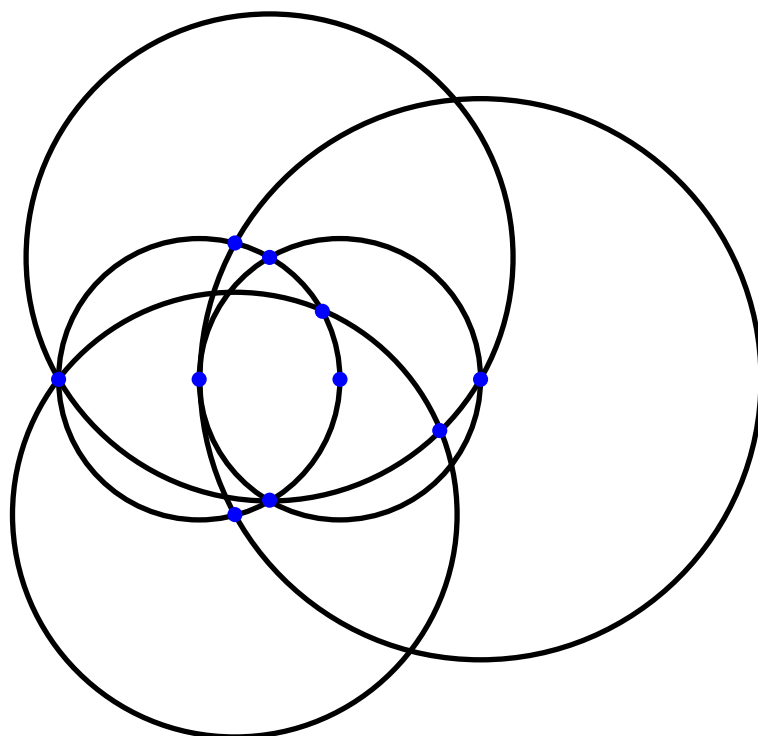
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} = \frac{d_4}{d_6} = \frac{d_5}{d_7} \quad (36 \text{ occurrences})$$

## Identical Constructions

RCC456 MM171

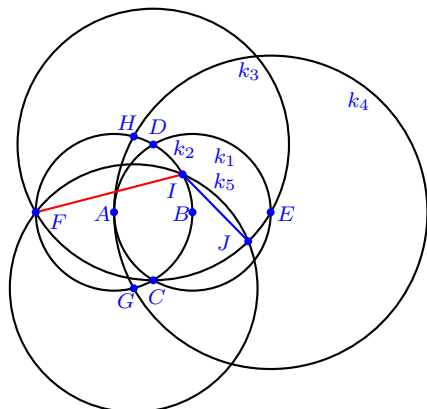
## 2.131 Class of Similar Constructions No. 120



### Contained Constructions

CCO53, CCO70, CCO95, CCO112

## 2.131.1 Construction CCO53



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, C)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, A)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
  5.  $k_5 = k(G, F)$   
 $I \in k_2 \cap k_5, J \in k_3 \cap k_5$

## Distances

$$d_0 = |FI| = |GH| \text{ (red)} \dots 2$$

$$d_1 = |IJ| \text{ (blue)} \dots 1$$

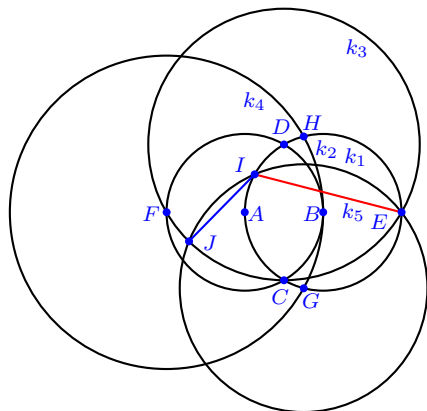
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (2 occurrences)}$$

## Identical Constructions

RCC360 MM353

## 2.131.2 Construction CCO70



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, C)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, B)$   
 $G \in k_1 \cap k_4, H \in k_1 \cap k_4$
  5.  $k_5 = k(G, E)$   
 $I \in k_1 \cap k_5, J \in k_3 \cap k_5$

## Distances

$$d_0 = |EI| = |GH| \text{ (red)} \dots 2$$

$$d_1 = |IJ| \text{ (blue)} \dots 1$$

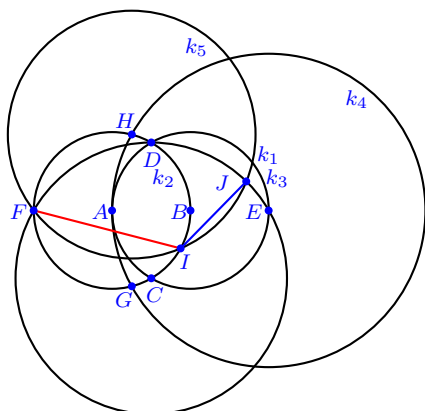
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (2 occurrences)}$$

## Identical Constructions

RCC389 MM307

## 2.131.3 Construction CCO95



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, D)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, A)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
  5.  $k_5 = k(H, F)$   
 $I \in k_2 \cap k_5, J \in k_3 \cap k_5$

## Distances

$$d_0 = |FI| = |GH| \text{ (red)} \dots 2$$

$$d_1 = |IJ| \text{ (blue)} \dots 1$$

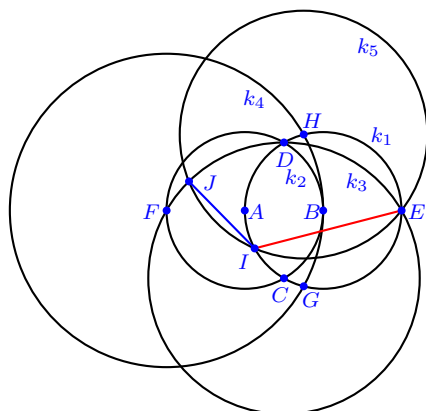
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (2 occurrences)}$$

## Identical Constructions

RCC426 MM243

## 2.131.4 Construction CCO112



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, D)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, B)$   
 $G \in k_1 \cap k_4, H \in k_1 \cap k_4$
  5.  $k_5 = k(H, E)$   
 $I \in k_1 \cap k_5, J \in k_3 \cap k_5$

## Distances

$$d_0 = |EI| = |GH| \text{ (red)} \dots 2$$

$$d_1 = |IJ| \text{ (blue)} \dots 1$$

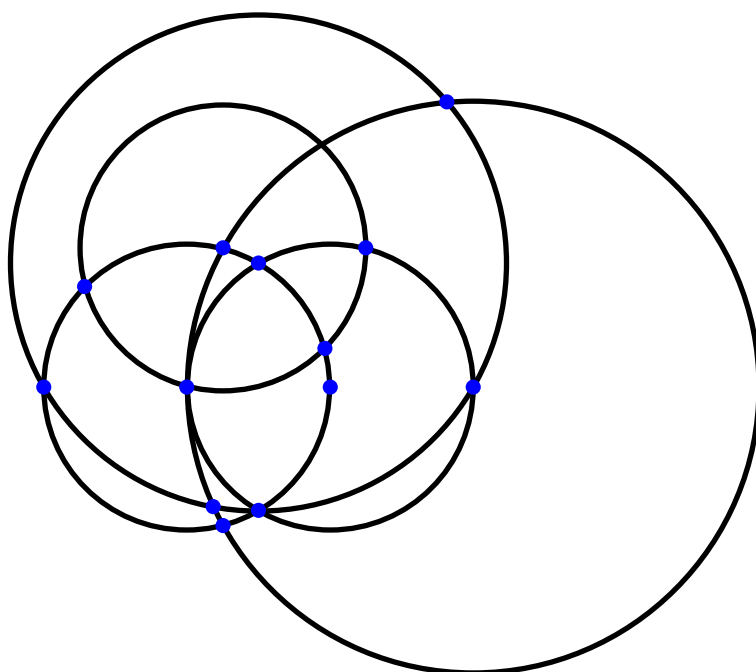
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (2 occurrences)}$$

## Identical Constructions

RCC455 MM197

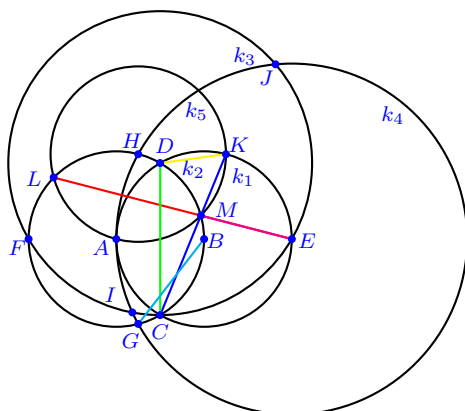
## 2.132 Class of Similar Constructions No. 121



### Contained Constructions

CCO54, CCO72, CCO82, CCO100

## 2.132.1 Construction CCO54



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, C)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, A)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
  5.  $k_5 = k(H, A)$   
 $K \in k_1 \cap k_5, L \in k_2 \cap k_5, M \in k_2 \cap k_5$

## Distances

$$\begin{aligned}
 d_0 &= |EL| \text{ (red)} \dots 1 \\
 d_1 &= |CK| = |CL| = |FM| = |KL| \text{ (blue)} \dots 4 \\
 d_2 &= |CD| = |CE| = |CF| = |DE| = |DF| = |DI| = |DJ| = |LM| \\
 &\text{ (green)} \dots 8 \\
 d_3 &= |BG| = |BH| = |CM| = |DL| = |EK| \text{ (cyan)} \dots 5 \\
 d_4 &= |EM| \text{ (magenta)} \dots 1 \\
 d_5 &= |DK| = |DM| = |FL| = |KM| \text{ (yellow)} \dots 4
 \end{aligned}$$

## Occurrences of the Golden Ratio

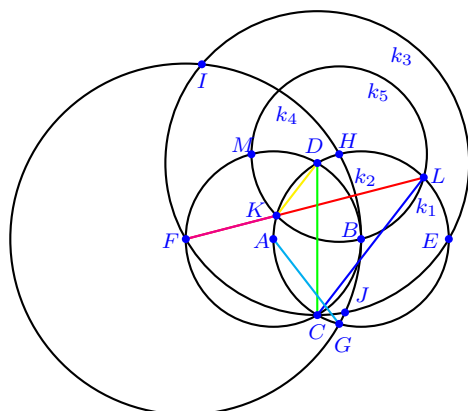
$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (56 occurrences)}$$

## Identical Constructions

RCC362 MM326



## 2.132.2 Construction CCO72



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, C)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, B)$   
 $G \in k_1 \cap k_4, H \in k_1 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
  5.  $k_5 = k(H, B)$   
 $K \in k_1 \cap k_5, L \in k_1 \cap k_5, M \in k_2 \cap k_5$

## Distances

- $d_0 = |FL|$  (red) ... 1
- $d_1 = |CL| = |CM| = |EK| = |LM|$  (blue) ... 4
- $d_2 = |CD| = |CE| = |CF| = |DE| = |DF| = |DI| = |DJ| = |KL|$  (green) ... 8
- $d_3 = |AG| = |AH| = |CK| = |DL| = |FM|$  (cyan) ... 5
- $d_4 = |FK|$  (magenta) ... 1
- $d_5 = |DK| = |DM| = |EL| = |KM|$  (yellow) ... 4

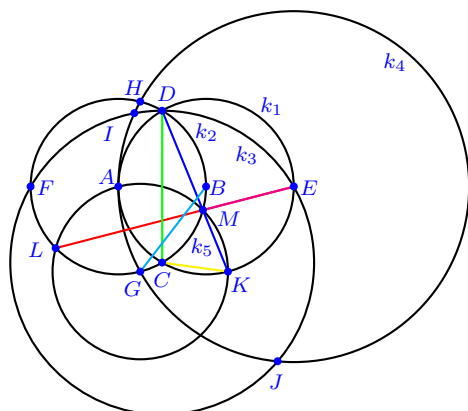
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \quad (56 \text{ occurrences})$$

## Identical Constructions

RCC392 MM280

## 2.132.3 Construction CCO82



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, D)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, A)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
  5.  $k_5 = k(G, A)$   
 $K \in k_1 \cap k_5, L \in k_2 \cap k_5, M \in k_2 \cap k_5$

## Distances

$$\begin{aligned}
 d_0 &= |EL| \text{ (red)} \dots 1 \\
 d_1 &= |DK| = |DL| = |FM| = |KL| \text{ (blue)} \dots 4 \\
 d_2 &= |CD| = |CE| = |CF| = |CI| = |CJ| = |DE| = |DF| = |LM| \\
 &\text{ (green)} \dots 8 \\
 d_3 &= |BG| = |BH| = |CL| = |DM| = |EK| \text{ (cyan)} \dots 5 \\
 d_4 &= |EM| \text{ (magenta)} \dots 1 \\
 d_5 &= |CK| = |CM| = |FL| = |KM| \text{ (yellow)} \dots 4
 \end{aligned}$$

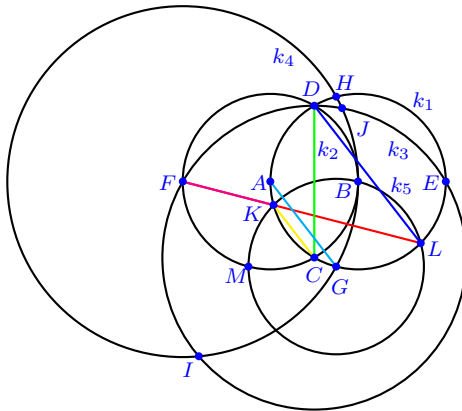
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (56 occurrences)}$$

## Identical Constructions

RCC406 MM212

## 2.132.4 Construction CCO100



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, D)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, B)$   
 $G \in k_1 \cap k_4, H \in k_1 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
  5.  $k_5 = k(G, B)$   
 $K \in k_1 \cap k_5, L \in k_1 \cap k_5, M \in k_2 \cap k_5$

## Distances

$$\begin{aligned}
 d_0 &= |FL| \text{ (red)} \dots 1 \\
 d_1 &= |DL| = |DM| = |EK| = |LM| \text{ (blue)} \dots 4 \\
 d_2 &= |CD| = |CE| = |CF| = |CI| = |CJ| = |DE| = |DF| = |KL| \text{ (green)} \\
 &\dots 8 \\
 d_3 &= |AG| = |AH| = |CL| = |DK| = |FM| \text{ (cyan)} \dots 5 \\
 d_4 &= |FK| \text{ (magenta)} \dots 1 \\
 d_5 &= |CK| = |CM| = |EL| = |KM| \text{ (yellow)} \dots 4
 \end{aligned}$$

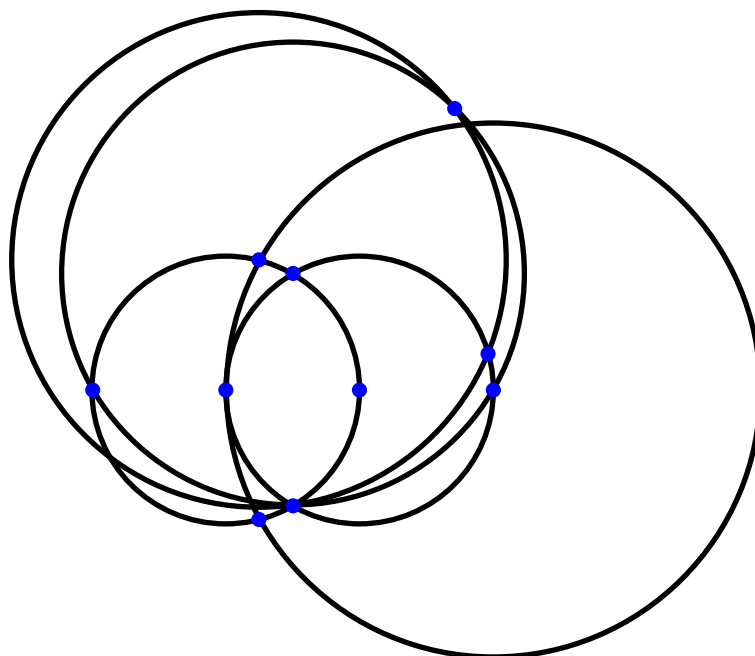
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (56 occurrences)}$$

## Identical Constructions

RCC436 MM166

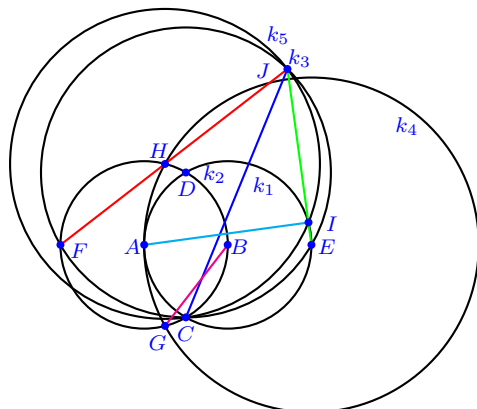
### 2.133 Class of Similar Constructions No. 122



#### Contained Constructions

CCO55, CCO73, CCO85, CCO103

## 2.133.1 Construction CCO55



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, C)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, A)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
  5.  $k_5 = k(H, C)$   
 $I \in k_1 \cap k_5, J \in k_3 \cap k_5$

## Distances

$$\begin{aligned}
 d_0 &= |FJ| \text{ (red)} \dots 1 \\
 d_1 &= |CJ| \text{ (blue)} \dots 1 \\
 d_2 &= |EJ| \text{ (green)} \dots 1 \\
 d_3 &= |AI| \text{ (cyan)} \dots 1 \\
 d_4 &= |BG| = |BH| \text{ (magenta)} \dots 2
 \end{aligned}$$

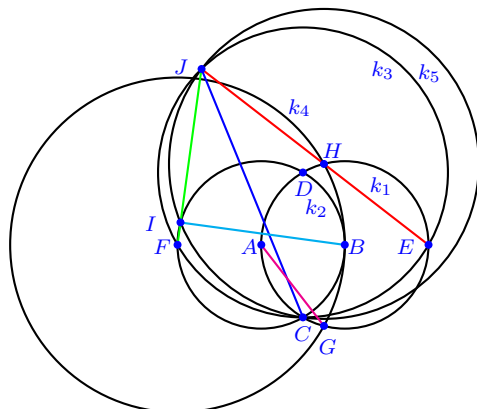
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_3}{d_4} \text{ (4 occurrences)}$$

## Identical Constructions

RCC364 MM348

## 2.133.2 Construction CCO73



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, C)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, B)$   
 $G \in k_1 \cap k_4, H \in k_1 \cap k_4$
  5.  $k_5 = k(H, C)$   
 $I \in k_2 \cap k_5, J \in k_3 \cap k_5$

## Distances

$$\begin{aligned}
 d_0 &= |EJ| \text{ (red)} \dots 1 \\
 d_1 &= |CJ| \text{ (blue)} \dots 1 \\
 d_2 &= |FJ| \text{ (green)} \dots 1 \\
 d_3 &= |BI| \text{ (cyan)} \dots 1 \\
 d_4 &= |AG| = |AH| \text{ (magenta)} \dots 2
 \end{aligned}$$

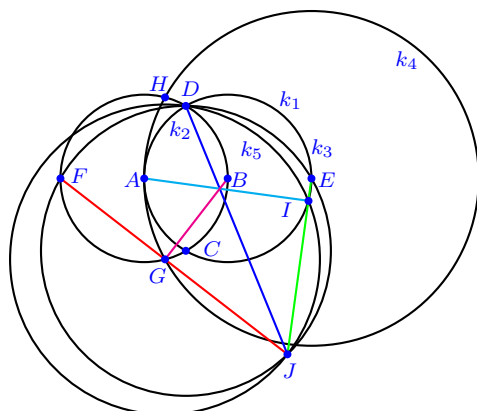
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_3}{d_4} \text{ (4 occurrences)}$$

## Identical Constructions

RCC394 MM302

## 2.133.3 Construction CCO85



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, D)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, A)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
  5.  $k_5 = k(G, D)$   
 $I \in k_1 \cap k_5, J \in k_3 \cap k_5$

## Distances

$$\begin{aligned}
 d_0 &= |FJ| \text{ (red)} \dots 1 \\
 d_1 &= |DJ| \text{ (blue)} \dots 1 \\
 d_2 &= |EJ| \text{ (green)} \dots 1 \\
 d_3 &= |AI| \text{ (cyan)} \dots 1 \\
 d_4 &= |BG| = |BH| \text{ (magenta)} \dots 2
 \end{aligned}$$

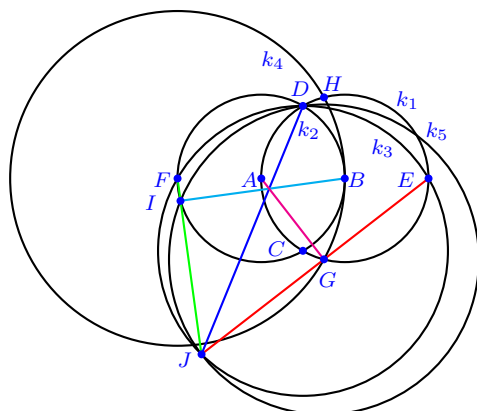
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_3}{d_4} \text{ (4 occurrences)}$$

## Identical Constructions

RCC411 MM236

## 2.133.4 Construction CCO103



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, D)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, B)$   
 $G \in k_1 \cap k_4, H \in k_1 \cap k_4$
  5.  $k_5 = k(G, D)$   
 $I \in k_2 \cap k_5, J \in k_3 \cap k_5$

## Distances

$$\begin{aligned}
 d_0 &= |EJ| \text{ (red)} \dots 1 \\
 d_1 &= |DJ| \text{ (blue)} \dots 1 \\
 d_2 &= |FJ| \text{ (green)} \dots 1 \\
 d_3 &= |BI| \text{ (cyan)} \dots 1 \\
 d_4 &= |AG| = |AH| \text{ (magenta)} \dots 2
 \end{aligned}$$

## Occurrences of the Golden Ratio

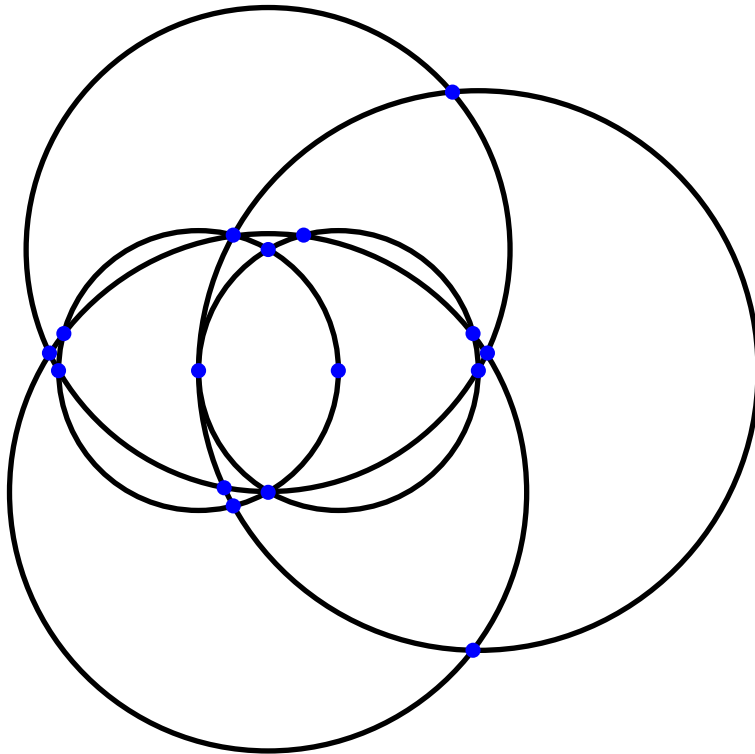
$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_3}{d_4} \text{ (4 occurrences)}$$

## Identical Constructions

RCC441 MM190



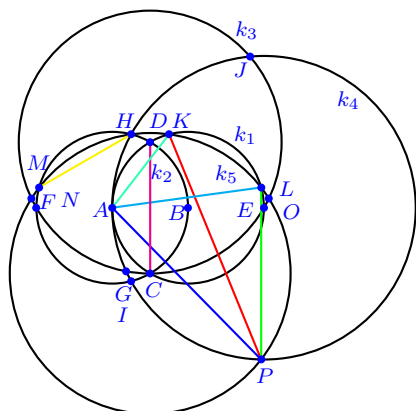
## 2.134 Class of Similar Constructions No. 123



### Contained Constructions

CCO56, CCO74, CCO86, CCO104

## 2.134.1 Construction CCO56



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, C)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, A)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
  5.  $k_5 = k(C, H)$   
 $K \in k_1 \cap k_5, L \in k_1 \cap k_5, M \in k_2 \cap k_5, N \in k_3 \cap k_5, O \in k_3 \cap k_5,$   
 $P \in k_4 \cap k_5$

## Distances

- $d_0 = |KP|$  (red) ... 1  
 $d_1 = |AP|$  (blue) ... 1  
 $d_2 = |LP|$  (green) ... 1  
 $d_3 = |AL| = |BM|$  (cyan) ... 2  
 $d_4 = |CD| = |CE| = |CF| = |DE| = |DF| = |DI| = |DJ| = |DN| = |DO| = |GM|$  (magenta) ... 10  
 $d_5 = |HM| = |KL|$  (yellow) ... 2  
 $d_6 = |AK| = |BG| = |BH|$  (grass) ... 3

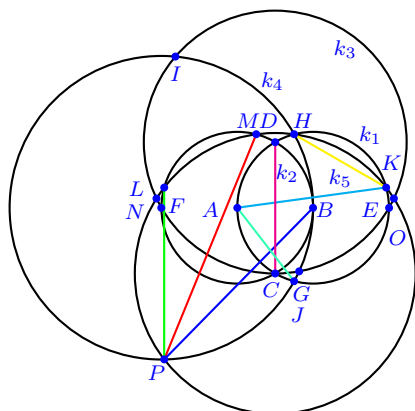
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_3} = \frac{d_1}{d_4} = \frac{d_2}{d_5} = \frac{d_3}{d_6} \quad (20 \text{ occurrences})$$

## Identical Constructions

RCC365 MM343

## 2.134.2 Construction CCO74



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, C)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, B)$   
 $G \in k_1 \cap k_4, H \in k_1 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
  5.  $k_5 = k(C, H)$   
 $K \in k_1 \cap k_5, L \in k_2 \cap k_5, M \in k_2 \cap k_5, N \in k_3 \cap k_5, O \in k_3 \cap k_5,$   
 $P \in k_4 \cap k_5$

## Distances

$$\begin{aligned}
 d_0 &= |MP| \text{ (red)} \dots 1 \\
 d_1 &= |BP| \text{ (blue)} \dots 1 \\
 d_2 &= |LP| \text{ (green)} \dots 1 \\
 d_3 &= |AK| = |BL| \text{ (cyan)} \dots 2 \\
 d_4 &= |CD| = |CE| = |CF| = |DE| = |DF| = |DI| = |DJ| = |DN| = |DO| = |GK| \text{ (magenta)} \dots 10 \\
 d_5 &= |HK| = |LM| \text{ (yellow)} \dots 2 \\
 d_6 &= |AG| = |AH| = |BM| \text{ (grass)} \dots 3
 \end{aligned}$$

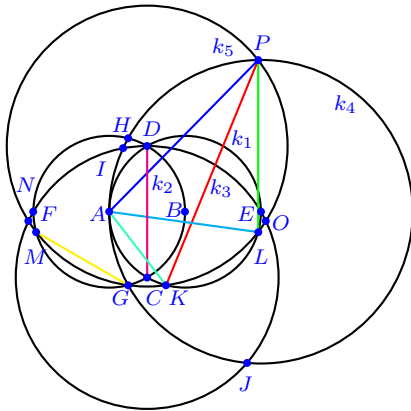
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_3} = \frac{d_1}{d_4} = \frac{d_2}{d_5} = \frac{d_3}{d_6} \text{ (20 occurrences)}$$

## Identical Constructions

RCC395 MM297

## 2.134.3 Construction CCO86



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, D)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, A)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
  5.  $k_5 = k(D, G)$   
 $K \in k_1 \cap k_5, L \in k_1 \cap k_5, M \in k_2 \cap k_5, N \in k_3 \cap k_5, O \in k_3 \cap k_5,$   
 $P \in k_4 \cap k_5$

## Distances

$$\begin{aligned}
 d_0 &= |KP| \text{ (red)} \dots 1 \\
 d_1 &= |AP| \text{ (blue)} \dots 1 \\
 d_2 &= |LP| \text{ (green)} \dots 1 \\
 d_3 &= |AL| = |BM| \text{ (cyan)} \dots 2 \\
 d_4 &= |CD| = |CE| = |CF| = |CI| = |CJ| = |CN| = |CO| = |DE| = \\
 &= |DF| = |HM| \text{ (magenta)} \dots 10 \\
 d_5 &= |GM| = |KL| \text{ (yellow)} \dots 2 \\
 d_6 &= |AK| = |BG| = |BH| \text{ (grass)} \dots 3
 \end{aligned}$$

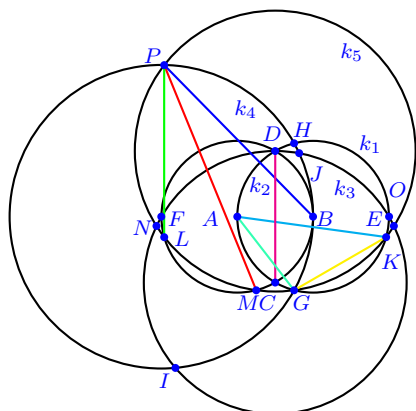
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_3} = \frac{d_1}{d_4} = \frac{d_2}{d_5} = \frac{d_3}{d_6} \text{ (20 occurrences)}$$

## Identical Constructions

RCC412 MM233

## 2.134.4 Construction CCO104



## Construction Process

- $A, B$  given initial points
- $k_1 = k(B, A)$
  - $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  - $k_3 = k(C, D)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  - $k_4 = k(F, B)$   
 $G \in k_1 \cap k_4, H \in k_1 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
  - $k_5 = k(D, G)$   
 $K \in k_1 \cap k_5, L \in k_2 \cap k_5, M \in k_2 \cap k_5, N \in k_3 \cap k_5, O \in k_3 \cap k_5, P \in k_4 \cap k_5$

## Distances

$$\begin{aligned}
 d_0 &= |MP| \text{ (red)} \dots 1 \\
 d_1 &= |BP| \text{ (blue)} \dots 1 \\
 d_2 &= |LP| \text{ (green)} \dots 1 \\
 d_3 &= |AK| = |BL| \text{ (cyan)} \dots 2 \\
 d_4 &= |CD| = |CE| = |CF| = |CI| = |CJ| = |CN| = |CO| = |DE| = \\
 &|DF| = |HK| \text{ (magenta)} \dots 10 \\
 d_5 &= |GK| = |LM| \text{ (yellow)} \dots 2 \\
 d_6 &= |AG| = |AH| = |BM| \text{ (grass)} \dots 3
 \end{aligned}$$

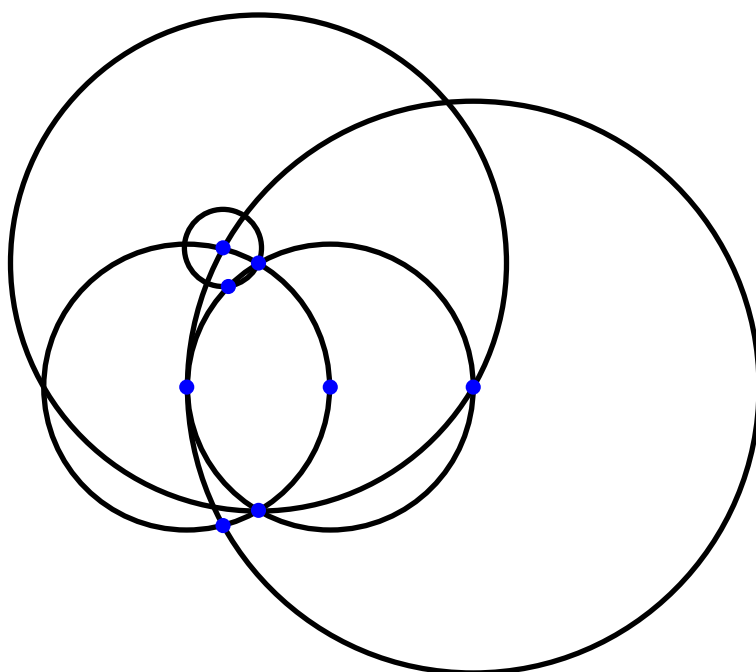
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_3} = \frac{d_1}{d_4} = \frac{d_2}{d_5} = \frac{d_3}{d_6} \text{ (20 occurrences)}$$

## Identical Constructions

RCC442 MM187

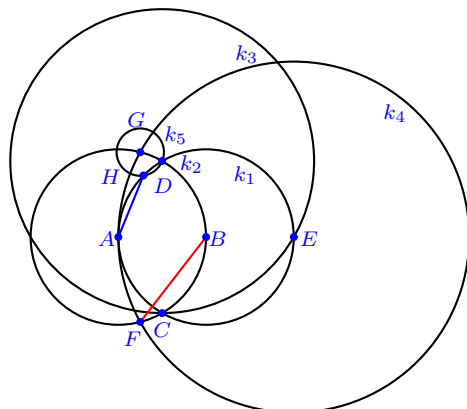
## 2.135 Class of Similar Constructions No. 124



### Contained Constructions

CCO57, CCO75, CCO83, CCO101

## 2.135.1 Construction CCO57



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, C)$   
 $E \in k_1 \cap k_3$
  4.  $k_4 = k(E, A)$   
 $F \in k_2 \cap k_4, G \in k_2 \cap k_4$
  5.  $k_5 = k(G, D)$   
 $H \in k_1 \cap k_5$

## Distances

$$d_0 = |BF| = |BG| \text{ (red) } \dots 2$$

$$d_1 = |AH| \text{ (blue) } \dots 1$$

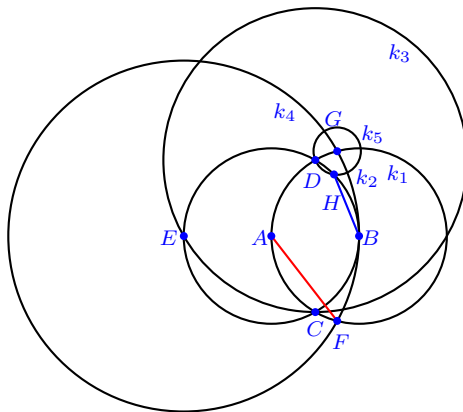
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (2 occurrences)}$$

## Identical Constructions

RCC367 MM340

## 2.135.2 Construction CCO75



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, C)$   
 $E \in k_2 \cap k_3$
  4.  $k_4 = k(E, B)$   
 $F \in k_1 \cap k_4, G \in k_1 \cap k_4$
  5.  $k_5 = k(G, D)$   
 $H \in k_2 \cap k_5$

## Distances

$$d_0 = |AF| = |AG| \text{ (red) } \dots 2$$

$$d_1 = |BH| \text{ (blue) } \dots 1$$

## Occurrences of the Golden Ratio

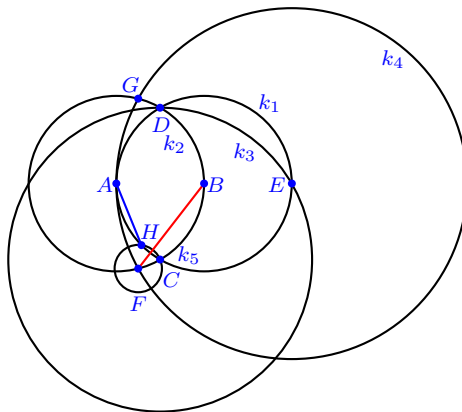
$$\phi = \frac{d_0}{d_1} \text{ (2 occurrences)}$$

## Identical Constructions

RCC397 MM294



## 2.135.3 Construction CCO83



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, D)$   
 $E \in k_1 \cap k_3$
  4.  $k_4 = k(E, A)$   
 $F \in k_2 \cap k_4, G \in k_2 \cap k_4$
  5.  $k_5 = k(F, C)$   
 $H \in k_1 \cap k_5$

## Distances

$$d_0 = |BF| = |BG| \text{ (red) } \dots 2$$

$$d_1 = |AH| \text{ (blue) } \dots 1$$

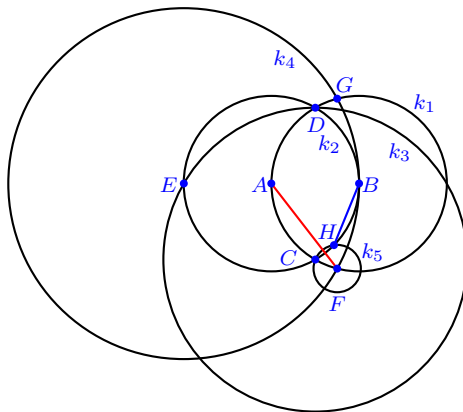
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (2 occurrences)}$$

## Identical Constructions

RCC408 MM228

## 2.135.4 Construction CCO101



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, D)$   
 $E \in k_2 \cap k_3$
  4.  $k_4 = k(E, B)$   
 $F \in k_1 \cap k_4, G \in k_1 \cap k_4$
  5.  $k_5 = k(F, C)$   
 $H \in k_2 \cap k_5$

## Distances

$$d_0 = |AF| = |AG| \text{ (red) } \dots 2$$

$$d_1 = |BH| \text{ (blue) } \dots 1$$

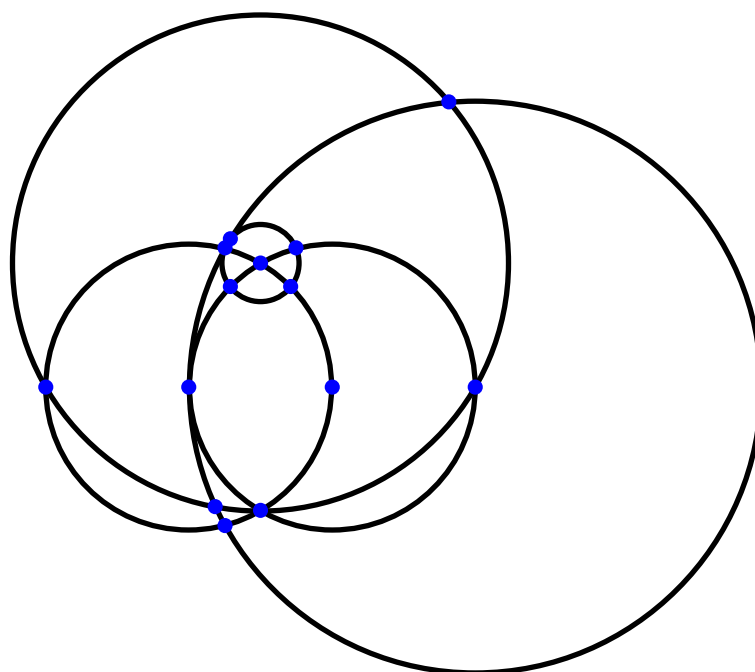
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (2 occurrences)}$$

## Identical Constructions

RCC438 MM182

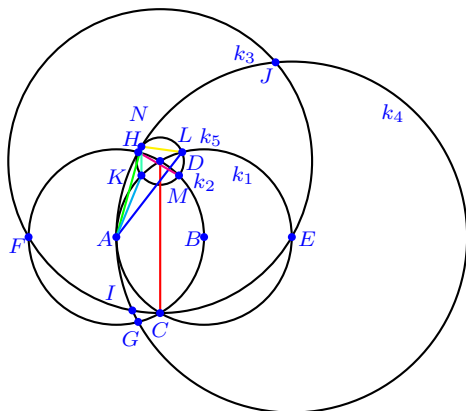
## 2.136 Class of Similar Constructions No. 125



### Contained Constructions

CCO58, CCO76, CCO84, CCO102

## 2.136.1 Construction CCO58



## Construction Process

- $A, B$  given initial points
- $k_1 = k(B, A)$
  - $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  - $k_3 = k(D, C)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  - $k_4 = k(E, A)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
  - $k_5 = k(D, H)$   
 $K \in k_1 \cap k_5, L \in k_1 \cap k_5, M \in k_2 \cap k_5, N \in k_4 \cap k_5$

## Distances

$$d_0 = |CD| = |CE| = |CF| = |DE| = |DF| = |DI| = |DJ| = |GM| \text{ (red)} \dots 8$$

$$d_1 = |AL| = |BG| = |BH| \text{ (blue)} \dots 3$$

$$d_2 = |AN| \text{ (green)} \dots 1$$

$$d_3 = |AK| = |BM| \text{ (cyan)} \dots 2$$

$$d_4 = |HM| = |KL| \text{ (magenta)} \dots 2$$

$$d_5 = |LN| \text{ (yellow)} \dots 1$$

$$d_6 = |KN| \text{ (grass)} \dots 1$$

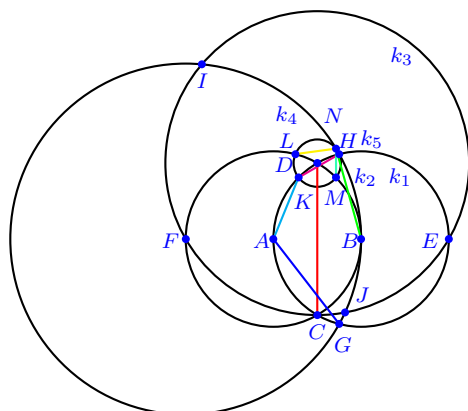
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_3}{d_5} = \frac{d_4}{d_6} \text{ (18 occurrences)}$$

## Identical Constructions

RCC368 MM336

## 2.136.2 Construction CCO76



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, C)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, B)$   
 $G \in k_1 \cap k_4, H \in k_1 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
  5.  $k_5 = k(D, H)$   
 $K \in k_1 \cap k_5, L \in k_2 \cap k_5, M \in k_2 \cap k_5, N \in k_4 \cap k_5$

## Distances

$$d_0 = |CD| = |CE| = |CF| = |DE| = |DF| = |DI| = |DJ| = |GK| \text{ (red)} \\ \dots 8$$

$$d_1 = |AG| = |AH| = |BL| \text{ (blue)} \dots 3$$

$$d_2 = |BN| \text{ (green)} \dots 1$$

$$d_3 = |AK| = |BM| \text{ (cyan)} \dots 2$$

$$d_4 = |HK| = |LM| \text{ (magenta)} \dots 2$$

$$d_5 = |LN| \text{ (yellow)} \dots 1$$

$$d_6 = |MN| \text{ (grass)} \dots 1$$

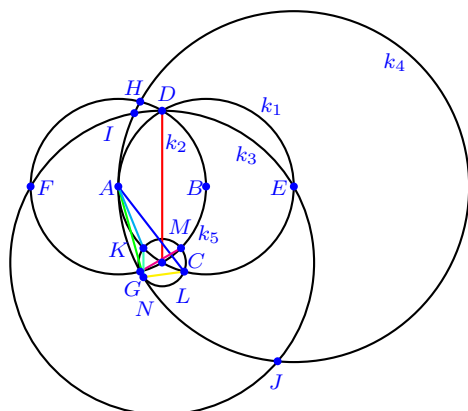
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_3}{d_5} = \frac{d_4}{d_6} \text{ (18 occurrences)}$$

## Identical Constructions

RCC398 MM290

## 2.136.3 Construction CCO84



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, D)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, A)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
  5.  $k_5 = k(C, G)$   
 $K \in k_1 \cap k_5, L \in k_1 \cap k_5, M \in k_2 \cap k_5, N \in k_4 \cap k_5$

## Distances

$$d_0 = |CD| = |CE| = |CF| = |CI| = |CJ| = |DE| = |DF| = |HM| \text{ (red)} \dots 8$$

$$d_1 = |AL| = |BG| = |BH| \text{ (blue)} \dots 3$$

$$d_2 = |AN| \text{ (green)} \dots 1$$

$$d_3 = |AK| = |BM| \text{ (cyan)} \dots 2$$

$$d_4 = |GM| = |KL| \text{ (magenta)} \dots 2$$

$$d_5 = |LN| \text{ (yellow)} \dots 1$$

$$d_6 = |KN| \text{ (grass)} \dots 1$$

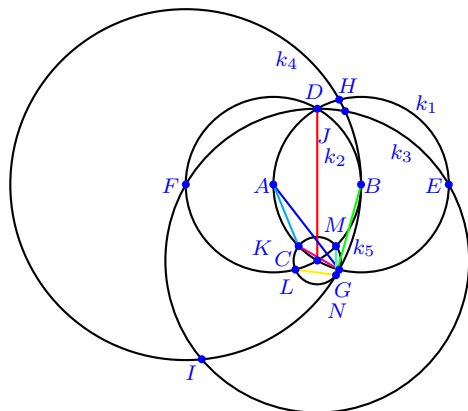
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_3}{d_5} = \frac{d_4}{d_6} \text{ (18 occurrences)}$$

## Identical Constructions

RCC409 MM224

## 2.136.4 Construction CCO102



## Construction Process

- $A, B$  given initial points
- $k_1 = k(B, A)$
  - $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  - $k_3 = k(C, D)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  - $k_4 = k(F, B)$   
 $G \in k_1 \cap k_4, H \in k_1 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
  - $k_5 = k(C, G)$   
 $K \in k_1 \cap k_5, L \in k_2 \cap k_5, M \in k_2 \cap k_5, N \in k_4 \cap k_5$

## Distances

$$\begin{aligned}
 d_0 &= |CD| = |CE| = |CF| = |CI| = |CJ| = |DE| = |DF| = |HK| \text{ (red)} \\
 &\dots 8 \\
 d_1 &= |AG| = |AH| = |BL| \text{ (blue)} \dots 3 \\
 d_2 &= |BN| \text{ (green)} \dots 1 \\
 d_3 &= |AK| = |BM| \text{ (cyan)} \dots 2 \\
 d_4 &= |GK| = |LM| \text{ (magenta)} \dots 2 \\
 d_5 &= |LN| \text{ (yellow)} \dots 1 \\
 d_6 &= |MN| \text{ (grass)} \dots 1
 \end{aligned}$$

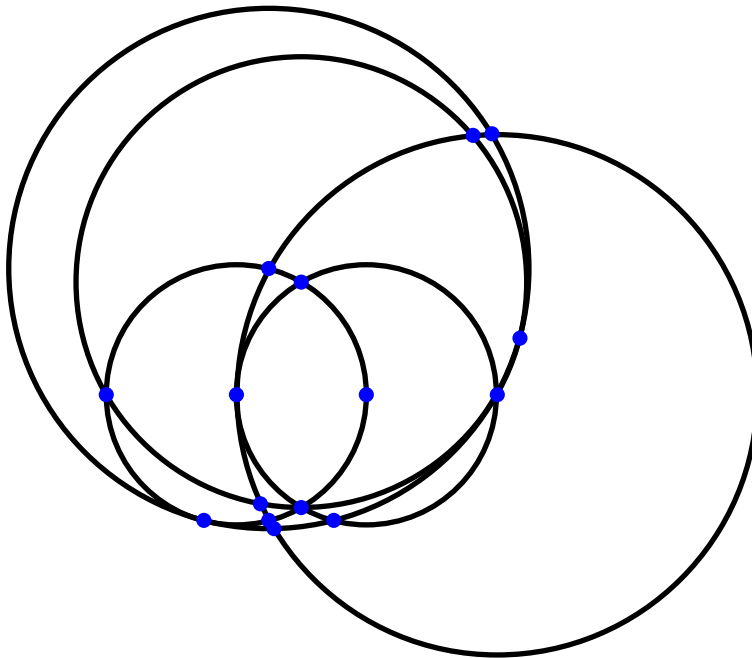
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_3}{d_5} = \frac{d_4}{d_6} \text{ (18 occurrences)}$$

## Identical Constructions

RCC439 MM178

**2.137 Class of Similar Constructions No. 126**

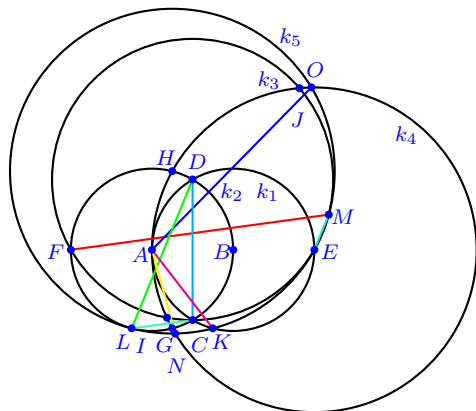


**Contained Constructions**

CCO59, CCO78, CCO87, CCO106



## 2.137.1 Construction CCO59



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, C)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, A)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
  5.  $k_5 = k(H, E)$   
 $K \in k_1 \cap k_5, L \in k_2 \cap k_5, M \in k_3 \cap k_5, N \in k_4 \cap k_5, O \in k_4 \cap k_5$

## Distances

$$\begin{aligned}
 d_0 &= |FM| = |KO| \text{ (red)} \dots 2 \\
 d_1 &= |AO| = |LM| \text{ (blue)} \dots 2 \\
 d_2 &= |DL| \text{ (green)} \dots 1 \\
 d_3 &= |CD| = |CE| = |CF| = |DE| = |DF| = |DI| = |DJ| = |DM| \\
 &\text{ (cyan)} \dots 8 \\
 d_4 &= |AK| = |BG| = |BH| = |FL| \text{ (magenta)} \dots 4 \\
 d_5 &= |AN| \text{ (yellow)} \dots 1 \\
 d_6 &= |CL| \text{ (grass)} \dots 1 \\
 d_7 &= |EM| = |KN| \text{ (sea)} \dots 2
 \end{aligned}$$

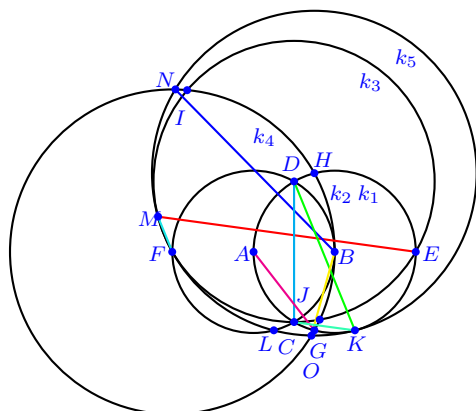
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} = \frac{d_4}{d_6} = \frac{d_6}{d_7} \text{ (36 occurrences)}$$

## Identical Constructions

RCC369 MM328

## 2.137.2 Construction CCO78



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, C)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, B)$   
 $G \in k_1 \cap k_4, H \in k_1 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
  5.  $k_5 = k(H, F)$   
 $K \in k_1 \cap k_5, L \in k_2 \cap k_5, M \in k_3 \cap k_5, N \in k_4 \cap k_5, O \in k_4 \cap k_5$

## Distances

- $d_0 = |EM| = |LN|$  (red) ... 2  
 $d_1 = |BN| = |KM|$  (blue) ... 2  
 $d_2 = |DK|$  (green) ... 1  
 $d_3 = |CD| = |CE| = |CF| = |DE| = |DF| = |DI| = |DJ| = |DM|$  (cyan) ... 8  
 $d_4 = |AG| = |AH| = |BL| = |EK|$  (magenta) ... 4  
 $d_5 = |BO|$  (yellow) ... 1  
 $d_6 = |CK|$  (grass) ... 1  
 $d_7 = |FM| = |LO|$  (sea) ... 2

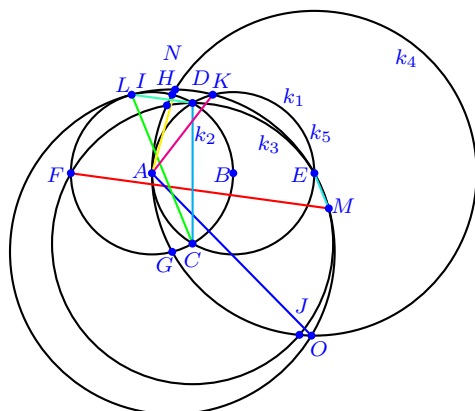
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} = \frac{d_4}{d_6} = \frac{d_6}{d_7} \quad (36 \text{ occurrences})$$

## Identical Constructions

RCC401 MM282

## 2.137.3 Construction CCO87



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, D)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, A)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
  5.  $k_5 = k(G, E)$   
 $K \in k_1 \cap k_5, L \in k_2 \cap k_5, M \in k_3 \cap k_5, N \in k_4 \cap k_5, O \in k_4 \cap k_5$

## Distances

$$\begin{aligned}
 d_0 &= |FM| = |KO| \text{ (red)} \dots 2 \\
 d_1 &= |AO| = |LM| \text{ (blue)} \dots 2 \\
 d_2 &= |CL| \text{ (green)} \dots 1 \\
 d_3 &= |CD| = |CE| = |CF| = |CI| = |CJ| = |CM| = |DE| = |DF| \text{ (cyan)} \\
 &\dots 8 \\
 d_4 &= |AK| = |BG| = |BH| = |FL| \text{ (magenta)} \dots 4 \\
 d_5 &= |AN| \text{ (yellow)} \dots 1 \\
 d_6 &= |DL| \text{ (grass)} \dots 1 \\
 d_7 &= |EM| = |KN| \text{ (sea)} \dots 2
 \end{aligned}$$

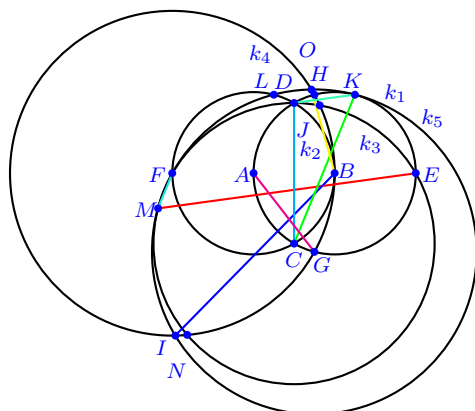
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} = \frac{d_4}{d_6} = \frac{d_6}{d_7} \text{ (36 occurrences)}$$

## Identical Constructions

RCC413 MM214

## 2.137.4 Construction CCO106



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, D)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, B)$   
 $G \in k_1 \cap k_4, H \in k_1 \cap k_4, I \in k_3 \cap k_4, J \in k_3 \cap k_4$
  5.  $k_5 = k(G, F)$   
 $K \in k_1 \cap k_5, L \in k_2 \cap k_5, M \in k_3 \cap k_5, N \in k_4 \cap k_5, O \in k_4 \cap k_5$

## Distances

$$\begin{aligned}
 d_0 &= |EM| = |LN| \text{ (red)} \dots 2 \\
 d_1 &= |BN| = |KM| \text{ (blue)} \dots 2 \\
 d_2 &= |CK| \text{ (green)} \dots 1 \\
 d_3 &= |CD| = |CE| = |CF| = |CI| = |CJ| = |CM| = |DE| = |DF| \text{ (cyan)} \\
 &\dots 8 \\
 d_4 &= |AG| = |AH| = |BL| = |EK| \text{ (magenta)} \dots 4 \\
 d_5 &= |BO| \text{ (yellow)} \dots 1 \\
 d_6 &= |DK| \text{ (grass)} \dots 1 \\
 d_7 &= |FM| = |LO| \text{ (sea)} \dots 2
 \end{aligned}$$

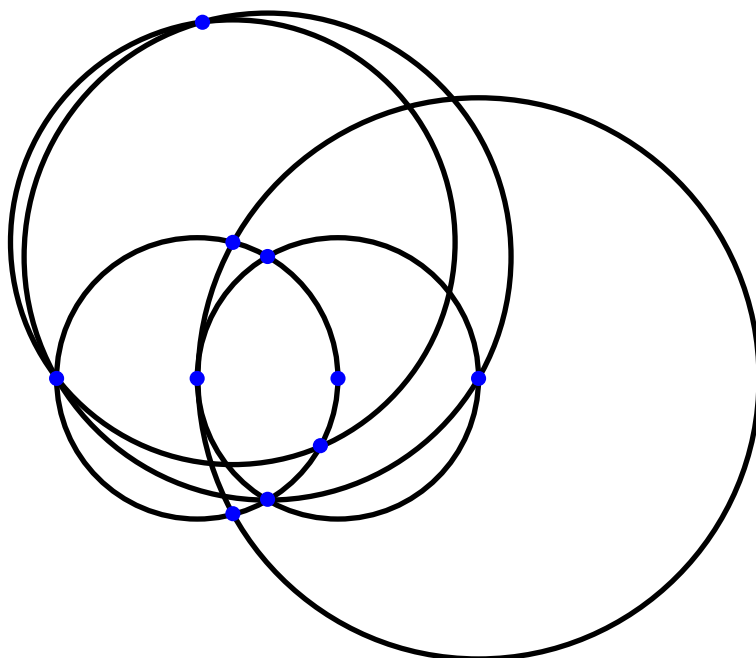
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} = \frac{d_4}{d_6} = \frac{d_6}{d_7} \text{ (36 occurrences)}$$

## Identical Constructions

RCC445 MM168

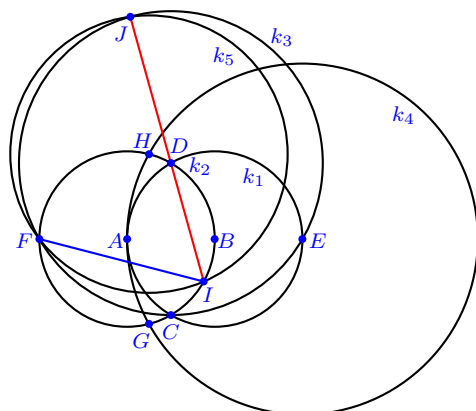
## 2.138 Class of Similar Constructions No. 127



### Contained Constructions

CCO60, CCO77, CCO88, CCO105

## 2.138.1 Construction CCO60



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, C)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, A)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
  5.  $k_5 = k(H, F)$   
 $I \in k_2 \cap k_5, J \in k_3 \cap k_5$

## Distances

$$d_0 = |IJ| \text{ (red)} \dots 1$$

$$d_1 = |FI| = |GH| \text{ (blue)} \dots 2$$

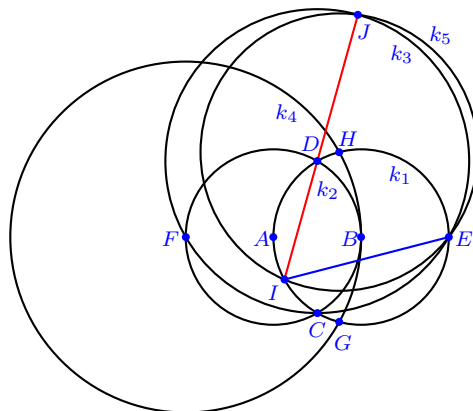
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (2 occurrences)}$$

## Identical Constructions

RCC371 MM354

## 2.138.2 Construction CCO77



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, C)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, B)$   
 $G \in k_1 \cap k_4, H \in k_1 \cap k_4$
  5.  $k_5 = k(H, E)$   
 $I \in k_1 \cap k_5, J \in k_3 \cap k_5$

## Distances

$$d_0 = |IJ| \text{ (red)} \dots 1$$

$$d_1 = |EI| = |GH| \text{ (blue)} \dots 2$$

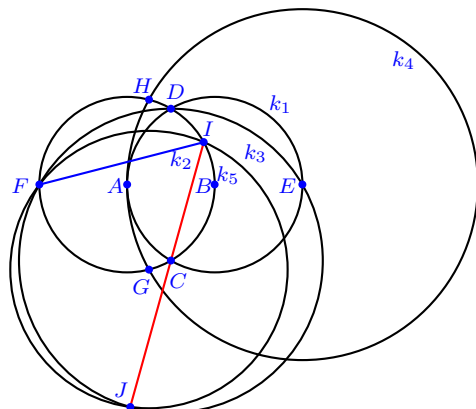
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (2 occurrences)}$$

## Identical Constructions

RCC400 MM308

## 2.138.3 Construction CCO88



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, D)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(E, A)$   
 $G \in k_2 \cap k_4, H \in k_2 \cap k_4$
  5.  $k_5 = k(G, F)$   
 $I \in k_2 \cap k_5, J \in k_3 \cap k_5$

## Distances

$$d_0 = |IJ| \text{ (red) } \dots 1$$

$$d_1 = |FI| = |GH| \text{ (blue) } \dots 2$$

## Occurrences of the Golden Ratio

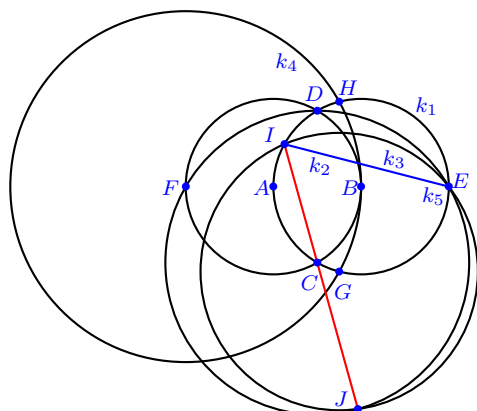
$$\phi = \frac{d_0}{d_1} \text{ (2 occurrences)}$$

## Identical Constructions

RCC415 MM242



## 2.138.4 Construction CCO105



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, D)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(F, B)$   
 $G \in k_1 \cap k_4, H \in k_1 \cap k_4$
  5.  $k_5 = k(G, E)$   
 $I \in k_1 \cap k_5, J \in k_3 \cap k_5$

## Distances

$$d_0 = |IJ| \text{ (red)} \dots 1$$

$$d_1 = |EI| = |GH| \text{ (blue)} \dots 2$$

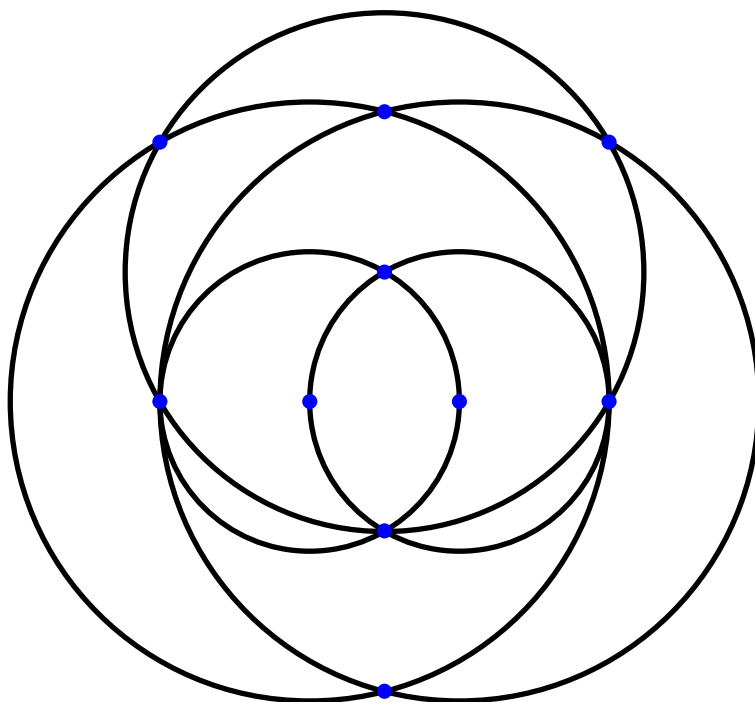
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} \text{ (2 occurrences)}$$

## Identical Constructions

RCC444 MM196

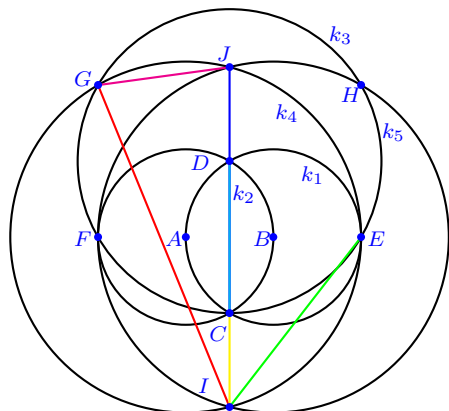
## 2.139 Class of Similar Constructions No. 128



### Contained Constructions

CCO61, CCO96

## 2.139.1 Construction CCO61



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, C)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(A, E)$   
 $G \in k_3 \cap k_4$
  5.  $k_5 = k(B, F)$   
 $H \in k_3 \cap k_5, I \in k_4 \cap k_5, J \in k_4 \cap k_5$

## Distances

$$\begin{aligned}
 d_0 &= |GI| = |HI| \text{ (red)} \dots 2 \\
 d_1 &= |CJ| = |DI| \text{ (blue)} \dots 2 \\
 d_2 &= |EI| = |EJ| = |FI| = |FJ| \text{ (green)} \dots 4 \\
 d_3 &= |CD| = |CE| = |CF| = |DE| = |DF| = |DG| = |DH| = |EH| = |FG| \text{ (cyan)} \dots 9 \\
 d_4 &= |GJ| = |HJ| \text{ (magenta)} \dots 2 \\
 d_5 &= |CI| = |DJ| \text{ (yellow)} \dots 2
 \end{aligned}$$

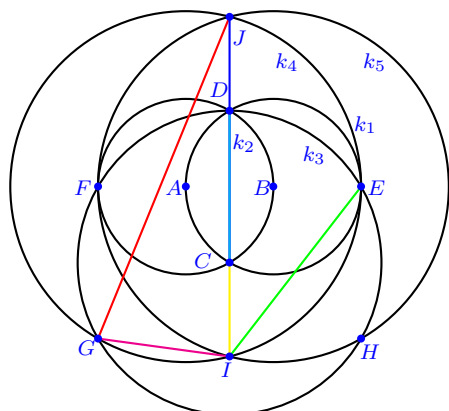
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (52 occurrences)}$$

## Identical Constructions

RCC372 MM260

## 2.139.2 Construction CCO96



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, D)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(A, E)$   
 $G \in k_3 \cap k_4$
  5.  $k_5 = k(B, F)$   
 $H \in k_3 \cap k_5, I \in k_4 \cap k_5, J \in k_4 \cap k_5$

## Distances

$$\begin{aligned}
 d_0 &= |GJ| = |HJ| \text{ (red)} \dots 2 \\
 d_1 &= |CJ| = |DI| \text{ (blue)} \dots 2 \\
 d_2 &= |EI| = |EJ| = |FI| = |FJ| \text{ (green)} \dots 4 \\
 d_3 &= |CD| = |CE| = |CF| = |CG| = |CH| = |DE| = |DF| = |EH| = |FG| \text{ (cyan)} \dots 9 \\
 d_4 &= |GI| = |HI| \text{ (magenta)} \dots 2 \\
 d_5 &= |CI| = |DJ| \text{ (yellow)} \dots 2
 \end{aligned}$$

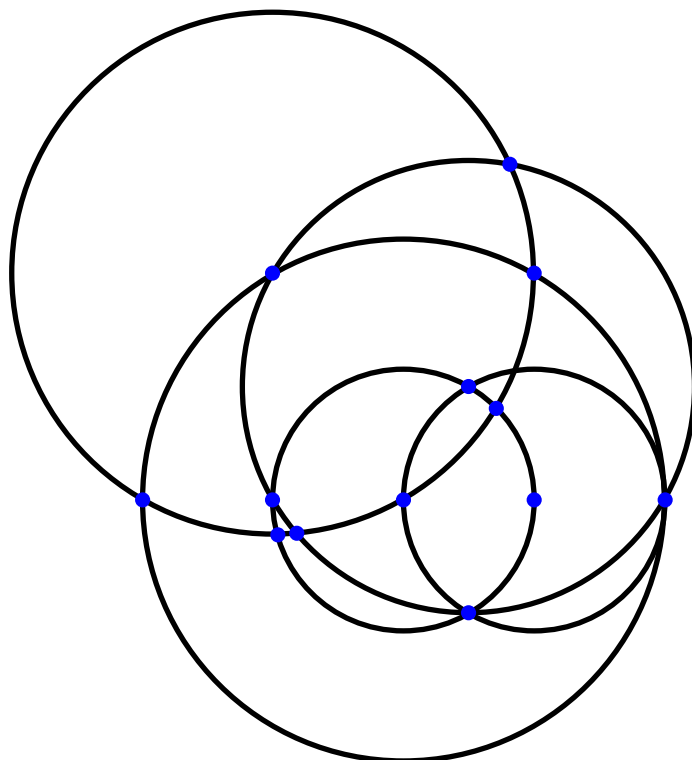
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_2} = \frac{d_1}{d_3} = \frac{d_2}{d_4} = \frac{d_3}{d_5} \text{ (52 occurrences)}$$

## Identical Constructions

RCC427 MM149

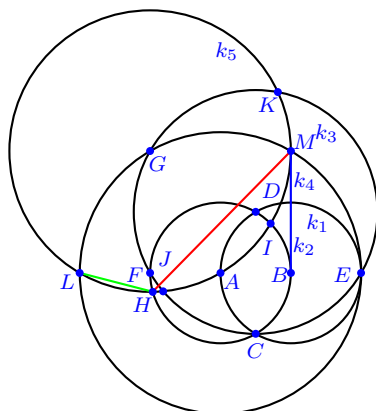
## 2.140 Class of Similar Constructions No. 129



### Contained Constructions

CCO62, CCO79, CCO97, CCO114

## 2.140.1 Construction CCO62



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, C)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(A, E)$   
 $G \in k_3 \cap k_4$
  5.  $k_5 = k(G, A)$   
 $H \in k_2 \cap k_5, I \in k_2 \cap k_5, J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5,$   
 $M \in k_4 \cap k_5$

## Distances

$$d_0 = |HM| = |IL| \text{ (red)} \dots 2$$

$$d_1 = |BM| = |CD| = |CE| = |CF| = |DE| = |DF| = |DG| = |DJ| = |DK| = |FG| \text{ (blue)} \dots 10$$

$$d_2 = |HL| = |IM| \text{ (green)} \dots 2$$

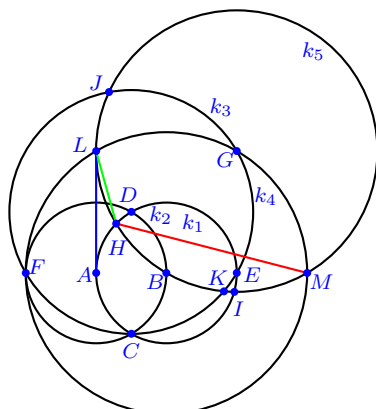
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (40 occurrences)}$$

## Identical Constructions

RCC375 MM264

## 2.140.2 Construction CCO79



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(D, C)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(B, F)$   
 $G \in k_3 \cap k_4$
  5.  $k_5 = k(G, B)$   
 $H \in k_1 \cap k_5, I \in k_1 \cap k_5, J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5,$   
 $M \in k_4 \cap k_5$

## Distances

$$d_0 = |HM| = |IL| \text{ (red)} \dots 2$$

$$d_1 = |AL| = |CD| = |CE| = |CF| = |DE| = |DF| = |DG| = |DJ| =$$

$$|DK| = |EG| \text{ (blue)} \dots 10$$

$$d_2 = |HL| = |IM| \text{ (green)} \dots 2$$

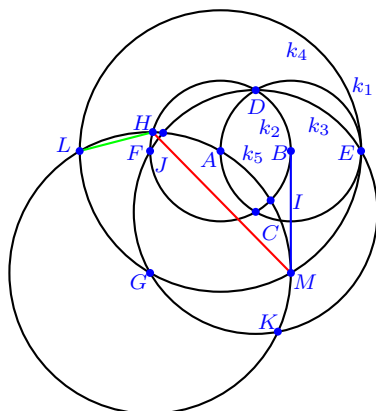
## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (40 occurrences)}$$

## Identical Constructions

RCC402 MM268

## 2.140.3 Construction CCO97



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, D)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(A, E)$   
 $G \in k_3 \cap k_4$
  5.  $k_5 = k(G, A)$   
 $H \in k_2 \cap k_5, I \in k_2 \cap k_5, J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5,$   
 $M \in k_4 \cap k_5$

## Distances

$$d_0 = |HM| = |IL| \text{ (red) } \dots 2$$

$$d_1 = |BM| = |CD| = |CE| = |CF| = |CG| = |CJ| = |CK| = |DE| = |DF| = |FG| \text{ (blue) } \dots 10$$

$$d_2 = |HL| = |IM| \text{ (green) } \dots 2$$

## Occurrences of the Golden Ratio

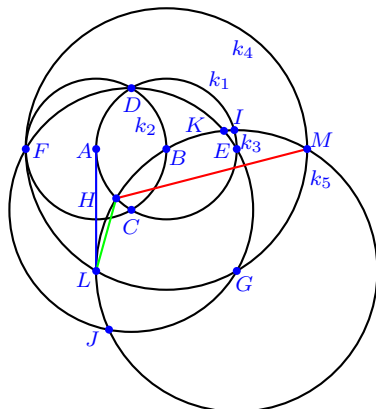
$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (40 occurrences)}$$

## Identical Constructions

RCC430 MM153



## 2.140.4 Construction CCO114



## Construction Process

- $A, B$  given initial points
1.  $k_1 = k(B, A)$
  2.  $k_2 = k(A, B)$   
 $C \in k_1 \cap k_2, D \in k_1 \cap k_2$
  3.  $k_3 = k(C, D)$   
 $E \in k_1 \cap k_3, F \in k_2 \cap k_3$
  4.  $k_4 = k(B, F)$   
 $G \in k_3 \cap k_4$
  5.  $k_5 = k(G, B)$   
 $H \in k_1 \cap k_5, I \in k_1 \cap k_5, J \in k_3 \cap k_5, K \in k_3 \cap k_5, L \in k_4 \cap k_5,$   
 $M \in k_4 \cap k_5$

## Distances

$$d_0 = |HM| = |IL| \text{ (red) } \dots 2$$

$$d_1 = |AL| = |CD| = |CE| = |CF| = |CG| = |CJ| = |CK| = |DE| =$$

$$|DF| = |EG| \text{ (blue) } \dots 10$$

$$d_2 = |HL| = |IM| \text{ (green) } \dots 2$$

## Occurrences of the Golden Ratio

$$\phi = \frac{d_0}{d_1} = \frac{d_1}{d_2} \text{ (40 occurrences)}$$

## Identical Constructions

RCC457 MM157

## 3 Proofs

### 3.1 The Golden Ratio – Really?

Only lists of lengths and golden ratios of these lengths in the constructions are provided in this book. Whether these ratios are really golden ratios is not immediately known. These statements need to be shown. Additionally, we do not support the stated congruence of listed lengths. Many are easy to show, others are not, necessitating this chapter. This chapter addresses proofs.

We are not going to prove all listed statements. This opportunity we leave to the reader. Additionally, many stated golden ratios (constructed using different sequences of steps) appear in multiple constructions and thus must be proven once. However, plenty of statements remain to be proven.

The goal of this chapter is to provide the reader with the main ideas and hints of how to construct the necessary proofs.

All constructions are based on few lines – straight lines and circles. Therefore, discovering some basic constructions and *patterns* in which the golden ratio can be found is possible. The term *pattern* indicates a particular couple or triple of segments (or distances of constructed points) in a specific geometric relation (for instance, the chords of one circle or of congruent circles intercepted by central, or inscribed, angles of particular values). We describe the *patterns* required for our proofs, indicate them by names and prove that the ratios of the lengths characterized with a given relation are the golden ratio. These *patterns* occur in the majority of the presented constructions. The other ratios must be proven individually by using known geometry and trigonometric identities.

In the following text, we present the *patterns* mentioned above and the main ideas as to how the desired ratios can be found (calculated).

### 3.2 The Properties of the Golden Ratio

As we indicated in the preface, the geometric definition of the "golden section" is that it divides a length into a larger and smaller part for which the length of the larger part forms the identical ratio with the length of the smaller part as the whole length does with the larger part. This rule implies the value of the ratio of the two lengths.

This number is usually denoted  $\phi$  and

$$\phi = \frac{\sqrt{5} + 1}{2}$$

which implies

$$\phi - 1 = \frac{1}{\phi} = \frac{\sqrt{5} - 1}{2}.$$

We will prove that the listed ratios for several constructions are equal to these numbers.

To avoid misunderstandings and confusion, we will not label any angle  $\varphi$  or  $\phi$  in this chapter.

### 3.3 Symbols Used

We denote the line segment with endpoints  $A$  and  $B$  by  $AB$ , and we denote the length of this segment and the distance of points  $A, B$  as  $|AB|$ . Not to consider various variants of the positions of points on the circle we use oriented angles. The value of the oriented angle  $\angle ABC$  is the value of the angle by which we have to rotate the ray  $BA$  counterclockwise to the position of ray  $BC$ . We denote the value of angle  $\angle ABC$  as  $|\angle ABC|$ . We consider arcs as oriented, too. In this text, we denote circle  $k$  with the center  $S$  passing through point  $A$  by  $k(S, A)$ , circle  $m$  with the center  $S$  and radius  $|PQ|$  by  $m(S, |PQ|)$ . Because in the proofs we do not need to distinguish the tool the circle was constructed with ("collapsible" compass from the "normal" compass), we will denote circle  $k(A, |AB|) = k(A, B)$  by  $k(A, B)$ .

In the figures that illustrate the proofs we use the identical captions for points and lines as in the original construction figures in the list. Therefore, the similar placed points and lines in the different figures in this text may not have identical captions. When different figures are linked, this fact is stressed in the text.

The illustrative figures may contain lines and points which are not a part of the discussed construction, but which are necessary for performing the proof. The initial points  $A$  and  $B$  of each construction are drawn as empty circles.

### 3.4 The Basic Length and Constructed Lengths

All constructions are based on segment  $AB$  (which is not drawn in the constructions). This segment can be considered the unit length,  $|AB| = 1$ . We do not require this premise in our proofs; however, if the reader wishes to construct their own proofs, then the unity length may be helpful.

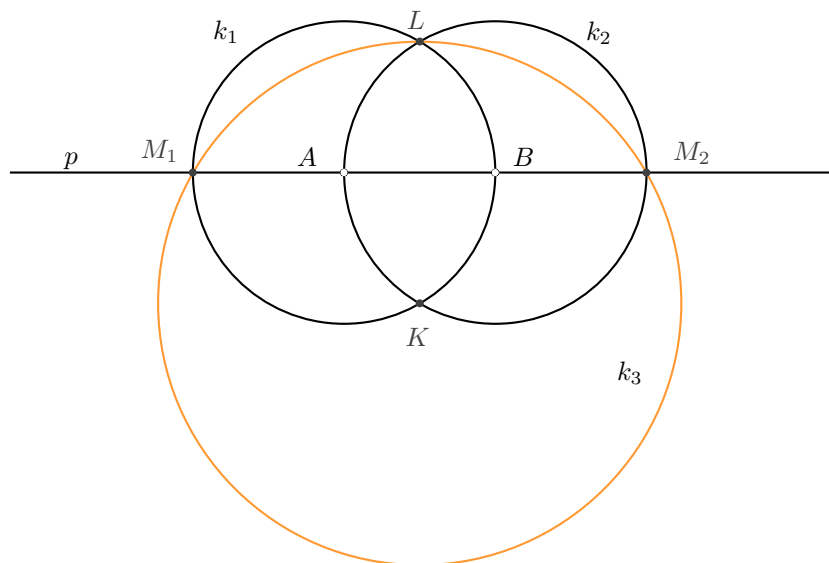


Figure 1

The first steps in constructions must construct any of the circles  $k_1(A, B)$  and  $k_2(B, A)$  or the straight line  $p(A, B)$ . These circles, with the radius  $r = |AB|$ , intersect, and their common chord is the side  $KL$  of two equilateral inscribed triangles;  $|KL| = \sqrt{3} \cdot |AB|$ . The vertices  $M_1$  and  $M_2$  of these equilateral

triangles are the endpoints of the diameters  $BM_1$  and  $AM_2$  of circles  $k_1$  and  $k_2$ , i.e., the segments of the length  $2 \cdot |AB|$ , (or  $2r$  in the case of circles with radius  $r$ ). The points  $M_1$  and  $M_2$  are the points of intersection of the circles  $k_1$  or  $k_2$  with any of the circles  $k_3(K, L)$  or  $k_4(L, K)$  or line  $p(A, B)$  (see Figure 1). The triangles  $\triangle KAB$  and  $\triangle LAB$  are equilateral. Any of the circles  $(K, A)$  and  $(L, A)$  (there is the circle  $(K, A)$  drawn in Figure 2) intersect the circles  $k_1$  and  $k_2$  (except for the points  $B$  and  $A$ ) at the endpoints of its diameter. Thus, we can determine the length  $2r$  as the length of the side of an equilateral triangle with medians in the sides of the triangle  $ABK$  (in Figure 2 triangle  $P_1P_2L$ ).

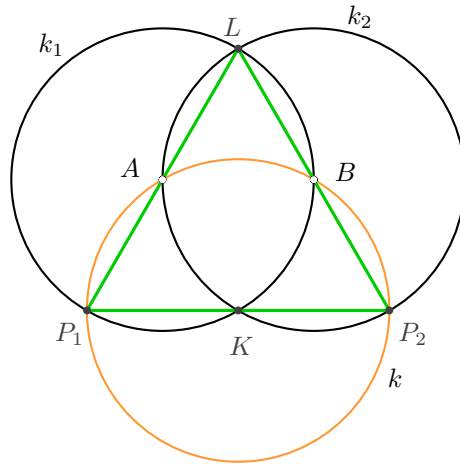


Figure 2

In case the first two steps in the construction are to draw the circles  $k_1(A, B)$  and  $k_2(B, A)$ , we can construct a perpendicular bisector of  $AB$  in the third step. This process does not create a new point (of intersection) in the construction. After constructing any triple from the circles and straight lines mentioned above, we continue the construction by adding more lines and circles. The new intersection points form new lengths. Because all of the constructions in this book consist of at most five lines (straight lines and circles), recurring *patterns* arise frequently.

### 3.5 Pattern A – Common Chord of Concentric Circles of Radii $r, 2r$

Well-known golden ratio we can find in following construction: A straight line intersects two concentric circles of radii  $r, 2r$ . The distance of the line from the common center of the circles is half the radius of the inner circle. The circles cut segments on the line, lengths of which provide the golden ratio. In Figure 3 we note intersection points  $K, L$  and  $Q$  that are the endpoints of such segments.

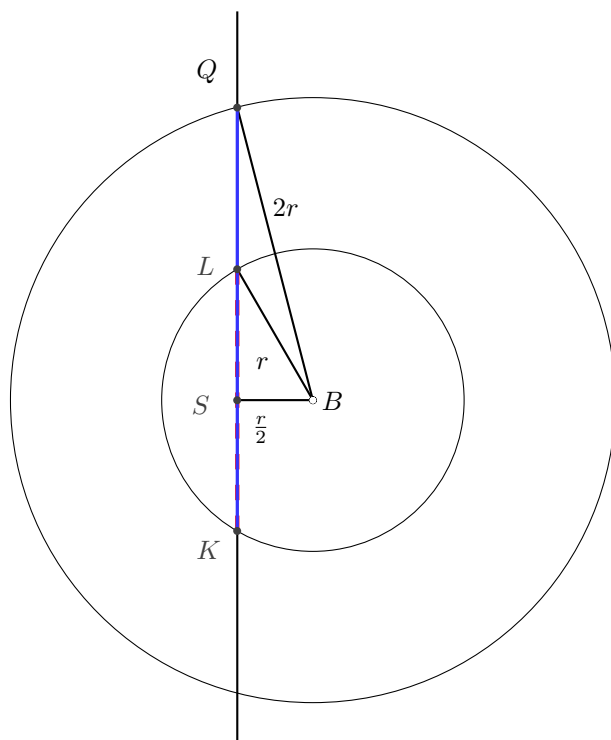


Figure 3

*Proof.* Because (by Pythagorean Theorem)

$$|KL| = r\sqrt{3}, |SL| = r\frac{\sqrt{3}}{2}, |SQ| = r\frac{\sqrt{15}}{2},$$

we have

$$\frac{|KQ|}{|KL|} = \frac{\sqrt{15} + \sqrt{3}}{2\sqrt{3}} = \frac{\sqrt{5} + 1}{2},$$

and from the definition of the golden section, it follows that

$$\frac{|KL|}{|QL|} = \frac{2\sqrt{3}}{\sqrt{15} - \sqrt{3}} = \frac{2}{\sqrt{5} - 1} = \frac{\sqrt{5} + 1}{2}.$$

This construction is obtained by starting from two given points,  $A$  and  $B$ , using the circles mentioned above. After constructing the length  $2r$ , we draw a concentric circle of this radius to any of circles  $k_1$  or  $k_2$  and intersect both the concentric circles with line  $KL$  (Figure 4), which is the common chord of circles  $k_1$  and  $k_2$ .

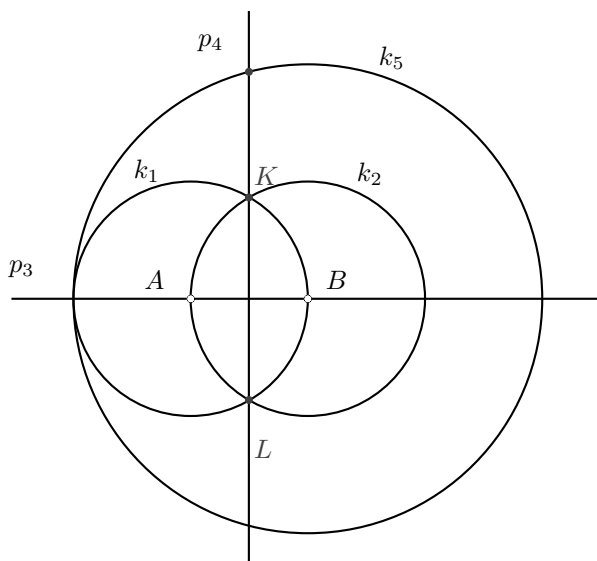


Figure 4

This pattern can be found in multiple constructions, e.g., in the constructions of classes 16, 18, 39, 50, 58, 59, 71 (e.g., in the constructions RCC27, RCC30, RCC44, MM33, MM125, MM96 and MM150).

*Remark 1.* This pattern can also be interpreted as a ratio of the lengths of the "longer" diagonals of a kite and a rhombus as can be seen in Figure 5.

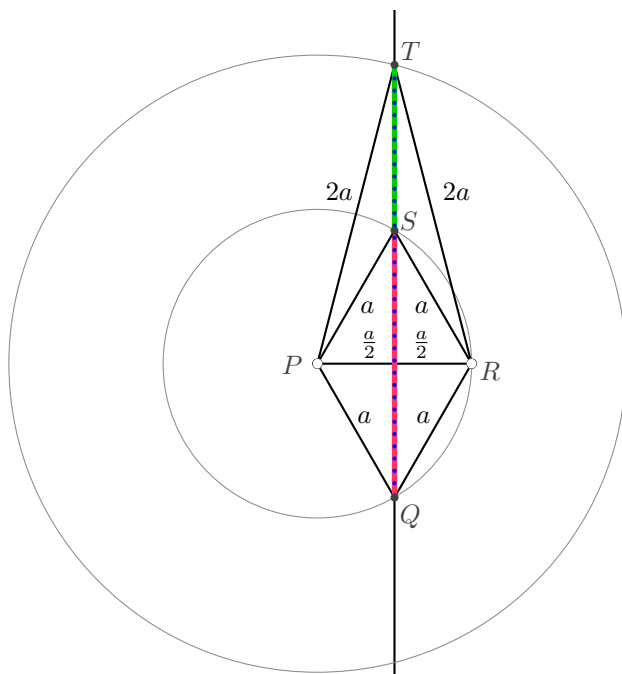


Figure 5

For a rhombus  $PQRS$ , in which  $a = |PQ|$  and a kite  $PQRT$ , in which  $|RT| = 2a$ ,  $|PR| = a$  is the common "shorter" diagonal of these quadrilaterals. Then, the

ratio of lengths of the "longer" diagonal of the kite to the "longer" diagonal of the rhombus is the golden ratio.

If we find the rhombus and the kite mentioned above in a particular construction, we can prove an occurrence of the golden ratio in this construction. In construction RCC25 (the class of similar constructions 15), we prove that  $\triangle FEG$  and  $\triangle FEH$  are equilateral triangles and that  $|ED| = 2|EG| = 2|EH|$  (see Figure 6).

*Proof.* Points  $H, E$  and  $G$  lie on one circle with center  $F$  and  $GH$  is the perpendicular bisector of segment  $EF$ ; therefore,  $\triangle FEG$  and  $\triangle FEH$  are equilateral triangles.

Angle  $\angle FCA$ ,  $|\angle FCA| = \alpha$  is a central angle and  $\angle FDA$  and  $\angle FEA$  are inscribed angles intercepting the arc  $FA$  in circle  $k_4(C, A)$ . Therefore,  $|\angle ADE| = |\angle FDA| = |\angle FEA| = \frac{\alpha}{2}$  and right triangles  $\triangle AED, \triangle EJD$  (and also  $\triangle JAE$  and many others) are similar. Therefore,  $|ED| : |EJ| = |AD| : |AE| = 4 : 1$ , hence  $|ED| = 4|EJ| = 2|EG| = 2|EH|$ . Therefore,

$$\frac{|DG|}{|GH|} = \frac{|GH|}{|DH|} = \phi.$$

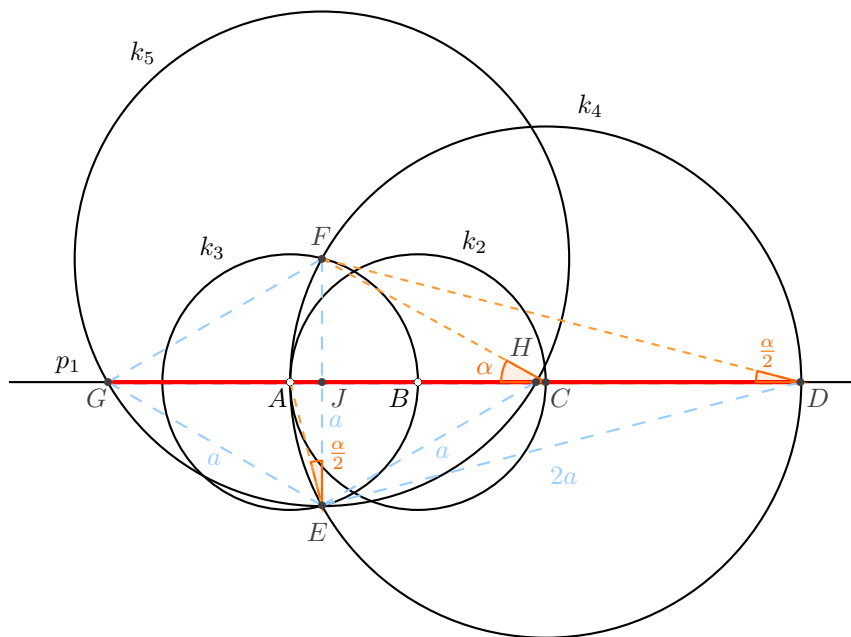


Figure 6

*Remark 2.* The identical pattern can be found in numerous clever constructions of the golden ratio. However, these constructions use other tools, not only the straightedge and compass.

[8] shows the construction displayed in Figure 7.

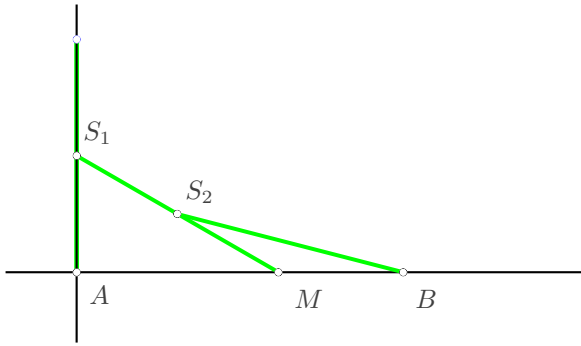


Figure 7

We start with two orthogonal straight lines in the drawing ("vertical" and "horizontal") and a strip of paper with the center marked.

We attach the strip of paper to one of the lines (in Figure 7, the line is the "vertical" one) so the "bottom" endpoint coincides with the intersection point of the drawn lines. We mark the position of the center of the strip on the drawing (point  $S_1$ ) and the position of the endpoint of the strip on the "horizontal" line – point  $A$ . We slide the strip. The "upper" endpoint moves along the "vertical" line to position  $S_1$ ; the "bottom" endpoint moves along the "horizontal" line. We mark the position of the center of the strip again (point  $S_2$ ) and the position of the endpoint of the strip on the "horizontal" line – point  $M$ .

We slide the strip again. The "upper" endpoint slides to point  $S_2$ ; the "bottom" slides along the "horizontal" line. We mark the position of the "bottom" endpoint on the "horizontal" line – point  $B$ .

Point  $M$  divides the segment  $AB$  in the golden section.

### 3.6 Pattern B – Chords on One Circle (construction using circles of radii $r$ , $2r$ )

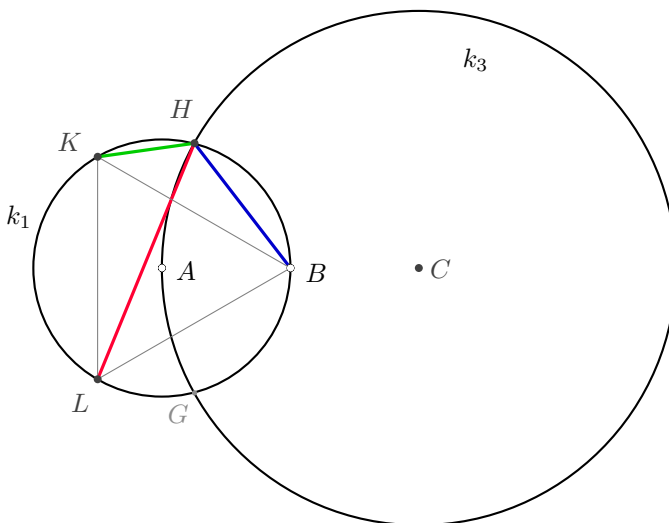


Figure 8



Draw a circle  $k_1(A, B)$  and intersect it with circle  $k_3(C, A)$ ,  $B$  being the midpoint of the  $AC$ . Denote the points of intersection  $G$  and  $H$ . Consider the equilateral triangle  $BKL$  inscribed in the circle  $k_1$ . Then, the ratios of the lengths of segments  $HB$ ,  $HK$  and  $HL$  (see Figure 8) provide the golden ratio

$$\frac{|HB|}{|HK|} = \frac{|HL|}{|HB|} = \phi.$$

*Proof.* See Figure 9. Denote  $|\angle BAH| = \alpha$ . Because of

$$|\angle BAH| = \alpha, |\angle HAK| = 120^\circ - \alpha, |\angle HAL| = 240^\circ - \alpha,$$

it follows that

$$\frac{|HB|}{|HK|} = \frac{\sin \frac{\alpha}{2}}{\sin \frac{120^\circ - \alpha}{2}}, \quad \frac{|HL|}{|HB|} = \frac{\sin \frac{240^\circ - \alpha}{2}}{\sin \frac{\alpha}{2}}.$$

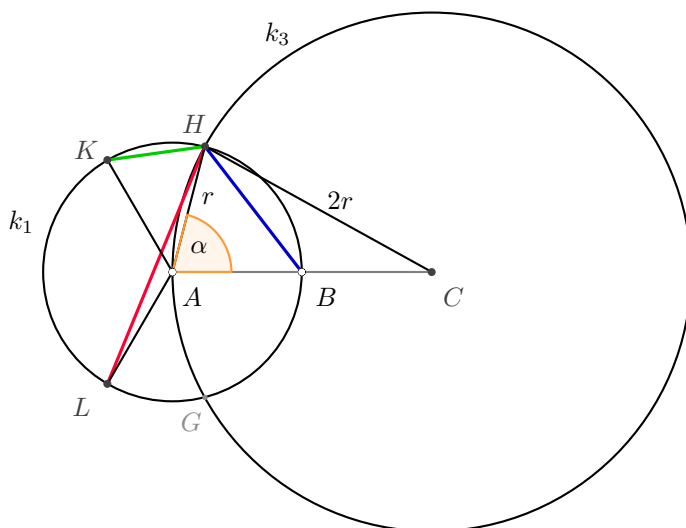


Figure 9

Because of  $\cos \alpha = \frac{|AH|}{2|AC|} = \frac{1}{4}$ , it follows that  $\sin \frac{\alpha}{2} = \sqrt{\frac{3}{8}}$ ,  $\cos \frac{\alpha}{2} = \sqrt{\frac{5}{8}}$ , hence

$$\frac{|HB|}{|HK|} = \frac{\sin \frac{\alpha}{2}}{\sin (60^\circ - \frac{\alpha}{2})} = \frac{\sqrt{\frac{3}{8}}}{\frac{\sqrt{3}}{2} \sqrt{\frac{5}{8}} - \frac{1}{2} \sqrt{\frac{3}{8}}} = \frac{1}{\frac{\sqrt{5}-1}{2}} = \frac{\sqrt{5}+1}{2},$$

$$\frac{|HL|}{|HB|} = \frac{\sin (120^\circ - \frac{\alpha}{2})}{\sin \frac{\alpha}{2}} = \frac{\frac{\sqrt{3}}{2} \sqrt{\frac{5}{8}} + \frac{1}{2} \sqrt{\frac{3}{8}}}{\sqrt{\frac{3}{8}}} = \frac{\sqrt{5}+1}{2}.$$

This pattern is frequent in the constructions, e.g., in constructions of class 3 (e.g., RCC2) and in constructions of class 20 (e.g., RCC64).

### 3.7 One Important Angle and Patterns A, B

The perpendicular bisector  $AB$  intersects circle  $k_1$  at points  $E, F$ . Consider an isosceles triangle  $ABQ$  with base  $AB$  and legs of length  $b = 2|AB|$ . Then,  $\triangle ABQ, \triangle AHC$  in Figure 10 are congruent triangles. Therefore, their angles are congruent and point  $H$  lies on segment  $AQ$ . Therefore, line  $AH$  intersects the perpendicular bisector of  $AB$  at point  $Q$ , for which the points  $A, B, F, E$  and  $Q$  create pattern **A**. Denote angle  $|\angle EAQ|$  in this pattern by  $\psi$ .

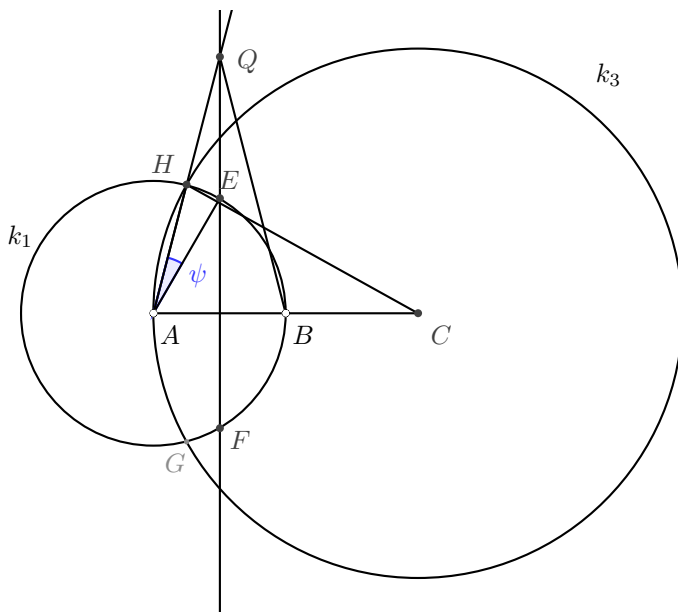


Figure 10

Circle  $k_2(B, A)$ , intersects circle  $k_1(A, B)$  at the identical intersection points  $E, F$  as above because  $\triangle ABE$  and  $\triangle ABF$  are equilateral triangles (see Figure 11).

Angle  $\angle EAQ = \angle EAH$  (and its value  $\psi = |\angle EAH|$ ) and angles whose values we are able to derive from value  $\psi$ , are frequently used in our proofs.

If we describe the geometric relations in pattern **B** using angle  $\psi$ , then values of the central angles corresponding to intercepted chords  $HK, HB$  and  $HL$  are equal to  $|\angle HAK| = 60^\circ - \psi$ ,  $|\angle BAH| = 60^\circ + \psi$  and  $|\angle HAL| = 180^\circ - \psi$ , respectively (see Figure 12). This proposition proven for pattern **B** can also be formulated as follows:

Chord lengths  $d_1, d_2$  and  $d_3$  are subtended on a circle by corresponding central angles of values  $180^\circ - \psi, 60^\circ + \psi$  and  $60^\circ - \psi$  respectively; these lengths provide ratios

$$\frac{d_1}{d_2} = \frac{d_2}{d_3} = \frac{\sqrt{5} + 1}{2} = \phi.$$

In the examined constructions, the chords mentioned above are the chords not only in one circle, but frequently in congruent circles.

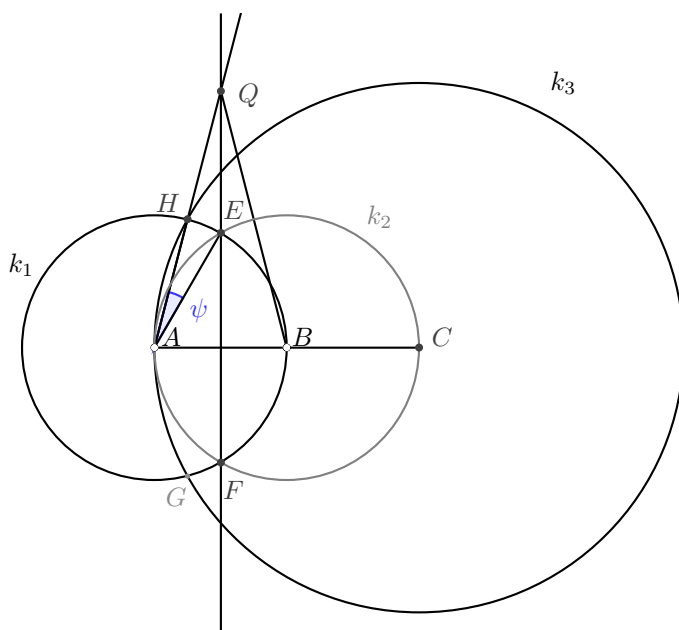


Figure 11

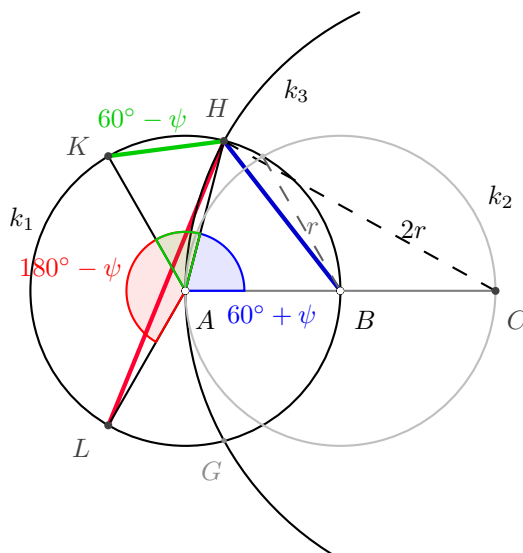


Figure 12

If we rotate segment  $FQ$  in rotation about point  $A$  by angle  $-\psi$  (see Figure 13), then point  $F$  rotates to point  $G$ , point  $Q$  rotates to point  $Q'$  on circle  $k_3(C, A)$ ,  $|AQ| = |AQ'|$ ,  $|\angle BAQ'| = 60^\circ$ , and point  $E$  rotates to point  $E'$ , for which  $|GE'| = |EF|$ .

*Remark.*  $\triangle AQ'C$  is an equilateral triangle, and point  $E$  is the midpoint of segment  $AQ'$ .

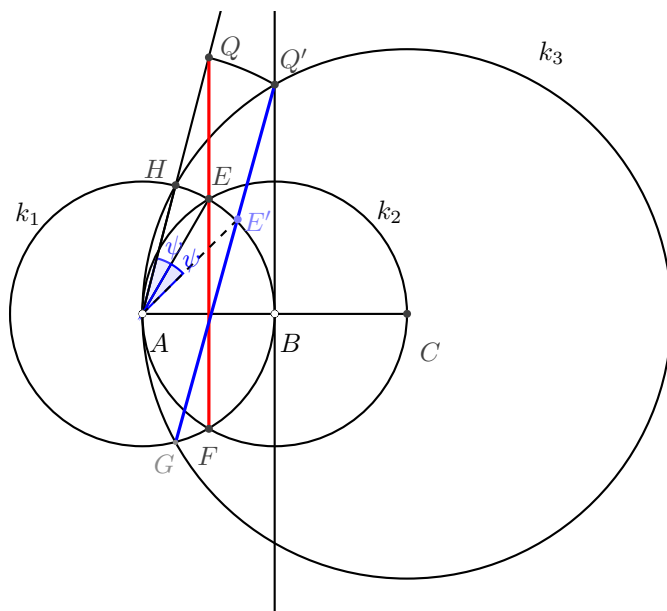


Figure 13

The ratios obtained in similar ways can be found in many constructions and configurations of circles and straight lines. These constructions are different at first sight. Figure 14 explains the pattern, in which segment  $PG$  (divided by point  $H$  on circle  $k_1$  in the golden section) is rotated in rotation about point  $A$  by angle  $-60^\circ$  to position  $CD$ . Point  $H$  must rotate to point  $K$  on circle  $k_1$ , which divides segment  $CD$  in the identical ratio as point  $H$  divides segment  $PG$ . Therefore,

$$\frac{|PG|}{|HG|} = \frac{|HG|}{|PH|} = \frac{|CD|}{|KD|} = \frac{|KD|}{|CK|} = \phi.$$

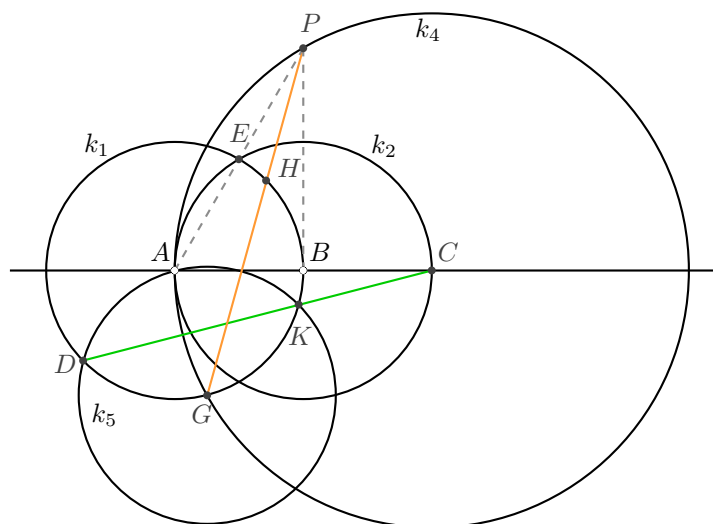


Figure 14

This pattern we denote as pattern **A2**.

The circles mentioned above and the pairs of points that correspond to pairs  $PG$  and  $CD$  in Figure 14 can be found in many constructions, but the lines are not part of these constructions – e.g., straight line (segment)  $PG$ . Therefore, point  $H$  which divides the segment  $PG$  in the golden section is not constructed. However some necessary length (in Figure 14 length  $|GH|$  or  $|HP|$ ) can be found as the distance of another pair of points in the construction. In Figure 14 we have pairs of points (segments)  $KD$  and  $EC$  and – denoting the next intersection point of circles  $k_1$  and  $k_2$  by  $F$  and the next intersection point of circles  $k_5$  and  $k_2$  by  $J$  – we have  $|EF| = |FC| = |AJ| = |GH| = \sqrt{3}r$ .

This implies that segments in the following pattern form the golden ratio.

### 3.8 Pattern C – Chords on a Circle of Radius 2r, a Doubled Golden Ratio

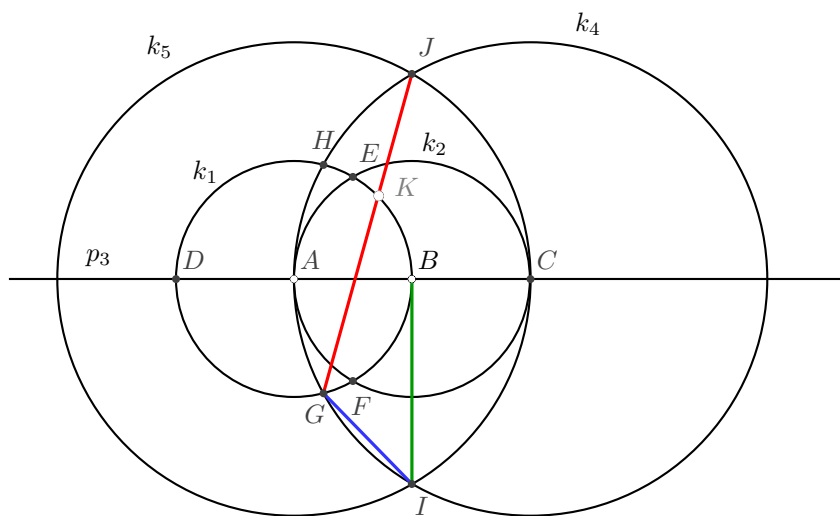


Figure 15

Figure 15 displays construction RCC1 (class 2), which (in various modifications) appears in other constructions, e.g., in construction RCC31 (class 19), in which two patterns can be found (pattern **B** and **C**) and in construction RCC3 (class 4), in which circle  $k_5$  is replaced by straight line  $AE$  (and point  $J$  is the intersection point of circle  $k_4$  with line  $AE$ ).

*Proposition.*

$$\frac{|GJ|}{|BI|} = \frac{|BI|}{|GI|} = \frac{\sqrt{5} + 1}{2}.$$

*Proof.* This statement is a corollary of the reasoning above. This statement follows from the congruence of segments  $|BI| = |FE| = |KG|$  (point  $K$  is not drawn in the construction; we have added the point for the proof) and  $|GI| = |HJ| = |KJ|$ . The first congruence results from the symmetry about line  $p_3$ , and the next congruence results from symmetry about line  $AJ$  ( $A$  is the center of arc  $HG$ ; thus,  $AJ$  is the angle bisector of  $\angle HJG$ ).

Therefore,  $|BI| = \frac{1}{2}|JI| = |FE| = |KG| = \sqrt{3}r$ , where  $r = |AB|$ .

Points  $J, K$  and  $G$  in Figure 15 correspond to points  $P, H$  and  $G$  in the pattern in Figure 14, which we have denoted by **A2**.

**Golden Ratio Squared (a Doubled Ratio)**

Only two chords are constructed in circle  $k_4(C, A)$  in Figure 15 and their lengths provide the golden ratio with a particular third length. The ratio of the lengths between these two chords is equal to the ratio of the length of the entire segment divided by the golden section to the length of its smaller part; thus, this ratio is the square of  $\phi$ .

$$\frac{|GJ|}{|GI|} = \frac{\sqrt{5} + 1}{\sqrt{5} - 1} = \left( \frac{\sqrt{5} + 1}{2} \right)^2 = \phi^2 = \frac{\sqrt{5} + 3}{2}.$$

**Description of Pattern C Using Central Angles**

*Proposition.*

Chords  $GI$  and  $GJ$  are intercepted on circle  $k_4$  in Figure 15 by central angles  $|\angle GCI| = 2\psi$  and  $|\angle JCG| = 120^\circ - 2\psi$ , respectively.

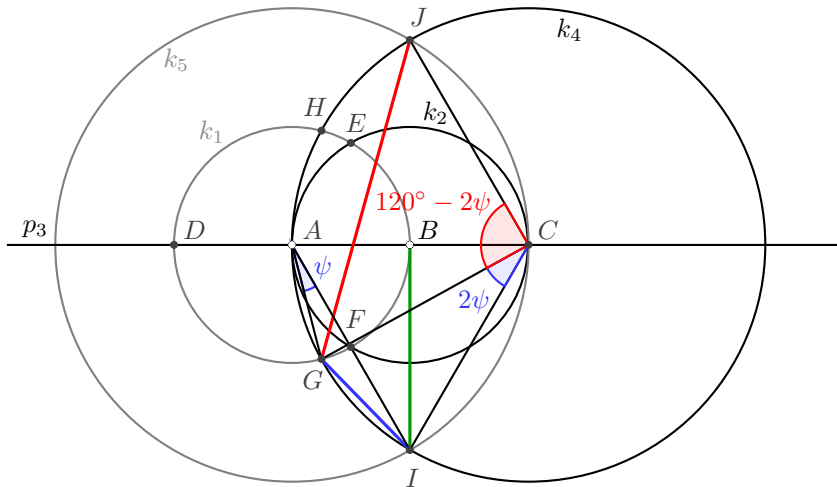


Figure 16

*Proof.* The triangles  $\triangle AJC$  and  $\triangle AIC$  are equilateral; thus  $|\angle IAJ| = 120^\circ$ , see Figure 16.

Point  $F$  is the midpoint of  $AI$ , thus  $|\angle GAI| = |\angle GAF| = \psi$ . Angle  $\angle GAI$  is an inscribed angle corresponding to the intercepted chord  $GI$  and to the central angle  $\angle GCI$ , therefore  $|\angle GCI| = 2\psi$ .

(Point  $F$  does not lie on segment  $GC$ .)

### 3.9 Pattern D – Chords on Circle of Radius $3r$ : Another Doubled Golden Ratio

The lengths of two chords on a circle of the radius  $3r$  that are constructed in Figure 17 give a square of the golden ratio

$$\frac{|HE|}{|EG|} = \phi^2 = \frac{\sqrt{5} + 3}{2}.$$

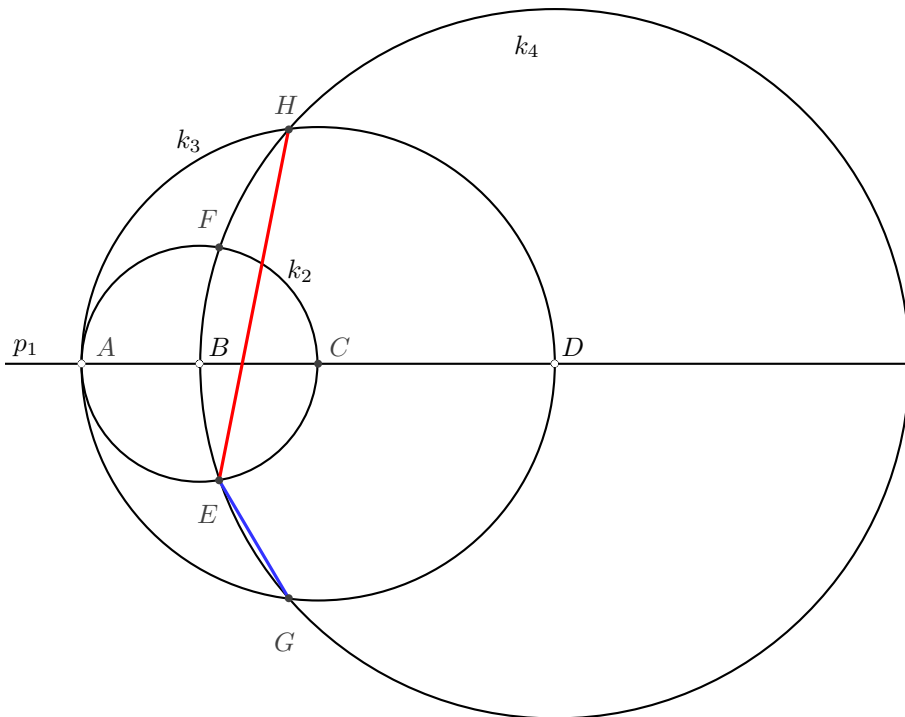


Figure 17

*Proof.* We illustrate the proof in Figure 18. Denote angles in the isosceles triangles  $\triangle HCD$  and  $\triangle BDE$  by  $|\angle HDC| = \delta$  and  $|\angle BDE| = \gamma$ , respectively. Then,

$$\cos \delta = \frac{3}{4}, \quad \sin \frac{\gamma}{2} = \frac{1}{6}$$

from which

$$\cos \frac{\delta}{2} = \sqrt{\frac{7}{8}}, \quad \sin \frac{\delta}{2} = \sqrt{\frac{1}{8}}, \quad \cos \frac{\gamma}{2} = \frac{\sqrt{35}}{6}.$$

Because

$$\frac{|HE|}{|EG|} = \frac{\sin \frac{\delta+\gamma}{2}}{\sin \frac{\delta-\gamma}{2}} = \frac{\sin \frac{\delta}{2} \cos \frac{\gamma}{2} + \cos \frac{\delta}{2} \sin \frac{\gamma}{2}}{\sin \frac{\delta}{2} \cos \frac{\gamma}{2} - \cos \frac{\delta}{2} \sin \frac{\gamma}{2}},$$

we have

$$\frac{|HE|}{|EG|} = \frac{\sqrt{35} + \sqrt{7}}{\sqrt{35} - \sqrt{7}} = \frac{\sqrt{5} + 1}{\sqrt{5} - 1} = \frac{\sqrt{5} + 3}{2}.$$

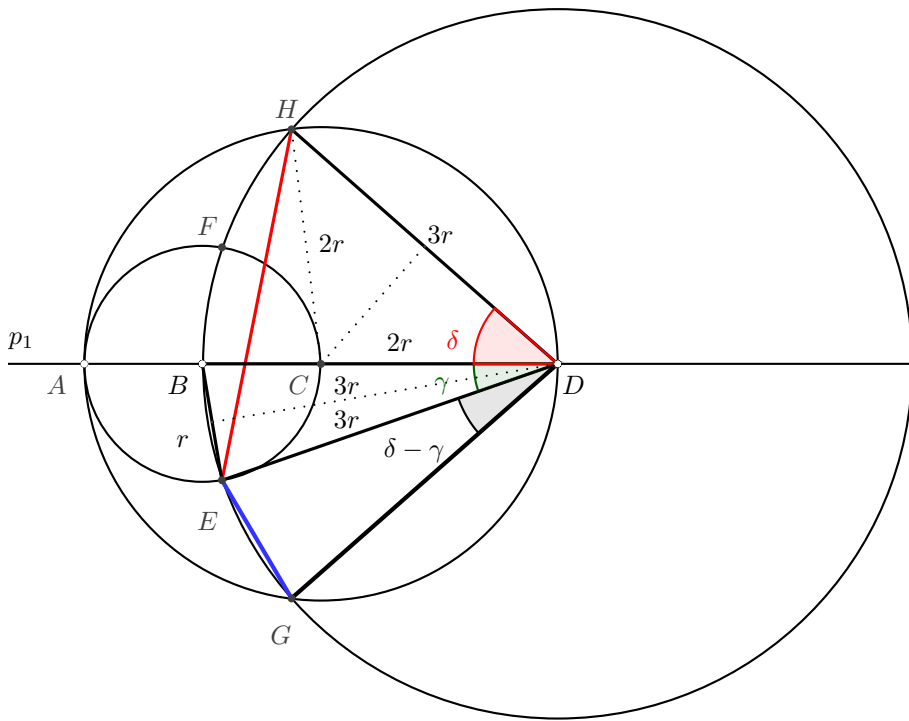


Figure 18

### Missing Length Providing the Golden Ratio

Similar to pattern C, we must find a third length that forms the golden ratio with the lengths of these chords. We will prove that this missing length is a length of segment  $EJ$  in Figure 19. Point  $J$  is the intersection point of segment  $HE$  with circle  $k_1(B, A)$ . Thus, this pattern is similar to pattern C.

The symmetry about line  $AB$  implies congruence  $|EG| = |FH|$ . Because  $\angle HBF$  and  $\angle HEF$  are inscribed angles that intercept the identical arc and chord  $HF$  on circle  $(D, B)$ , they are equal. Because the central angle  $\angle JBF$  corresponds to the inscribed angle  $\angle JEF$ , it follows that  $|\angle JBF| = 2|\angle JEF| = 2|\angle HEF|$ . Thus, straight line  $HB$  is the bisector of  $\angle JBF$ . Because  $B$  is the midpoint of arc  $FE$ , line  $HB$  is the bisector of  $\angle FHE$ , see Figure 19.

Thus, line  $BH$  is the perpendicular bisector of  $FJ$ . Therefore,  $|FH| = |JH|$ , and point  $J$  divides segment  $EH$  in the golden section.



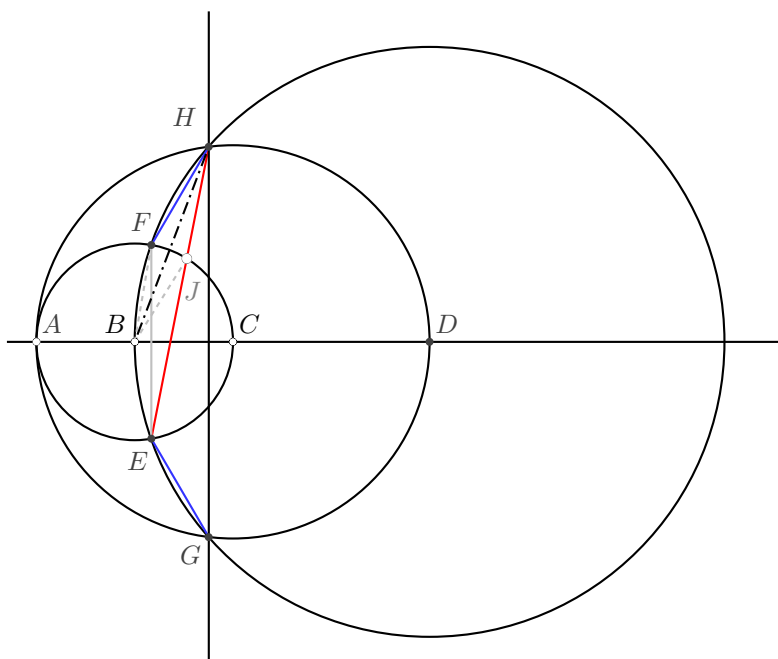


Figure 19

In Figures 20 and 21, we can see other segments of identical length (equal to  $|EJ|$ ). Hints to the proofs are also displayed in these Figures.

In Figure 20, we find construction RCC86 from class 28.

Because the right triangles  $\triangle ADH$ ,  $\triangle HDM$ ,  $\triangle AHM$  and  $\triangle IBM$  are similar, we have

$$\frac{|AI|}{2r} = \sin \frac{180^\circ - \delta}{2} = \sin \left( 90^\circ - \frac{\delta}{2} \right) = \cos \frac{\delta}{2}.$$

Therefore,

$$\frac{|HE|}{|AI|} = \frac{3 \sin \frac{\delta + \gamma}{2}}{\cos \frac{\delta}{2}} = \frac{\frac{\sqrt{35}}{2\sqrt{8}} + \frac{\sqrt{7}}{2\sqrt{8}}}{\sqrt{\frac{7}{8}}} = \frac{\sqrt{5} + 1}{2} = \phi.$$

Figure 21 displays construction RCC81 from class 23.

Denote  $|\angle IDC| = \delta$  (the labels for the points follow the labels in the original construction). Then,

$$|\angle DCI| = 180^\circ - 2\delta \quad \text{and} \quad |\angle IEB| = \frac{1}{2}|\angle IDB| = \frac{\delta}{2} \quad (\text{central and inscribed angle}).$$

Because  $|\angle DCI| = 180^\circ - 2\delta$ , we have

$$|\angle CIJ| = |\angle IJC| = \frac{3\delta}{2}, \quad \text{from which } |\angle JCI| = 180^\circ - 3\delta.$$

Therefore, 
$$\frac{|IF|}{\frac{3r}{2}} = \frac{\sin \frac{\delta + \gamma}{2}}{\sin \left( 90^\circ - 3\frac{\delta}{2} \right)} = \frac{\sin \frac{\delta + \gamma}{2}}{\cos \frac{3\delta}{2}}, \quad \text{which results in } \frac{|IF|}{|IJ|} = \frac{\sqrt{5} + 1}{2}.$$

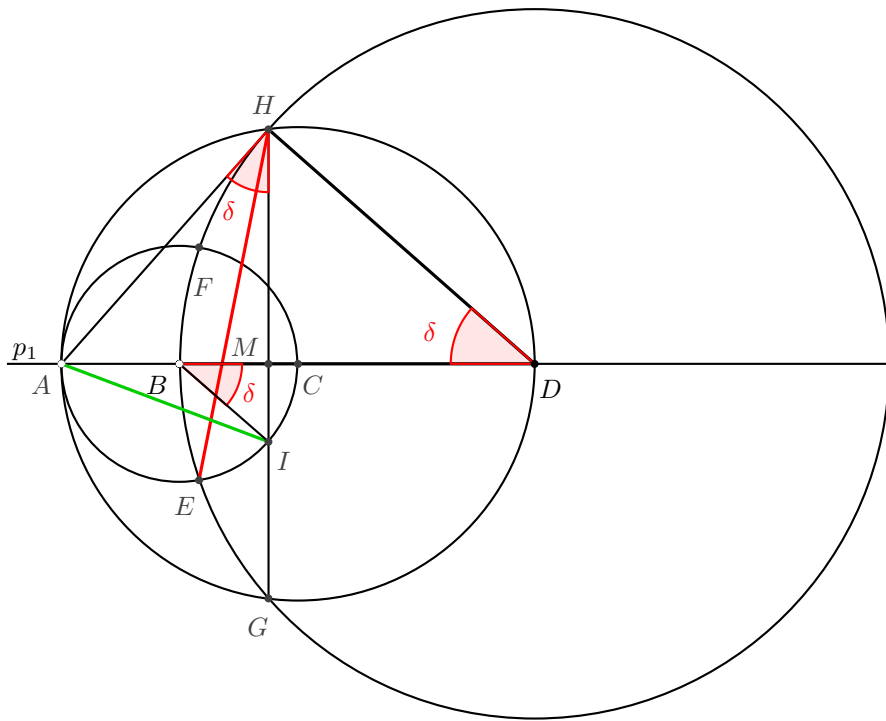


Figure 20

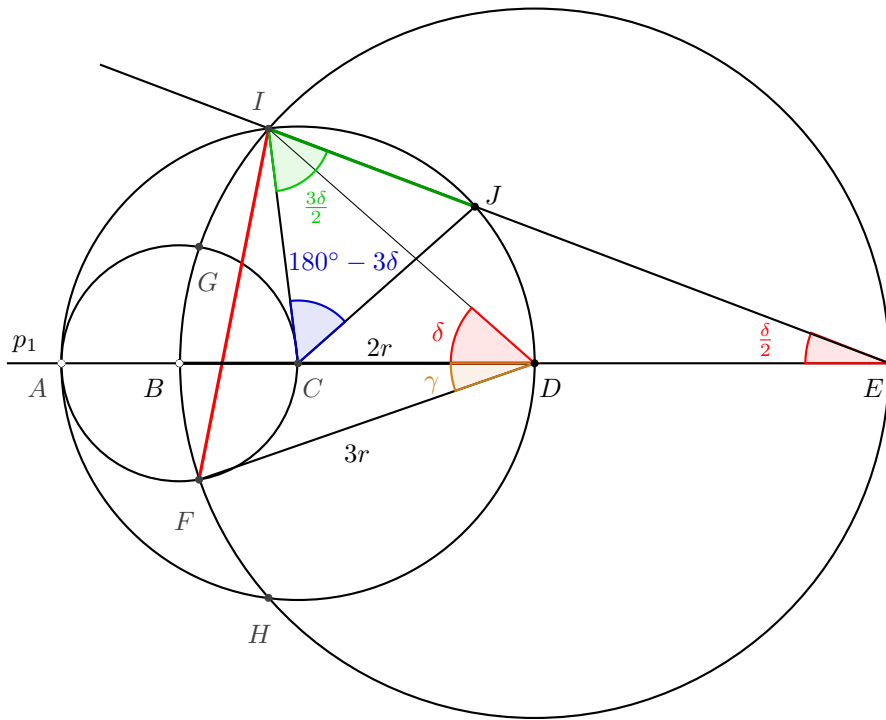


Figure 21

### 3.10 More Patterns and Approaches – Circles

We must often find angles in the circles (central and inscribed angles) and triangles (isosceles, equilateral and scalene) in our proofs.

Figure 22 illustrates the process to find the values of angles, which can be derived from the value  $|\angle EAH| = \psi$  (patterns **A**, **B**) in constructions:

- $\angle EAH$  is a central angle in circle  $k_1(A, B)$  corresponding to inscribed angles  $\angle EFH$  and  $\angle EGH$ ; therefore,  $|\angle EFH| = |\angle EGH| = \frac{\psi}{2}$ .
- $\angle ACG$  is a central angle corresponding to any angle  $\angle AMG$ , where  $M$  is any point on arc  $GA$  on circle  $k_3(C, A)$ ; thus,  $\angle ACG$  is corresponding to inscribed angle  $\angle AKG$ .
- Because  $|\angle GAF| = \psi$ , we have  $|\angle GAC| = 60^\circ + \psi$  and in isosceles triangle  $ACG$  we have  $|\angle ACG| = 60^\circ - 2\psi$ . Therefore,  $|\angle AKG| = 30^\circ - \psi$ .
- Because equal chords subtend angles that are equal, we can write  $|\angle HGA| = |\angle AHG| = |\angle AKG| = |\angle AMG| = 30^\circ - \psi$ .
- $\angle HGA = \angle HGJ$ , which is an inscribed angle corresponding to central angle  $\angle HAJ$ ; therefore  $|\angle HAJ| = 60^\circ - 2\psi$ .  $|\angle EAJ| = 60^\circ - \psi$  then can be written.
- In the isosceles triangle  $HAE$ , we find  $|\angle AHE| = |\angle HEA| = 90^\circ - \frac{\psi}{2}$  (not drawn in Figure 22).

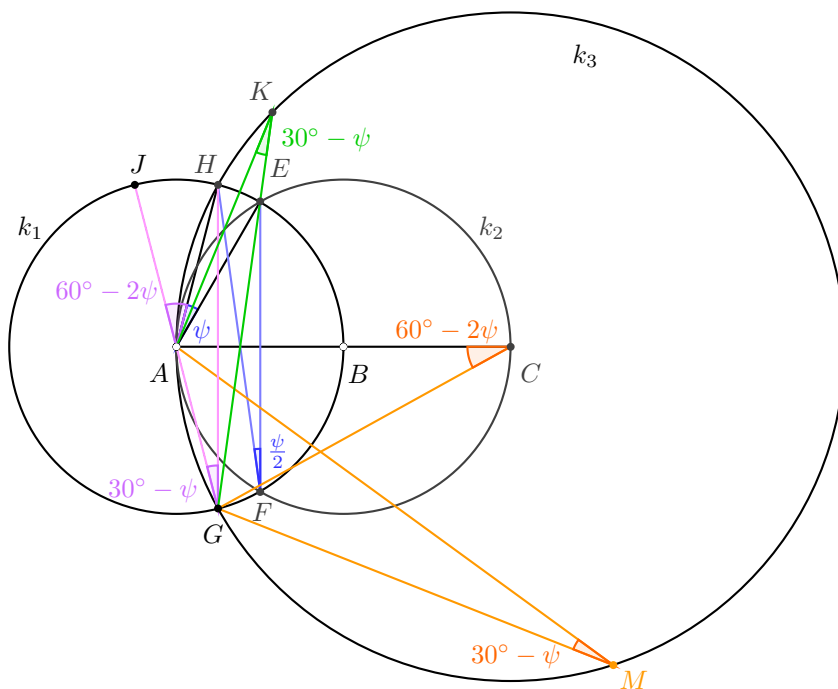


Figure 22

In a similar manner and using the properties of the angles in the triangles, we derive the values of angles required for performing the proofs.

## I Rotation of Circles

Each pair of congruent circles forms a preimage and an image in infinitely many rotations. Each circle centered in the center of such a rotation (and intersecting the given circles) intersects them in congruent chords that are mapped (the first to the second chord) in this rotation.

*Example:* Triangle  $FHJ$  in Figure 23 is equilateral.

*Proof.* Circle  $k_5(F, H)$  intersects circles  $k_1$  and  $k_2$ . A rotation about center  $F$  by  $60^\circ$  rotates  $k_2$  to  $k_1$ . The "first" of the intersection points of  $k_5$  with  $k_1$  (when moving counterclockwise) we denote by  $H$ ; the "first" of the intersection points of  $k_5$  with  $k_2$  we denote by  $J$ . Thus,  $J$  rotates to  $H$  in the identical rotation. Therefore,  $|\angle JFH| = 60^\circ$ .

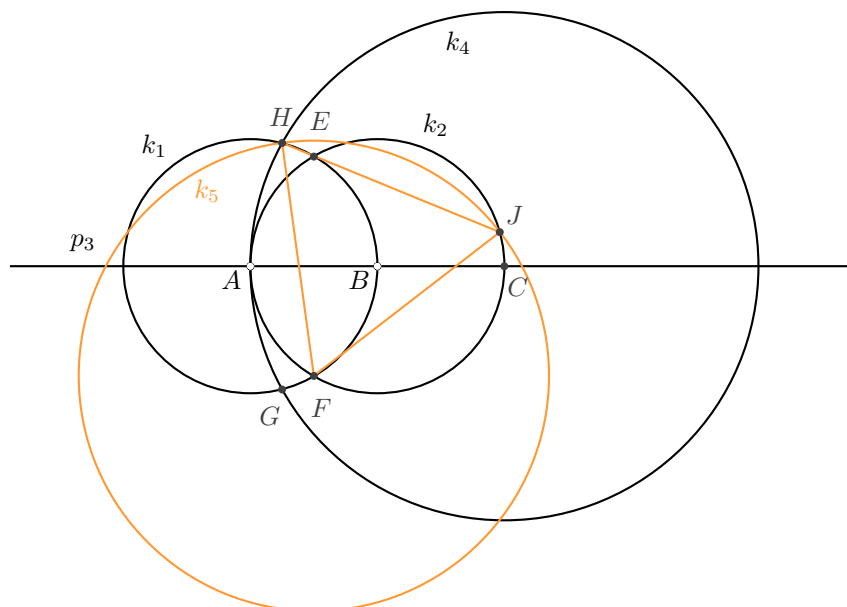


Figure 23

Point  $E$  "seems to lie" on side  $HJ$  of triangle  $FJH$  in Figure 23. We will prove this statement.

*Proof.* Triangle  $FCE$  is an equilateral triangle. From the rotation mentioned above, we have  $|\angle EFH| = |\angle CFJ|$  and  $|\angle EAH| = |\angle CBJ|$ .

$|\angle EAH| = \psi$ , thus  $|\angle FAH| = \psi + 120^\circ$  (central angle in  $k_1(A, B)$ ),

thus  $|\angle HEF| = 120^\circ - \frac{\psi}{2}$  (complementary inscribed angle in  $k_1(A, B)$ ).

$|\angle CBJ| = \psi$ , thus  $|\angle FBJ| = \psi + 120^\circ$  (central angle in  $k_2(B, A)$ ),

thus  $|\angle FEJ| = 60^\circ + \frac{\psi}{2}$  (inscribed angle in  $k_2(B, A)$ ).

Therefore,  $|\angle HEF| + |\angle FEJ| = 180^\circ$ , and point  $E$  lies on segment  $HJ$ .

The relations explained above imply that circles  $e(E, H)$ ,  $c(C, J)$  and  $f(F, G)$  in Figure 24 are congruent, rotated about center  $B$  by  $120^\circ$  and the values of the central angles in circle  $e(E, H)$  are  $|\angle HEL| = |\angle KEI| = 60^\circ$ .

Points  $G$  and  $H$  do not map to each other in any of these rotations.

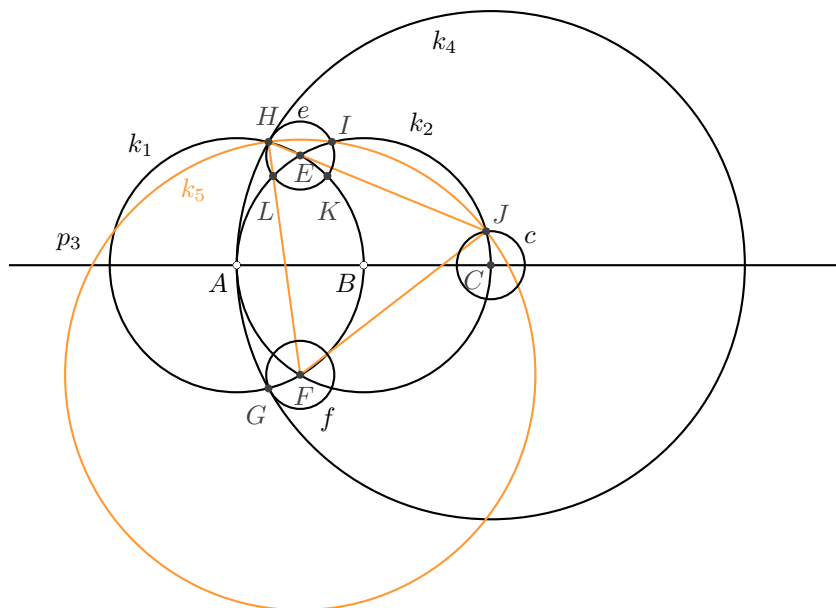


Figure 24

Each of the circles  $e = (E, H)$  and  $f = (F, G)$  intersects circle  $k_4$  at one more point, see circle  $e(E, H)$  in Figure 25.

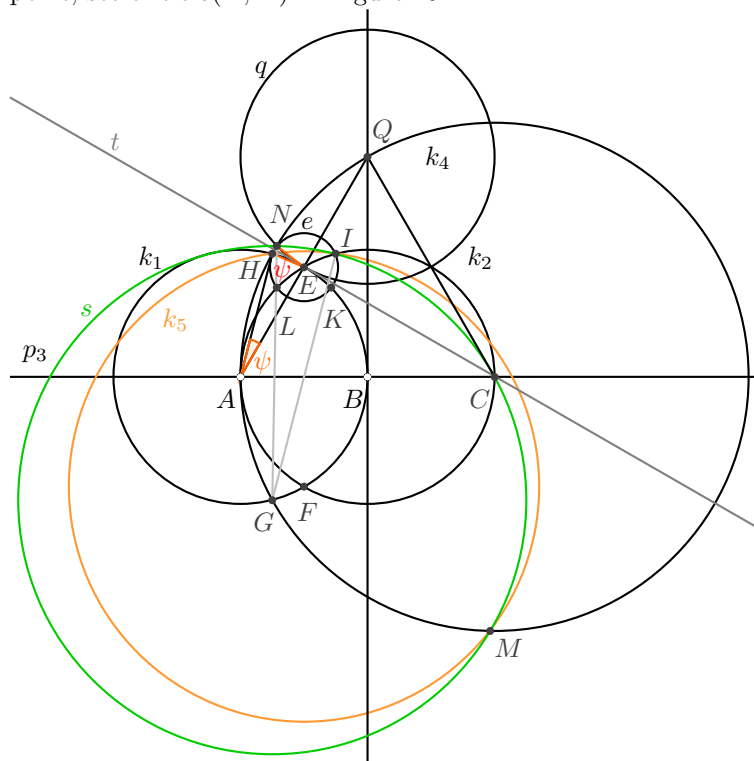


Figure 25

Because  $\triangle ACQ$  in Figure 25 is an equilateral triangle, circle  $k_1$  is mapped to circle  $q(Q, E)$  in the reflection about the straight line  $CE$  that is the common tangent of these circles, in the reflection about point  $E$  and in the rotation about center  $C$  by  $-60^\circ$ . This mapping implies the following corollaries:

- Point  $G$  maps to point  $N$  in rotation about center  $C$  by  $-60^\circ$ ; thus  $|AQ| = |GN| = 2r$ , and  $\triangle GCN$  is an equilateral triangle.
- Because the value of the angle between chord  $HE$  and tangent  $t = CE$  to the circle is equal to half the central angle  $\angle EAH$ ,  $\frac{\angle EAH}{2} = \frac{\psi}{2}$ . Thus  $|\angle NEH| = \psi$ .
- $NK$  is the diameter of circle  $e(E, H)$  (central symmetry of  $k_1$  and  $q$ ).
- The values of the central angles in circle  $e$  are  $|\angle NEH| = \psi$ ,  $|\angle HEL| = 60^\circ$ ,  $|\angle LEK| = 120^\circ - \psi$ ,  $|\angle KEI| = 60^\circ$ ,  $|\angle IEN| = 120^\circ$ .
- Circle  $s(G, C)$  of radius  $2r$  passes through point  $N$ .
- Line  $GA$  (not drawn in Figure 25) is the common diameter of circles  $k_1$  and  $s$  and intersects the circles at the point of tangency.
- Circles  $k_2(B, A)$ ,  $k_5(F, H)$ ,  $e(E, H)$  and  $s(G, C)$  meet at point  $I$ .
- The intersection point  $M$  of circles  $k_5(F, H)$  and  $s(G, C)$  lies on circle  $k_4$ . This statement results from the rotation of circle  $k_2$  about center  $F$  by  $-60^\circ$ , which is not drawn in Figure 25.  $|\angle MFI| = 120^\circ$ .

Figure 26 (without explanation) illustrates the relations in constructions containing line  $AB$ , circles  $k_1(A, B)$ ,  $k_2(B, A)$  and  $k_4(C, A)$  and some additional lines and circles. Circles  $f_C(F, C)$ ,  $f_H(F, H)$ ,  $g_C(G, C)$  and  $g_E(G, E)$  are also constructed.

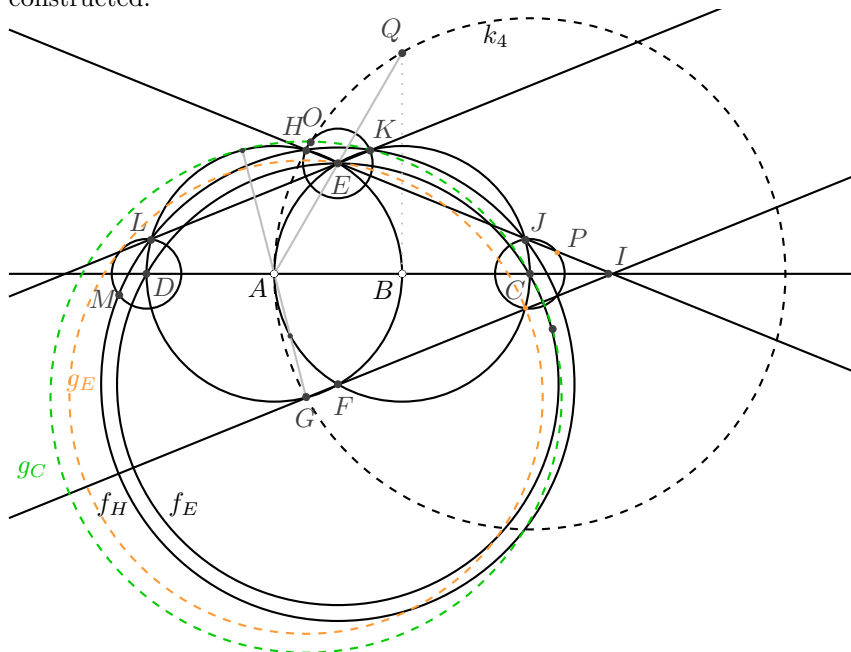


Figure 26

Figures 25 and 26 do not display all of the straight lines and circles that can be found in any construction. These Figures only illustrate the rules and relations that can assist us in finding angles in circles that we will use to calculate ratios of chords lengths or find the values of angles and lengths of sides in triangles.

## II Examples – Chords on One Circle

### Example 1.

To prove the ratio of lengths, we must (except for the known patterns) examine the ratios of lengths of chords on congruent circles.

In construction RCC28 from class 17 (see Figure 27), we get

$$\frac{|DH|}{|CE|} = \frac{|CE|}{|DG|} = \frac{|CG|}{|FG|} = \frac{|FG|}{|CH|} = \phi.$$

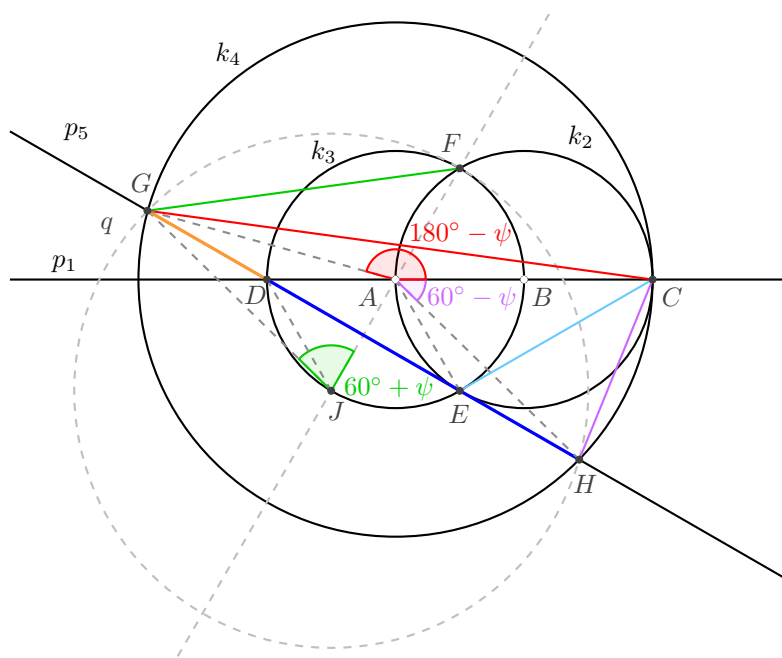


Figure 27

Because of  $|CE| = |DE|$  and  $|EH| = |GD|$ , we find that two ratios

$$\frac{|DH|}{|CE|} = \frac{|CE|}{|DG|} = \phi$$

satisfy the relations in pattern **A**.

From the symmetry about line  $p_5$ ,  $GF$  is a chord of a circle of radius  $|JF| = |AC| = 2|AB|$ . From the values of the angles in pattern **A**, we obtain  $|\angle DJG| = \psi$  and  $|\angle EAH| = \psi$ ; therefore,  $|\angle HAC| = 60^\circ - \psi$ ,  $|\angle FJG| = 60^\circ + \psi$  and  $|\angle CAG| = 180^\circ - \psi$ . Chords  $CH, FG$  and  $CG$ , are in pattern **B** in circles of radius  $2|AB|$ . Therefore,

$$\frac{|CG|}{|FG|} = \frac{|FG|}{|CH|} = \phi.$$

**Example 2.**

In construction MM200 from class 97, we find pattern **C** (see Figure 28). However, this statement must be proven. Point  $M$  is the intersection point of circle  $k_3(C, D)$  that we have already examined with circle  $k_5(J, |IB|)$  previously unknown.

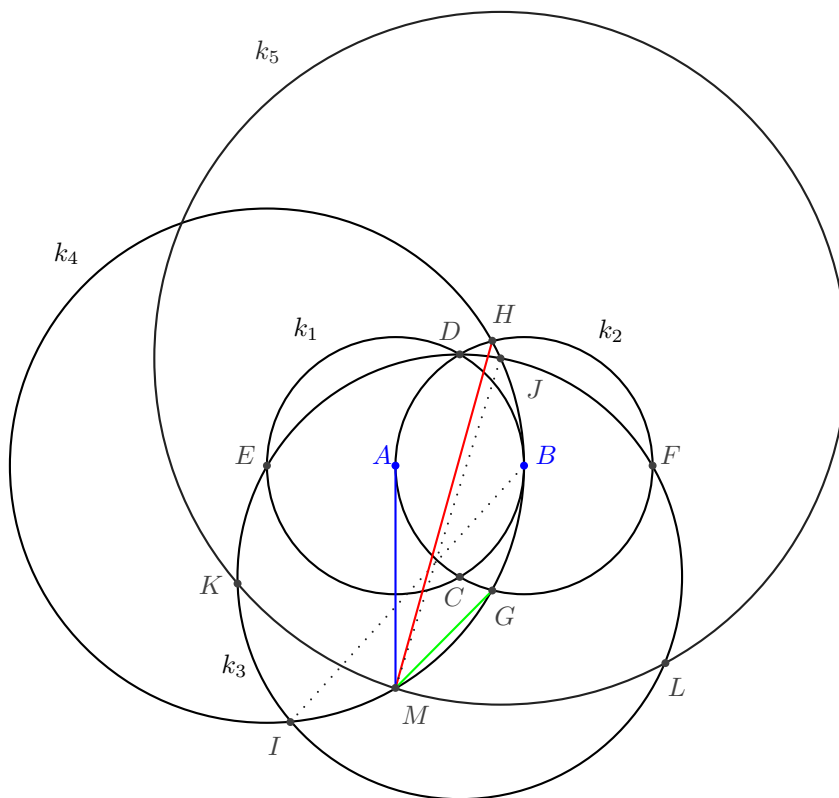


Figure 28

To prove that the configuration of circles, lines and points satisfies pattern **C**, we can prove (except for other possible proofs) any of following statements. Others follow from relations in pattern **C**.

- $C$  is the midpoint of  $BM$  ( $C$  lies on  $BM$ )
- $AM$  is perpendicular to  $AB$
- $|AM| = |CD|$
- $|\angle MEG| = 2\psi$
- $|\angle MEH| = 120^\circ - 2\psi$ .

The first statement can be seen in Figures 28 and 29;  $EC$  is the bisector of  $\angle IEJ$  and  $|\angle BCE| = 90^\circ$ .

To show another proof, we prove the statement  $|\angle MEG| = 2\psi$ .

We will examine the central angles for arcs (chords) subtended on circle  $k_4(E, B)$ . Their values can be seen in Figure 29.



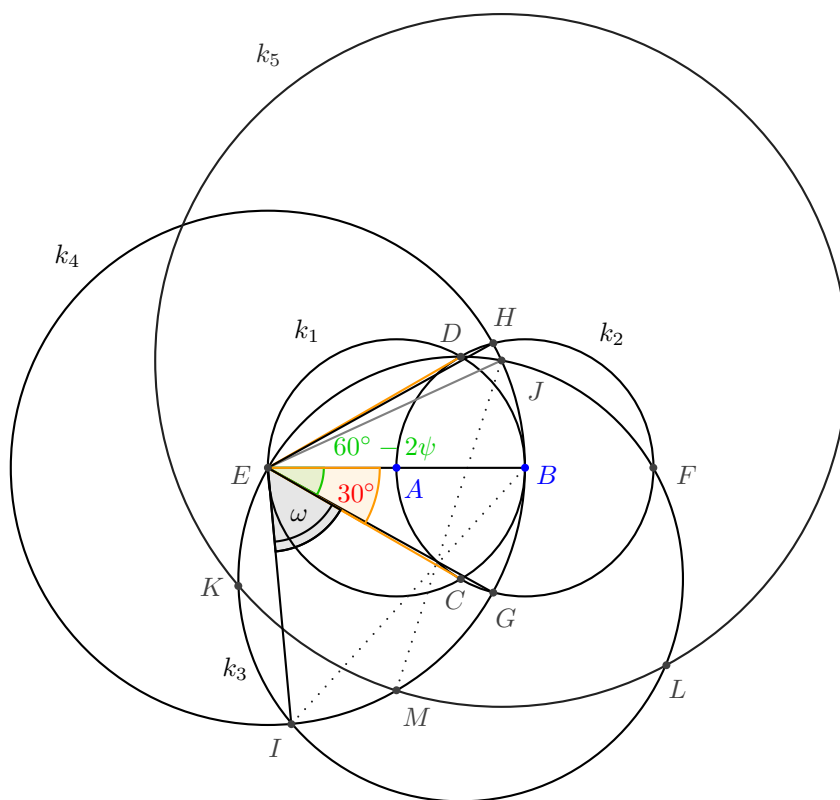


Figure 29

We have already proven

$$|\angle CEB| = |\angle BED| = 30^\circ, \quad |\angle GEB| = |\angle BEH| = 60^\circ - 2\psi, \quad \text{hence} \\ |\angle CEG| = |\angle HED| = 2\psi - 30^\circ.$$

Denote  $|\angle IEC| = \omega$ . Points  $I$  and  $J$  are intersection points of circles  $k_3(C, E)$  and  $k_4(E, B)$ ; thus, the congruent triangles  $\triangle ECI$  and  $\triangle ECJ$  are isosceles with bases  $EI, EJ$ ,  $|EI| = |EB| = 2|AB|$  and  $|EC| = \sqrt{3}|AB|$ . Therefore, we can calculate the value of  $\omega$ . However, this calculation is not necessary.

We must find  $|\angle MEG|$ . Because

$$|IB| = |MJ|, \text{ thus, } |\angle IEB| = |\angle MEJ| \text{ (} M \text{ is the intersection point of } k_4 \text{ and } k_5\text{)}.$$

Therefore,

$$|\angle IEM| = |\angle BEJ|, \quad |\angle MEC| = |\angle CEB| \quad (EC \text{ is the angle bisector of } \angle IEJ).$$

Thus, we have

$$|\angle MEG| = |\angle MEC| + |\angle CEG| = |\angle CEB| + |\angle CEG| = 30^\circ + (2\psi - 30^\circ) = 2\psi.$$

Similarly, we can examine the central angles for chords in circle  $k_4(A, B)$  (or its analogy in any symmetrical construction) in constructions of classes 95–108. These constructions imply each other, as seen in the following example.



Because of  $\cos(60^\circ + \psi) = \sin(30^\circ - \psi) = \frac{1}{4}$   
with trigonometric identities, we have

$$\begin{aligned}\sin(60^\circ + \psi) &= \cos(30^\circ - \psi) = \frac{\sqrt{15}}{4}, \\ \cos\left(30^\circ + \frac{\psi}{2}\right) &= \sin\left(60^\circ - \frac{\psi}{2}\right) = \sqrt{\frac{5}{8}}, \\ \sin\left(30^\circ + \frac{\psi}{2}\right) &= \cos\left(60^\circ - \frac{\psi}{2}\right) = \sqrt{\frac{3}{8}}, \\ \tan\left(30^\circ + \frac{\psi}{2}\right) &= \sqrt{\frac{3}{5}}.\end{aligned}$$

The values of the trigonometric functions of  $\psi$  and  $\frac{\psi}{2}$  can be calculated using known values of trigonometric functions of angle  $\nu = 60^\circ + \psi$ , which results in  $\psi = \nu - 60^\circ$ .

$$\begin{aligned}\sin \psi &= \sin(\nu - 60^\circ) = \frac{\sqrt{3}}{8}(\sqrt{5} - 1), \\ \cos \psi &= \cos(\nu - 60^\circ) = \frac{1}{8}(3\sqrt{5} + 1), \\ \cos \frac{\psi}{2} &= \frac{\sqrt{9 + 3\sqrt{5}}}{4} = \frac{\sqrt{6}}{8}(\sqrt{5} + 1), \quad \sin \frac{\psi}{2} = \frac{\sqrt{7 - 3\sqrt{5}}}{4} = \frac{\sqrt{2}}{8}(3 - \sqrt{5})\end{aligned}$$

and finally

$$\tan \frac{\psi}{2} = \frac{\sqrt{5} - 2}{\sqrt{3}}, \quad \cot \frac{\psi}{2} = \sqrt{3}(\sqrt{5} + 2).$$

#### IV Examples – Chords in Circles of Different Radii

Numerous golden ratios in our constructions are ratios of lengths of chords in circles of different radii. Several have been examined in the denoted *patterns*. Others remain to be calculated. We will use properties and relations in triangles, notably relations between angles and legs in isosceles triangles.

##### Example 4.

The distances of points in construction RCC9 from class 8 form several golden ratios. A few can be explained using known patterns; to prove others, we must calculate the ratios of lengths of chords in circles of different radii.

In construction RCC9, we obtain

$$\frac{|IL|}{|EJ|} = \frac{|EJ|}{|AI|} = \frac{|AI|}{|FJ|} = \frac{|FJ|}{|IK|} = \frac{|AL|}{|CE|} = \frac{|CE|}{|AK|} = \phi.$$

We will first show the known patterns (see Figure 31).

The equality

$$\frac{|AL|}{|CE|} = \frac{|CE|}{|AK|} = \phi$$

follows from pattern **C** in circle  $k_4(C, A)$  because  $|GL| = |GK| = 2r$  and  $|AH| = |AG| = r$  and because  $k_4$  passes through the center  $A$  of circle  $k_3(A, B)$ .

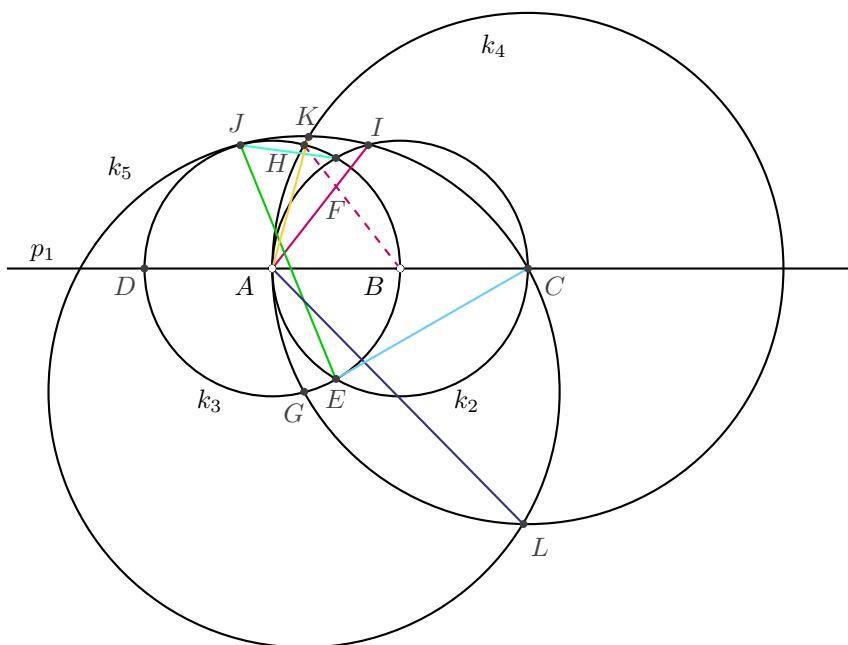


Figure 31

Because of  $|AI| = |BH|$ , the equality

$$\frac{|EJ|}{|AI|} = \frac{|AI|}{|FJ|} = \phi$$

follows from pattern **B** in circle  $k_3(A, B)$ . Segment  $GJ$  is the diameter of circle  $k_3$ ; thus,  $|\angle FAJ| = 60^\circ - \psi$ .

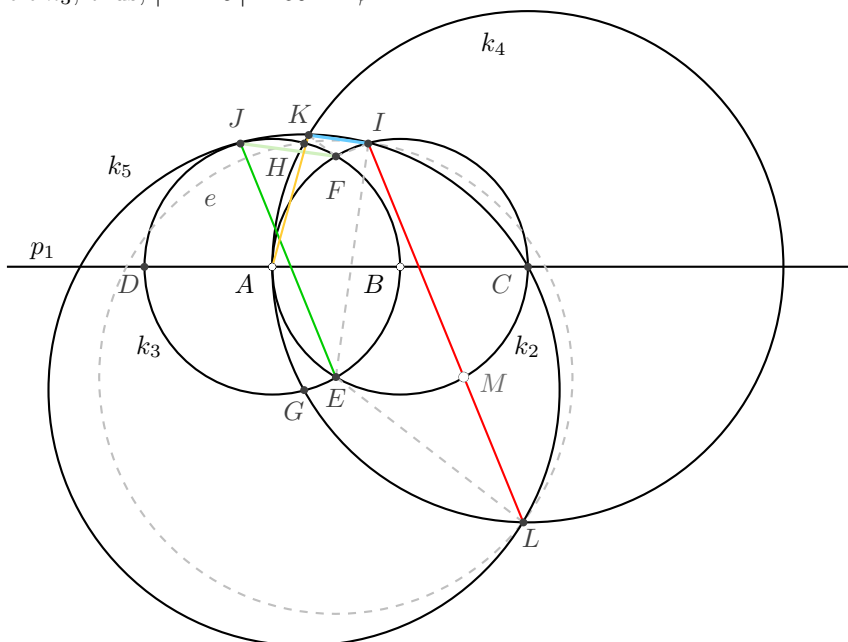


Figure 32

Proofs for the next two equalities require several calculations.

Let us prove

$$\frac{|IL|}{|EJ|} = \phi.$$

In Figure 32, we have drawn the circle  $e(E, H)$  that intersects circle  $k_5(G, C)$ , and (following the rotation of circles and corollaries in Figures 25 and 26) these circles have the common chord  $IL$ . Furthermore,  $|\angle LEI| = 120^\circ$ . Therefore,

$$|IL| = \sqrt{3}|EI| = \sqrt{3}|EH|.$$

Because of  $|\angle EAJ| = 180^\circ - \psi$  and  $|\angle EAH| = 120^\circ + \psi$ , we obtain

$$\frac{|IL|}{|EJ|} = \frac{\sqrt{3}|EI|}{|EJ|} = \frac{\sqrt{3}|EH|}{|EJ|} = \frac{\sqrt{3} \sin \frac{120^\circ + \psi}{2}}{\sin \frac{180^\circ - \psi}{2}} \quad \text{and}$$

$$\frac{|IL|}{|EJ|} = \frac{\sqrt{3} \sin (60^\circ + \frac{\psi}{2})}{\sin (90^\circ - \frac{\psi}{2})} = \frac{\sqrt{3} \sin (60^\circ + \frac{\psi}{2})}{\cos \frac{\psi}{2}} = \sqrt{3} \frac{\sin 60^\circ \cos \frac{\psi}{2} + \cos 60^\circ \sin \frac{\psi}{2}}{\cos \frac{\psi}{2}}.$$

After manipulation and substitution of known values, we obtain

$$\frac{|IL|}{|EJ|} = \frac{1}{2} \left( 3 + \sqrt{3} \tan \frac{\psi}{2} \right) = \frac{\sqrt{5} + 1}{2} = \phi.$$

In Figure 32, we have drawn the intersection point  $M$  of segment  $IL$  with circle  $k_2$  (not drawn in construction RCC9 because segment  $IL$  is not constructed; we have added  $IL$  for the proof).

Because  $IEL$  is an isosceles triangle, we have  $|\angle EIL| = 30^\circ$ .

The properties of the central and inscribed angles in circles  $k_2$  and  $k_3$  give  $|\angle IEJ| = |\angle IEF| + |\angle FEJ| = \frac{1}{2}|\angle IBF| + \frac{1}{2}|\angle FAJ| = 30^\circ$ . Therefore,  $JE$  and  $IL$  are parallel. Because  $JI$  and  $AB$  are parallel as well (an easy proof for the reader), we obtain  $|JE| = |IM|$ . Thus, point  $M$  divides segment  $IL$  in the golden ratio.

Further, let us prove

$$\frac{|FJ|}{|IK|} = \phi.$$

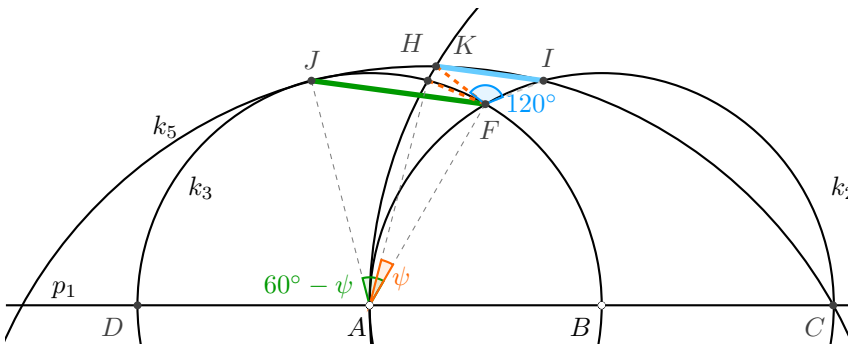


Figure 33

We have already shown (see Figures 24 and 25)  $|\angle IFK| = 120^\circ$  and  $|FK| = |FH|$ .

Similar to the previous proof

$$|IK| = \sqrt{3}|FK| = \sqrt{3}|FH|,$$

where segments  $FJ$  and  $FH$  are chords in circle  $k_3$  (see zoomed Figure 33).

Because  $|\angle FAJ| = 60^\circ - \psi$  and  $|\angle FAH| = \psi$ ,

$$\frac{|FJ|}{|IK|} = \frac{|FJ|}{\sqrt{3}|FH|} = \frac{\sin \frac{60^\circ - \psi}{2}}{\sqrt{3} \sin \frac{\psi}{2}} = \frac{\sin(30^\circ - \frac{\psi}{2})}{\sqrt{3} \sin \frac{\psi}{2}}.$$

After trigonometric manipulation (see 3.10–III) we obtain

$$\frac{|FJ|}{|IK|} = \frac{1}{2\sqrt{3}} \left( \cot \frac{\psi}{2} - \sqrt{3} \right) = \frac{\sqrt{5} + 1}{2} = \phi.$$

**Example 5.**

Individual constructions often differ only in a single line (a straight line or a circle). In this case, the proofs may be similar. This example shows construction RCC15, from class 14. By examining the ratios, we can find them based on previous calculations.

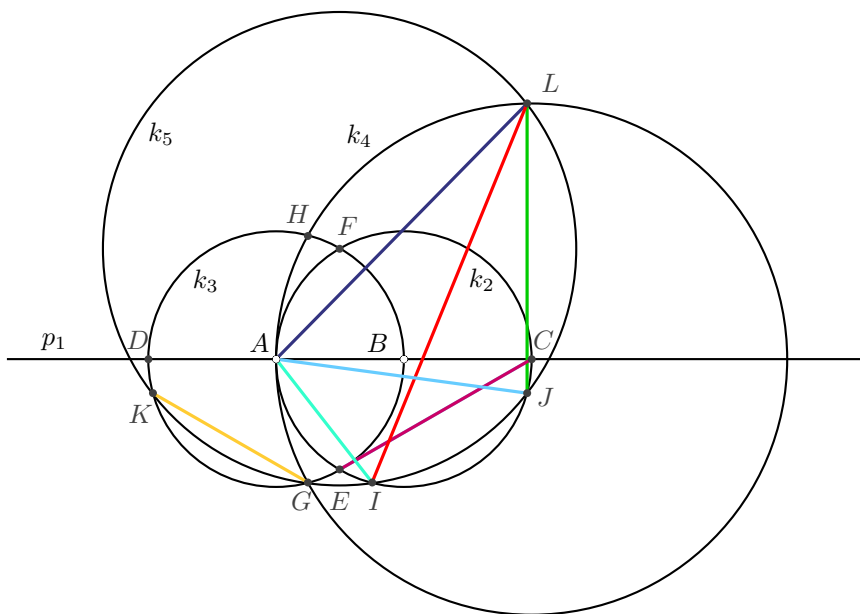


Figure 34

In construction RCC15 (see Figure 34), we have

$$\frac{|AJ|}{|AI|} = \frac{|IL|}{|AJ|} = \frac{|AL|}{|CE|} = \frac{|JL|}{|GK|} = \phi.$$

The equality

$$\frac{|AJ|}{|AI|} = \phi$$

is given by pattern **B** in circle  $k_2(B, A)$ .

The equality

$$\frac{|JL|}{|GK|} = \phi$$

is given by pattern **B** in circle  $k_5(F, G)$ . Because  $|IJ| = |GK|$  (intercepted chords on rotated circles) and because triangles  $\triangle FEC$  and  $\triangle FLC$  are equilateral,  $|\angle EFL| = 120^\circ$ .

Because  $|\angle EFC| = 60^\circ$  and  $|\angle JFC| = |\angle EFI| = |\angle GFE| = \frac{1}{2}|\angle GAE| = \frac{\psi}{2}$ , we have  $|\angle IFJ| = 60^\circ - 2\frac{\psi}{2} = 60^\circ - \psi$ .

Therefore,  $|\angle JFL| = 120^\circ - (60^\circ - \psi) = 60^\circ + \psi$ .

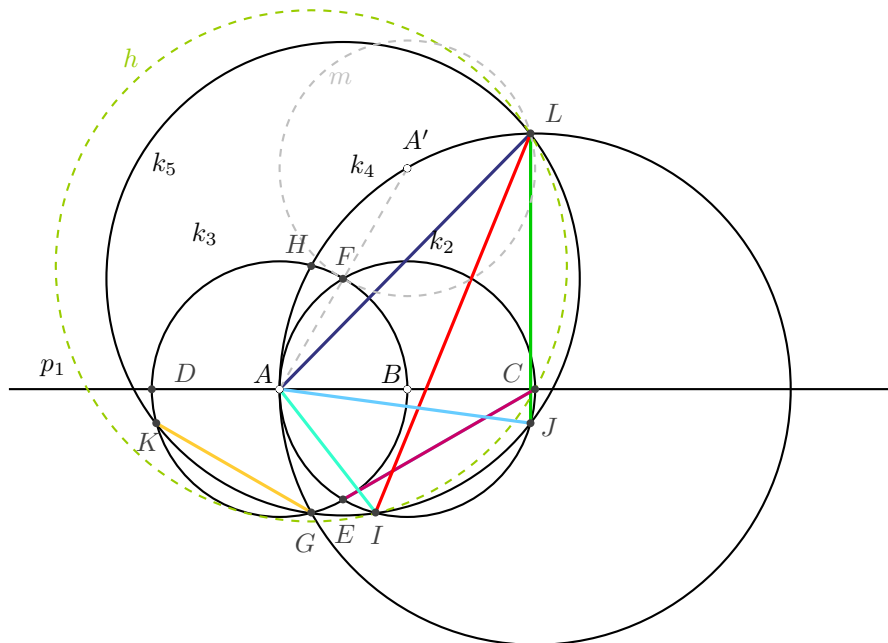


Figure 35

Constructions RCC15 and RCC9 differ in one circle and in reflection about line  $AB$ . Compare Figures 32 (circles  $k_5(G, C)$  and dashed  $e(E, H)$ ) and 35 (dashed circle  $h(H, I)$  and circle  $k_5(F, G)$ ). Thus, the equality

$$\frac{|IL|}{|AJ|} = \phi$$

was already proven in Example 4.

The equality

$$\frac{|AL|}{|CE|} = \phi$$

is given by pattern **C** in circle  $k_4(C, A)$ . We draw segment  $AA'$  with the midpoint  $F$ . Then,  $|AA'| = 2r$  and  $|FA'| = |LA'| = r$  (see circle  $m(A', F)$  that completes pattern **C** in Figure 35).

**Example 6.**

We prove construction RCC8 from class 7. In RCC8 (in Figure 36), we have

$$\frac{|DL|}{|BG|} = \frac{|BG|}{|DK|} = \frac{|CK|}{|CE|} = \frac{|CE|}{|CL|} = \frac{|IK|}{|EI|} = \frac{|EI|}{|IL|} = \phi.$$

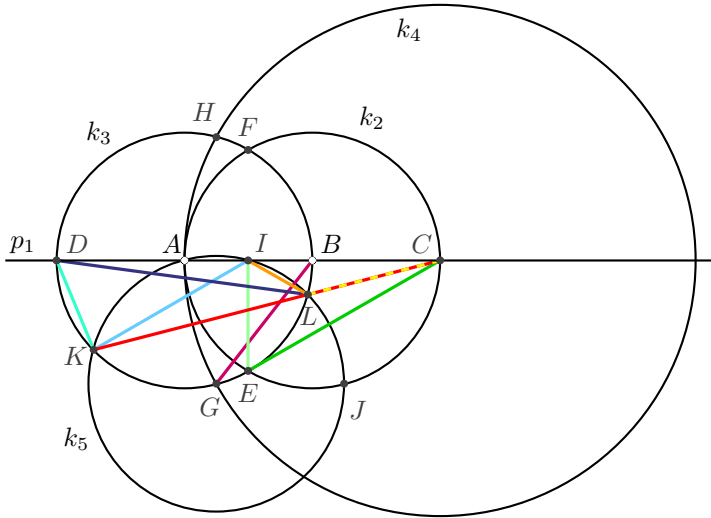


Figure 36

We first show the equalities given by known patterns.

The equality

$$\frac{|DL|}{|BG|} = \frac{|BG|}{|DK|} = \phi$$

is given by pattern **B** in circle  $k_3(A, B)$  because circle  $k_5(G, A)$  is rotated to circle  $k_2(B, A)$  about center  $A$  by angle  $\angle GAB$ ;  $|\angle GAB| = 60^\circ + \psi$ .

The equality

$$\frac{|CK|}{|CE|} = \left( \frac{|CK|}{|KL|} \right) = \frac{|CE|}{|CL|} = \phi$$

is given by patterns **A2** and **C**.

*Proposition.* Circle  $k_5(G, A)$  intersects segment  $AB$  at its midpoint.

*Proof.* Let  $S$  be the midpoint of segment  $AB$ . Therefore,  $|AH| : |AC| = 1 : 2 = |AS| : |AG|$  and  $|\angle CAH| = |\angle GAB| = |\angle GAS|$ . Thus, triangles  $\triangle ACH$  and  $\triangle AGS$  are similar; both are isosceles triangles, and point  $S$  lies on circle  $k_5(G, A)$ . Therefore,  $S = I$ .

Now, we can prove the equality

$$\frac{|IK|}{|EI|} = \phi.$$



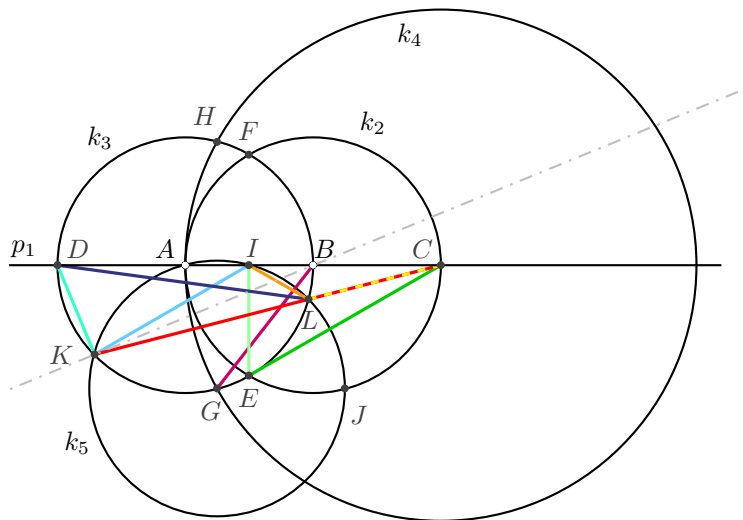


Figure 37

The central and inscribed angles in circles  $k_5$  and  $k_3$  in Figure 37 provide

$$|\angle LKB| = |\angle LDB| = \frac{1}{2}|\angle LAB| = \frac{1}{2}|\angle LAI| = \frac{1}{2}|\angle LKI|;$$

thus  $|\angle CKB| = \frac{1}{2}|\angle CKI|$ . Therefore, line  $KB$  is a bisector of  $\angle CKI$ . The bisector of the internal angle in the triangle intersects the opposite side at the point that divides that side into segments with lengths of the identical ratio as the lengths of the sides on the legs of the angle. Thus, in triangle  $CKI$  we have

$$\frac{|IK|}{|CK|} = \frac{|IB|}{|CB|} = \frac{1}{2}.$$

Then,  $|IK| = \frac{1}{2}|CK|$  and  $|EI| = \frac{1}{2}|EF| = \frac{1}{2}|CE|$ , and thus

$$\frac{|IK|}{|EI|} = \frac{|CK|}{|CE|} = \phi.$$

The equality

$$\frac{|EI|}{|IL|} = \phi$$

can be similarly proven.

We have  $|LC| = 2|IL|$  because point  $L$  lies on circle  $k_3(A, B)$ , which is the locus of point  $M$  for which  $|CM| : |IM| = 2$  (Apollonius circle). Therefore,

$$\frac{|EI|}{|IL|} = \frac{|EC|}{|CL|} = \phi.$$



and the ratio of lengths of the longer and shorter part of the divided segment. We have selected to examine the ratio  $|BI| : |AB|$ .

The equality

$$\frac{|BI|}{|AB|} = \frac{|BI|}{|BJ|} = \phi$$

can be proven using angle values and the lengths of the sides of triangle  $BIJ$  in Figure 39.

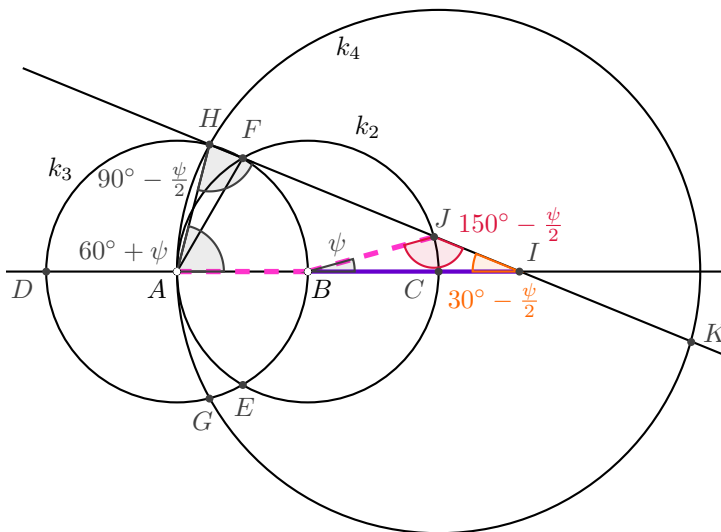


Figure 39

$\triangle FAH$  is an isosceles triangle with vertex angle  $\psi$  at vertex  $A$ .

Thus  $|\angle AHF| = 90^\circ - \frac{\psi}{2}$ .

In triangle  $AIH$ , we have  $|\angle IAH| = 60^\circ + \psi$  and  $|\angle AHI| = |\angle AHF| = 90^\circ - \frac{\psi}{2}$ , thus  $|\angle HIA| = 30^\circ - \frac{\psi}{2}$ .

In triangle  $BIJ$ , we have  $|\angle IBJ| = \psi$  and because  $|\angle JIB| = |\angle HIA| = 30^\circ - \frac{\psi}{2}$ , we have  $|\angle BJI| = 150^\circ - \frac{\psi}{2}$ .

Because  $|BJ| = |AB|$ , the law of sines in triangle  $BIJ$  can be used to find the ratio of the given lengths:

$$\frac{|BI|}{|BJ|} = \frac{\sin\left(150^\circ - \frac{\psi}{2}\right)}{\sin\left(30^\circ - \frac{\psi}{2}\right)}.$$

Using trigonometric identities, we obtain

$$\frac{\cos \frac{\psi}{2} + \sqrt{3} \sin \frac{\psi}{2}}{\cos \frac{\psi}{2} - \sqrt{3} \sin \frac{\psi}{2}} = \frac{1 + \sqrt{3} \tan \frac{\psi}{2}}{1 - \sqrt{3} \tan \frac{\psi}{2}} = \frac{\sqrt{5} + 1}{2} = \phi.$$

This calculation is not necessary. Using known formulae, we obtain

$$\frac{|BI|}{|BJ|} = \frac{\sin\left(150^\circ - \frac{\psi}{2}\right)}{\sin\left(30^\circ - \frac{\psi}{2}\right)} = \frac{\sin\left(30^\circ + \frac{\psi}{2}\right)}{\sin\left(30^\circ - \frac{\psi}{2}\right)},$$

which expresses the geometric principle of pattern **B**.

We could prove this equality using the law of sines in triangle  $BIE$ .

Because  $|FI| = |EI|$ , we must prove only one of the following pair of equalities

$$\frac{|FK|}{|EI|} = \frac{|EI|}{|IK|} = \phi.$$

Let us prove  $\frac{|FK|}{|EI|} = \frac{|FK|}{|FI|} = \phi$  using the ratios of lengths of the sides in triangles  $\triangle AIF$  and  $\triangle AKF$  in Figure 40.

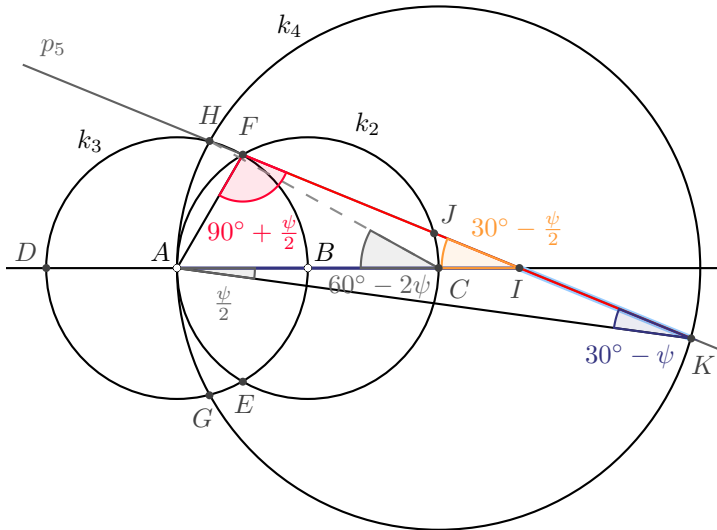


Figure 40

We must find the values of the internal angles in the triangles.

We have already found the following angles in triangle  $AIF$  :

$$|\angle IAF| = 60^\circ, |\angle FIA| = 30^\circ - \frac{\psi}{2} \text{ and } |\angle AFI| = 90^\circ + \frac{\psi}{2}.$$

For internal angles in triangle  $AKF$ , we have:

$$|\angle AFK| = |\angle AFI| = 90^\circ + \frac{\psi}{2}.$$

From the proof of pattern **B**, we obtain  $|\angle HCA| = 60^\circ - 2\psi$ . This angle is a central angle in circle  $k_4(C, A)$ ; thus, for the corresponding inscribed angle  $\angle HKA$ , we have  $|\angle HKA| = 30^\circ - \psi$ . Thus  $|\angle KAF| = 60^\circ + \frac{\psi}{2}$ .

From the law of sines in triangle  $AIF$ , we obtain

$$\frac{|FI|}{|FA|} = \frac{\sin 60^\circ}{\sin \left(30^\circ - \frac{\psi}{2}\right)}.$$

From the law of sines in triangle  $AKF$ , we obtain

$$\frac{|FK|}{|FA|} = \frac{\sin \left(60^\circ + \frac{\psi}{2}\right)}{\sin (30^\circ - \psi)}.$$

Therefore,

$$\frac{|FK|}{|FI|} = \frac{\sin \left(60^\circ + \frac{\psi}{2}\right)}{\sin (30^\circ - \psi)} \cdot \frac{\sin \left(30^\circ - \frac{\psi}{2}\right)}{\sin 60^\circ}.$$

After substitution of known values, we obtain

$$\frac{4}{\sqrt{3}} \cdot 2 \cos \left( 30^\circ - \frac{\psi}{2} \right) \cdot \sin \left( 30^\circ - \frac{\psi}{2} \right) = \frac{4}{\sqrt{3}} \cdot \sin (60^\circ - \psi),$$

and using trigonometric identities and known values (see 3.10–III), we have

$$\frac{|FK|}{|FI|} = \frac{\sqrt{5} + 1}{2}.$$

The last equality in this construction,

$$\frac{|GI|}{|DG|} = \phi,$$

can be similarly proven. One length in the ratio is the length of the chord in circle  $k_3(A, B)$ , the other is the length of the side of triangle  $AGI$  (see Figure 41). Internal angles of the triangle can be easily found.

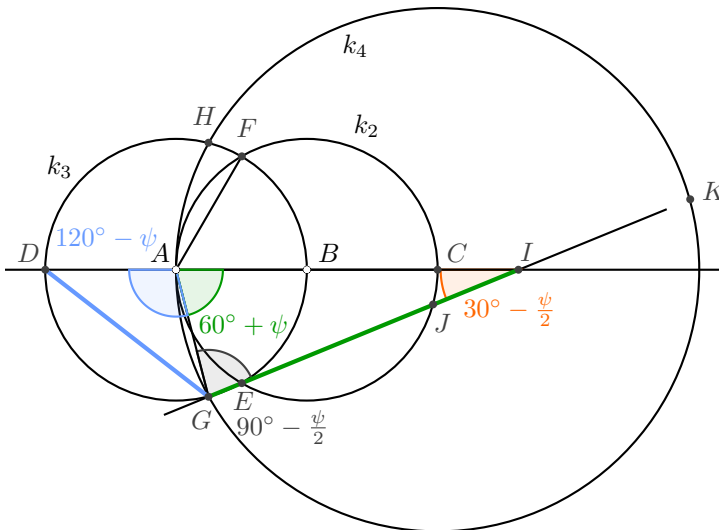


Figure 41

We have:  $|\angle GAI| = 60^\circ + \psi$ ,  $|\angle AIG| = 30^\circ - \frac{\psi}{2}$  and  $|\angle IGA| = 90^\circ - \frac{\psi}{2}$ .  
 $|\angle DAG| = 120^\circ - \psi$ .

From the isosceles triangle  $DAG$  and the law of sines in triangle  $GAI$  we obtain

$$\frac{|DG|}{2|GA|} = \sin \left( 60^\circ - \frac{\psi}{2} \right),$$

$$\frac{|GI|}{|GA|} = \frac{\sin (60^\circ + \psi)}{\sin \left( 30^\circ - \frac{\psi}{2} \right)}.$$

Therefore,

$$\frac{|GI|}{|DG|} = \frac{\sin (60^\circ + \psi)}{\sin \left( 30^\circ - \frac{\psi}{2} \right)} \cdot \frac{1}{2 \sin \left( 60^\circ - \frac{\psi}{2} \right)} = \frac{\sqrt{15}}{4} \cdot \frac{1}{2 \cdot \sin \left( 30^\circ - \frac{\psi}{2} \right) \cdot \sin \left( 60^\circ - \frac{\psi}{2} \right)},$$



follows from pattern **B** in congruent circles  $k_2(B, A)$  and  $k_3(A, B)$  (Figure 43). To prove this equality, we must find the central angles that subtend chords  $BG$  and  $AJ$ .

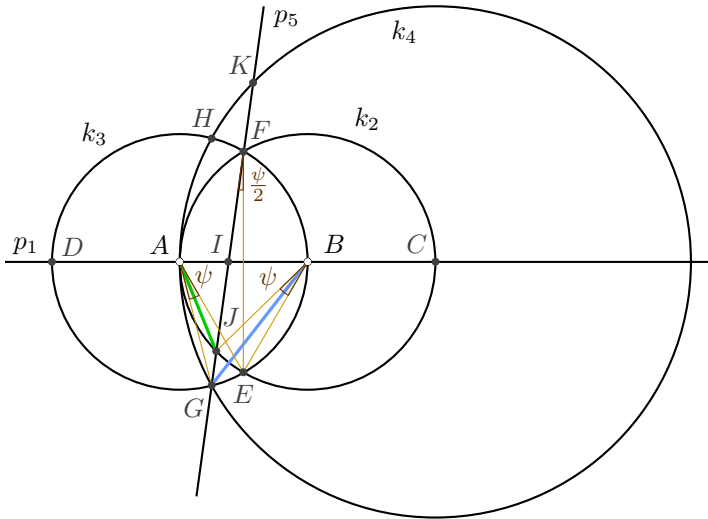


Figure 43

The chord  $GB$  in circle  $k_3(A, B)$  is intercepted by the central angle  $\angle GAB$ ;  $|\angle GAB| = 60^\circ + \psi$ .

The central angle  $\angle GAE$  in circle  $k_3(A, B)$  corresponds to the inscribed angle  $\angle GFE$ . Because  $\angle GFE = \angle JFE$  (which is the inscribed angle in circle  $k_2(B, A)$  corresponding to central angle  $\angle JBE$ ), we have  $|\angle JBE| = \psi$  and  $|\angle ABJ| = 60^\circ - \psi$ .

*Remark.* Because circle  $k_2(B, A)$  rotates to circle  $k_3(A, B)$  when rotated about point  $E$  by angle  $60^\circ$  and from the congruence of arcs  $GE$  and  $JE$  that we have just proven, the triangle  $GEJ$  in Figure 43 is therefore an equilateral triangle.

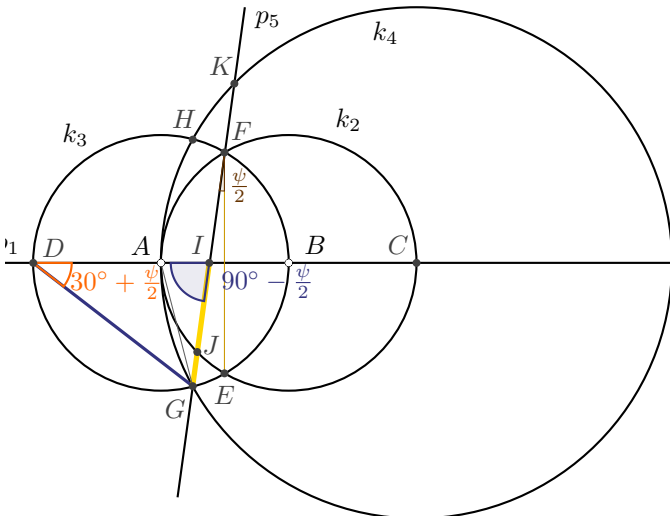


Figure 44

The equality

$$\frac{|DG|}{|GI|} = \phi$$

is proven using the law of sines in triangle  $DIG$  (see Figure 44).

Because  $|\angle IFE| = \frac{\psi}{2}$  (as we have already shown) and line  $FE$  is perpendicular to line  $IB$ , we have  $|\angle BIF| = |\angle DIG| = 90^\circ - \frac{\psi}{2}$ .

In the isosceles triangle  $DAG$ , we have  $|\angle GDA| = 30^\circ + \frac{\psi}{2}$ .

From the law of sines in triangle  $DIG$ , we have

$$\frac{|DG|}{|GI|} = \frac{\sin\left(90^\circ - \frac{\psi}{2}\right)}{\sin\left(30^\circ + \frac{\psi}{2}\right)}.$$

From which

$$\frac{|DG|}{|GI|} = 2 \cdot \frac{\cos \frac{\psi}{2}}{\cos \frac{\psi}{2} + \sqrt{3} \sin \frac{\psi}{2}} = 2 \cdot \frac{1}{1 + \sqrt{3} \tan \frac{\psi}{2}};$$

after substitution of known values, we obtain

$$\frac{|DG|}{|GI|} = \frac{2}{\sqrt{5} - 1} = \frac{\sqrt{5} + 1}{2} = \phi.$$

Because  $|EI| = |FI|$  and because of the definition of the golden section, only one from following equalities must be proven:  $\frac{|IK|}{|EI|} = \frac{|EI|}{|FK|} = \phi$ .

The equality

$$\frac{|IK|}{|EI|} = \frac{|IK|}{|FI|} = \phi$$

can be proven using the ratios of lengths in triangles  $\triangle AIK$  and  $\triangle AIF$  (see Figure 45).

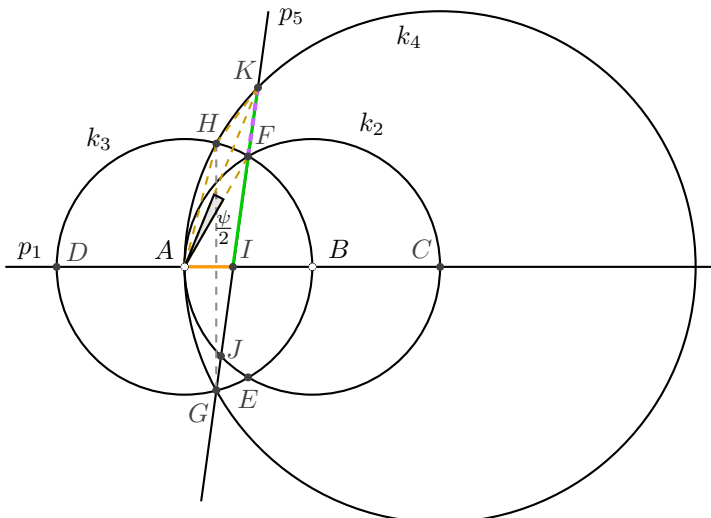


Figure 45



We must find the internal angles in triangles  $\triangle AIK$  and  $\triangle AIF$ .

Because line  $AK$  passes through the midpoint  $A$  of arc  $HG$ ,  $AK$  is the angle bisector of  $\angle HKG$ .

Because  $|\angle KAH| = |\angle KGH|$  and lines  $HG$  and  $FE$  are parallel, we have  $|\angle HGK| = |\angle HGF| = |\angle GFE| = \frac{\psi}{2}$ . Thus,  $|\angle KAF| = \frac{\psi}{2}$ .

Because  $|\angle IAF| = 60^\circ$  then  $|\angle IAK| = 60^\circ + \frac{\psi}{2}$ .

$|\angle FIA| = |\angle KIA| = 90^\circ + \frac{\psi}{2}$ , thus  $|\angle AFI| = 30^\circ - \frac{\psi}{2}$  and  $|\angle AKI| = 30^\circ - \psi$ .

From the law of sines in triangle  $AIK$ , we obtain

$$\frac{|KI|}{|AI|} = \frac{\sin\left(60^\circ + \frac{\psi}{2}\right)}{\sin\left(30^\circ - \psi\right)}.$$

From the law of sines in triangle  $AIF$ , we obtain

$$\frac{|FI|}{|AI|} = \frac{\sin 60^\circ}{\sin\left(30^\circ - \frac{\psi}{2}\right)}.$$

Thus

$$\frac{|KI|}{|FI|} = \frac{\sin\left(60^\circ + \frac{\psi}{2}\right)}{\sin\left(30^\circ - \psi\right)} \cdot \frac{\sin\left(30^\circ - \frac{\psi}{2}\right)}{\sin 60^\circ} = \frac{\cos\left(30^\circ - \frac{\psi}{2}\right)}{\frac{1}{4}} \cdot \frac{\sin\left(30^\circ - \frac{\psi}{2}\right)}{\frac{\sqrt{3}}{2}},$$

and after substitution of known values and simple manipulation, we have

$$\frac{|KI|}{|FI|} = \frac{4}{\sqrt{3}} \cdot \sin\left(60^\circ - \psi\right) = \frac{1}{2} \left(\sqrt{5} + 1\right) = \phi.$$

### The Golden Section of $AB$

Following equalities are formulae of golden ratios of segment  $AB$  and its parts and corollaries. Again, we must prove only one of the equalities

$$\frac{|CI|}{|AB|} = \frac{|AB|}{|BI|} = \frac{|BI|}{|AI|} = \phi.$$

The law of sines in triangle  $AIF$  allows us to show

$$\frac{|AB|}{|AI|} = \frac{|AF|}{|AI|} = \phi^2.$$

We have decided to prove the formula

$$\frac{|BI|}{|AI|} = \phi$$

using the ratios of lengths of sides in triangles  $\triangle AIF$  and  $\triangle BIF$  in Figure 46.

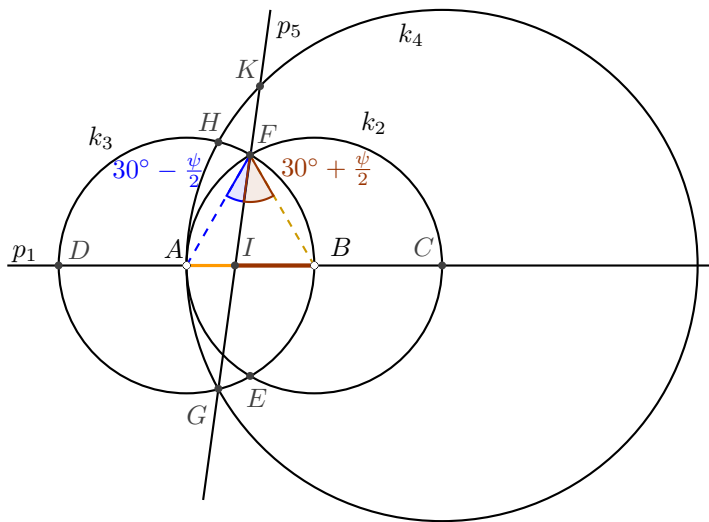


Figure 46

We will prove that line  $FG$  meets the side  $AB$  of an equilateral triangle  $ABF$  at point  $I$  that divides the segment  $AB$  in the golden section. For the internal angles in the triangles  $\triangle AIF$  and  $\triangle BIF$  we have  $|\angle IAF| = |\angle FBI| = 60^\circ$ ,  $|\angle FIA| = 90^\circ + \frac{\psi}{2}$ ,  $|\angle AFI| = 30^\circ - \frac{\psi}{2}$ ,  $|\angle BIF| = 90^\circ - \frac{\psi}{2}$ ,  $|\angle IFB| = 30^\circ + \frac{\psi}{2}$ .

Thus from the law of sines in triangle  $AIF$ , we can write

$$\frac{|FI|}{|AI|} = \frac{\sin 60^\circ}{\sin\left(30^\circ - \frac{\psi}{2}\right)},$$

and from the law of sines in triangle  $BIF$ , we can write

$$\frac{|BI|}{|FI|} = \frac{\sin\left(30^\circ + \frac{\psi}{2}\right)}{\sin 60^\circ}.$$

Therefore,

$$\frac{|BI|}{|AI|} = \frac{\sin\left(30^\circ + \frac{\psi}{2}\right)}{\sin\left(30^\circ - \frac{\psi}{2}\right)} = \frac{\sqrt{5} + 1}{2} = \phi.$$

However, it is not necessary to calculate and prove the last equality  $\frac{\sin\left(30^\circ + \frac{\psi}{2}\right)}{\sin\left(30^\circ - \frac{\psi}{2}\right)} = \frac{\sqrt{5}+1}{2} = \phi$ . This equality is the ratio of the lengths of chords intercepted in the circle by the central angles  $60^\circ + \psi$  and  $60^\circ - \psi$  (shown in pattern **B**).

The last two equalities

$$\frac{|AB|}{|BI|} = \frac{|CI|}{|AB|} = \phi$$

are corollaries of the proof above, equalities  $|CI| = |CB| + |BI| = |AB| + |BI|$  and from the definition of the golden section.

Construction RCC13 was described by and the golden section of segment  $AB$  was proved by Kurt Hofstetter in [6].

### 3.12 Finally...

We have already mentioned that the proofs we present in this part of the book provide hints of how to prove the other constructions in this book. Let us stress that other possibilities of how the necessary proofs could be performed are available, and our approach is only one of many.

In our method, we purposely avoided calculating explicit lengths, using algebraic equations and employing analytic geometry.

Moreover, the proofs shown can be similarly performed using properties in other triangles. The reader can certainly use their own ideas and inspiration.

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## 4 Conclusion

We have described four different types of constructions, and for each type of construction, we have presented a list of all of the shortest constructions of the golden ratio (715 in total).

To simplify the search, we added an image table of all 129 classes of similarity; the clickable link to this table can be found in the headline of every page. We also described a method to proof why ratio of those and these distances should be the golden ratio.

We (arrogantly) imagine that you, the reader, could find similar enjoyment in browsing and investigating of individual constructions as we have found in searching them. Whether you are teacher, student or somebody who found this book accidentally.

If you would like to provide comments or report errors, please write to [Sarka@gbn.cz](mailto:Sarka@gbn.cz) or to [Tomas.Holan@mff.cuni.cz](mailto:Tomas.Holan@mff.cuni.cz).

Alternatively, please visit the webpage dedicated to this book at <http://ksvi.mff.cuni.cz/~holan/GoldenRatio>. Thank you!

P.S.: Try to estimate how many different occurrences of the golden ratio is in all presented constructions; the answer can be found on the next page after a list of references.

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Answer:  $32 + 8308 + 9080 + 2624 = 20044$